*Macroeconomic Dynamics*, **21**, 2017, 862–888. Printed in the United States of America.
doi:10.1017/S1365100515000759

# PUBLIC RESEARCH SPENDING IN AN ENDOGENOUS GROWTH MODEL

**KUNIHIKO KONISHI**
*Kyoto University*

This study constructs a variety expansion growth model with public research spending. Public researchers financed by taxes on asset income, consumption, and corporate profits raise the productivity of private research and development. We show that the welfare-maximizing level of public research spending is below the growth-maximizing level. With regard to tax policy, a zero-profit tax maximizes the welfare of households. In addition, the study analyzes the dynamics of the economy, showing that equilibrium is indeterminate when the government's revenue source depends on an asset income tax.

**Keywords:** Public Research Spending, Endogenous Growth, Indeterminacy

## 1. INTRODUCTION

It is acknowledged that productive government spending is important for economic growth. As the first to study this issue using growth models, Arrow and Kurz (1970) focused on infrastructure (e.g., highways, airports, railroads, electrical facilities) and introduced the notion of productive public capital. However, in their model, growth is determined by exogenous factors. Among endogenous growth models, Barro's (1990) seminal model includes productive government spending. Under his assumption that public services raise the productivity of private firms, the social rate of return on private capital becomes constant, and the long-run growth rate is determined endogenously. As a result, we have been able to investigate the relationship between productive government spending and the long-run growth rate. Whereas Barro (1990) treats government spending as a flow, many studies extend the Barro model by introducing the stock of public capital and examining its effects [Futagami et al. (1993); Turnovsky (1997); Fisher and Turnovsky (1998)].[1]

I would like to express my sincere gratitude to the associate editor and the anonymous referees for their constructive comments and suggestions. I am especially grateful to Koichi Futagami for helpful discussions and suggestions. I also thank Shinsuke Ikeda, Tatsuro Iwaisako, Noritaka Maebayashi, Yoshiyasu Ono, Asuka Oura, and participants at the 2013 Autumn Annual Meeting of the Japanese Economic Association at Kanagawa University for their comments. The financial support from Grants for Excellent Graduate Schools, MEXT, Japan and the Research Fellowship for Young Scientists of the Japan Society for the Promotion of Science (JSPS) are gratefully acknowledged. Any errors are my responsibility. Address correspondence to: Kunihiko Konishi, Institute of Economic Research, Kyoto University, Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan; [Research Fellow of the Japan Society for the Promotion of Science (JSPS)]; e-mail: k.konishi.econ@gmail.com.

 1365-1005/16 

Although many studies analyze the effects of productive government spending on economic growth, Schumpeterian growth models developed by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) focus mainly on activities of private research and development (R&D) firms motivated by monopoly profit. However, many empirical studies argue that the public research sector contributes significantly to economic growth. Mansfield (1991) finds that 10% of innovations in the United States between 1975 and 1985 could not have been developed or would have been greatly delayed without academic research. Caloghirou et al. (2001) analyze over 6,000 research joint ventures in 42 nations that received funding from the European Commission during 1983–1996. They find that 65% of joint research ventures involved one or more universities. Based on the Swedish Community Innovation Survey, Lööf and Broström (2008) find that university collaboration positively influences the innovative activity of large manufacturers. Notwithstanding the documented benefits of publicly funded R&D, however, evidence also suggests that it hinders employment for private sector researchers [Lichtenberg (1984)] and crowds out private inventive activity [Goolsbee (1998)]. David et al. (2000) extensively survey empirical literature indicating that public research spending is a complement or substitute for private R&D.

Few theoretical studies consider public research policy from a macroeconomic perspective. Glomm and Ravikumar (1994) present a model in which the stock of technological knowledge depends upon public research, but it does not allow for private R&D. Park (1998) considers both public and private research. Public research contributes indirectly to market production by influencing the knowledge accumulation of private R&D. However, his study is concerned more with open economy issues and international spillovers than public research policy. Our study relates closely to Arnold (1997), who considers the first-best allocation but does not investigate the second-best policy. In addition, he disregards the possibility of indeterminacy because the government sets the public research spending constant and finances spending only by lump-sum taxation.[2]

To examine analytically the implications of public research spending, this study incorporates public researchers who raise the productivity of private R&D in a variety expansion model, following Arnold (1997). The government expends public research spending and interest from government debt but raises funds from taxes on asset income, consumption, and corporate profits.[3] This study focuses on the effects of public research spending regarding the number of public researchers. According to the National Science Foundation (2011) analysis of U.S. R&D spending, 46.7% goes to wages of R&D personnel, 10.1% to employer-sponsored benefits for R&D personnel, 11.7% to materials and supplies, 3.9% to depreciation, and 27.6% to other costs. Hence, we can see that the majority of public research spending goes toward hiring R&D personnel.

This study obtains two main results. First, the welfare-maximizing level of public research spending is lower than the growth-maximizing level. In this study, welfare is driven by household consumption expenditures and growth in the number of differentiated goods (i.e., love of variety). Whereas public research spending and the growth rate follow an inverted U shape, household consumption expenditures decrease with an increase in public research spending. Therefore, there is a trade-off between household consumption expenditures and growth in the number of differentiated goods. This trade-off leads to a welfare-maximizing level of public research spending below the growth-maximizing level. With regard to tax policy, exempting corporate profits from taxation maximizes welfare because doing so frees funds for investment, which contributes to growth and thereby maximizes welfare.

Second, an increase in asset income tax raises the possibility of indeterminacy, whereas an increase in government borrowing, an increase in household consumption tax, and an increase in taxes on corporate profits reduce it. Related studies also indicate the possibility that indeterminacy depends on fiscal policies [Guo and Harrison (2008); Kamiguchi and Tamai (2011)]. Guo and Harrison (2008) show that the presence of productive government spending and distortion by taxes affect the possibility of indeterminacy. This finding implies that financing government spending using distortionary taxes might provoke equilibrium indeterminacy. Kamiguchi and Tamai (2011) show that financing via income taxes has greater influence on indeterminacy than the presence of productive government spending. Findings in these studies seems to parallel ours. However, this study assumes that household labor supply is inelastic; that is, distortion through asset income/consumption taxes is not generated. Thus, the mechanism of indeterminacy differs from those in Guo and Harrison (2008) and Kamiguchi and Tamai (2011).[4]

This paper proceeds as follows. Section 2 establishes the model. Section 3 derives the dynamic system and the steady state of the economy. Section 4 examines the dynamics of the economy. Section 5 analyzes the policy effect on welfare. Section 6 concludes.

## 2. MODEL

There is a unit continuum of identical households. Each household supplies one unit of skilled labor and $L$ units of unskilled labor inelastically. The factor market is perfectly competitive, and the goods market is monopolistically competitive, as explained in the following. Households have perfect foresight.

### 2.1. Households

Households maximize the following lifetime utility:

$$U_t \equiv \int_t^{\infty} e^{-\rho(s-t)} \log C_s ds, \qquad (1)$$

where $C_t$ represents instantaneous utility derived from consuming a composite good and $\rho > 0$ is a rate of time preference. $C_t$ is given by

$$C_t = \left[\int_0^{N_t} c_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where $c_t(j)$ denotes the consumption of good $j$ and $N_t$ denotes the number of available varieties. We assume that $\varepsilon > 1$. $\varepsilon$ is the elasticity of substitution between any two products. Denoting the consumption expenditure of households as $E_t = \int_0^{N_t} P_t(j)c_t(j)dj$, we obtain the demand function for good $j$ as follows:

$$c_t(j) = \frac{P_t(j)^{-\varepsilon} E_t}{\int_0^{N_t} P_t(i)^{1-\varepsilon} di}, \tag{3}$$

where $P_t(j)$ is the price of good $j$, and $P_{D,t}$ is the price index, defined as

$$P_{D,t} = \left[\int_0^{N_t} P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}. \tag{4}$$

The maximization problem for households is as follows:

$$\max \ U_t$$

$$\text{subject to } \dot{A}_t = (1 - t_A) r_t A_t + w_t^s + w_t^u L - (1 + t_E) E_t,$$

where $A_t, r_t, w_t^s$, and $w_t^u$ respectively represent households' asset holdings, the rate of return on assets, the wage rate for skilled labor, and the wage rate for unskilled labor. The government taxes asset income at a rate $t_A \in [0, 1)$ and consumption at a rate $t_E \geq 0$. Substituting (3) into (2), we obtain the indirect subutility function as follows:

$$C_t = \frac{E_t}{P_{D,t}}. \tag{5}$$

The maximization subject to the intertemporal budget constraint yields the Euler equation

$$\frac{\dot{E}_t}{E_t} = (1 - t_A) r_t - \rho. \tag{6}$$

## 2.2. Firms

This subsection considers producer behavior. Producers undertake two distinct activities. They create blueprints for new varieties of differentiated goods and they manufacture the differentiated goods created by R&D.

