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Numerical and experimental characterization of singularities of a six-wire parallel architecture

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SUMMARY
A characterization of singularities for a six-wire parallel architecture is presented as a result of numerical and experimental analyses. Numerical analysis has been developed through geometrical and analytical considerations. The study is based on a classification that has been derived on the basis of the geometry of tetrahedra, and singular configurations have been classified as a function of the tetrahedron volume. Experimental characterization has been carried out by considering the wire parallel architecture Cassino tracking system (CATRASYS). Experimental results are reported to characterize the performance of the CATRASYS chain in different operating conditions as an illustrative practical example.

KEYWORDS: Experimental robotics; Wire parallel architectures; Measuring systems; Singularity characterization.

1. Introduction

Measuring systems determining the position and orientation of a moving object can be cameras, theodolites, laser tracking systems, and wire-based tracking systems. Most of these have a trilateration- or triangulation-based process of measurement.

In this paper, we address the wire-based tracking systems, which appear to be interesting since they show a good combination of accuracy, resolution, cost, measurement range, portability, and calibration procedure. Wire-based tracking systems consist of a fixed base and a moving platform connected by at least six wires whose tension is maintained by pulleys and spiral springs on the base. They can be modeled as six-degrees-of-freedom (DOF) parallel manipulators because wires can be considered as extensible legs connecting the platform and the base by means of spherical and universal joints, respectively.

The problem of pose identification of a rigid body in space has drawn great attention as can be seen in theoretical approaches and numerical algorithms.[1–3,4]

At the Laboratory of Robotics and Mechatronics (LARM), University of Cassiono, we have approached the problem looking at practical application with robust easy-operation device. Thus, the Cassino tracking system (CATRASYS) came into existence in 1994,[5] which uses an algebraic formulation via trilateration in order to identify the pose of a rigid body during its motion through an online computation of the position kinematics of the designed 3-2-1 wire parallel architecture. The wire-tracking systems have also been studied elsewhere.[6]

Here, a wire-based parallel architecture measuring system is considered to determine the position and orientation of moving objects. The architecture is modeled as a Gough–Stewart platform.[7]

In particular, the proposed parallel architecture measuring system can be modeled as a special type of Gough–Stewart platform having three attachment points at the moving platform and six attachment points at the base. For the above-mentioned model, only two wire clusters are possible, namely, the 2-2-2 and the 3-2-1 configurations. The parallel architecture measuring system with the 3-2-1 configuration has been chosen for the measuring system, since for this special type 3-2-1 Gough–Stewart platform, the direct kinematics can be solved in close form using trilateration.[8]

An important feature in the use of a measuring system is its accuracy, which can be related to the presence of singular configurations. In fact, the accuracy of measurements is pose-dependent and is strongly influenced by the closeness to singular configurations. Therefore, the identification and characterization of singularities of wire-parallel measuring systems is of great interest.[8]

Here, we propose a method that is based on characteristic tetrahedra, which describes the occurrence of singularity in parallel architectures qualitatively and qualitatively. In this paper, the characterization of singularities of the proposed wire parallel architecture for a measuring system is addressed and experimental results are reported not only to show the engineering significance of the proposed analysis, but also to obtain an experimental characterization of manipulator operation into singularity.

2. A six-wire parallel architecture

CATRASYS is a measuring system that has been conceived and designed at the LARM, to determine the pose (position and orientation) of a moving object during large displacements by using trilateration. Details of CATRASYS and its evolution are reported elsewhere.[9–11]

The CATRASYS system is composed of a mechanical part, an electronics/informatics interface unit, and a software package. The mechanical part consist of a fixed base, which has been named as the trilateral sensing platform, and a moving platform, which has been named as the end-effector

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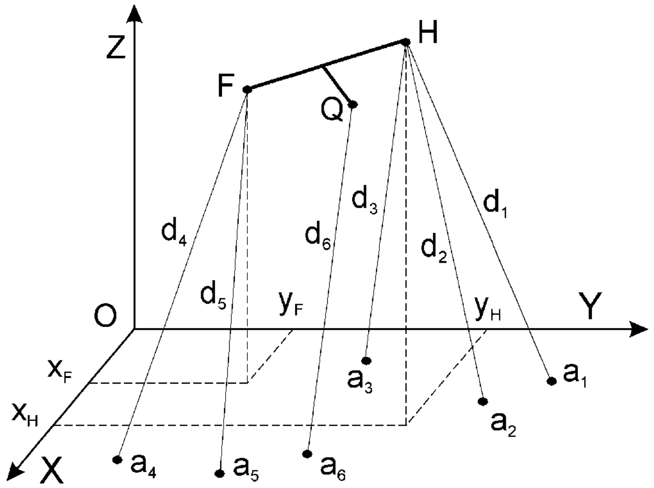

Fig. 1. A scheme of six-wire parallel architecture measuring system CATRASYS.

for CATRASYS. The two platforms are connected by six wires, whose tension is maintained by pulleys and spiral springs that are fixed on the base. The peculiarity of the mechanical structure is that it can be modeled as a six-DOFs parallel manipulator because the wires can be considered as extensible legs connecting the platform and the base by means of spherical and universal joints, respectively. The end-effector for CATRASYS is the moving platform operating as a coupling device: It connects the wires of the six transducers to the extremity of a movable system. It allows the wires to track the system while it moves. Signals from wire transducers are fed though an amplified connector to the electronic interface unit, which consists of a PC for data analysis. In particular, referring to Fig. 1, H is the reference point of the moving platform and a general scheme is shown as 3-2-1 parallel manipulator, whose platform is determined by points H, F, and Q. Points F and Q have been used to determine the orientation of the moving platform. The base reference frame is O-XYZ. The prototype of the measuring system is shown in Figs. 2 and 3.

