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Global stabilization for constrained robot motions with constraint uncertainties

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SUMMARY

In this paper, the global stability problem for constrained robot motions in the presence of constraint uncertainties is investigated. We focus on the uncertainties in the constraint functions and their effects on the global stability. PD type controllers are used and conditions for global stability are developed using Lyapunov's direct approach. In the presence of the constraint uncertainties under investigation, the desired position and constraint force can be guaranteed with global asymptotic convergence. The developed conditions for feedback gain selections clearly show the effects of the constraint uncertainties. For the case when the velocity measurements are not available, conditions for global stability regulation are also established and the robot controller uses only the measurements of the position angles. Finally, we consider the case where the robot joints are flexible and global stability conditions are given.

KEYWORDS: Constrained robot motion; Global stability; Uncertainties; Lyapunov's direct approach.

1. INTRODUCTION

In order to deploy robot manipulators in many advanced machine operations, it is necessary to control both the position and velocity of the end-effector and the constraint force between the end-effector and the environment. The class of dextrous robot motions has a wide range of potential applications, such as a robot manipulator whose end-effector is in contact with a constraint surface, multirobots holding a common object, etc.[1,2] With the progressive deployment of robotic manipulators for more sophisticated tasks, the dynamics and control issues of dextrous robot motions have attracted a lot of attentions and research efforts in the past years.

We focus our attention on the studies for the constrained robot motions where the constraints are rigid and always in effect. Recent research has generated many results on simultaneous position and force control. Since the early work by Whitney,[1] many control schemes have been proposed. Raibert and Craig have proposed a hybrid control method,[3] Yoshikawa has extended it to dynamic hybrid control[4] and Khatib has proposed an operational space formulation.[5] A careful stability analysis for such closed loop systems and a position force tracking controller have been given by McClamroch and Wang.[6] Conditions have been developed for local stabilization using a linear feedback controller.[7] Wang and McClamroch have further presented the analysis of effects of constraint functions on constrained system stability in paper[8] and developed stability conditions for constrained robot systems using Lyapunov's direct approach.[9] Mills and Goldenberg applied the theory for linear descriptor system to achieve stable position and force control.[10] More recently, Arimoto surveyed the developments for dextrous robot motions in papers[11,12] and also proposed the joint-space orthogonalization method.[13] Passivity properties for these dextrous robot motions have also been analyzed in papers[11–13] and used for control design.[14]

Most of the developed methodologies require the exact knowledge of the algebraic constraint functions in order to carry out the stability analyses and control designs. The reason is that most methods utilize the constraint functions to construct local coordinates and/or transformations in one way or another. In the presence of uncertainties in the constraint functions, methods relying on coordinates and transformation are not guaranteed to work well. The constrained robot motions in the presence of constraint uncertainties remain an open field for research. The uncertainties in the constraint functions change the steady states, in general, and may affect the stability of the dextrous robot motions as the example shown in paper.[8] In most practical operations, exact knowledge of the constraint functions is not easy to obtain or requires a lot of careful robotic workcell setup and surface measurements. Hence it is of practical interest to establish that, with uncertainties in the constraint functions due to the modeling inaccuracy, stability on the actual constraint surfaces will remain and the constrained robot motions will perform as expected.

In this paper, the focus of the investigation is on the stability of constrained robot motions in the presence of uncertainties in the constraint functions. The constraint functions define the admissible motion space and thus uncertainties in these functions will change the admissible motion space. The uncertainties are described and imbedded in the steady state error analysis and global stability conditions. We use Lyapunov's direct approach because it does not depend on the local coordinate systems or transformations at the contact point. It is shown that the Lyapunov's direct approach can form the basis for position and force control design

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in the presence of the constraint uncertainties as well as the development of stability conditions that guarantee global regulation.

The organization of this paper is as follows: In section 2, constrained robot motions are modelled using Lagrangian formulation, objectives are defined for position and force control, and uncertainties in the constraint functions are described. In section 3, constrained robot motions with rigid link and rigid joints are investigated and global stability conditions in the presence of the constraint uncertainties are given. In section 4, we investigate the case where only position angle measurements are available for feedback. The velocity measurements are not required in the feedback control and the global stability conditions are developed. In section 5, robots with flexible joints are considered. The constrained motions of robots with flexible joints are globally stabilized using an affine linear controller and conditions are given for feedback gain selections. In section 6, concluding remarks are made.

2. ROBOT DYNAMICS AND CONSTRAINT UNCERTAINTIES

It has been understood that the constrained robot systems can be modeled using a Lagrangian formulation expressed by a set of differential-algebraic equations. Let $\mathbf{q} \in \mathrm{R}^n$ be a generalized velocity vector and $\dot{\mathbf{q}} \in \mathrm{R}^n$ be a generalized velocity vector. Suppose the holonomic constraints on the motion are described by the following m algebraic equations

$$\mathbf{\Phi}(\mathbf{q}) = 0, \tag{1}$$

where the constraint functions $\mathbf{\Phi}^{\mathrm{T}} = [\phi_1, \ldots, \phi_m]$ are at least twice differentiable. The kinetic and potential energy functions are denoted by $K(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{M}(\mathbf{q})\dot{\mathbf{q}}$ and $P(\mathbf{q})$, respectively, where $\mathbf{M}: \mathrm{R}^n \to \mathrm{R}^{n\times n}$ is a symmetric positive definite inertia matrix, and the potential energy function $P: \mathrm{R}^n \to \mathrm{R}$ is at least twice differentiable. A Lagrangian function is defined for this constrained robot system as

$$L(\mathbf{q}, \dot{\mathbf{q}}) = K(\mathbf{q}, \dot{\mathbf{q}}) - P(\mathbf{q}). \tag{2}$$

so that

$$\frac{d}{dt}\left[\frac{\partial}{\partial \dot{\mathbf{q}}} L(\mathbf{q}, \dot{\mathbf{q}})\right] - \frac{\partial}{\partial \mathbf{q}} L(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{J}^{\mathrm{T}}(\mathbf{q})\lambda + \mathbf{u}, \tag{3}$$

where $\lambda = (\lambda_1, \ldots, \lambda_m)^{\mathrm{T}} \in \mathrm{R}^m$ is a vector of m Lagrangian multipliers and $\mathbf{u} \in \mathrm{R}^n$ is a vector of control inputs. $\mathbf{J}(\mathbf{q})$ is the Jacobian matrix of the constraint function $\mathbf{\Phi}(\mathbf{q})$. Using the definition of $L(\mathbf{q}, \dot{\mathbf{q}})$, the equations of constrained motion can be expressed as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \beta(\mathbf{q}, \dot{\mathbf{q}}) + \frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}) = \mathbf{J}^{\mathrm{T}}(\mathbf{q})\lambda + \mathbf{u} \tag{4}$$

where $\beta(\mathbf{q}, \dot{\mathbf{q}}) = \left[\frac{\mathrm{d}}{\mathrm{dt}}\mathbf{M}(\mathbf{q})\right]\dot{\mathbf{q}} - \frac{\partial}{\partial \mathbf{q}}[\frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{M}(\mathbf{q})\dot{\mathbf{q}}]$.