We assume that each differentiated good is produced by a single firm because the good is infinitely protected by a patent. We further assume that the production function of good $j$ is Cobb–Douglas in form, as follows:

$$X_t(j) = \theta\left[l_t^s(j)\right]^{\alpha}\left[l_t^u(j)\right]^{1-\alpha},\ \theta > 0,\ \text{and}\ \alpha \in (0, 1),$$

where $X_t(j)$ is the output of good $j$ and $l_t^s(j)$ and $l_t^u(j)$ denote the amounts of skilled and unskilled labor devoted to producing good $j$. From cost minimization, the unit cost function $z(w_t^s, w_t^u)$ is

$$z(w_t^s, w_t^u) = \theta^{-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}\left(w_t^s\right)^{\alpha}\left(w_t^u\right)^{1-\alpha}. \quad (7)$$

Applying Shephard's Lemma, we obtain demand functions for skilled and unskilled labor as follows:

$$l_t^s(j) = \frac{\alpha z(w_t^s, w_t^u)}{w_t^s} X_t(j), \quad (8)$$

$$l_t^u(j) = \frac{(1-\alpha) z(w_t^s, w_t^u)}{w_t^u} X_t(j). \quad (9)$$

The manufacturer of good $j$ (firm $j$) maximizes its after-tax profit: $(1-t_\pi)\Pi_t(j) = (1-t_\pi)\left[P_t(j)X_t(j) - z(w_t^s, w_t^u)X_t(j)\right]$, where $t_\pi \in [0, 1)$ represents the tax rate on profit. Then firm $j$ charges the following price:

$$P_t(j) = P_t = \frac{\varepsilon}{\varepsilon - 1} z(w_t^s, w_t^u). \quad (10)$$

Therefore, all goods are priced equally. Pricing rules (10) and (3) yield

$$X_t(j) = X_t = \frac{\varepsilon - 1}{\varepsilon} \frac{E_t}{z(w_t^s, w_t^u) N_t}. \quad (11)$$

From (8) and (9), we obtain $l_t^s(j) = l_t^s$ and $l_t^u(j) = l_t^u$. Then per-brand operating profits are as follows:

$$\Pi_t = \frac{E_t}{\varepsilon N_t}. \quad (12)$$

The no-arbitrage condition is given by

$$(1 - t_A) r_t = (1 - t_\pi)\frac{\Pi_t}{v_t} + \frac{\dot{v}_t}{v_t}, \quad (13)$$

where $v_t$ denotes the value of a firm.[5]

Next, we consider the technology involved in developing a new good.[6] R&D firms create blueprints and expand the varieties of goods available for consumption. We assume that R&D needs skilled labor. Further, we incorporate public researchers into the model. Public research enhances the productivity of private

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

R&D through numerous channels (e.g., publications, scientific reports, conferences, research joint ventures, and university collaboration). We assume the following production function for R&D:

$$\dot{N}_t = f(G_t)N_t L_{R,t}, \tag{14}$$

where $G_t$ and $L_{R,t}$ represent the number of public researchers and the amount of skilled labor devoted to R&D, respectively. Equation (14) implies that one unit of R&D activity needs $1/f(G_t)N_t$ units of skilled labor. Because the knowledge already produced includes all that is needed for invention, greater knowledge entails further invention. Because knowledge is nonrival and nonexcludable, expansion in the number of varieties reduces the skilled labor input. In addition, we postulate that $f$ satisfies these conditions:

$$f(0) > 0 \text{ and } f'(G_t) > 0 \text{ and } f''(G_t) < 0.$$

We assume that firms enter the R&D race freely. The free entry condition is given by

$$v_t = \frac{w_t^s}{f(G_t)N_t} \Leftrightarrow \dot{N}_t > 0. \tag{15}$$

Equation (15) shows that an increase in $G_t$ reduces $v_t$. The reasoning is as follows: Greater R&D productivity creates an excess supply of blueprints. In a competitive market, the Walrasian adjustment mechanism reduces the patent value, $v_t$.

### 2.3. Government

The government taxes asset income, consumption, and corporate profits. We assume the respective tax rates and government indebtedness $\bar{B}$ are held constant over time. Therefore, the government finances public research spending and interest on its debt via taxation. By doing so, it satisfies the budget constraint

$$w_t^s G_t + r_t \bar{B} = t_A r_t A_t + t_E E_t + t_\pi \Pi_t N_t. \tag{16}$$

Note that $\bar{B} = 0$ implies a balanced budget.[7,8]

## 3. EQUILIBRIUM

### 3.1. Dynamic System

We normalize the wage rate for skilled labor at unity, and thus, $w_t^s = 1$. Skilled labor is used for production, private R&D, and the employment of public researchers.[9] The market clearing condition for skilled labor becomes

$$N_t l_t^s + L_{R,t} + G_t = 1. \tag{17}$$

The market-clearing condition for unskilled labor is

$$N_t l_t^u = L.$$

Using (9) and (11) makes this condition $(1-\alpha)(\varepsilon-1)E_t/\varepsilon = w_t^u L$. From (15), the asset market equilibrium is

$$A_t = \bar{B} + N_t v_t = \bar{B} + \frac{1}{f(G_t)}. \tag{18}$$

From (8), (11), (12), (14), (15), and (17), the no-arbitrage condition (13) becomes

$$(1-t_A)r_t = \frac{\mu}{\varepsilon} f(G_t)E_t - f(G_t)(1-G_t) - \frac{f'(G_t)}{f(G_t)}\dot{G}_t, \tag{19}$$

where $\mu \equiv 1 - t_\pi + \alpha(\varepsilon - 1)$. Using (12), (16), (18), and (19), we obtain

$$\frac{f'(G_t)}{f(G_t)}\dot{G}_t = \frac{\mu}{\varepsilon} f(G_t)E_t - f(G_t)(1-G_t) + \frac{1}{\Delta_t}\left[\left(t_E + \frac{t_\pi}{\varepsilon}\right)E_t - G_t\right], \tag{20}$$

where $\Delta_t \equiv t_A/[(1-t_A)f(G_t)] - \bar{B}$. From (15), (19), and (20), the Euler equation (6) becomes

$$\frac{\dot{E}_t}{E_t} = \frac{1}{\Delta_t}\left[G_t - \left(t_E + \frac{t_\pi}{\varepsilon}\right)E_t\right] - \rho. \tag{21}$$

Equations (20) and (21) formulate the autonomous dynamic system with respect to $E_t$ and $G_t$.

## 3.2. Steady State

This subsection examines the steady state of the economy, defined by the condition where $E_t$, $G_t$, and the innovation rate are constant. Imposing $\dot{E}_t = \dot{G}_t = 0$ in (20) and (21) results in

$$\left(t_E + \frac{t_\pi}{\varepsilon}\right)E_t = G_t - \rho\Delta_t, \tag{22}$$

$$\rho = \frac{\mu}{\varepsilon} f(G_t)E_t - f(G_t)(1-G_t). \tag{23}$$

By eliminating $E_t$ from equations (22) and (23), we have

$$\bar{B} = \left(\frac{t_A}{1-t_A} + \frac{\varepsilon t_E + t_\pi}{\mu}\right)\frac{1}{f(G_t)} + \frac{1}{\rho\mu}\left[\varepsilon t_E + t_\pi - \left(\varepsilon t_E + t_\pi + \mu\right)G_t\right]$$
$$\equiv \Lambda(G_t). \tag{24}$$

As shown in Figure 1, the intersection of the left- and right-hand sides (LHS and RHS) of (24) determines the steady state value, $G^*$. Asterisks represent variables in the steady state.

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

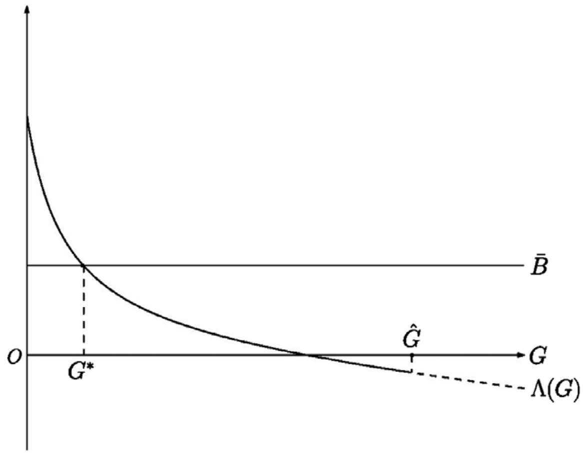


**FIGURE 1.** Determination of the steady state.

The steady state value $E^*$ is obtained from (23) as follows:

$$E^* = \frac{\varepsilon}{\mu}\left[(1 - G^*) + \frac{\rho}{f(G^*)}\right]. \tag{25}$$

From (8), (11), (14), (17), and (25), the growth rate in the steady state is given by

$$\gamma^* \equiv \left(\frac{\dot{N}_t}{N_t}\right)^* = \frac{1}{\mu}\Big[(1 - t_\pi)f(G^*)(1 - G^*) - \alpha(\varepsilon - 1)\rho\Big]. \tag{26}$$

Moreover, we assume a positive growth rate at $G^* = 0$, as follows:

$$\gamma^*|_{G^*=0} = \frac{1}{\mu}\Big[(1 - t_\pi)f(0) - \alpha(\varepsilon - 1)\rho\Big] > 0. \tag{27}$$

Differentiating $\gamma^*$ with respect to $G^*$, we obtain

$$\frac{\partial \gamma^*}{\partial G^*} = \frac{1 - t_\pi}{\mu}\Big[f'(G^*)(1 - G^*) - f(G^*)\Big]. \tag{28}$$

From the assumption of $f$, $\partial\gamma^*/\partial G^*$ is decreasing in $G^*$.[10] Assuming that $f'(0) > f(0)$, there is a level of public research spending that satisfies $f'(G^*)(1 - G^*) - f(G^*) = 0$. We define $G^*$ as $G_g$, which yields the following relation:

$$\frac{\partial \gamma^*}{\partial G^*} \gtreqless 0 \Leftrightarrow G^* \lesseqgtr G_g. \tag{29}$$

Therefore, we can illustrate the graph of (26) as shown in Figure 2. As shown by (29), the relationship between the growth rate and $G^*$ follows an inverted U shape, and growth-maximizing public research spending is $G_g$. Note that the level of $G_g$ is independent of tax rates applicable to asset income, consumption, and corporate

