CANADIAN JOURNAL OF PHILOSOPHY
2019, VOL. 49, NO. 8, 1083–1103
https://doi.org/10.1080/00455091.2019.1584936

Routledge
Taylor & Francis Group

ARTICLE

Debunking, supervenience, and Hume's Principle

Mary Leng

Department of Philosophy, University of York, York, UK

ABSTRACT
Debunking arguments against both moral and mathematical realism have been pressed, based on the claim that our moral and mathematical beliefs are insensitive to the moral/mathematical facts. In the mathematical case, I argue that the role of Hume's Principle as a conceptual truth speaks against the debunkers' claim that it is intelligible to imagine the facts about numbers being otherwise while our evolved responses remain the same. Analogously, I argue, the conceptual supervenience of the moral on the natural speaks presents a difficulty for the debunker's claim that, had the moral facts been otherwise, our evolved moral beliefs would have remained the same.

ARTICLE HISTORY Received 27 November 2017; Accepted 23 October 2018

KEYWORDS Mathematics; metaethics; evolutionary debunking; Hume's principle; supervenience

Introduction

Our beliefs about a domain of facts are plausibly undermined if we can show that, had the relevant facts been otherwise, our beliefs would still have been just as they are. Establishing this kind of insensitivity of our moral beliefs to the moral facts (on a realist construal) is, according to Clarke-Doane (2012), central to evolutionary debunking challenges to moral realism (such as that offered by Street (2006)), where it is suggested that, no matter how things might have been with the realist's domain of independent normative truths, we would still have been subject to the very same evolutionary forces and would still have formed the same normative beliefs. Furthermore, according to Clarke-Doane, parallel considerations can be used to launch an analogous 'debunking' argument against the mathematical Platonist, who claims that our mathematical beliefs reflect mind-independent mathematical facts, since the arithmetic beliefs that it is evolutionary advantageous for us to develop are those that correspond to relevant logical truths, and this is so regardless of how things might have been with the Platonist's domain of numbers as abstract objects. I argue

CONTACT Mary Leng mary.leng@york.ac.uk Department of Philosophy, University of York, York YO10 5DD, UK

that attention to the role of *Hume's Principle* as characterizing the Platonist's concept of number in the mathematical case shows that Clarke-Doane is mistaken to think that we can conceive of the mathematical facts about numbers being other than they are while the logical truths that shape our evolved arithmetic beliefs remain the same, since the function of Hume's Principle is precisely to line up facts about numbers with the logical truths about numerosities that plausibly make it evolutionary advantageous to believe that 1 + 1 = 2. This may at first glance look like good news for those who wish to use evolutionary considerations to debunk our *moral* beliefs while avoiding what might seem like a slippery slope to debunking mathematical beliefs too, since there is no straightforward moral analogue to Hume's Principle, a conceptual truth that serves to line up particular moral facts with evolutionarily relevant natural facts. However, I argue, no such specific principle is required; the supervenience of the moral on the natural, viewed as a conceptual truth about moral truths realistically construed, suffices to play a role analogous to Hume's Principle in creating an obstacle for the debunkers' insensitivity claim.

If debunking challenges to realism about a domain D require us to show that, had the D-facts been different, nevertheless our D-beliefs would have remained the same, then it might look as though there are slim prospects for raising a genuine debunking challenge for either mathematical or moral realism. According to many mathematical Platonists, the facts about pure mathematical objects, as well as the facts about what *impure* mathematical objects exist given what non-mathematical objects there are (such as the fact that, *if* Socrates exists, then so does {Socrates}), are *metaphysically necessary*, so that it is not metaphysically possible for the mathematical facts to be different while the nonmathematical facts are held fixed. Similarly, Clarke-Doane points out,

> Moral realists typically allege that truths that link moral properties to "descriptive" ones are metaphysically necessary. If so, then it is not metaphysically possible for the moral truths to be very different while the descriptive truths are held fixed. (Clarke-Doane 2012, 320)

However, Clarke-Doane claims, this does not undermine the debunkers' attempt to argue that, in some epistemically relevant sense of 'had', *had* the D-facts been different, nevertheless our D-beliefs would have remained the same. Rather, Clarke-Doane clarifies, the modality involved must be *conceptual possibility* rather than *metaphysical possibility*. Thus, while realists in both domains may claim that the relevant moral and mathematical truths are *metaphysically* necessary, nevertheless it is *intelligible to imagine* those truths being otherwise than they are, and that is all that is needed to get the debunking challenge off the ground. In what follows, then, we will assume with Clarke-Doane (2012) that the relevant sense of 'had the D-facts been different' required for the debunker's argument is *conceptual*, rather than metaphysical, and will consider whether debunkers can launch a challenge to the sensitivity of our D-beliefs to the D-facts in either the mathematical or moral case by showing that we can conceive of the D-facts being otherwise, and that in the scenario we are conceiving, this difference in the D-facts is not reflected by a corresponding difference in our D-beliefs.