The constrained dynamics are described by n second order differential equations (4) and m algebraic equations (1) in terms of $n+m$ variables $\mathbf{q}$ and λ. The m-vector of variables λ determines the constraint force vector $\mathbf{f} = \mathbf{J}^{\mathrm{T}}(\mathbf{q})\lambda$ which is normal to the constraint surface (1) at the contact point $\mathbf{q}$.

For dextrous robot motions, the equations (1) and (4) clearly show that the constraint functions and their differentiations are part of the dynamics and, more importantly, the constraint equation (1) defines the admissible motion space. Uncertainties in the constraint functions will change the admissible motion space as well as the dynamics of the dextrous robot motions. Here we describe the uncertainties in the constraint functions under investigation in this paper.

Equation (1) represents the actual constraint functions which we do not know exactly. Instead, a set of m nominal constraint functions $\mathbf{\Phi}_{\mathrm{n}}(\mathbf{q})$ are used to model the constraints for control design purpose. They are described as follows:

$$\mathbf{\Phi}_{\mathrm{n}}(\mathbf{q}) = 0. \tag{5}$$

Associated with this set of nominal constraints, the nominal Jacobian matrix is given as

$$\mathbf{J}_{\mathrm{n}}^{\mathrm{T}}(\mathbf{q}) \in \mathrm{R}^{n\times m}. \tag{6}$$

Note that at any position $\mathbf{q}$, a rotation matrix $R(\mathbf{q}, \mathbf{x}) \in \mathrm{R}^{n\times n}$ exists and satisfies

$$\mathbf{J}_{\mathrm{n}}^{\mathrm{T}}(\mathbf{q})\mathbf{x} = \mathrm{R}(\mathbf{q}, \mathbf{x})\mathbf{J}^{\mathrm{T}}(\mathbf{q})\mathbf{x}, \tag{7}$$

for all $\mathbf{x} \in \mathrm{R}^m$. This rotational matrix $\mathrm{R}(\mathbf{q}, \mathbf{x})$ has the properties of being full rank, $\mathrm{R}^{-1}(\mathbf{q}, \mathbf{x}) = \mathrm{R}^{\mathrm{T}}(\mathbf{q}, \mathbf{x})$ and $\|\mathrm{R}(\mathbf{q}, \mathbf{x})\| = 1$ for all $\mathbf{q}$. $\mathrm{R}(\mathbf{q}, \mathbf{x}) = \mathrm{I}$ indicates that the normal directions of the actual and nominal constraints coincide at $\mathbf{q}$. Equality (7) is always valid except a scalar multiplier which can be eliminated by normalizations of constraint functions.

In this paper, we study the control and stability problem when there are uncertainties in the constraint functions. The control design objective is to achieve global stability regulation of position and the constraint force to desired vectors. In particular, a desired constant position vector $\mathbf{q}_{\mathrm{d}}$ is specified as the target position and a desired constant constraint force $\mathbf{f}_{\mathrm{d}}$ is given as the desired force. The objective is to achieve global regulation at $(\mathbf{q}_{\mathrm{d}}, \mathbf{f}_{\mathrm{d}})$ using controller with feedback of displacement, velocity and constraint force errors. As will be seen, force feedback is not necessary to achieve global regulation. Global convergence of displacements and forces towards the desired position and force values can be guaranteed by appropriate selection of the feedback gains. Because robot motion considered in this paper is under constraints, the global stability at $\mathbf{q}_{\mathrm{d}}$ is equivalent to the convergence to $\mathbf{q}_{\mathrm{d}}$ for all $\mathbf{q}$ satisfying constraint equation (1).

The desired position $\mathbf{q}_{\mathrm{d}}$ and constraint force $\mathbf{f}_{\mathrm{d}}$ must be specified to be consistent with nominal constraints in the sense that they satisfy $\mathbf{\Phi}_{\mathrm{n}}(\mathbf{q}_{\mathrm{d}}) = 0$ and $\mathbf{f}_{\mathrm{d}} = \mathbf{J}_{\mathrm{n}}^{\mathrm{T}}(\mathbf{q}_{\mathrm{d}})\lambda_{\mathrm{d}}$ for some constant Lagrangian multiplier vector $\lambda_{\mathrm{d}} \in R^m$. To tackle the control design problem, some basic assumptions on constraint modelling are needed as follows.

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Assumption 1: Both actual and nominal constraint sets contain m independent constraint functions, respectively. Moreover, the given target position $\mathbf{q}_d$ satisfies both sets of constraints, i.e.,

$$\mathbf{\Phi}(\mathbf{q}_d) = \mathbf{\Phi}_n(\mathbf{q}_d) = 0. \tag{8}$$

The desired position $\mathbf{q}_d$ is chosen based on the nominal constraints but must also be on the actual constraints. In other words, the nominal motion space is required to intersect the actual motion space at least at the desired position. In practice, this assumption is not restrictive because only one position point needs to be modelled accurately using nominal constraint functions.

Assumption 2: Given a desired constraint force vector $\mathbf{f}_d$, a solution λ_e exists for the following equation

$$\mathbf{f}_d = \mathbf{J}^T(\mathbf{q}_d)\lambda_e. \tag{9}$$

In this case, a solution is given as, using $\mathbf{f}_d = \mathbf{J}_n^T(\mathbf{q}_d)\lambda_d$, (7) and (9),

$$\lambda_e = (\mathbf{J}^T(\mathbf{q}_d))^*\mathbf{f}_d = (\mathbf{J}^T(\mathbf{q}_d))^*R(\mathbf{q}_d, \lambda_d)\mathbf{J}^T(\mathbf{q}_d)\lambda_d \tag{10}$$

where A^* is a pseudo-inverse of the matrix A.

In the special case of $m=1$, both λ_d and λ_e are scalars. Equation (9) implies that nominal gradient vector $\mathbf{J}_n^T(\mathbf{q}_d)$ must be parallel to the actual gradient vector $\mathbf{J}^T(\mathbf{q}_d)$ at $\mathbf{q}_d$.

Assumption 3: The second order derivatives, or the Hessian matrices of $\phi_j(\mathbf{q})$, exist and are bounded in the sense of

$$\left\|\frac{\partial^2}{\partial \mathbf{q}^2}\phi_j(\mathbf{q})\right\| \le h \tag{11}$$

for a finite positive constant h, for all $j = 1, \ldots, m$, and for $\mathbf{q}$ satisfying $\mathbf{\Phi}(\mathbf{q}) = 0$.