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

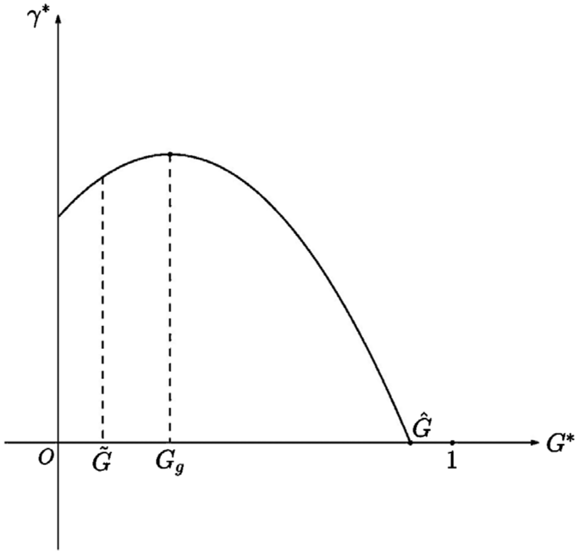


**FIGURE 2.** The relationship between $\gamma^*$ and $G^*$.

profits. From Figure 2, the growth rate becomes 0 if $G^*$ is sufficiently large. In this case, R&D is not undertaken, and the skilled labor input to private R&D becomes 0. Here, $\hat{G}$ is defined as $\gamma^*|_{G^*=\hat{G}} = 0$. In Figure 1, we assume that the growth rate is positive under a balanced budget ($\bar{B} = 0$). That is, $G^*|_{\bar{B}=0} < \hat{G}$. Further, we impose the following condition to focus on the positive growth rate and $G^* \geq 0$:[11]

$$\Lambda(\hat{G}) < \bar{B} \leq \Lambda(0).$$

These results are summarized in the following proposition:

PROPOSITION 1. *There is a positive-growth steady state in this economy if* $\Lambda(\hat{G}) < \bar{B} \leq \Lambda(0)$. *In addition, there is a positive growth-maximizing level of public research spending,* $G_g$, *if* $f'(0) > f(0)$.

We now study the detailed relationship between the growth rate and public research spending in the steady state. From (14), the growth rate, $\gamma^* = f(G^*)L_R^*$, is determined by $f(G^*)$ and $L_R^*$. Differentiating $\gamma^*$ with respect to $G^*$ yields

$$\frac{\partial \gamma^*}{\partial G^*} = f'(G^*)L_R^* + f(G^*)\frac{\partial L_R^*}{\partial G^*}.$$

Although R&D productivity exerts a positive effect, $f'(G^*) > 0$, its effect on private R&D labor input, $\partial L_R^*/\partial G^*$, is ambiguous. From (8), (11), (17), and (25),

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

skilled labor input into private R&D in the steady state is given by

$$L_R^* = \frac{1-t_\pi}{\mu}(1-G^*) - \frac{\alpha(\varepsilon-1)\rho}{\mu}\frac{1}{f(G^*)}.$$

The first term represents a crowding-out effect on private R&D labor input. The second term implies that public research spending raises firms' incentive to conduct R&D because R&D productivity increases. Hence, a trade-off exists between these two effects. Differentiating $L_R^*$ with respect to $G^*$ yields

$$\frac{\partial L_R^*}{\partial G^*} = \frac{1}{\mu}\left\{-(1-t_\pi) + \alpha(\varepsilon-1)\rho\frac{f'(G^*)}{[f(G^*)]^2}\right\}.$$

From the assumption of $f$, $f'(G)/[f(G)]^2$ is decreasing in $G$.[12] We define $\tilde{G}$ as $f'(\tilde{G})/[f(\tilde{G})]^2 = (1-t_\pi)/\alpha(\varepsilon-1)\rho$. Then we obtain the following relation:

$$\frac{\partial L_R^*}{\partial G^*} \gtreqless 0 \Leftrightarrow G^* \lesseqgtr \tilde{G}.$$

When $G^* < \tilde{G}$, the effect of an increase in incentives for R&D exceeds the crowding-out effect, and $\partial L_R^*/\partial G^* > 0$ holds.[13] In contrast, when $G^* > \tilde{G}$, the crowding-out effect is sufficiently large; so $\partial L_R^*/\partial G^* < 0$ holds.

Next, we compare $G_g$ with $\tilde{G}$. Substituting $G^* = \tilde{G}$ into (28) yields

$$\left.\frac{\partial \gamma^*}{\partial G^*}\right|_{G^*=\tilde{G}} = f'(\tilde{G})L_R^*|_{G^*=\tilde{G}} > 0.$$

Therefore, from (29), we have $\tilde{G} < G_g$.

We summarize these results as follows. If $G^* < \tilde{G}$, public research spending is a complement for private R&D, and thus, an increase in $G^*$ raises the growth rate. If $\tilde{G} < G^* < G_g$, public research spending is a substitute for private R&D. However, the effect of R&D productivity growth exceeds the crowding-out effect, and an increase in $G^*$ raises the growth rate. If $G^* > G_g$, the crowding-out effect exceeds the effect of R&D productivity growth, and an increase in $G^*$ reduces the growth rate.

Finally, we consider the condition of $G_g > 0$ that corresponds to $f'(0) > f(0)$. From the foregoing discussion, if the productivity effect exceeds the crowding-out effect at $G^* = 0$, there exists a positive growth-maximizing level of public research spending. On the other hand, if crowding out exceeds the productivity effect at $G^* = 0$, public research spending crowds out private R&D input. That is, there is no need for public research spending.

## 4. DYNAMICS

We examine the local dynamics of the economy. Approximating (20) and (21) linearly in the neighborhood of the steady states, we obtain

$$\begin{pmatrix} \dot{G}_t \\ \dot{E}_t \end{pmatrix} = \begin{pmatrix} J_{GG} & J_{GE} \\ J_{EG} & J_{EE} \end{pmatrix} \begin{pmatrix} G_t - G^* \\ E_t - E^* \end{pmatrix}. \tag{30}$$

Here, $J_{ij}$ $(i, j = G, E)$ denotes entities in the Jacobian matrix of this system (see Appendix A.1 for details):

$$J_{GG} = \frac{f(G^*)}{f'(G^*)}\left[f(G^*) + \rho\frac{f'(G^*)}{f(G^*)}\right] - \frac{f(G^*)}{f'(G^*)\Delta^*}\left\{1 + \rho\frac{t_A}{1-t_A}\frac{f'(G^*)}{[f(G^*)]^2}\right\},$$

$$J_{GE} = \frac{f(G^*)}{f'(G^*)\Delta^*}\left[\frac{\mu}{\varepsilon}f(G^*)\Delta^* + \left(t_E + \frac{t_\pi}{\varepsilon}\right)\right],$$

$$J_{EG} = \frac{E^*}{\Delta^*}\left\{1 + \rho\frac{t_A}{1-t_A}\frac{f'(G^*)}{[f(G^*)]^2}\right\},$$

$$J_{EE} = -\left(t_E + \frac{t_\pi}{\varepsilon}\right)\frac{E^*}{\Delta^*},$$

where $\Delta^* = t_A/[(1-t_A)f(G^*)] - \bar{B}$.[14] The eigenvalues of the Jacobian matrix, $J$, are defined as $\lambda_i$ $(i = 1, 2)$. Here, $\lambda_1$ and $\lambda_2$ are the roots of the characteristic equation $\lambda^2 - (J_{GG} + J_{EE})\lambda + J_{GG}J_{EE} - J_{GE}J_{EG} = 0$. To check the sign of $\det J = J_{GG}J_{EE} - J_{GE}J_{EG}$, we obtain

$$\det J = -\frac{f(G^*)E^*}{f'(G^*)\Delta^*}\left[\left(1 + t_E + \frac{t_\pi}{\varepsilon}\right)f(G^*) + \left(\frac{t_A}{1-t_A} + t_E + \frac{t_\pi}{\varepsilon}\right)\rho\frac{f'(G^*)}{f(G^*)}\right].$$

Because the sign of the expression in square brackets is positive, the sign of $\det J$ is determined by the sign of $\Delta^*$.

As claimed in the literature of perfect foresight equilibrium models [e.g., Buiter (1984)], having the same number of unstable roots and jump variables (nonpredetermined variables) implies there is a unique perfect foresight equilibrium path. On the other hand, if the number of unstable roots is less than the number of jump variables, then the dynamic system generates multiple converging paths leading to the steady state. We refer to this case as *indeterminacy*.

In this study, $E_t$ and $G_t$ are free to change instantaneously regardless of their past trajectories, and thus, $E_t$ and $G_t$ are jump variables. As a result, when $\Delta^* > 0$, the sign of $\det J$ is negative. Therefore, the dynamic system (30) has one positive and one negative root. That is, equilibrium is indeterminate. In contrast, when $\Delta^* < 0$, the sign of $\det J$ is positive. In this case, we obtain $\text{tr}J = J_{GG} + J_{EE} > 0$ because $J_{GG} > 0$ and $J_{EE} > 0$. The eigenvalues of $J$ have positive real parts and thus equilibrium is determinate. Given these results, we state the following proposition:

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

PROPOSITION 2. *Equilibrium is indeterminate if* $\Delta^* > 0$ *and determinate if* $\Delta^* \leq 0$.