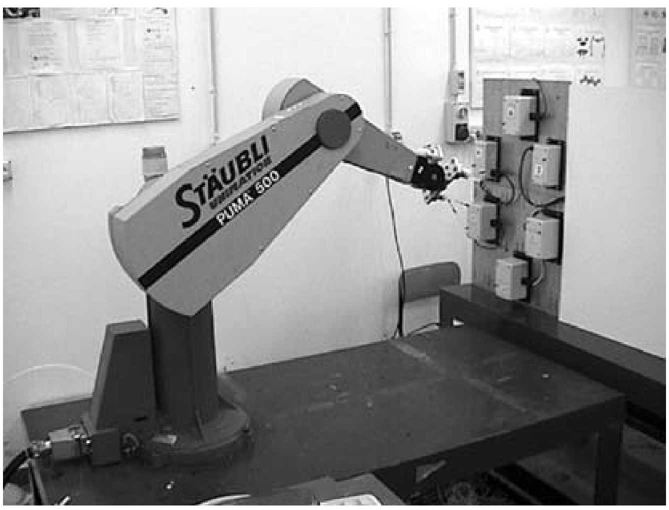

Fig. 2. The CATRASYS measuring system at LARM in Cassino: Lay-out for experimental tests with a PUMA robot.

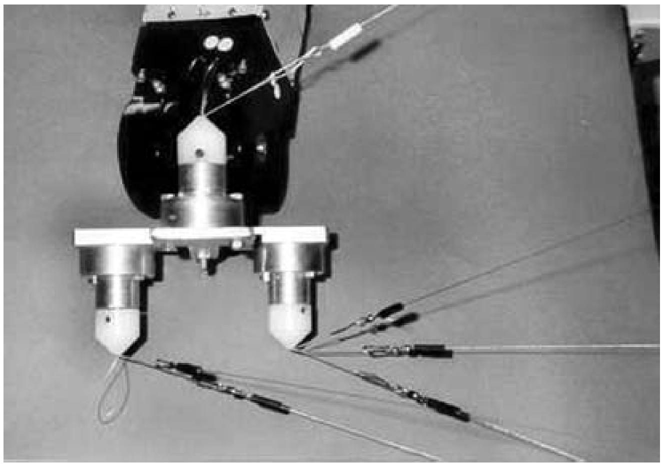

Fig. 3. The end-effector for CATRASYS.

The trilateral sensing platform (Fig. 2) has been designed such that it remains portable and easy to locate. Furthermore, it allows changing the position of the transducers on it and the usage of other measurement instruments. The end-effector for CATRASYS allows a suitable connection to the wires to measure position and orientation of the movable system. The single module for the end-effector for CATRASYS has a fully rotating shaft, which permits an easy connection to the wires and avoids wrapping of the wires. The prototype, shown in Fig. 3, is composed of three identical modules for the end-effector for CATRASYS in order to give an easy connection to all the six wires. The wire transducers in the built prototype are of a potentiometric type. They have a working range of 2500 mm and have a continuous resolution. A torsional spring, a pulley for the wire, and a potentiometer are fixed on a common shaft. The output transducer signal is proportional to the length of the wire and is expressed in volt. Tension in the wire is maintained through the torsional spring.

CATRASYS has been used successfully for the determination of the workspace and kinematic parameters of complex mechanical systems, as reported in refs. 11 and 12, and in the experimental analysis of the kinematic parameters of human arm. Kinematics of the measuring system CATRASYS has been solved in closed-form due to the special arrangement of the system by using the trilateration technique as reported in ref. 3.

3. Singularity determination

A singularity of the parallel architecture measuring system can be defined as the configuration at which the measure cannot be determined or it is not reliable.

In general, the analysis of singularities of parallel architectures is an important issue since they set the limits of the kinematic motion and static load equilibrium. Several authors have extensively studied singularities of parallel architectures. For example, Merlet has used Plucker line coordinates, Grassmann line geometry, and the screw theory.[13] Line geometry, wrench singularity analyses for parallel architectures have been presented in refs. 14–16.

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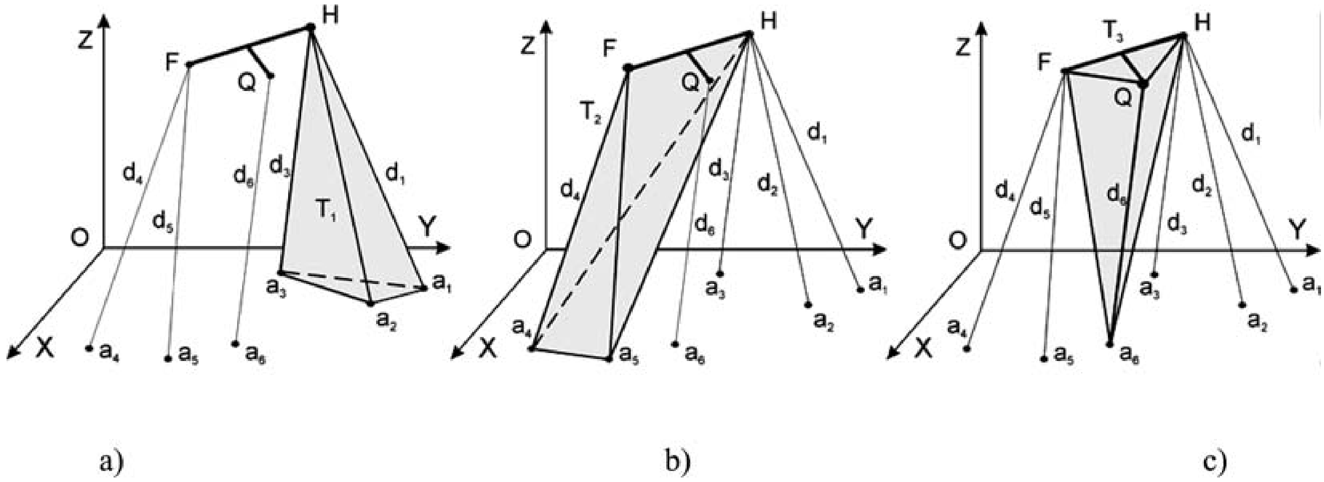

Fig. 4. Nonsingular characteristic tetrahedra for the parallel architecture CATRASYS: (a) T_1; (b) T_2; and (c) T_3.

Based on their nature, singularities of a measuring system can be classified into three categories: configuration, architecture, and formulation singularities. The above-mentioned classification has been proposed in ref. 17, for parallel manipulators.

The first type of singularity is an inherent property of the device, which occurs at some points within its workspace. Architecture singularities are due to the architecture and can prevail over the entire workspace. Formulation singularities are caused due to the adopted analysis formulation and can be avoided easily by changing the formulation method.