Assumption 4: The contact between the robot end effector and the constraint surface is always maintained and free of friction.

3. STABILITY IN THE PRESENCE OF CONSTRAINT UNCERTAINTIES

The effectiveness of Lyapunov's direct approach has been recognized for stability analysis and control design for constrained robot motions, as can be seen in papers.[9,15–17] In paper,[15] the constrained robot motions are decoupled using transformation and Lyapunov's direct approach to study the stability of the closed loop systems. In papers,[9,16] Lyapunov's direct approach is used directly to the constrained robot motions without any local coordinates or transformations. However, the systems considered in these results are assumed to be known exactly. Wang and McClamroch consider the effects of uncertainties in the constraint functions in paper[9] but the analysis and results are valid in local sense.

In this section, the stability problem is studied for the constrained robots with rigid joints and rigid links. The presence of the uncertainties in the constraint functions as described in the previous section is taken into account in the stability conditions for control designs. We consider a nonlinear controller to illustrate the idea and a linear PD type controller for feedback gain determination.

(A) A nonlinear feedback controller

To achieve regulation of position and constraint force to the specified vectors $(\mathbf{q}_d, \mathbf{f}_d)$, it is necessary to guarantee the desired values are an equilibrium of the closed loop system. This can be achieved by the following nonlinear controller:

$$\mathbf{u} = \frac{\partial}{\partial \mathbf{q}}P(\mathbf{q}) - \frac{\partial}{\partial \mathbf{q}}P_d(\mathbf{q}) - \mathbf{C}\dot{\mathbf{q}} \tag{12}$$

where $P_d(\mathbf{q})$ is any function that satisfies

$$\frac{\partial}{\partial \mathbf{q}}P_d(\mathbf{q}_d) = \mathbf{f}_d \tag{13}$$

and the gain matrix $\mathbf{C}$ is chosen symmetric and positive definite.

Using controller (12) in equation (4), the closed loop equations are given as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \left[\frac{d}{dt}\mathbf{M}(\mathbf{q})\right]\dot{\mathbf{q}} - \frac{1}{2}\frac{\partial}{\partial \mathbf{q}}[\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}}] + \frac{\partial}{\partial \mathbf{q}}P_d(\mathbf{q}) = \mathbf{J}^T(\mathbf{q})\lambda - \mathbf{C}\dot{\mathbf{q}}. \tag{14}$$

A Lyapunov function for the constrained robot system can be constructed to guarantee global stability at the desired position $\mathbf{q}_d$. In particular, we introduce a function

$$P_{ed}(\mathbf{q}) = P_d(\mathbf{q}) - P_d(\mathbf{q}_d) - \mathbf{\Phi}^T(\mathbf{q})\lambda_e. \tag{15}$$

This function can be used to form a Lyapunov function for the constrained system as

$$V(\mathbf{q}, \dot{\mathbf{q}}) = \tfrac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} + P_{ed}(\mathbf{q}). \tag{16}$$

This is a global positive definite function of $(\mathbf{q}, \dot{\mathbf{q}})$ at the desired position, if the Hessian matrix of $P_{ed}(\mathbf{q})$ is positive definite for all $\mathbf{q}$ satisfying equation (1). The Hessian matrix of P_{ed} is obtained as

$$\mathbf{N}(\mathbf{Q}, \lambda_e) = \left[\frac{\partial^2}{\partial \mathbf{q}^2}P_d(\mathbf{q}) - \sum_{j=1}^{m}\lambda_{je}\frac{\partial^2}{\partial \mathbf{q}^2}\phi_j(\mathbf{q})\right] \tag{17}$$

These developments lead to the following theorem:

Theorem 3.1: *The closed loop constrained robot system described by* (1), (4) *and* (12), *with the constraint uncertainties satisfying Assumptions* 1–4, *is globally asymptotically stable at the specified position* $\mathbf{q}_d$ *in the sense that*

$$\mathbf{q}(t) \to \mathbf{q}_d \quad \textit{and} \quad \dot{\mathbf{q}}(t) \to 0$$

as $t \to \infty$ *for any* $(\mathbf{q}(0), \dot{\mathbf{q}}(0))$ *satisfying equation* (1) *and* $\mathbf{J}(\mathbf{q}(0))\dot{\mathbf{q}}(0) = 0$ *if the* $n \times n$ *symmetric matrix* $\mathbf{N}(\mathbf{q}, \lambda_e)$ *in*

(17) *is positive definite for all* $\mathbf{q}$ *satisfying equation* (1). *Furthermore, the constraint force asymptotically converges to the desired vector in the sense that*

$$\mathbf{f}(t) \to \mathbf{f}_d.$$

Proof: We examine the time derivative of the Lyapunov function $V(\mathbf{q}, \dot{\mathbf{q}})$ given by (16) along the solutions of (14) and (1). It is easily shown that

$$\frac{d}{dt} V(\mathbf{q}, \dot{\mathbf{q}}) = -\dot{\mathbf{q}}^T \mathbf{C} \dot{\mathbf{q}} \le 0,$$

where we have used the identities

$$\mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = 0,$$

and

$$\dot{\mathbf{q}}^T \left[\frac{d}{dt} \mathbf{M}(\mathbf{q})\right] \dot{\mathbf{q}} = \dot{\mathbf{q}}^T \frac{\partial}{\partial \mathbf{q}} [\dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}}].$$

Suppose the solution satisfies, on some time interval, $\dot{\mathbf{q}}^T \mathbf{C} \dot{\mathbf{q}} = 0$; then $\dot{\mathbf{q}} \equiv 0$ and $\ddot{\mathbf{q}} \equiv 0$ since the matrix $\mathbf{C}$ is symmetric and positive definite.

From (14) and (1), a steady state solution $(\mathbf{q}_c, \lambda_c)$ must satisfy the following $n + m$ equations

$$\frac{\partial}{\partial \mathbf{q}} P_d(\mathbf{q}_c) - \mathbf{J}^T(\mathbf{q}_c)\lambda_c = 0, \tag{18}$$

$$\mathbf{\Phi}(\mathbf{q}_c) = 0. \tag{19}$$

It is easy to verify that the pair $(\mathbf{q}_c, \lambda_c) = (\mathbf{q}_d, \lambda_c)$ satisfies the above equations. Use equations (13), (9) and Mid-value theorem of calculus, the above two stationary equations (18) and (19) can be rewritten as

$$\mathbf{N}(\mathbf{q}_1, \lambda_c)(\mathbf{q}_c - \mathbf{q}_d) - \mathbf{J}^T(\mathbf{q}_c)(\lambda_c - \lambda_c) = 0 \tag{20}$$

$$\mathbf{J}(\mathbf{q}_1)(\mathbf{q}_c - \mathbf{q}_d) = 0. \tag{21}$$

where $\mathbf{q}_1 \in [\mathbf{q}_d, \mathbf{q}_c]$. Since $\mathbf{N}(\mathbf{q}, \lambda_c)$ is positive definite and $\mathbf{J}(\mathbf{q})$ is of full rank for all $\mathbf{q}$ satisfying equation (1), the above equations (20) and (21) have the unique solution $\mathbf{q}_c = \mathbf{q}_d$ and $\lambda_c = \lambda_c$. Consequently, $(\mathbf{q}_d, 0, \lambda_c)$ is the only solution of (1) and (14) which satisfies $\frac{dV}{dt} = 0$. Thus according to LaSalle's Theorem,[18] $\mathbf{q}(t) \to \mathbf{q}_d$, $\dot{\mathbf{q}}(t) \to 0$, $\lambda(t) \to \lambda_e$ as $t \to \infty$. Finally, by noting equation (9), we have $\mathbf{f}(t) \to \mathbf{f}_d$. This completes the proof. ▽▽▽