In this study, $G^*$ is determined by the government budget constraint. That is, $G^*$ is affected by fiscal variables. Therefore, from the definition of $\Delta^*$, fiscal variables influence the value of $\Delta^*$. We next examine the relationship between $\Delta^*$ and the fiscal variables. To begin with, we investigate the effects of the fiscal variables on $G^*$ through changes in the fiscal variables. As shown in Appendix A.3, we can state the following lemma:

LEMMA 1. *The effects of changes in policy variables* $\bar{B}$ *and* $t_i$ $(i = A, E, \pi)$ *are as follows:*

$$\frac{\partial G^*}{\partial \bar{B}} < 0, \frac{\partial G^*}{\partial t_i} > 0.$$

An increase in $\bar{B}$ raises interest payment on the debt, and $G^*$ decreases. Meanwhile, an increase in $t_i$ $(i = A, E, \pi)$ raises the government revenue, and $G^*$ increases. Using Lemma 1, we examine the effects of $\Delta^*$ through changes in fiscal variables. As shown in Appendix A.4, we obtain the following relation:

$$\frac{\partial \Delta^*}{\partial \bar{B}} < 0, \frac{\partial \Delta^*}{\partial t_A} > 0, \frac{\partial \Delta^*}{\partial t_i} < 0 (i = E, \pi).$$

From these results and Proposition 2, an increase in $t_A$ increases the possibility of indeterminacy, whereas an increase in $t_E$, $t_\pi$, and $\bar{B}$ reduces it. We also consider the following extreme cases:

$$\Delta^* = \frac{t_A}{1 - t_A} \frac{1}{f(G^*)} - \bar{B} > 0,\ \bar{B} \leq 0, t_A, t_E, t_\pi > 0,$$

$$\Delta^* = \frac{1}{\rho} G^* > 0, \quad t_E = t_\pi = 0, \bar{B}, t_A > 0,$$

$$\Delta^* = -\bar{B} < 0, \quad t_A = 0, \bar{B}, t_E, t_\pi > 0.$$

When $\bar{B} = t_A = 0$ is a special case, because $\Delta^* = 0$. In this instance we must reconsider the equilibrium dynamics. Appendix A.2 shows that the equilibrium is determinate. We can state the following corollary in summary:

COROLLARY. *When* $t_A$ *is sufficiently large and* $t_E$, $t_\pi$, *and* $\bar{B}$ *are sufficiently small, equilibrium is indeterminate. When* $t_A$ *is sufficiently small and* $t_E$, $t_\pi$, *and* $\bar{B}$ *are sufficiently large, equilibrium is determinate.*

To investigate the intuitive explanation of indeterminacy, suppose the initial level of $G_t$ exceeds the steady state level of public research spending, $G^*$, which increases the productivity of R&D.[15] From (15), the value of a firm, $v_t$, declines. From (14) and (15), the no-arbitrage condition is given by

$$(1 - t_A)r_t = (1 - t_\pi)\frac{\Pi_t}{v_t} - f(G_t)L_{R,t} - \frac{f'(G_t)}{f(G_t)}\dot{G}_t.$$

In this case, the first term on the RHS of this condition increases. The second term's variation is small because an increase in $G_t$ reduces the skilled labor input to R&D, $L_{R,t}$, which partly offsets the increase in $f(G_t)$. If the government does not change the tax rate, $\dot{G}_t > 0$ cannot satisfy the budget constraint in the long run. Therefore, $\dot{G}_t < 0$ holds, and the third term on the RHS becomes positive. From these results, an increase in $G_t$ raises the interest rate, $r_t$.

Using this result, we provide an intuitive explanation of indeterminacy. Suppose the economy is in a steady state and households expect the government to raise $G_t$. This expectation implies that households also expect an increase in $r_t$. Households then have greater incentive to save. As a result, households' asset holdings, $A_t$, increase, and their expenditures, $E_t$, decline. Under these circumstances, we can think of the government budget constraint as

$$G_t = \underbrace{t_A r_t A_t - r_t \bar{B}}_{(1-t_A)r_t\Delta_t} + \left(t_E + \frac{t_\pi}{\varepsilon}\right)E_t. \quad \textbf{(31)}$$

From (31), when the government's interest payments exceed revenue from the income tax on assets, $\Delta^* \leq 0$ holds.[16] This implies that the government's revenue source depends on consumption/profit tax financing. In this case, because $t_A$ is sufficiently small, the effect of a reduction in $E_t$ exceeds that of an increase in $A_t$, and the government then decreases $G_t$. Thus, households' expectations are not self-fulfilling; that is, the equilibrium is determinate. On the other hand, $\Delta^* > 0$ implies that the asset income tax revenue is sufficient to finance the interest payments and that the government's revenue source depends on asset income tax financing. In this case, because $t_A$ is sufficiently large, an increase in $A_t$ will raise $G_t$. Households' expectations can be self-fulfilling, and equilibrium is indeterminate.[17]

As shown in Appendix A.5, we can depict the phase diagram in $(G_t, E_t)$ space. Figure 3 illustrates the case where equilibrium is indeterminate. The saddlepath slopes downward in $(G_t, E_t)$ space.[18] As described earlier, one would expect paths starting either from a high $G_t$ and a low $E_t$ or from a low $G_t$ and a high $E_t$ to constitute equilibrium paths. At a low $E_t$ and a high $G_t$, the amount of skilled labor devoted to production, $N_t l_t^s = \alpha(\varepsilon - 1)E_t/\varepsilon$, is small, and private firms' R&D expenditures can be high. Public research spending, $G_t$, is high along this path, so the economy can attain the high growth rate. By contrast, at a high $E_t$ and a low $G_t$, private R&D can be low and the growth rate can also be low. The indeterminacy result implies that the relationship between private and public R&D can be complementary.[19]

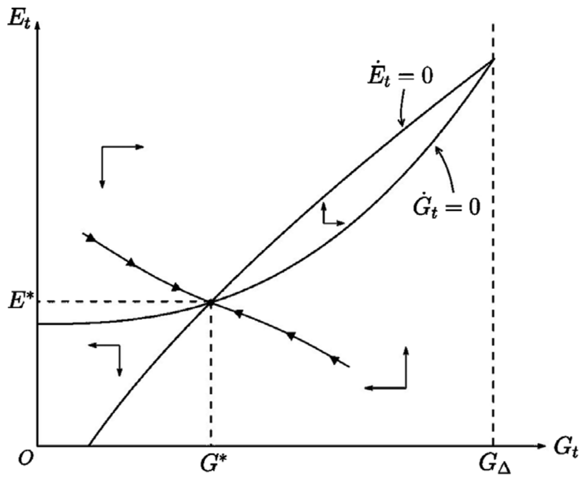


**FIGURE 3.** The case where equilibrium is indeterminate.

## 5. WELFARE-MAXIMIZING POLICY

This section investigates the welfare level of the steady state. Equations (4), (5), (7), and (10) yield

$$\log C_t = \alpha \log E_t + \frac{1}{\varepsilon - 1} \log N_t + \log\left[\theta\left(\frac{\varepsilon - 1}{\varepsilon}\alpha\right)^{\alpha} L^{1-\alpha}\right]. \quad (32)$$

If indeterminacy occurs, the effect of the welfare-maximizing policy is ambiguous. To focus on the determinate equilibrium, we impose the following condition using the definition of $\Delta^*$ and (A.3):

$$t_A \le \frac{\bar{B}}{\frac{1}{f(G^*)} + \bar{B}} \text{ or } \varepsilon t_E + t_\pi \ge \frac{\varepsilon t_E + t_\pi + \mu}{\frac{\rho}{f(G^*)} + 1} G^*.$$

This condition implies that $\Delta^* \le 0$. Because the economy initially jumps to the steady state, $E_t = E^*$ and $N_t = N_0 \exp\left(\int_0^t \gamma^* ds\right) = N_0 \exp\left(\gamma^* t\right)$ hold. Using (1) and (32), the welfare level at the steady state is calculated by

$$U^* = \frac{\alpha}{\rho} \log E^* + \frac{1}{\rho^2(\varepsilon - 1)}\gamma^* + \frac{1}{\rho} \log\left[\theta\left(\frac{\varepsilon - 1}{\varepsilon}\alpha\right)^{\alpha} L^{1-\alpha} N_0^{\frac{1}{\varepsilon-1}}\right]. \quad (33)$$

Without loss of generality, we set $\theta\left(\frac{\varepsilon-1}{\varepsilon}\alpha\right)^{\alpha} L^{1-\alpha} N_0^{1/(\varepsilon-1)} = 1$. Differentiating $U^*$ with respect to $G^*$, we obtain

$$\frac{\partial U^*}{\partial G^*} = \frac{\alpha}{\rho E^*} \frac{\partial E^*}{\partial G^*} + \frac{1}{\rho^2(\varepsilon - 1)} \frac{\partial \gamma^*}{\partial G^*}. \quad (34)$$

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

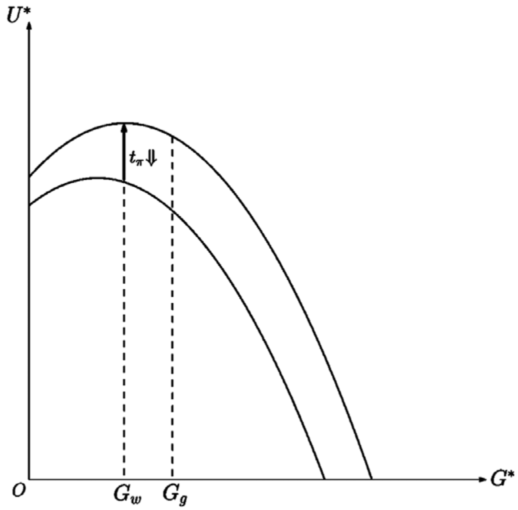


**FIGURE 4.** The growth effect is large.

To consider the effect of $G^*$ on $E^*$, we differentiate $E^*$ with respect to $G^*$ as follows:

$$\frac{\partial E^*}{\partial G^*} = \frac{\varepsilon}{\mu}\left\{-1 - \rho\frac{f'(G^*)}{[f(G^*)]^2}\right\} < 0. \tag{35}$$

Thus, $E^*$ is decreasing in $G^*$. Note that the relationship between $\gamma^*$ and $G^*$ is an inverted U shape (Figure 2). From these results, we can depict equation (33) in $(G^*, U^*)$ space, as shown in Figures 4 and 5. If the growth effect, $\partial\gamma^*/\partial G^*$, is sufficiently large, $U^*$ and $G^*$ follow an inverted U shape (see Figure 4).[20] In contrast, if the growth effect is sufficiently small, $U^*$ is decreasing in $G^*$ (see Figure 5).