In order to analyze the singularities of the six-wire parallel architecture, the characteristic tetrahedron has been introduced here. The parallel architecture measuring system can be thought as being composed of three nonsingular characteristic tetrahedra, as shown in Fig. 4. By observing the singularity of the three tetrahedra, one can deduce the singularity of the parallel architecture measuring system, as outlined in refs. 18 and 19 in a preliminary manner. For the following analysis of each tetrahedron, the base is identified by three points, whose positions are known or determined, as shown in Fig. 4; the position of the apex is unknown.

Indeed, by considering Fig. 4, tetrahedron T_1 is defined by its base formed by connecting a_1, a_2, and a_3, with three edges d_1, d_2, and d_3, and its apex which is H. Tetrahedron T_2 is identified by its base formed by H, a_4, and a_5, with three edges d_4, d_5, and HF, and its apex which is F. Tetrahedron T_3 is identified by its base formed by H, F, and a_6, with three edges d_6, HF, and FQ, and its apex which is Q. All the three nonsingular characteristic tetrahedra for the parallel architecture measuring system are shown in Fig. 4.

A tetrahedron is nonsingular if and only if it does not collapse to give zero volume. By considering the definition, singular tetrahedra have a great variety of forms. We do not consider tetrahedra with infinite elements, since they do not have a physical meaning. For a singular tetrahedron, the position of the apex cannot be univocally determined when the base points lie on a line. Therefore, the possibilities for zero-volume tetrahedron are only the following: The apex collapse into the base plane of the tetrahedron, or the base points are aligned, as shown in Fig. 5.

When the tetrahedron is singular, its volume is zero, i.e., the tetrahedron degenerates in a triangle in a plane. In addition, for a singular tetrahedron the position of the apex cannot be univocally determined when the base points lie on a line. In this case, the apex can be on a circle in the space, and therefore, its position cannot be univocally determined. Indeed, one of the basic requirements to obtain a

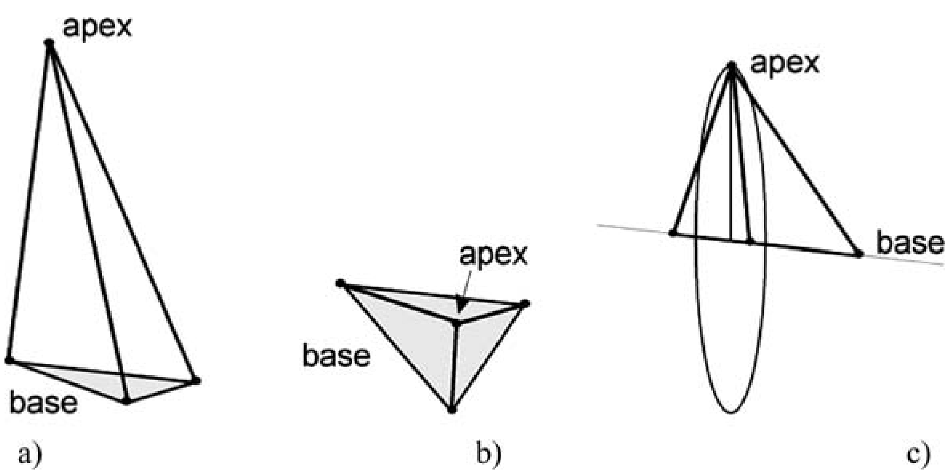

Fig. 5. Characteristic tetrahedra: (a) nonsingular tetrahedron; (b) singularity of zero high; and (c) singularity of zero area.

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measure is that the parallel architecture does not meet special configurations, which will be explored experimentally in the next section.

Applying the above-mentioned considerations to the CATRASYS measuring system, a singularity of one or more tetrahedra arises for particular arrangement of anchor points of the parallel architecture. Indeed, a study for the positioning of the wire transducers on the trilateral sensing platform (points a_i, for $i = 1, \ldots, 6$), can be also carried out in order to avoid possible singularities of the three tetrahedra. In particular, by considering the Cayley–Menger determinant,[20] which has been introduced and used to study basic geometry, one can also derive the analytical conditions to determine singularities of the characteristic tetrahedra.[18,19] In fact, the Cayley–Menger determinant is equal to zero if the three points of the base and the apex of a tetrahedron lie on a plane. This case occurs if the height is zero, or if the three points of the base are aligned, i.e., the base area is equal to zero. Analysis that is based on the geometry of the tetrahedra have been also presented[21] for a class of parallel manipulators with three legs.

A general nonsingular tetrahedron, which is depicted in Fig. 5(a), and singularities of zero high and zero area types are shown in Fig. 5(b) and (c), respectively.

Indeed, the singular configurations for the parallel architecture CATRASYS can be determined by observing the three tetrahedra T_1, T_2, and T_3 defined by points (a_1, a_2, a_3, H); (a_4, a_5, H, F); and (a_6, H, F, Q), respectively, as shown in Fig. 4.

Singular configurations can occur and their classification type can be determined as in the following.

1. Singularity of tetrahedron T_1.
   (a) a_1, a_2, a_3 are aligned: This can be considered as an architecture singularity.
   (b) a_1, a_2, a_3, and b_1 lie on a plane: This is a configuration singularity.
2. Singularity of tetrahedron T_2.
   (a) a_4, a_5, F are aligned: This is configuration singularity.
   (b) a_4, a_5, H, F lie on a plane: This is configuration singularity.
3. Singularity of tetrahedron T_3.
   (a) a_6, H, F are aligned: This is configuration singularity.
   (b) a_6, H, F, and Q lie on a plane: This is configuration singularity.
4. Combined singularities.

Special cases can arise if T_1, T_2, and T_3 or a combination of any two of them are contemporaneously singular.

Singular configurations of the parallel architecture CATRASYS can be determined by using the above-mentioned analysis for tetrahedra, as shown in Figs. 6–16. These are all the possible singular configurations for CATRASYS. By looking at the geometry of the tetrahedra, basic conditions can be determined for proper design and operation of CATRASYS. Thus, condition 1(a) can be avoided by considering an arrangement of the transducers on the trilateral sensing platform to avoid the alignment of points a_1, a_2, and a_3. The above-mentioned singular configuration is shown in Fig. 6.