The condition for the matrix in (17) clearly demonstrates the effects of the constraints and on the stability of the dextrous robot motions with constraint uncertainties. The Hessians of the actual constraint functions play a critical role in determining the stability of the constrained system. The desired potential energy function $P_d(\mathbf{q})$ which guarantees $\mathbf{N}(\mathbf{q}, \lambda_c) > 0$ exists because of equation (11) in assumption 3.

(B) An affine linear feedback controller

The controller (12) is in general a nonlinear feedback controller. The conditions in Theorem 1 serve as general guidelines for control design. In the following, we choose a particular function $P_d(\mathbf{q})$ which results in a simple affine linear feedback control law. The control law is also shown to achieve simultaneous regulation of position and force to the specified vector $(\mathbf{q}_d, \mathbf{f}_d)$.

In this case, we choose

$$P_d(\mathbf{q}) = P(\mathbf{q}) - P(\mathbf{q}_d) - \left[\frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_d) - \mathbf{f}_d\right]^T (\mathbf{q} - \mathbf{q}_d) + \tfrac{1}{2}(\mathbf{q} - \mathbf{q}_d)^T \mathbf{W}(\mathbf{q} - \mathbf{q}_d). \tag{22}$$

It is easy to check that $P_d(\mathbf{q})$ satisfies the requirement (13). The modified energy function is

$$P_{cd}(\mathbf{q}) = P_d(\mathbf{q}) - \lambda_c^T \mathbf{\Phi}(\mathbf{q}) \tag{23}$$

It is easy to verify that $P_{cd}(\mathbf{q}_d) = 0$ and $\frac{\partial}{\partial \mathbf{q}} P_{cd}(\mathbf{q}_d) = 0$. Also the matrix $\mathbf{W}$ is chosen to be symmetric such that

$$\mathbf{Q}(\mathbf{q}, \lambda_c) = \left[\mathbf{W} + \frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) - \sum_{j=1}^{m} \lambda_{jc} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q})\right] \tag{24}$$

is positive definite for all $\mathbf{q}$. This is always achievable if $P(\mathbf{q})$ and $\mathbf{\Phi}(\mathbf{q})$ are twice continuously differentiable and their Hessian matrices are bounded for all $\mathbf{q}$ satisfying equation (1).

With this choice, the controller (12) takes the following specific form

$$\mathbf{u} = \frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_d) - \mathbf{f}_d - \mathbf{W}(\mathbf{q} - \mathbf{q}_d) - \mathbf{C}\dot{\mathbf{q}}. \tag{25}$$

This is an affine linear feedback control law of PD type. The first two terms represent a constant bias, and the third and fourth terms represent the feedback of position and velocity errors. Note that if the matrices $\mathbf{W}$ and $\mathbf{C}$ are diagonal matrices, then the feedback controller is decentralized. Such feedback is useful in implementation of robot control systems where each joint actuator depends only on feedback of its local joint displacement and velocity.

This controller is an extension of the PD controller which was proposed by Takegaki and Arimoto[15] for the rigid joint and free end effector robot motion. The main difference is the force feedforward term f_d.

The closed loop equations with controller (25) are given as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \beta_d(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{C}\dot{\mathbf{q}} + \mathbf{W}(\mathbf{q} - \mathbf{q}_d) = \mathbf{J}^T(\mathbf{q})\lambda - \mathbf{J}_n^T(\mathbf{q}_d)\lambda_d \tag{26}$$

where

$$\beta_d(\mathbf{q}, \dot{\mathbf{q}}) = \left[\frac{d}{dt} \mathbf{M}(\mathbf{q})\right] \dot{\mathbf{q}} - \frac{1}{2} \frac{\partial}{\partial \mathbf{q}} [\dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}}] + \frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}) - \frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_d).$$

Furthermore, the stationary closed loop equations of (26) and (1) can be rewritten as follows.

$$\mathbf{Q}(\mathbf{q}_1, \lambda_c)(\mathbf{q}_c - \mathbf{q}_d) - \mathbf{J}^T(\mathbf{q}_c)(\lambda_c - \lambda_c) = 0 \tag{27}$$

$$\mathbf{J}(\mathbf{q}_1)(\mathbf{q}_c - \mathbf{q}_d) = 0. \tag{28}$$

where $\mathbf{q}_1 \in [\mathbf{q}_d, \mathbf{q}_c]$. Because $\mathbf{Q}(\mathbf{q}, \lambda_c)$ and $\mathbf{J}(\mathbf{q})$ have full

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ranks, $(\mathbf{q}_c, \lambda_c) = (\mathbf{q}_d, \lambda_c)$ is the unique solution for all $\mathbf{q}$ satisfying equation (1). Following Theorems 3.1, we have

Theorem 3.2: *Consider the closed loop constrained system* (26) *and* (1) *with the constraint uncertainties satisfying Assumptions* 1–3. *Position asymptotically converges to the specified position* $\mathbf{q}_d$ *if the matrix* $\mathbf{W}$ *is chosen such that the matrix in* (24) *is symmetrically and positive definite. Furthermore, the constraint force asymptotically converges to a constant vector in the sense that*

$$\mathbf{f}(t) \to \mathbf{f}_d.$$

4. STABILIZATION WITHOUT MOTOR VELOCITY FEEDBACK

Most robotic manipulators used today are not equipped with velocity transducers. Hence the angular velocity is obtained by numerically differentiating the measured angular positions and thus the velocity data is very noisy. To tackle this problem, many efforts have been put in, such as in papers.[19–23] The results given in these papers are dealing with unconstrained robot systems. In this section, we consider the constrained robot systems without the velocity measurements. That is, only position measurements are used to achieve global asymptotic stability for constrained robot systems in the presence of uncertainties in the constraint functions.