We next consider the welfare-maximizing policy. From (25), (26), and (33), only the profit tax is distorting, and the asset income/consumption taxes are equivalent to lump-sum taxes. Thus, differentiating $U^*$ with respect to $t_\pi$, under given $G^*$, we obtain

$$\left.\frac{\partial U^*}{\partial t_\pi}\right|_{G^*\text{given}} = \frac{\alpha}{\rho^2(\varepsilon - t_\pi)^2}\Phi(G^*),$$

where

$$\Phi(G) \equiv \rho[\alpha(\varepsilon - 1) - t_\pi] - f(G)(1 - G).$$

We then investigate the sign of $\Phi(G^*)$ under $\dot{N}_t > 0$ (i.e., $0 \le G^* < \hat{G}$). Appendix

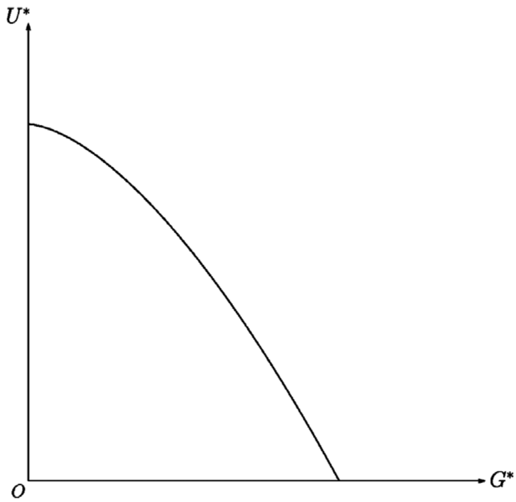


**FIGURE 5.** The growth effect is small.

A.6 shows that the sign of $\Phi(G^*)$ is negative when $0 \leq G^* < \hat{G}$. Thus, the sign of $\partial U^*/\partial t_\pi\big|_{G^*\text{given}}$ is also negative. A decrease in the profit tax shifts the curve in Figure 4 [RHS of (33)] up, and the optimum can be attained by setting the profit tax to zero.

Finally, we derive the level of welfare-maximizing public research spending. By setting $t_\pi = 0$ and using (28), (34), and (35), we obtain

$$\frac{\partial U^*}{\partial G^*} = \frac{\alpha}{\rho E^*}\left\{-1 - \rho\frac{f'(G^*)}{[f(G^*)]^2}\right\} + \frac{f'(G^*)(1-G^*) - f(G^*)}{\rho^2(\varepsilon - 1)[1+\alpha(\varepsilon-1)]} \equiv \frac{1}{\rho}\Omega(G^*). \tag{36}$$

From Figures 4 and 5, when the welfare-maximizing level of public research spending exists, the sign of $\partial U^*/\partial G^*$ at $G^* = 0$ becomes positive (i.e., the growth effect is sufficiently large at $G^* = 0$). Hence, the existence condition of a positive welfare-maximizing level of public research spending becomes as follows:

$$\Omega(0) = \frac{\alpha}{1+\frac{\rho}{f(0)}}\left\{-1 - \rho\frac{f'(0)}{[f(0)]^2}\right\} + \frac{f'(0) - f(0)}{\rho^2(\varepsilon-1)[1+\alpha(\varepsilon-1)]} > 0. \tag{37}$$

When $\varepsilon$ and $\rho$ are sufficiently small, and $f'(0)$ is sufficiently large, $\Omega(0) > 0$ holds (see Appendix A.7 for more detail). Let us define $G_w$ by $\Omega(G_w) = 0$.

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

In this case, $\partial U^*/\partial G^* \gtreqless 0$ holds when $G^* \lesseqgtr G_w$ (Figure 4), and thus, $G_w$ represents the welfare-maximizing level of public research spending. However, if $\Omega(0) \leq 0$, then $\partial U^*/\partial G^* < 0$ holds (Figure 5). In this case, $G^* = 0$ maximizes welfare. Further, we can compare the welfare-maximizing level of public research spending with the growth-maximizing level. Substituting $G^* = G_g$ into (36) yields

$$\left.\frac{\partial U^*}{\partial G^*}\right|_{G^*=G_g} = \frac{\alpha}{\rho E^*}\left\{-1 - \rho\frac{f'(G_g)}{[f(G_g)]^2}\right\} < 0.$$

Therefore, we see that the welfare-maximizing level of public research spending is below the growth-maximizing level. We can state the following proposition in summary:

PROPOSITION 3. *If $\Omega(0) > 0$, the policy mix of $t_\pi = 0$ and $\bar{B} = \Lambda(G_w)$ maximizes welfare. In addition, the welfare-maximizing level of public research spending is below the growth-maximizing level.*

Proposition 3 states that the government should not levy a tax on corporate profit and should finance spending with taxes on asset income/consumption. From (33), welfare is driven by household consumption expenditures and growth in the number of differentiated goods. Thus, the tax rate on profit should be zero to maximize the growth effect.

From (29), (34), and (35), we summarize the result of Proposition 3 as follows. If $G^* < G_w$, the growth-enhancing effect is sufficiently large, and thus, an increase in $G^*$ raises $U^*$. If $G_w < G^* < G_g$, the negative effect on household consumption expenditures is sufficiently large, and an increase in $G^*$ reduces $U^*$. When the government increases public research spending to maximize the growth rate, it impairs household consumption expenditures; that is, the welfare-maximizing level of public research spending, $G_w$, is lower than the growth-maximizing level, $G_g$. In contrast, when the production function of final output is a Cobb–Douglas form and the government imposes only a tax on household income in the Barro model, the growth-maximizing policy is equivalent to the welfare-maximizing policy.[21] In this setup, the relationship between consumption and productive government spending follows an inverted U shape and the growth-maximizing level of productive government spending also maximizes the consumption level. Hence, the growth-maximizing level of productive government spending equals the welfare-maximizing level.

The reason that the result of Proposition 3 is different from Barro's result is as follows. In the Barro model, when the government raises productive government spending, the production of final output increases. An increase in the available resources can be devoted to further consumption if the level of productive government spending is sufficiently small. However, because sufficiently large productive government spending crowds out consumption, the relationship between consumption and productive government spending is an inverted U shape. Turning to this study, public research spending, private R&D, and production require only labor input. Because the total labor supply is constant over time, an increase in public research spending decreases the amount of skilled labor devoted to production and private R&D. For this reason, household consumption expenditures are decreasing in public research spending. As discussed in the preceding, this leads to a welfare-maximizing level of public research spending below the growth-maximizing level.[22]

## 6. CONCLUSION

This study has developed an R&D-based growth model to examine the effects of public research spending on private R&D. We found that a zero profit tax maximizes welfare in the steady state. Furthermore, the welfare-maximizing level of public research spending is below the growth-maximizing level. We also found that equilibrium is indeterminate when government debt, consumption, and the profit tax are sufficiently small and the asset income tax is sufficiently large.

This study has the potential for several extensions. First, it is a first-generation R&D-based growth model that exhibits scale effects, and scale effects are not generally supported by empirical studies. It would be interesting to consider a nonscale growth model.[23,24] Second, the stipulation that the supply of skilled and unskilled labor is exogenous seems unrealistic. To address this, we could incorporate endogenous skill acquisition following Dinopoulos and Segerstrom (1999). Third, we could consider the labor–leisure decision. In this context, taxes on asset income and consumption are distortionary because they affect the trade-off between consumption and leisure. Future research should examine how these extensions affect the results of welfare-maximizing policy and indeterminacy.

## *NOTES*

1. Irmen and Kuehnel (2009) provide an extensive survey of the literature. Recent contributions include, for instance, Chatterjee (2007), Marrero (2008), Economides et al. (2011), Misch et al. (2013), and Angyridis (2015).

2. Peretto (2007) considers productive government spending, following the Barro model in an R&D-based growth model. However, his main focus is upon welfare effects of revenue-neutral changes in tax structure.

3. Many theoretical studies investigate a fiscal policy of productive public spending with debt financing. For example, see Bruce and Turnovsky (1999), Greiner and Semmler (2000), Ghosh and Mourmouras (2004), Futagami et al. (2008), Yakita (2008), Minea and Villieu (2012, 2013), Morimoto et al. (2013), and Maebayashi et al. (2014).

4. Moreover, these studies do not consider the effect of government debt. In fact, few studies examine the relationship between the debt policy rule and equilibrium indeterminacy. Futagami et al. (2008) show that, in a closed-economy model of endogenous growth with public services and a debt policy, the high growth steady state can be equilibrium indeterminate when the long-run

debt-private capital ratio is sufficiently high. Maebayashi et al. (2014) show that equilibrium indeterminacy never arises in a closed-economy model of endogenous growth with public capital and a debt policy. Employing a small open-economy model of endogenous growth with public capital and a debt policy, Morimoto et al. (2013) show that equilibrium indeterminacy arises when the long-run debt-GDP ratio is sufficiently high. In contrast to these studies, this study shows that equilibrium indeterminacy arises when government debt is sufficiently small.