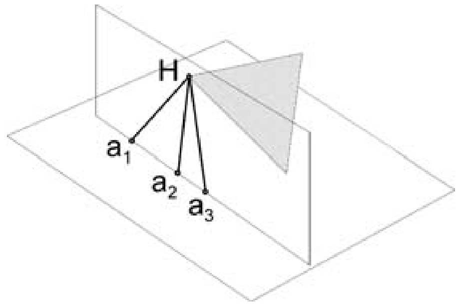

Fig. 6. Singular configuration of tetrahedron T_1 with zero area.

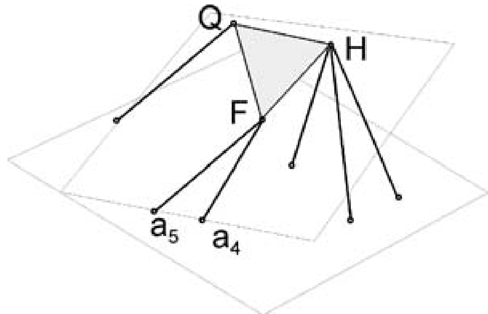

Fig. 7. Singular configuration of tetrahedron T_2 with zero height.

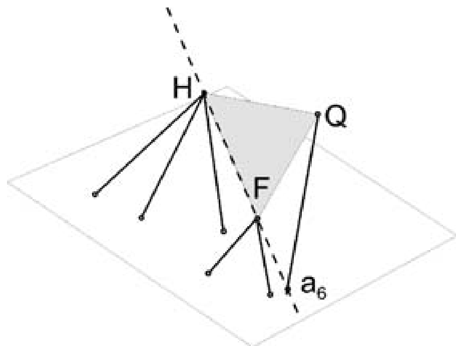

Fig. 8. Singular configuration of tetrahedron T_3 with zero area.

Conditions 1(b), 2(a), and 2(b) can be avoided by choosing the working area of the system outside the trilateral sensing platform. This means that points H, F, and Q should never reach the plane of the trilateral sensing platform.

Conditions 3(a) and 3(b) and combined singularities can occur in the working area of CATRASYS, and they should be avoided by properly choosing the working area of the measuring system. In particular, Fig. 7 shows the singularity 2(b) of T_2, in which the tetrahedron has zero height. Figure 8 shows the singularity of T_3, which has been classified as 3(a). Most of the configuration singularities can be avoided only by redesigning the manipulative task.

Figures 9–12 show the possible configurations for the singularities of tetrahedron T_3, when it has zero height.

Figures 13–16 show four possible configurations, which are given as combined singularities of the three tetrahedra. Of course, all the three give singular configurations of CATRASYS.

It is worth noting that the above-mentioned analysis, which is based on descriptive geometry is an alternative method to

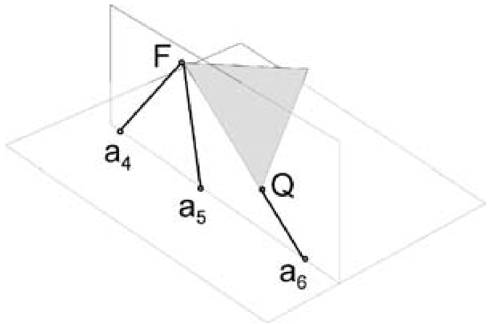

Fig. 9. Singular configuration of tetrahedron T_3 with zero height.

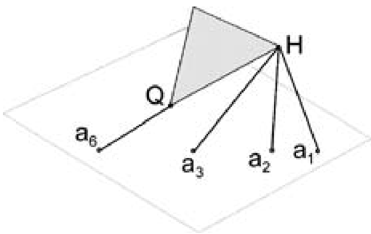

Fig. 10. Singular configuration of tetrahedron T_3 with zero height.

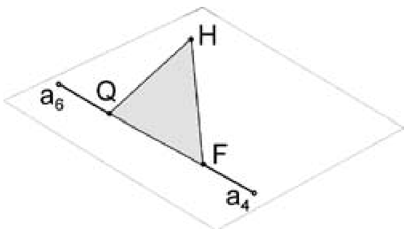

Fig. 11. Singular configuration of tetrahedron T_3 with zero height.

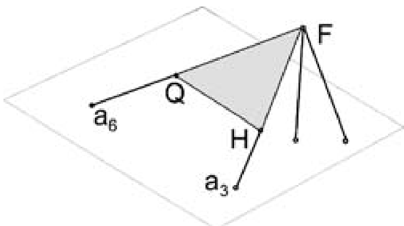

Fig. 12. Singular configuration of tetrahedron T_3 with zero height.

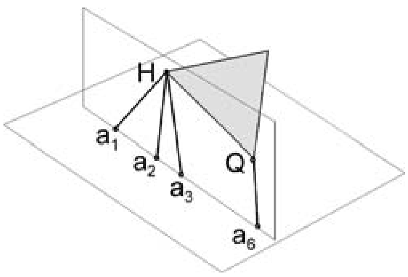

Fig. 13. Combined singular configuration of tetrahedra T_1 and T_2.

the Grassmann line geometry and the screw theory in order to obtain singular configurations for a class of both parallel and serial manipulators named as trilaterable manipulators, as pointed out in ref. 19. Indeed, the above-mentioned

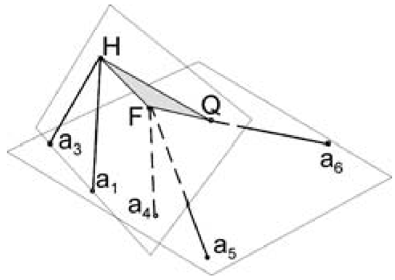

Fig. 14. Combined singular configuration of tetrahedra T_2 and T_3.

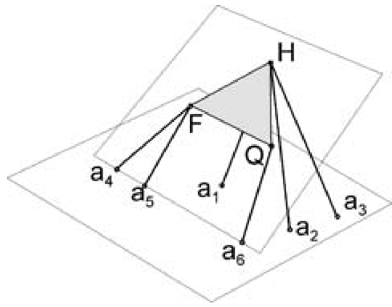

Fig. 15. Combined singular configuration of tetrahedra T_2 and T_3.