To avoid using velocity measurements, the following controller is used to replace the controller (25)

$$\mathbf{u} = \frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_d) - \mathbf{f}_d - \mathbf{W}(\mathbf{q} - \mathbf{q}_d) - \mathbf{C}\mathbf{z}. \qquad (29)$$

The variable vector $\mathbf{z}$ is defined by

$$\mathbf{z} = \text{diag}\left\{\frac{a_i s}{s + b_i}\right\}\mathbf{q}, \qquad (30)$$

where the parameters a_i and b_i, $i = 1, \ldots, n$, are positive constants. We define the diagonal matrixes $\mathbf{A} = \text{diag}\{a_i\}$ and $\mathbf{B} = \text{diag}\{b_i\}$.

For the closed loop systems formed by equations (1), (4), (29) and (30), an equilibrium point is found at

$$(\mathbf{q}^T, \dot{\mathbf{q}}^T, \mathbf{z}^T)^T = (\mathbf{q}_d^T, 0^T, 0^T)^T. \qquad (31)$$

We can state a global stability condition without the requirement of the motor velocity feedback as follows.

Theorem 4.1: *Consider the closed loop system* (1), (4), (29) *and* (30) *with the constraint uncertainties satisfying Assumptions* 1–4. *The desired configuration* $\mathbf{q}_d$ *is globally asymptotically stable if* $\mathbf{C}$, $\mathbf{A}$ *and* $\mathbf{B}$ *are chosen diagonal and positive definite, and* $\mathbf{W}$ *is chosen symmetric and such that*

$$Q(q, \lambda_c) = \left[\mathbf{W} + \frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) - \sum_{j=1}^{m} \lambda_{jc} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q})\right] \qquad (32)$$

is positive definite for all $\mathbf{q}$ *satisfying equation* (1). *At the same time, the constraint force* $\mathbf{f}(t)$ *converges to the desired vector* $\mathbf{f}_d$.

Proof: We define the function V the same as that in Theorem 3.2. In addition, we define the following function

$$V_a = \tfrac{1}{2}\mathbf{z}^T\mathbf{C}\mathbf{A}^{-1}\mathbf{z}. \qquad (33)$$

Note that this function is positive definite in $\mathbf{z}$ at $\mathbf{z} = 0$. Thus, we define the Lyapunov function for this case as

$$U = V + V_a. \qquad (34)$$

This Lyapunov function is globally positive definite at the equilibrium point defined by equation (31) because the V is globally positive definite at $(\mathbf{q}^T, \dot{\mathbf{q}}^T)^T = (\mathbf{q}_d^T, 0^T)^T$, as shown in the previous section, and V_a is globally positive definite at the equilibrium $\mathbf{z} = 0$.

We also note the filter (30) is equivalent to the following dynamic controller

$$\dot{\mathbf{z}} = -\mathbf{B}\mathbf{z} + \mathbf{A}\dot{\mathbf{q}}. \qquad (35)$$

The time derivative of the function U along the closed loop motion described by equations (1), (4), (29) and (30) is, after similar manipulation as in the proof of Theorem 3.2,

$$\begin{aligned}\dot{U} &= \dot{\mathbf{V}} + \mathbf{z}^T\mathbf{C}\mathbf{A}^{-1}\dot{\mathbf{z}}\\ &= -\mathbf{z}^T\mathbf{C}\mathbf{A}^{-1}\mathbf{B}\mathbf{z}\\ &\le 0\end{aligned}$$

This proves stability of the equilibrium. To establish that it is globally asymptotically stable, we need to invoke LaSalle's invariance theorem with the following arguments. From equation (35), $\mathbf{z} \equiv 0$ implies $\dot{\mathbf{q}} \equiv 0$. Similar to the arguments for Theorem 3.2, we can show that the equilibrium point defined by equation (31) is the unique equilibrium point and it is thus globally asymptotically stable. At the same time, the constraint force $\mathbf{f}(t) \to \mathbf{f}_d$ by the same arguments as before. This completes the proof. ∇∇∇

Compared with Theorem 3.2, the selection condition of position feedback gain $\mathbf{W}$ is the same, but the selection condition of "velocity" feedback gain $\mathbf{C}$ is slightly more restrictive.

5. STABILITY FOR DEXTROUS ROBOTS WITH FLEXIBLE JOINTS

Now we consider the case where the joints of the manipulator are flexible and characterized by mechanisms illustrated in Fig. 1. The stiffness of the flexible

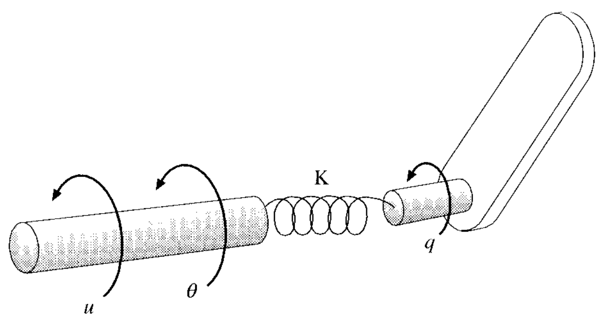

Fig. 1. Flexible joint transmission.

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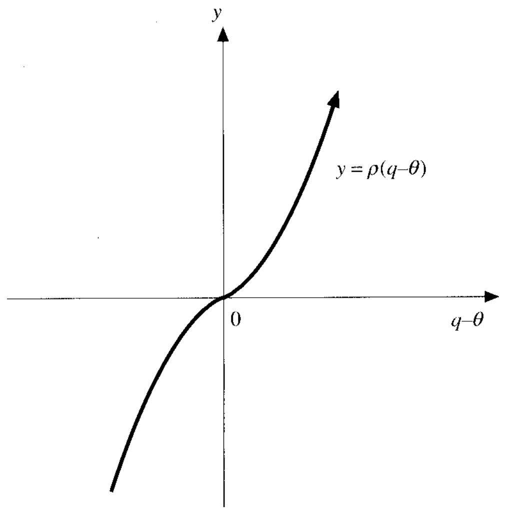

Fig. 2. A typical profile of the stiffening spring characteristics $y = \rho(q-\theta)$.

joints are characterized by the n-vector $\rho(\mathbf{q}-\theta) = [\rho_1(q_1-\theta_1), \ldots, \rho_n(q_n-\theta_n)]^T$ where each $\rho_i(\cdot)$ is the stiffness function of a flexible joint transmission and its characteristics is described by Fig. 2. The variables θ_i, $i = 1, \ldots, n$, are the motor angles and the variables q_i, $i = 1, \ldots, n$, are the link angles.