5. Note that the profit tax can be translated as a dividend tax.

6. See Grossman and Helpman (1991) for details of the R&D process.

7. Our main results remain unaltered if the government follows a debt policy rule per Futagami et al. (2008). In that case, the government budget constraint is

$$w_t^s G_t + r_t B_t = t_A r_t A_t + t_E E_t + t_\pi \Pi_t N_t + \dot{B}_t.$$

The government adjusts its debt by the following rule:

$$\dot{B}_t = -\phi(B_t - \bar{B}),$$

where $\bar{B}$ and $\phi$ denote the target level of government debt and the adjustment coefficient of the rule, respectively.

8. If $\bar{B} < 0$, the government lends to households and earns interest.

9. As Goolsbee (1998) discusses, the supply of R&D workers is inelastic. Therefore, an increase in the number of public sector researchers reduces the number of private sector researchers. We adopt this fact and further assume that skilled labor in the R&D sector displaces that in the production sector. In this study, the degree of displacement of workers betweenthe production and R&D sectors is $\alpha$. In the extreme case ($\alpha \to 0$), researchers do not displace workers in the production sector.

10. Differentiating $\partial\gamma^*/\partial G^*$ with respect to $G^*$ yields

$$\frac{\partial^2 \gamma^*}{\partial G^{*2}} = \frac{1-t_\pi}{\mu}\Big[f''(G^*)(1-G^*) - 2f'(G^*)\Big] < 0.$$

11. If $\Lambda(1) < \bar{B} \leq \Lambda(\hat{G})$, $\hat{G} < G^* < 1$ holds. In this case, $L_R^* = 0$ and the market-clearing condition for skilled labor becomes $(N_t l_t^s)^* + G^* = 1$.

12. Differentiating $f'(G)/[f(G)]^2$ with respect to $G$ yields $\{f''(G)f(G) - 2[f'(G)]^2\}/[f(G)]^3 < 0$.

13. Note that $\tilde{G}$ can be a negative value. In this case, there is no complementarity between public and private R&D investments.

14. If $\Delta^* = 0$, the economy jumps to the steady state immediately. See Appendix A.2 for details.

15. This increase in $G_t$ is not caused by the fiscal policy change.

16. We can derive $t_A r_t A_t - r_t \bar{B} = (1 - t_A) r_t \Delta_t$ from (18) and the definition of $\Delta_t$.

17. We modify the model as follows. The government also employs labor for other governmental activities. Suppose this employment does not affect production and R&D. Under this modification, we obtain the following result: When public employees other than public researchers are larger, the equilibrium path tends to be indeterminate. The reasoning is as follows. If public employees other than public researchers are sufficiently large, government spending becomes sufficiently large. To satisfy the budget constraint, the government must depend on asset income tax revenue. In this case, households' expectations can be self-fulfilling, and equilibrium is indeterminate. The detailed discussion is included in the Technical Appendix available on request.

18. From Figure 3, it seems to imply that $G_t = G_\Delta$ is also the steady state. However, we show in Appendix A.2 that $G_t = G_\Delta$ is not the steady state unless $G^* = G_\Delta$.

19. We thank an anonymous referee for suggesting this discussion.

20. We derive this condition later.

21. If the government can use consumption or lump-sum taxes, the growth rate is increasing in productive government spending. In this set-up, we cannot compare the welfare-maximizing level of productive government spending with the growth-maximizing level.

22. If we employ the *lab-equipment* specification in Rivera-Batiz and Romer (1991), the result of Proposition 3 can be equivalent to Barro's result. Under the lab-equipment specification, public research spending and private R&D use final output. However, as mentioned in the fourth paragraph of the Introduction, the majority of public research spending goes toward hiring R&D personnel. Hence, the setup of this study is more realistic.

23. See Jones (1995) for a detailed discussion of scale effects in R&D-based growth models.

24. With regard to the result of indeterminacy, we show that a similar condition yields equilibrium indeterminacy in a growth model without scale effects. The proof is included in the Technical Appendix available on request.

## REFERENCES

Aghion, Philippe and Peter Howitt (1992) A model of growth through creative destruction. *Econometrica* 60, 323–351.

Angyridis, Constantine (2015) Endogenous growth with public capital and progressive taxation. *Macroeconomic Dynamics* 19(6), 1220–1239.

Arnold, Lutz (1997) Basic and applied research. *FinanzArchiv: Public Finance Analysis* 54, 169–186.

Arrow, Kenneth J. and Mordecai Kurz (1970) *Public Investment, the Rate of Return and Optimal Fiscal Policy*. Baltimore, MD: Johns Hopkins University Press.

Barro, Robert J. (1990) Government spending in a simple model of endogenous growth. *Journal of Political Economy* 98, 103–125.

Bruce, Neil and Stephen J. Turnovsky (1999) Budget balance, welfare, and the growth rate: "Dynamic scoring" of the long-run government budget. *Journal of Money, Credit and Banking* 31, 162–186.

Buiter, Willem H. (1984) Saddlepoint problems in continuous time rational expectations models: A general method and some macroeconomic examples. *Econometrica* 52, 665–680.

Caloghirou,Yannis, Aggelos Tsakanikas, and Nicholas S. Vonortas (2001) University–industry cooperation in the context of the European Framework Programmes. *Journal of Technology Transfer* 26, 153–161.

Chatterjee, Santanu (2007) Should the private sector provide public capital? *Macroeconomic Dynamics* 11, 318–346.

David, Paul A., Bronwyn H. Hall, and Andrew A. Toole (2000) Is public R&D a complement or substitute for private R&D? A review of the econometric evidence. *Research Policy* 29, 497–529.

Dinopoulos, Elias and Paul Segerstrom (1999) A Schumpeterian model of protection and relative wages. *American Economic Review* 89, 450–472.

Economides, George, Hyun Park, and Apostolis Philippopoulos (2011) How should the government allocate its tax revenues between productivity-enhancing and utility-enhancing public goods? *Macroeconomic Dynamics* 15, 336–364.

Fisher, Walter H. and Stephen J. Turnovsky (1998) Public investment, congestion, and private capital accumulation. *Economic Journal* 108, 399–413.

Futagami, Koichi, Tatsuro Iwaisako, and Ryoji Ohdoi (2008) Debt policy rule, productive government spending, and multiple growth paths. *Macroeconomic Dynamics* 12, 445–462.

Futagami, Koichi, Yuichi Morita, and Akihisa Shibata (1993) Dynamic analysis of an endogenous growth model with public capital. *Scandinavian Journal of Economics* 95, 607–625.

Ghosh, Sugata and Iannis A. Mourmouras (2004) Endogenous growth, welfare and budgetary regimes. *Journal of Macroeconomics* 26, 623–635.

Glomm, Gerhard and B. Ravikumar (1994) Growth-inequality trade-offs in a model with public sector R&D. *Canadian Journal of Economics* 27, 484–493.

Goolsbee, Austan (1998) Does government R&D policy mainly benefit scientists and engineers? *American Economic Review* 88, 298–302.

Greiner, Alfred and Willi Semmler (2000) Endogenous growth, government debt and budgetary regimes. *Journal of Macroeconomics* 22, 363–384.

Grossman, Gene M. and Elhanan Helpman (1991) *Innovation and Growth in the Global Economy*. Cambridge, MA: MIT Press.

Guo, Jang-Ting and Sharon G. Harrison (2008) Useful government spending and macroeconomic (in)stability under balanced-budget rules. *Journal of Public Economic Theory* 10, 383–397.

Irmen, Andreas and Johanna Kuehnel (2009) Productive government expenditure and economic growth. *Journal of Economic Surveys* 23, 692–733.

Jones, Charles I. (1995) R&D-based models of economic growth. *Journal of Political Economy* 103, 759–784.

Kamiguchi, Akira and Toshiki Tamai (2011) Can productive government spending be a source of equilibrium indeterminacy? *Economic Modeling* 28, 1335–1340.

Lichtenberg, Frank R. (1984) The relationship between federal contract R&D and company R&D. *American Economic Review* 74, 73–78.

Lööf, Hans and Andres Broström (2008) Does knowledge diffusion between university and industry increase innovativeness? *Journal of Technology Transfer* 33, 73–90.

Maebayashi, Noritaka, Takeo Hori, and Koichi Futagami (2014) Dynamic Analysis of Reductions in Public Debt in an Endogenous Growth Model with Public Capital. Discussion paper in economics and business 12-08-Rev.2, Osaka University.

Mansfield, Edwin (1991) Academic research and industrial innovation. *Research Policy* 20, 1–12.

Marrero, Gustavo A. (2008) Revisiting the optimal stationary public investment policy in endogenous growth economies. *Macroeconomic Dynamics* 12, 172–194.

Minea, Alexandru and Patrick Villieu (2012) Persistent deficit, growth, and indeterminacy. *Macroeconomic Dynamics* 16, 267–283.

Minea, Alexandru and Patrick Villieu (2013) Debt policy rule, productive government spending, and multiple growth paths: A note. *Macroeconomic Dynamics* 17, 947–954.

Misch, Florian, Norman Gemmell, and Richard Kneller (2013) Growth and welfare maximization in models of public finance and endogenous growth. *Journal of Public Economic Theory* 15, 939–967.

Morimoto, Keiichi, Takeo Hori, Noritaka Maebayashi, and Koichi Futagami (2013) Fiscal Sustainability, Macroeconomic Stability, and Welfare under Fiscal Discipline in a Small Open Economy. Discussion papers in economics and business 13-07, Osaka University.

National Science Foundation (2011) Research and Development in Industry: 2006–07. Detailed Statistical Tables NSF 11-301. Arlington, VA: National Science Foundation. Available at http://www.nsf.gov/statistics/nsf11301/.