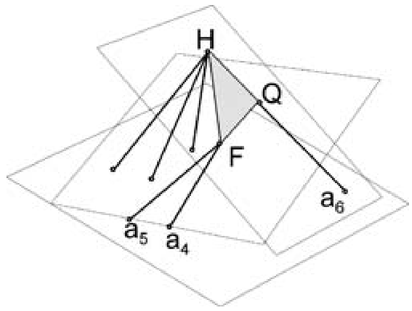

Fig. 16. Combined singular configuration of tetrahedra T_2 and T_3.

analysis can be considered general for six-legged trilaterable manipulators, with or without wires, although it has been applied specifically to CATRASYS architecture.

In addition, the use of characteristic tetrahedra allows one to determine their volume and obtain analytical conditions for the singularity analysis by considering the Cayley–Menger determinant. This characterization has been attempted elsewhere.[18,19]

The forward kinematics for 3-2-1 parallel chains can be solved in closed form either using the formulation for trilateration,[9–11] or by using the Cayley–Menger determinants.[19] Thus, given three points in space whose positions are given by vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$, the trilateration problem consists in finding the location of another point $\mathbf{H}$, whose distance to these three points is known (Fig. 1). According to the results presented in ref. 19, $\mathbf{H}$ can be expressed as

$$\mathbf{H} = \mathbf{a}_1 + k_1\mathbf{v}_1 + k_2\mathbf{v}_2 \pm k_3(\mathbf{v}_1 \times \mathbf{v}_2) \qquad (1)$$

where $\mathbf{v}_1 = (\mathbf{a}_2 - \mathbf{a}_1)$; $\mathbf{v}_2 = \mathbf{a}_3 - \mathbf{a}_1$; the $\pm$ sign takes into account the two mirror symmetric solutions with respect to

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the plane defined by points a_1, a_2, and a_3 in Fig. 1 and

$$k_1 = -\frac{D(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3; \mathbf{a}_1, \mathbf{a}_3, \mathbf{H})}{D(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)};$$

$$k_2 = \frac{D(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3; \mathbf{a}_1, \mathbf{a}_2, \mathbf{H})}{D(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)}; \qquad (2)$$

$$k_3 = \frac{\sqrt{D(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{H})}}{D(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)}$$

in which $D(\mathbf{p}_1, \ldots, \mathbf{p}_n; \mathbf{q}_1, \ldots, \mathbf{q}_n)$ denotes the Cayley–Menger determinant of the two sequences of n points $\mathbf{p}_1, \ldots, \mathbf{p}_n$ and $\mathbf{q}_1, \ldots, \mathbf{q}_n$.[20] It can be expressed as

$$D(\mathbf{p}_1, \ldots, \mathbf{p}_n; \mathbf{q}_1, \ldots, \mathbf{q}_n) = 2\left(\frac{-1}{2}\right)^n \times \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & D(\mathbf{p}_1, \mathbf{q}_1) & D(\mathbf{p}_1, \mathbf{q}_2) & \cdots & D(\mathbf{p}_1, \mathbf{q}_n) \\ 1 & D(\mathbf{p}_2, \mathbf{q}_1) & D(\mathbf{p}_2, \mathbf{q}_2) & \cdots & D(\mathbf{p}_2, \mathbf{q}_n) \\ \vdots & \vdots & \vdots & \ddots & \cdots \\ 1 & D(\mathbf{p}_n, \mathbf{q}_1) & D(\mathbf{p}_n, \mathbf{q}_2) & \cdots & D(\mathbf{p}_n, \mathbf{q}_n) \end{vmatrix}. \qquad (3)$$

Advantages of the above-mentioned formulation can be that it is coordinate-free and all the involved determinants have geometric meaning, as outlined in refs. 19 and 20.

By applying the above-mentioned formulation for three consecutive trilateration operations, the forward kinematics of the 3-2-1 parallel structure can be solved. Indeed, according to Fig. 1, giving the wire lengths d_1, d_2, and d_3, there are two possible mirror locations for point H with respect to the plane defined by points a_1, a_2, and a_3. Once one of these two solutions is chosen, a_4, a_5, and H define the second tetrahedron with known edge lengths. Again, there are two possible mirror locations for F, in this case with respect to the plane defined by a_4, a_5, and H, as shown in Fig. 4(b). Finally, after choosing one of the two solutions, a_6, H, and F define another tetrahedron with known edge lengths. In this case, there are two possible mirror locations for Q with respect to the plane defined by a_6, H, and F [Fig. 4(c)].

Once the points H, F, and Q have been located, they can be used to define a reference frame on the moving object, which is useful to determine the orientation of the moving rigid body that is attached to the CATRASYS end-effector with respect to the base frame of CATRASYS.

To summarize, the CATRASYS singular configurations can be detected by analyzing the three determinants for the characteristic tetrahedra as

$$D(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{H}); \quad D(\mathbf{a}_4, \mathbf{a}_5, \mathbf{H}, \mathbf{F}); \quad D(\mathbf{a}_6, \mathbf{H}, \mathbf{F}, \mathbf{Q}). \qquad (4)$$

Determinants in Eq. (4) are proportional to the volumes of the tetrahedra T_1, T_2, and T_3. Indeed, it is possible to track the pose of a moving object that is attached to the mobile frame HFQ by using Eqs. (1)–(3) and detect the presence and/or closeness to singularity by using Eq. (4). Furthermore, a sensitivity index has been proposed[19] to give a measure on the pose error as a function of the wire length errors. The above-mentioned sensitivity index has also been defined as a function of the tetrahedra volumes that are computed by Eq. (4).

The results that are obtained by using Eq. (4) are in accordance with the characterization of singularities given in ref. 22.

4. An experimental characterization of singularities

Experimental activity for investigating singularities in Figs. 6–16 has been carried out with CATRASYS at the LARM by using a PUMA 562 robot. The layout with the PUMA robot is shown in Figs. 2 and 3. The wire transducers in the built prototype have a working range of 2500 mm. The output transducer signal is proportional to the length of the wire and is expressed in volt. The tension of the wire (0.08 N) is maintained through the torsional spring in the transducers. The used PC IBM 486 with 32 MB of RAM is equipped with an acquisition card AT-MIO16F5 and has been used for programming and monitoring the system performance. The resolution of the acquisition card is 2^8 bit. The scan rate for the following experiments has been set equal to 1500 scan/s. The calibration of the wire system is obtained with another suitable measuring system. The above-mentioned singularity classification has been useful to determine singular configurations of CATRASYS system. It is worth noting that only singular configurations with zero area can be considered as singularities of CATRASYS operation, i.e., they are configurations in which the measure cannot be determined. For the experimental tests, the PUMA robot has been used to repeat the experiments with suitable trajectories. In Fig. 17, the experimental results have been reported during a regular operation for a robot movement, which involves configurations of the measuring system far from singularities.