Define $\mathbf{x} = \mathbf{q} - \theta$ and $\mathbf{x}_d$ as a constant vector, then $x_i = q_i - \theta_i$ and x_{id} for $i = 1, \ldots, n$. Also, define the following function

$$\mathbf{h}(\mathbf{x}) = \rho(\mathbf{x}) - \mathbf{K}\mathbf{x}, \tag{36}$$

where the diagonal matrix $\mathbf{K}$ is chosen so that $\frac{\partial \rho}{\partial \mathbf{x}}(\mathbf{x}) \geq \mathbf{K} > 0$ for $\mathbf{x} \neq 0$. Such a $\mathbf{K}$ implies $\mathbf{x}^T\rho(\mathbf{x}) \geq \mathbf{x}^T\mathbf{K}\mathbf{x} > 0$. In the case of linear springs, $\rho(\mathbf{x}) = \mathbf{K}\mathbf{x}$ or $\mathbf{h}(\mathbf{x}) = 0$. Based on the definition of $\mathbf{h}(\mathbf{x})$, $\mathbf{x}^T\mathbf{h}(\mathbf{x}) \geq 0$ holds for all $\mathbf{x}$ and, furthermore, for all $i = 1, \ldots, n$,

$$F_i(x_i) = \int_{x_{id}}^{x_i} (h_i(p) - h_i(x_{id}))dp \geq 0. \tag{37}$$

The control input is applied to the motor and the motor torque is passed through the spring to turn the link on the other side. The contact force is applied to the end-effector at the last link of the manipulator. Based on Lagrangian formulation, the dynamics of the constrained robot with flexible joints can be modeled by the following equations.

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \beta(\mathbf{q}, \dot{\mathbf{q}}) + \frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}) = -\rho(\mathbf{q}-\theta) + \mathbf{J}^T(\mathbf{q})\lambda \tag{38}$$

$$R\ddot{\theta} + \mathbf{D}\dot{\theta} = \rho(\mathbf{q}-\theta) + \mathbf{u} \tag{39}$$

$$\mathbf{\Phi}(\mathbf{q}) = 0 \tag{40}$$

In these $2n$ differential equations and m algebraic equations, $\mathbf{q} \in R^n$ is the vector containing n link angles, $\dot{\mathbf{q}}$ is the link angular velocity vector, $\theta \in R^n$ is the vector containing motor angles, $\dot{\theta}$ is the rotor angular velocity vector, and $\mathbf{u} \in R^n$ is the control inputs applied at the n joint motors. Furthermore, $\mathbf{R}$ is the inertia matrix of the joint motors. The $n \times n$ matrix $\mathbf{D} = \mathrm{diag}[d_1, \ldots, d_n]$ is the joint friction coefficient matrix. In the link dynamic equation (38), $\rho(\mathbf{q}-\theta)$ and $\mathbf{J}^T(\mathbf{q})\lambda$ are, respectively, from the flexible-joint transmissions and the interaction between the end-effector and the constraints. Other terms are the same as those defined in section 2.

With the definition of the function $\mathbf{h}(\mathbf{q}-\theta)$, the dynamic equations (38)–(40) can be rewritten as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \beta(\mathbf{q}, \dot{\mathbf{q}}) + \frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}) + \mathbf{K}(\mathbf{q}-\theta) + \mathbf{h}(\mathbf{q}-\theta) = \mathbf{J}^T(\mathbf{q})\lambda \tag{41}$$

$$R\ddot{\theta} + \mathbf{D}\dot{\theta} - \mathbf{K}(\mathbf{q}-\theta) - \mathbf{h}(\mathbf{q}-\theta) = \mathbf{u} \tag{42}$$

$$\mathbf{\Phi}(\mathbf{q}) = 0. \tag{43}$$

Suppose that the manipulators considered here are of articulated types and that only the motor angle and velocity variables are measurable. The control objective is the same as that in the previous sections. For such a regulation objective, we consider a simple biased proportional plus derivative (PD) controller as follows:

$$\mathbf{u} = \frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_d) - \mathbf{f}_d - \mathbf{C}\dot{\theta} - \mathbf{W}(\theta - \theta_d) \tag{44}$$

where θ_d satisfies the equilibrium equation

$$\frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_d) + \rho(\mathbf{q}_d - \theta_d) = \mathbf{f}_d \tag{45}$$

In the presence of uncertainties described in section 2, the stability conditions for feedback gain selections are given in the following theorem.

Theorem 5.1: *Consider the closed loop system* (38)–(40) *and* (44) *with the constraint uncertainties satisfying Assumptions* 1–4. *The desired configuration* $\mathbf{q}_d$ *with desired constraint force* $\mathbf{f}_d$ *is globally asymptotically stable in the sense that* $\mathbf{q}(t) \to \mathbf{q}_d$, $\dot{\mathbf{q}}(t) \to 0$ *and* $\mathbf{f}(t) \to \mathbf{f}_d$ *as* $t \to \infty$ *if* $\mathbf{C}$ *is chosen so that* $\mathbf{D} + \mathbf{C}$ *is symmetric and positive definite, and* $\mathbf{W}$ *is chosen symmetric and*

$$\mathbf{H}(\mathbf{q}) = \mathbf{K} - \mathbf{K}(\mathbf{K}+\mathbf{W})^{-1}\mathbf{K} + \frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) - \sum_{j=1}^{m} \lambda_{jc} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q}) \tag{46}$$

is positive definite for all $\mathbf{q}$ *satisfying equation* (1).

Proof: Let us choose the Lyapunov function as

$$V = \tfrac{1}{2}\dot{q}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} + \tfrac{1}{2}\dot{\theta}^T\mathbf{R}\dot{\theta} + V_1(\mathbf{q}) + V_2(\mathbf{q}, \theta) + V_h(\mathbf{q} - \theta)$$

where

$$V_1(\mathbf{q}) = P(\mathbf{q}) - P(\mathbf{q}_d) - \mathbf{\Phi}^Y(\mathbf{q})\lambda_d - \left(\frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_d) - \mathbf{f}_d\right)^T (\mathbf{q} - \mathbf{q}_d) + \tfrac{1}{2}(\mathbf{q}-\mathbf{q}_d)^T\left(\tfrac{1}{2}\mathbf{H}(\mathbf{q}) - \frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) + \sum_{j=1}^{m} \lambda_{jc} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q})\right)(\mathbf{q} - \mathbf{q}_d),$$

$$V_2(\mathbf{q}, \theta) = \frac{1}{2}\begin{pmatrix} \mathbf{q} - \mathbf{q}_d \\ \theta - \theta_d \end{pmatrix}^T \times$$

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$$\begin{pmatrix} \mathbf{K} - \frac{1}{2}\mathbf{H}(\mathbf{q}) + \frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) - \sum_{j=1}^{m} \lambda_{je} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q}), & -\mathbf{K} \\ -\mathbf{K}, & \mathbf{W} + \mathbf{K} \end{pmatrix} \times \begin{pmatrix} \mathbf{q} - \mathbf{q}_d \\ \theta - \theta_d \end{pmatrix}$$

and

$$V_h(\mathbf{q} - \theta) = \sum_{i=1}^{n} F_i(q_i - \theta_i).$$

Clearly, $V_1(\mathbf{q})$ is positive definite at $\mathbf{q}_d$ because

$$V_1(\mathbf{q}_d) = 0$$

$$\frac{\partial}{\partial \mathbf{q}} V_1(\mathbf{q}_d) = 0$$

and

$$\frac{\partial^2}{\partial \mathbf{q}^2} V_1(\mathbf{q}) = \tfrac{1}{2}\mathbf{H}(\mathbf{q}) > 0$$