Park, Walter G. (1998) A theoretical model of government research and growth. *Journal of Economic Behavior and Organization* 34, 69–85.

Peretto, Pietro F. (2007) Schumpeterian growth with productive public spending and distortionary taxation. *Review of Development Economics* 11, 699–722.

Rivera-Batiz, Luis A. and Paul M. Romer (1991) Economic integration and endogenous growth. *Quarterly Journal of Economics* 106, 531–555.

Romer, Paul M. (1990) Endogenous technological change. *Journal of Political Economy* 98, 71–102.

Turnovsky, Stephen J. (1997) Fiscal policy in a growing economy with public capital. *Macroeconomic Dynamics* 1, 615–639.

Yakita, Akira (2008) Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. *Journal of Public Economics* 92, 897–914.

# APPENDIX

## A.1. DERIVATION OF THE JACOBIAN MATRIX

From (20) and (21), we obtain

$$\dot{G}_t = \frac{f(G_t)}{f'(G_t)}\left\{\frac{\mu}{\varepsilon}f(G_t)E_t - f(G_t)(1-G_t) + \frac{1}{\Delta_t}\left[\left(t_E + \frac{t_\pi}{\varepsilon}\right)E_t - G_t\right]\right\},$$

$$\dot{E}_t = E_t\left\{\frac{1}{\Delta_t}\left[G_t - \left(t_E + \frac{t_\pi}{\varepsilon}\right)E_t\right] - \rho\right\}.$$

When these equations are approximated linearly in the neighborhood of the steady states, the following elements of the Jacobian matrix are obtained:

$$J_{GG} = \frac{f(G^*)}{f'(G^*)}\left[\frac{\mu}{\varepsilon}f'(G^*)E^* - f'(G^*)(1-G^*) + f(G^*)\right]$$

$$+\frac{f(G^*)}{f'(G^*)(\Delta^*)^2}\left\{-\Delta^* + \left[\left(t_E + \frac{t_\pi}{\varepsilon}\right)E^* - G^*\right]\frac{t_A}{1-t_A}\frac{f'(G^*)}{[f(G^*)]^2}\right\},$$

$$J_{GE} = \frac{f(G^*)}{f'(G^*)}\left[\frac{\mu}{\varepsilon}f(G^*) + \frac{1}{\Delta^*}\left(t_E + \frac{t_\pi}{\varepsilon}\right)\right],$$

$$J_{EG} = \frac{E^*}{(\Delta^*)^2}\left\{\Delta^* + \left[G^* - \left(t_E + \frac{t_\pi}{\varepsilon}\right)E^*\right]\frac{t_A}{1-t_A}\frac{f'(G^*)}{[f(G^*)]^2}\right\},$$

$$J_{EE} = -\left(t_E + \frac{t_\pi}{\varepsilon}\right)\frac{E^*}{\Delta^*}.$$

Using (22) and (25), we can rewrite $J_{ij}(i, j = G, E)$ as follows:

$$J_{GG} = \frac{f(G^*)}{f'(G^*)}\left[f(G^*) + \rho\frac{f'(G^*)}{f(G^*)}\right] - \frac{f(G^*)}{f'(G^*)\Delta^*}\left\{1 + \rho\frac{t_A}{1-t_A}\frac{f'(G^*)}{[f(G^*)]^2}\right\},$$

$$J_{GE} = \frac{f(G^*)}{f'(G^*)\Delta^*}\left[\frac{\mu}{\varepsilon}f(G^*)\Delta^* + \left(t_E + \frac{t_\pi}{\varepsilon}\right)\right],$$

$$J_{EG} = \frac{E^*}{\Delta^*}\left\{1 + \rho\frac{t_A}{1-t_A}\frac{f'(G^*)}{[f(G^*)]^2}\right\},$$

$$J_{EE} = -\left(t_E + \frac{t_\pi}{\varepsilon}\right)\frac{E^*}{\Delta^*}.$$

## A.2. DYNAMICS WHEN $\Delta^* = 0$

From the definition of $\Delta^*$, the case where $\Delta^* = 0$ implies that $\bar{B} > 0$ and $t_A > 0$ or $\bar{B} = t_A = 0$. First, we examine the case where $\bar{B} > 0, t_A > 0$. We define $G_\Delta$ as $f(G_\Delta) = t_A/(1-t_A)\bar{B}$. That is, $G_\Delta$ satisfies $\Delta_t = 0$. When $G_t = G_\Delta$, (12) and (16) yield $E_\Delta \equiv G_\Delta/(t_E + \frac{t_\pi}{\varepsilon})$. Therefore, $\dot{G}_t = \dot{E}_t = 0$ holds, and the economy jumps to the steady state immediately. This result seems to imply multiple steady states, $G^*$ and $G_\Delta$. However,

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

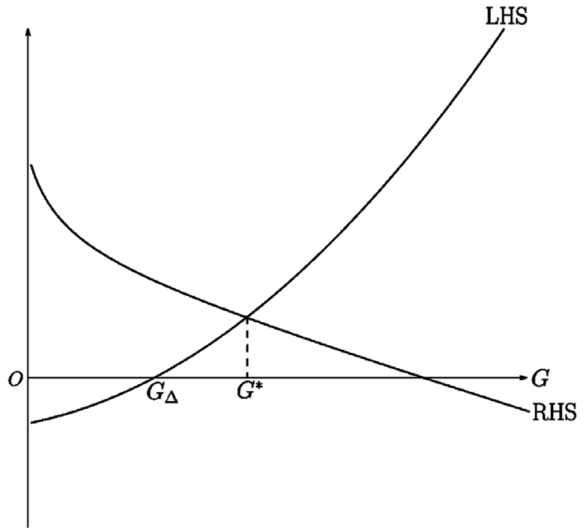


**FIGURE A.1.** Determination of $G^*$.

$G_\Delta$ cannot hold the no-arbitrage condition unless $G^* = G_\Delta$. We rearrange equation (24) as follows:

$$\underbrace{\bar{B} - \frac{t_A}{1-t_A}\frac{1}{f(G_t)}}_{-\Delta_t} = \frac{\varepsilon t_E + t_\pi}{\mu}\frac{1}{f(G_t)} + \frac{1}{\rho\mu}\Big[\varepsilon t_E + t_\pi - \big(\varepsilon t_E + t_\pi + \mu\big)G_t\Big]. \quad \textbf{(A.1)}$$

As shown in Figure A.1, the intersection of the LHS and RHS of (A.1) determines $G^*$. When $G^* \neq G_\Delta$, the following inequality holds:

$$\frac{\varepsilon t_E + t_\pi}{\mu}\frac{1}{f(G_\Delta)} + + \frac{1}{\rho\mu}\Big[\varepsilon t_E + t_\pi - \big(\varepsilon t_E + t_\pi + \mu\big)G_\Delta\Big] > 0.$$

Thus, this inequality implies that the no-arbitrage condition cannot hold.

Second, we investigate the case wherein $\bar{B} = t_A = 0$. From (12) and (16), we obtain

$$G_t = \left(t_E + \frac{t_\pi}{\varepsilon}\right)E_t. \quad \textbf{(A.2)}$$

Equations (6), (19), and (A.2) yield the following autonomous dynamic system with respect to $G_t$:

$$\left[1 + \frac{f'(G_t)G_t}{f(G_t)}\right]\frac{\dot{G}_t}{G_t} = \frac{\mu}{\varepsilon t_E + t_\pi} f(G_t)G_t - f(G_t)(1-G_t) - \rho \equiv \psi(G_t).$$

Note that $\psi(0) < 0$ and $\psi(G_t)$ is increasing in $G_t$ (or U-shaped). As shown in Figure A.2, the economy jumps to the steady state immediately.

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

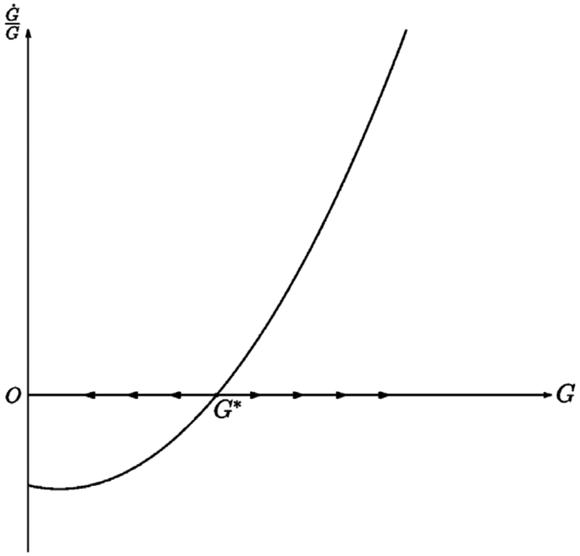


**FIGURE A.2.** Dynamics when $\Delta^* = 0$.

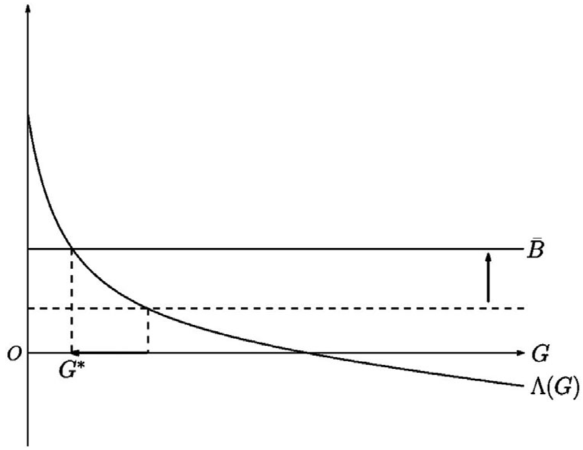


**FIGURE A.3.** The effect of $\bar{B}$ on $G^*$.

### A.3. PROOF OF LEMMA 1

We examine the effects of the fiscal variables on $G^*$ through changes in the fiscal variables. The effect of $\bar{B}$ can be examined easily using (24). When $\bar{B}$ increases, the horizontal line rises, as shown in Figure A.3. Thus, $G^*$ decreases.