The robot movement consists of a translation and a rotation of the end-effector.

Singular configurations of CATRASYS that are expressed by conditions 2(b), 3(a), 3(b), and the combined singularity have been experimentally detected using the PUMA robot, and the results are shown in Figs. 18–21. In particular, Fig. 18 shows a singular configuration when the tetrahedron T_2 is with zero high, which is shown in the scheme of Fig. 7.

Figures 18 and 19 show the layout and experimental results that have been obtained by using the CATRASYS system during a robot movement, in which the end-effector passes trough a zero-high singularity for the tetrahedron T_2. By analyzing Fig. 19, it should be noted that the coordinates of point F have been determined but their value cannot be considered correct. Close to the singular configuration, an amplification of the noise for the x- and y-coordinates of F give problems on the determination of point Q in the recursive trilateration operation, as shown in Fig. 19. The singular configuration can be clearly determined at $t = 87$ s in the plots for Q coordinates. Thus, the singularity occurrence can be detected both by an increase of noise in coordinate values and an irregular tracking of the measured values, as shown in the experimental tests of Fig. 19. A measure of singularity

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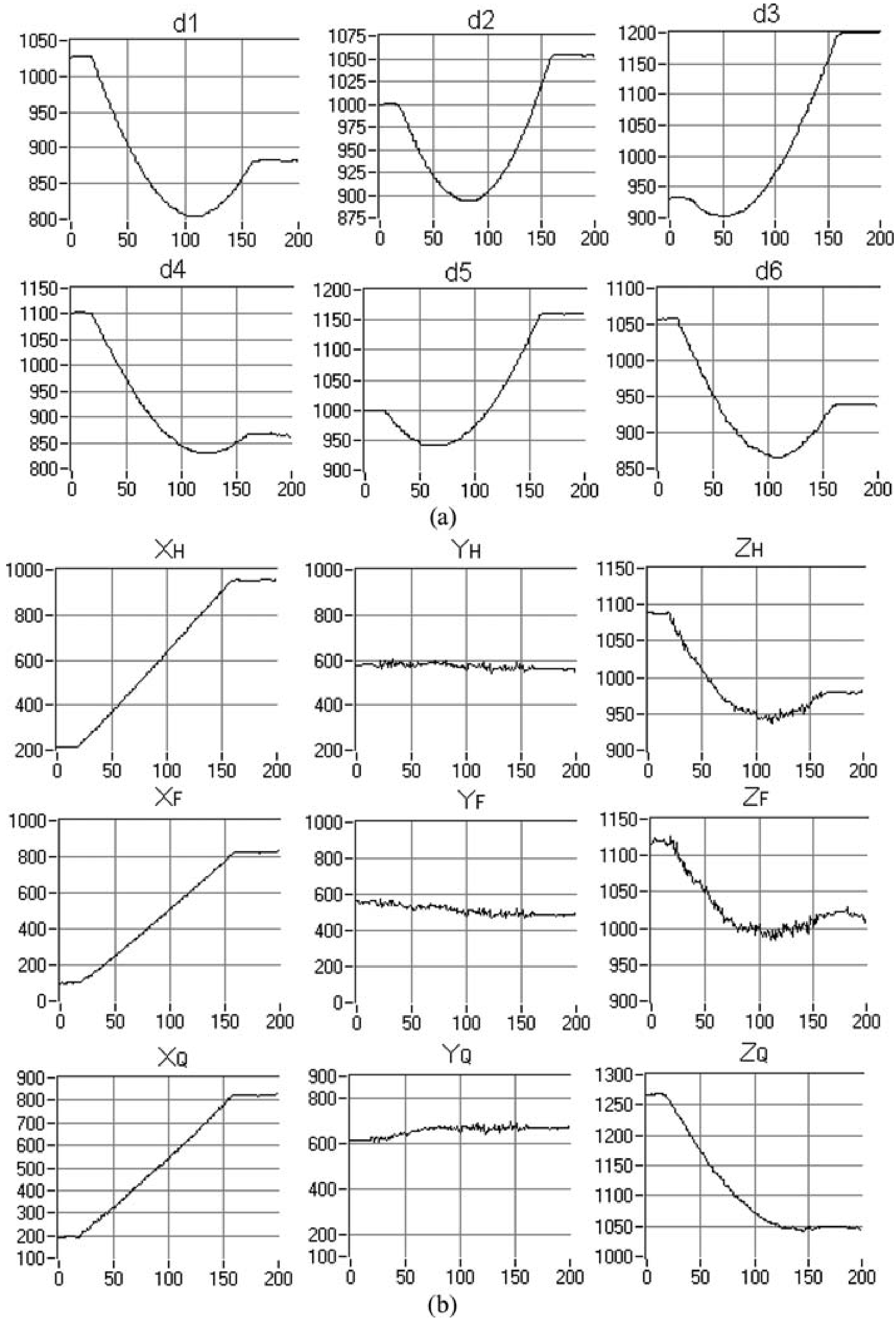

Fig. 17. A measure of the CATRASYS system during a robot movement, which involves configurations far from singularities: (a) plots of the measures for d_i ($i = 1, \ldots, 6$); (b) computed coordinates of points H, F, and Q (time unit for the *x*-axis of the plots is second; distance unit for the *y*-axis of the plots is millimeter).