Also, $V_2(\mathbf{q}, \theta)$ is positive definite at $(\mathbf{q}_d, \theta_d)$ because

$$V_2(\mathbf{q}_d, \theta_d) = 0$$

$$\frac{\partial}{\partial \mathbf{q}} V_2(\mathbf{q}_d, \theta_d) = 0$$

$$\frac{\partial}{\partial \theta} V_2(\mathbf{q}_d, \theta_d) = 0$$

and its Hessian matrix is

$$\begin{pmatrix} \mathbf{K} - \frac{1}{2}\mathbf{H}(\mathbf{q}) + \frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) - \sum_{j=1}^{m} \lambda_{je} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q}), & -\mathbf{K} \\ -\mathbf{K}, & \mathbf{W} + \mathbf{K} \end{pmatrix}$$

which is congruent to

$$\begin{pmatrix} \frac{1}{2}\mathbf{H}(\mathbf{q}), & 0 \\ 0, & \mathbf{W} + \mathbf{H} \end{pmatrix}$$

and the later is symmetric and positive definite.

The function $V_h(\mathbf{q} - \theta)$ is positive semidefinite at $\mathbf{q}_d - \theta_d$ because each $F_i(q_i - \theta_i)$ is positive semidefinite at $q_{id} - \theta_{id}$.

The above shows that V is positive definite at $(\mathbf{q}, \dot{\mathbf{q}}, \theta, \dot{\theta}) = (\mathbf{q}_d, 0, \theta_d, 0)$. Now, the time derivative of V along the closed loop dextrous robot motion trajectories can be derived as

$$\begin{aligned} \dot{\mathbf{V}}(t) &= \left[\frac{\partial T}{\partial \mathbf{q}}\right]^T \dot{\mathbf{q}} + \left[\frac{\partial T}{\partial \dot{\mathbf{q}}}\right]^T \ddot{\mathbf{q}} + \left[\frac{\partial P}{\partial \mathbf{q}}\right]^T \dot{\mathbf{q}} \\ &- \left[\frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_d) - \mathbf{f}_d\right]^T \dot{\mathbf{q}} + \dot{\mathbf{q}}^T \mathbf{J}^T(\mathbf{q}) \lambda_d \\ &+ \dot{\theta}^T \mathbf{R} \ddot{\theta} + \begin{pmatrix} \mathbf{q} - \mathbf{q}_d \\ \theta - \theta_d \end{pmatrix}^T \begin{pmatrix} \mathbf{K} & -\mathbf{K} \\ -\mathbf{K}, & \mathbf{W} + \mathbf{K} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{q}} \\ \dot{\theta} \end{pmatrix} \\ &+ \dot{V}_h(q - \theta). \end{aligned}$$

where $T = \frac{1}{2}\dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q})\dot{\mathbf{q}}$. Using equations (41)–(43), (44) and equality $\frac{d}{dt}\mathbf{\Phi}(\mathbf{q}) = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = 0$, the above equation can be reduced to

$$\begin{aligned} \dot{\mathbf{V}}(t) &= -\dot{\theta}^T(\mathbf{D} + \mathbf{C})\dot{\theta} + \dot{V}_h(q - \theta) - (\dot{q} - \dot{\theta})^T[\mathbf{h}(\mathbf{q} - \theta) \\ &+ K(q_d - \theta_d) + \frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_d) - \mathbf{f}_d]. \end{aligned}$$

Also,

$$\begin{aligned} \dot{\mathbf{V}}_h(t) &= \frac{d}{dt}\left[\sum_{i=1}^{n} F_i(x_i)\right] = \sum_{i=1}^{n} \frac{\partial F_i(x_i)}{\partial x_i} \dot{x}_i \\ &= \sum_{i=1}^{n} (h_i(x_i) - h_i(x_{id}))\dot{x}_i \\ &= (\dot{\mathbf{q}} - \dot{\theta})^T(\mathbf{h}(\mathbf{q} - \theta) - \mathbf{h}(\mathbf{q}_d - \theta_d)) \end{aligned}$$

With the above equation, $\dot{\mathbf{V}}(t)$ reduces to

$$\begin{aligned} \dot{\mathbf{V}}(t) &= -\dot{\theta}^T(\mathbf{D} + \mathbf{C})\dot{\theta} - (\dot{\mathbf{q}} - \dot{\theta})^T \\ &\times \left[\mathbf{h}(\mathbf{q}_d - \theta_d) + \mathbf{K}(\mathbf{q}_d - \theta_d) + \frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_d) - \mathbf{f}_d\right]. \end{aligned}$$

But from equation (45), the second term is zero. Hence

$$\dot{\mathbf{V}}(t) = -\dot{\theta}^T(\mathbf{D} + \mathbf{C})\dot{\theta} \le 0.$$

In the invariant set, $\dot{\mathbf{V}}(t) \equiv 0$ implies $\dot{\theta}(t) \equiv 0$ since $\mathbf{D} + \mathbf{C}$ is symmetric and positive definite. This fact implies that $\ddot{\theta}(t) = 0$ and $\theta(t) = \theta_c$, where θ_c is a constant vector. Substitute these results in equations (44) and (39) to yield

$$-\rho(\mathbf{q}_c - \theta_c) + \mathbf{W}(\theta_c - \theta_d) - \frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_d) - \mathbf{f}_d = 0, \quad (48)$$

where $\mathbf{q}$ is set to a constant value vector $\mathbf{q}_c$ since the rest in the identity is a constant vector. Therefore, $\dot{\mathbf{q}} \equiv 0$ and $\ddot{\mathbf{q}} \equiv 0$. Hence equation (38) becomes

$$\frac{\partial}{\partial \mathbf{q}} P(\mathbf{q}_c) + \rho(\mathbf{q}_c - \theta_c) - \mathbf{J}^T(\mathbf{q}_c)\lambda_c = 0. \quad (49)$$

and the constraint equation

$$\mathbf{\Phi}(\mathbf{q}_c) = 0 \quad (50)$$

The Jacobian matrix of equations (48)–(50) is obtained as

$$\begin{pmatrix} \frac{\partial}{\partial x}\rho(\mathbf{x}_d) + \frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) - \sum_{j=1}^{m} \lambda_{je} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q}), & -\frac{\partial}{\partial \mathbf{x}}\rho(\mathbf{x}), & -\mathbf{J}^T(\mathbf{q}) \\ -\frac{\partial}{\partial \mathbf{x}}\rho(\mathbf{x}), & \frac{\partial}{\partial \mathbf{x}}\rho(\mathbf{x}) + \mathbf{W}, & 0 \\ -\mathbf{J}(\mathbf{q}), & 0, & 0 \end{pmatrix}. \quad (51)$$