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

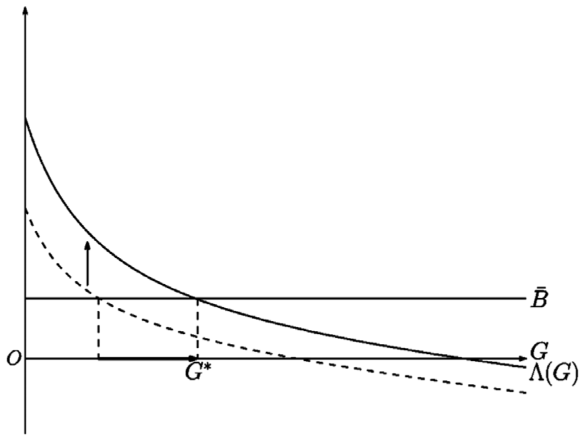


**FIGURE A.4.** The effects of taxes on $G^*$.

Differentiating $\Lambda(G)$ with respect to $t_A$, $t_E$, and $t_\pi$, we obtain

$$\left.\frac{\partial \Lambda}{\partial t_A}\right|_{G_t \text{ given}} = \frac{1}{(1-t_A)^2}\frac{1}{f(G_t)} > 0,$$

$$\left.\frac{\partial \Lambda}{\partial t_E}\right|_{G_t \text{ given}} = \frac{\varepsilon}{\mu}\left[\frac{1}{f(G_t)} + \frac{1}{\rho}(1-G_t)\right] > 0,$$

$$\left.\frac{\partial \Lambda}{\partial t_\pi}\right|_{G_t \text{ given}} = \frac{\varepsilon t_E + t_\pi + \mu}{\mu^2}\left[\frac{1}{f(G_t)} + \frac{1}{\rho}(1-G_t)\right] > 0.$$

When tax rates $t_i$ $(i = A, E, \pi)$ increase, the graph of the RHS of (24) rises (Figure A.4). Therefore, $G^*$ increases.

## A.4. EFFECTS OF $\Delta^*$ THROUGH CHANGES IN FISCAL VARIABLES

Differentiating $\Delta^*$ with respect to $t_i$ $(i = E, \pi)$ yields

$$\frac{\partial \Delta^*}{\partial t_i} = -\frac{t_A}{1-t_A}\frac{f'(G^*)}{[f(G^*)]^2}\frac{\partial G^*}{\partial t_i}\ (i = E, \pi).$$

Using Lemma 1, we obtain $\partial \Delta^*/\partial t_i < 0$ $(i = E, \pi)$. To study the effect of $\bar{B}$ and $t_A$, we rewrite $\Delta^*$ using (24) as follows:

$$\Delta^* = \frac{1}{\rho\mu}\left\{-\left(\varepsilon t_E + t_\pi\right)\left[\frac{\rho}{f(G^*)} + 1\right] + \left(\varepsilon t_E + t_\pi + \mu\right)G^*\right\}. \quad \textbf{(A.3)}$$

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

Differentiating $\Delta^*$ with respect to $\bar{B}$ and $t_A$, we obtain

$$\frac{\partial \Delta^*}{\partial \bar{B}} = \frac{1}{\rho\mu}\left\{\left(\varepsilon t_E + t_\pi\right)\rho\frac{f'(G^*)}{[f(G^*)]^2} + \left(\varepsilon t_E + t_\pi + \mu\right)\right\}\frac{\partial G^*}{\partial \bar{B}},$$

$$\frac{\partial \Delta^*}{\partial t_A} = \frac{1}{\rho\mu}\left\{\left(\varepsilon t_E + t_\pi\right)\rho\frac{f'(G^*)}{[f(G^*)]^2} + \left(\varepsilon t_E + t_\pi + \mu\right)\right\}\frac{\partial G^*}{\partial t_A}.$$

From Lemma 1, $\partial\Delta^*/\partial\bar{B} < 0$ and $\partial\Delta^*/\partial t_A > 0$ both hold.

## A.5. PHASE DIAGRAM OF THE EQUILIBRIUM INDETERMINACY

From Proposition 2, equilibrium indeterminacy occurs if $\Delta^* > 0$. We first investigate the condition that $\Delta_t > 0$ holds. Suppose $t_A > (1 - t_A)\bar{B}f(0)$. From the definition of $\Delta_t$ and $f(G_t)$, $\Delta_t > 0$ holds if $0 < G_t < G_\Delta$. In this analysis, we restrict the case where $0 < G_t < G_\Delta$. Using (20) and (21), we obtain

$$\dot{G}_t \gtreqless 0 \Leftrightarrow E_t \gtreqless \frac{(1 - G_t)\Delta_t f(G_t) + G_t}{\frac{\mu}{\varepsilon}\Delta_t f(G_t) + t_E + \frac{t_\pi}{\varepsilon}} \equiv \Gamma(G_t),$$

$$\dot{E}_t \gtreqless 0 \Leftrightarrow E_t \lesseqgtr \frac{1}{t_E + \frac{t_\pi}{\varepsilon}}\left(G_t - \rho\Delta_t\right) \equiv \Upsilon(G_t).$$

In addition, we obtain the following results:

$$\Gamma(0) = \frac{\frac{t_A}{1-t_A} - \bar{B}f(0)}{\frac{\mu}{\varepsilon}\left[\frac{t_A}{1-t_A} - \bar{B}f(0)\right] + t_E + \frac{t_\pi}{\varepsilon}} > 0,$$

$$\Upsilon(0) = \frac{\rho}{t_E + \frac{t_\pi}{\varepsilon}}\left[\bar{B} - \frac{t_A}{(1 - t_A)f(0)}\right] < 0,$$

$$\Gamma(G_\Delta) = \Upsilon(G_\Delta) = \frac{G_\Delta}{t_E + \frac{t_\pi}{\varepsilon}} > 0,$$

$$\Upsilon'(G_t) = \frac{1}{t_E + \frac{t_\pi}{\varepsilon}}\left\{1 + \frac{\rho t_A}{1 - t_A}\frac{f'(G_t)}{\left[f(G_t)\right]^2}\right\} > 0.$$

From these results and the uniqueness of the steady state, we can depict the phase diagram in $(G_t, E_t)$ space.

## A.6. SIGN OF $\Phi(G^*)$

We investigate the sign of $\Phi(G^*)$ under $\dot{N}_t > 0$ (i.e., $0 \leq G^* < \hat{G}$). Differentiating $\Phi(G^*)$ with respect to $G^*$, we obtain

$$\Phi'(G^*) = -f'(G^*)(1 - G^*) + f(G^*) \lesseqgtr 0 \Longleftrightarrow G^* \lesseqgtr G_g.$$

https://doi.org/10.1017/S1365100515000759 Published online by Cambridge University Press

Therefore, $\Phi(G^*)$ and $G^*$ follow a U shape. Assumption (27) yields $f(0) > \alpha(\varepsilon - 1)\rho/(1 - t_\pi)$, and thus, we obtain

$$\Phi(0) < -\rho t_\pi - \alpha(\varepsilon - 1)\rho \frac{t_\pi}{1 - t_\pi} < 0.$$

The definition of $\hat{G}$ yields

$$f(\hat{G})(1 - \hat{G}) = \frac{\alpha(\varepsilon - 1)\rho}{1 - t_\pi}. \qquad \textbf{(A.4)}$$

Using (A.6), we obtain

$$\Phi(\hat{G}) = -\rho t_\pi - \alpha(\varepsilon - 1)\rho \frac{t_\pi}{1 - t_\pi} < 0.$$

Thus, the sign of $\Phi(G^*)$ is negative when $0 \leq G^* < \hat{G}$.

## A.7. CONDITION $\Omega(0) > 0$

From (37), the necessary condition for $\Omega(0) > 0$ is $f'(0) > f(0)$. We assume this condition to ensure the existence of $G_g$. Hence, $\partial\Omega(0)/\partial\varepsilon < 0$ apparently holds. Differentiating $\Omega(0)$ with respect to $\rho$ and $f'(0)$ yields

$$\frac{\partial\Omega(0)}{\partial\rho} = -\left\{\frac{\alpha}{\left[f(0) + \rho\right]^2} + \frac{1}{\rho(\varepsilon - 1)[1 + \alpha(\varepsilon - 1)]}\right\}\left[f'(0) - f(0)\right] < 0,$$

$$\frac{\partial\Omega(0)}{\partial f'(0)} = \frac{-\alpha\rho^2(\varepsilon - 1)[1 + \alpha(\varepsilon - 1)] + [f(0)]^2 + \rho f(0)}{\rho(\varepsilon - 1)[1 + \alpha(\varepsilon - 1)]\left\{[f(0)]^2 + \rho f(0)\right\}}.$$

Using the assumption in (27), we obtain $-\rho^2(\varepsilon - 1)[1 + \alpha(\varepsilon - 1)] + [f(0)]^2 + \rho f(0) > 0$. Thus, $\partial\Omega(0)/\partial f'(0) > 0$ holds. From these results, when $\varepsilon$ and $\rho$ are sufficiently small and $f'(0)$ is sufficiently large, $\Omega(0) > 0$ holds.