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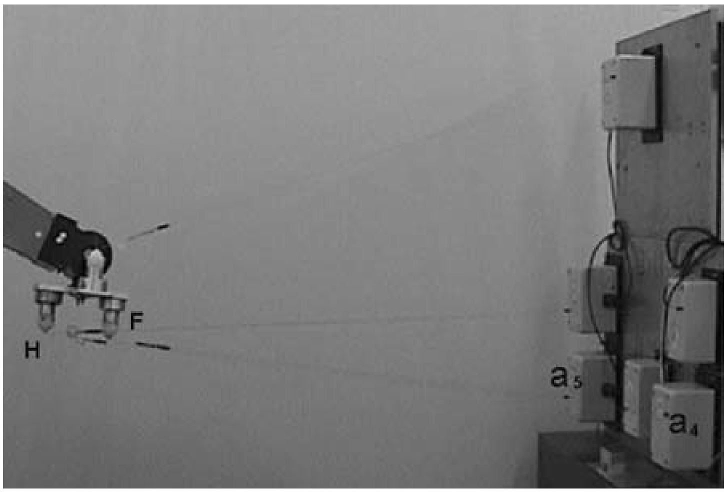

Fig. 18. Experimental layout of CATRASYS at zero high singular configuration of tetrahedron T_2 in Fig. 7.

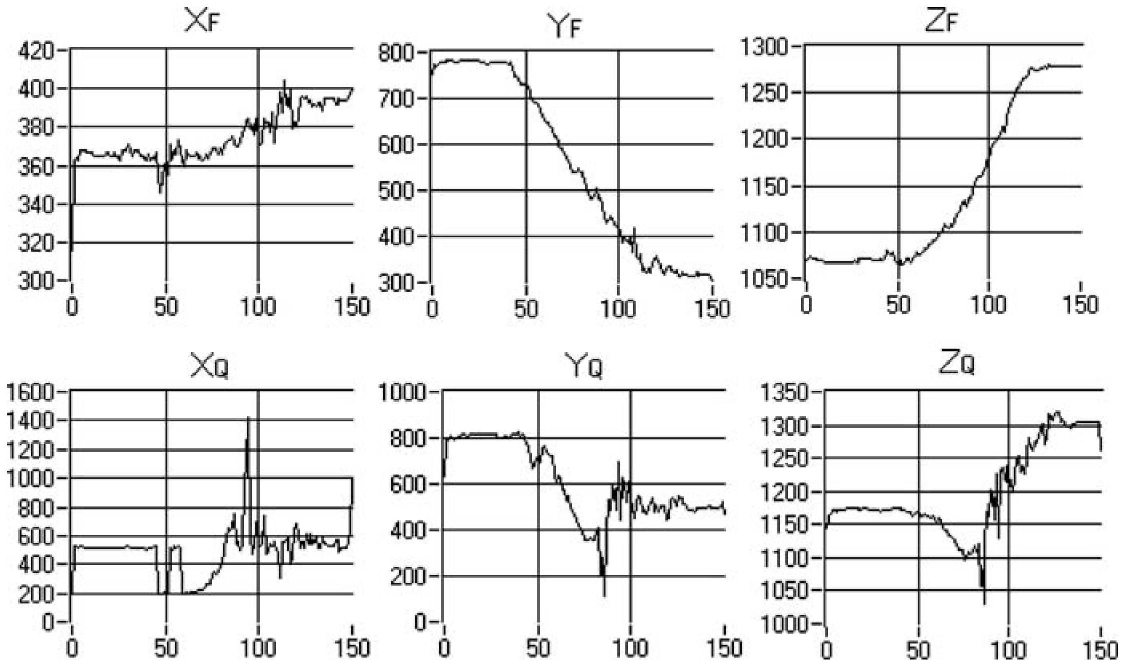

Fig. 19. Experimental determination of the zero high singular configuration of tetrahedron T_2 in Fig. 7, through the computation of coordinates of points F and Q in Fig. 18 (time unit for the x-axis of the plots is second; distance unit for the y-axis of the plots is millimeter).

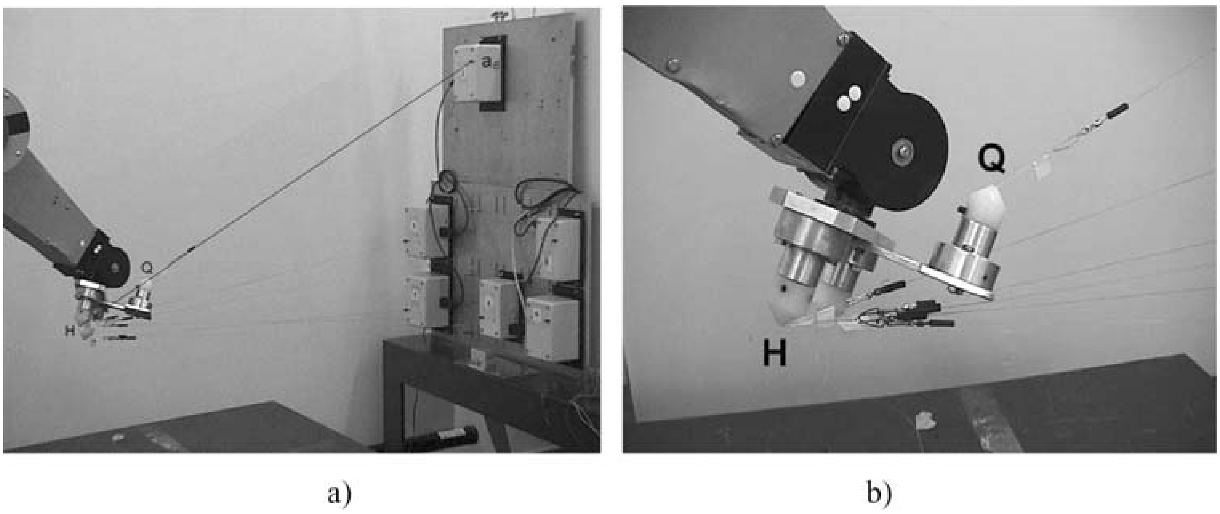

Fig. 20. Experimental layout of CATRASYS at the zero height singular configuration of the tetrahedron T_3 in Fig. 10: (a) overall view and (b) a detailed view of the end-effector.