This matrix is symmetric and congruent to

$$\begin{pmatrix} \mathbf{Y} & 0, & 0 \\ 0, & \frac{\partial}{\partial \mathbf{x}}\rho(\mathbf{x}) + \mathbf{W}, & 0 \\ 0, & 0, & \mathbf{J}(\mathbf{q})\mathbf{Y}^{-1}\mathbf{J}^T(\mathbf{q}) \end{pmatrix}$$

where

$$\mathbf{Y} = \frac{\partial}{\partial \mathbf{x}} \rho(\mathbf{x}) - \frac{\partial}{\partial \mathbf{x}} \rho(\mathbf{x}) \left(\frac{\partial}{\partial \mathbf{x}} \rho(\mathbf{x}) + \mathbf{W} \right)^{-1} \frac{\partial}{\partial \mathbf{x}} \rho(\mathbf{x}) + \frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) - \sum_{j=1}^{m} \lambda_{jc} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q})$$

is symmetric and positive definite. The positive definiteness of this matrix follows from the given conditions of $\frac{\partial}{\partial \mathbf{x}} \rho(\mathbf{x}) \geq \mathbf{K} > 0$ for all $\mathbf{x}$ and $\mathbf{H} > 0$. (See Lemma 2 in Appendix.) Therefore, the $(2n+m) \times (2n+m)$ Jacobian matrix (51) has full rank and this implies that $\mathbf{q}_c = \mathbf{q}_d$, $\theta_c = \theta_d$ and $\lambda_c = \lambda_e$ are the unique solution in the invariant set. According to LaSalle's invariance principle, the equilibrium $(\mathbf{q}_d, \theta_d, \lambda_e)$, or equivalently $(\mathbf{q}_d, \theta_d, \mathbf{f}_d)$, is globally asymptotically stable.[18] This completes the proof. ∇∇∇

For industrial robot systems, there always exists a feedback gain matrix $\mathbf{W}$ such that the matrix $\mathbf{H}$ in (46) is symmetric and positive definite. One such choice is

$$\mathbf{W} = \frac{1}{\alpha} \mathbf{K} \tag{52}$$

where α is a positive number. With this selection, we have

$$\mathbf{H} = \mathbf{K} - \frac{1}{1 + \frac{1}{\alpha}} \mathbf{K} + \frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) - \sum_{j=1}^{m} \lambda_{jc} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q})$$
$$= \frac{1}{1+\alpha} \mathbf{K} + \frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) - \sum_{j=1}^{m} \lambda_{jc} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q}). \tag{53}$$

For industrial manipulators, $\frac{\partial}{\partial \mathbf{x}} \rho(\mathbf{x})$ is much larger than $\frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) - \Sigma_{j=1}^{m} \lambda_{jc} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q})$. We can choose $\mathbf{K}$ as the infimum of $\frac{\partial}{\partial \mathbf{x}} \rho(\mathbf{x})$ for all $\mathbf{x}$. Then such a positive number α exists to guarantee $\mathbf{H} > 0$ and, therefore, the chosen $\mathbf{W}$ can guarantee asymptotic stability. If $\frac{\partial^2}{\partial \mathbf{q}^2} P(\mathbf{q}) - \Sigma_{j=1}^{m} \lambda_{jc} \frac{\partial^2}{\partial \mathbf{q}^2} \phi_j(\mathbf{q})$ is indefinite or negative definite, α must not be too large to keep the positive definiteness of matrix $\mathbf{H}$.

Furthermore, if the velocity measurements are not available for feedback purpose, the controller can be modified similarly to that in equation (29). The proof of the global asymptotic stability for the case where no velocity measurements can be easily carried out by combining the Lyapunov function for the flexible joint robot and the V_a defined in section 4.

6. CONCLUSIONS

Global stability of the constrained robot motions with uncertainties in the constraint functions has been studied. Feedback controllers have been proposed and conditions have been established for feedback gain selections. It is recognized that the uncertainties in the constraint functions must be taken into account in the feedback gains for the closed loop to remain stable. The cases with proved results include the rigid robots with rigid joints or with flexible joints, with complete state measurements of angular positions and velocities or with partial state measurements of angular position only. It has been shown that force feedback is not necessary to achieve position and force control, but it can be useful to improve closed loop robustness properties. In the developments, Lyapunov's direct approach has been the vehicle for analyses and control design. It does not require the local coordinate systems and transformations at the contact point and thus is appropriate for the study of constrained robot motions in the presence of uncertainties in the constraint functions.

APPENDIX

Lemma 1: *Suppose that* $\mathbf{P}_1 \geq \mathbf{P}_2 > 0$ *and* $\mathbf{V} > 0$. *If* $(\mathbf{P}_1 + \mathbf{V})^{-1} + \mathbf{B} > 0$, *then* $(\mathbf{P}_2 + \mathbf{V})^{-1} + \mathbf{B} > 0$.

Proof: Since

$$\mathbf{P}_1 + \mathbf{V} \geq \mathbf{P}_2 + \mathbf{V} > 0,$$
$$(\mathbf{P}_2 + \mathbf{V})^{-1} \geq (\mathbf{P}_1 + \mathbf{V})^{-1} > 0.$$

Hence the result. ∇∇∇

Lemma 2: *Suppose that* $\mathbf{K}_2 \geq \mathbf{K}_1 > 0$. *If*

$$\mathbf{K}_1 - \mathbf{K}_1(\mathbf{K}_1 + \mathbf{W})^{-1}\mathbf{K}_1 + \mathbf{B} > 0,$$

then

$$\mathbf{K}_2 - \mathbf{K}_2(\mathbf{K}_2 + \mathbf{W})^{-1}\mathbf{K}_2 + \mathbf{B} > 0.$$

Proof:

$$\mathbf{K}_1 - \mathbf{K}_1(\mathbf{K}_1 + \mathbf{W})^{-1}\mathbf{K}_1 = (\mathbf{K}_1^{-1} + \mathbf{W}^{-1})^{-1}$$

then

$$(\mathbf{K}_1^{-1} + \mathbf{W}^{-1})^{-1} + \mathbf{B} > 0$$

implies from Lemma 1,

$$(\mathbf{K}_2^{-1} + \mathbf{W}^{-1})^{-1} + \mathbf{B} > 0$$

or

$$\mathbf{K}_2 - \mathbf{K}_2(\mathbf{K}_2 + \mathbf{W})^{-1}\mathbf{K}_2 + \mathbf{B} > 0.$$

∇∇∇

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