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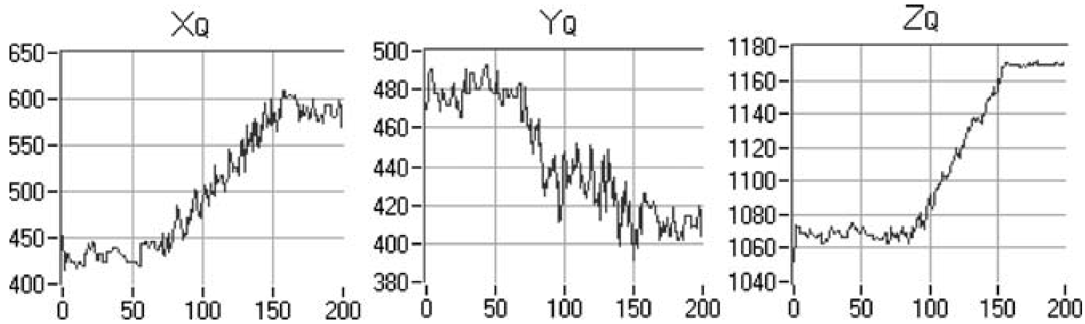

Fig. 21. Experimental determination of the zero high singular configuration of tetrahedron T_3 in Fig. 10, through the computation of coordinates of point Q in Fig. 20 (time unit for the *x*-axis of the plots is second; distance unit for the *y*-axis of the plots is millimeter).

detection can be seen in the plots of Fig. 19, where the noise oscillation has a value that is comparable with the measure of the wire length itself. Even the magnitude of the jump in the measure is considerable and it can be easily detected and understood as an indication of singularity presence.

Experimental results of Figs. 20 and 21 refer to a robot movement of the end-effector crossing a singularity that is zero high, as shown in the scheme of Fig. 10. It is worth noting that the measuring systems still work properly, but the determination of coordinates of point Q is not accurate, since it is affected by considerable noise, as shown in Fig. 21.

Figures 22 and 23 show the experimental results that have been obtained by considering a robot movement of the end-effector involving the singularity of Fig. 8 concerning tetrahedron T_3 with zero area. In this configuration, the measure cannot be determined and what experimentally happens to the measuring system is that the measured coordinates of point Q are not physically correct. In particular, it is worth noting that even if a smooth trajectory is imposed and no noise is detected, the related measure presents a jump that is not the real measure but due to the numerical instability of the trilateration calculation for the system close to the above-mentioned singular configuration.

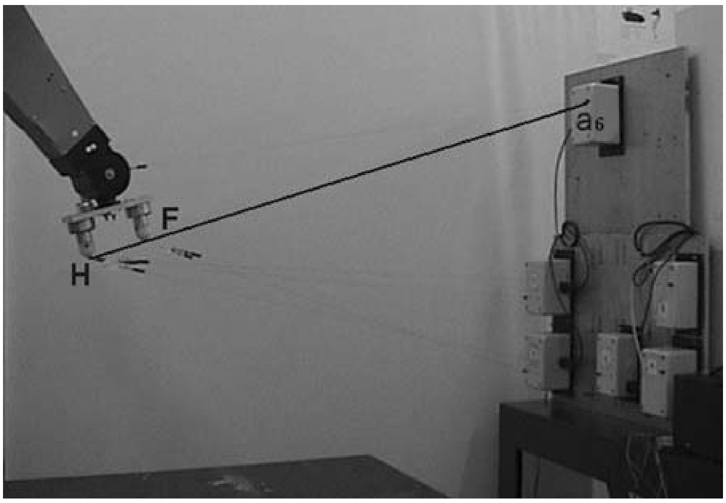

Fig. 22. Experimental layout of CATRASYS at zero area singular configuration of tetrahedron T_3 in Fig. 8.

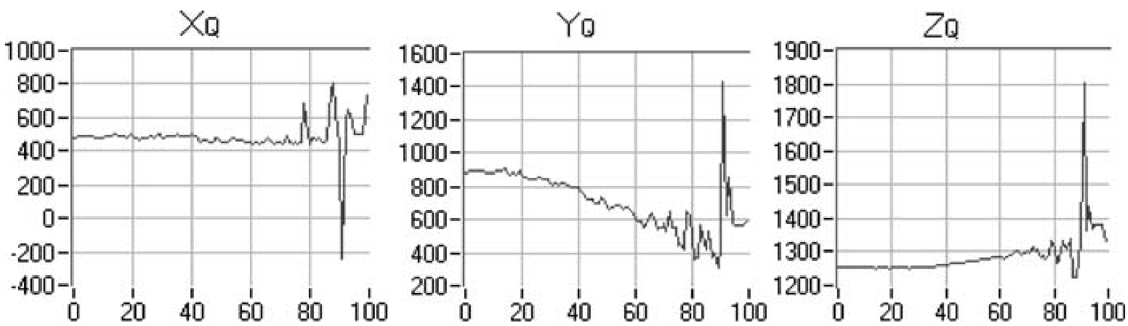

Fig. 23. Experimental determination of CATRASYS at the zero area singular configuration of the tetrahedron T_3 in Fig. 8, through the computation of coordinates of point Q in Fig. 22 (time unit for the *x*-axis of the plots is second; distance unit for the *y*-axis of the plots is millimeter).

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The singular configuration can be clearly determined at $t = 92$ s in the plots of Q coordinates.

The wrong computation can be easily detected by an increase of the numerical noise of computation in the neighborhood of the singularity, and finally by an irregular tracking of the computed values with a jump at the singularity configuration. To summarize, a singularity configuration for CATRASYS can be identified and characterized by the numerical noise and the computed coordinates, and the presence of sudden jumps in the tracked values. This paper is mainly focused on practical aspect for experimental analysis, characterization, and identification of singularities in wire-based tracking systems. Previous works[17,18] were mainly focused on the formulation for singularity detection and the definition of indices based on numerical simulation.

5. Conclusion

In this paper, a characterization of singularities of a wire-based parallel architecture has been performed by considering the geometry of three characteristic tetrahedra. In particular, the analysis is based on the singularities of these tetrahedra. The obtained results can be used in future work by considering the Cayley–Menger determinant in order to have an analytical description based on the tetrahedral volumes, which gives a tool for the practical use of the measuring system. The main and novel goal of the paper is an experimental characterization of singularities for detecting their existence in a straightforward simple experiment. A quantitative measure of singularity detection can be considered through the noise oscillation of the measure itself. Experimental results have been reported to show the engineering significance of the proposed formulation and analysis.

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