The mathematical case

In the mathematical case, then, Clarke-Doane's challenge to the sensitivity of our mathematical beliefs to the mathematical facts involves a thought experiment where we imagine the mathematical truths about natural numbers being different while relevant logical truths remain the same. Thus, Clarke-Doane asks us to imagine the natural number 1 bearing the plus relation to itself and 0 (i.e. to imagine that 1 + 1 = 0). To the extent that beliefs concerning numbers have any effect on our behaviour, would having the true belief that 1 + 1 = 0 be evolutionarily advantageous? Not at all, Clarke-Doane claims, since if we were to believe that 1 + 1 = 0, we would also be inclined to believe that, if there is exactly one lion behind bush A and exactly one lion behind bush B, and no lions behind both, then there are *no* lions behind the bushes, and would therefore be more likely to be eaten when the two lions emerged from the nearby bushes and attacked. On the other hand, if we were to believe (mistakenly, in the situation we are being asked to imagine) that 1 + 1 = 2, then we would also be likely to believe that there are two lions behind the bushes in this situation, and to modify our behaviour accordingly (i.e. run away). Thus, Clarke-Doane tells us:

> If our ancestors who believed that 1 + 1 = 2 had an advantage over our ancestors who believed that 1 + 1 = 0, the reason that they did is that corresponding (first-order) logical truths obtained. In particular, ancestor P, who believed that 1 + 1 = 2, had an advantage over ancestor Q, who believed that 1 + 1 = 0, in the above scenario intuitively because if there is exactly one lion behind bush A, and there is exactly one lion behind bush B, and no lion behind bush A is a lion behind bush B, then there are exactly two lions behind bush A or B. (Clarke-Doane 2012, 330)

Even if, as a matter of fact, 1 + 1 = 0 was true of the natural numbers, it would still be advantageous to believe that 1 + 1 = 2 in this scenario, because of its relation to the corresponding logical truth.

Is it really intelligible to imagine that 1 + 1 = 0? Certainly, there are nominalists who believe that there are no numbers, so do not believe that 1 + 1 = 2 is true, when taken literally as a claim about the relations between abstract objects. But Clarke-Doane rightly refuses to rest his case for it being intelligible to imagine the mathematical facts being otherwise on the intelligibility of nominalism. Of course, a skeptic who does not believe that there

https://doi.org/10.1080/00455091.2019.1584936 Published online by Cambridge University Press

are any (non-trivial) D-facts *could* challenge the D-realist by arguing that, even if (as is conceptually possible) there were no D-facts they would still have the very same D-beliefs that they have. But it is not at all clear that the D-realist should accept this argument, since it is not clear that they should accept the intelligibility of the D-skeptic's position, and therefore accept that it's conceptually possible that there are no D-facts.[1] Given that the D-realist may plausibly refuse to concede the intelligibility of the D-skeptic's position, resting the debunking challenge on the claim that it is intelligible to imagine there being *no* D-facts weakens the debunker's case. The challenge from sensitivity presents a stronger challenge than that presented by the D-skeptic, since it suggests that even if the D-realist is right that there are some (non-trivial) D-facts, modal tracking considerations *still* serve to undermine the D-realist's beliefs. The debunker can argue that *even if we assume that there are some D-facts*, we still have no reason to think that our D-beliefs would vary with variations in those facts.

For this reason, the debunker's case is strengthened if it is possible to argue that it is intelligible to imagine the D-facts being otherwise *on the assumption that there are some (non-trivial) D-facts*. In the moral case, Clarke-Doane points out, the phenomenon of reasonable disagreement amongst those who are competent with moral concepts can be used to support the claim that it is intelligible to imagine the moral facts being otherwise, even if both sides agree that there are some (non-trivial) moral facts. These are disagreements that that do not 'seem to bottom out in disagreement over whether there are any (substantive) moral truths at all' (Clarke-Doane 2012, 335). Thus Clarke-Doane ventures that,

> As long as there has been some disagreement among apparently conceptually competent people with respect to a moral sentence 's', this affords evidence that it is intelligible to imagine both that s and that not-s. (Clarke-Doane 2012, 335)

With the example of moral disagreement in mind, Clarke-Doane asks whether there has been 'analogous disagreement over a wide variety of mathematical claims', that similarly does not bottom out in disagreement over whether there are any substantive mathematical truths at all. If there is such disagreement, this could be used to support the claim that it is intelligible to imagine the mathematical facts being otherwise, independently of the intelligibility of nominalism.

To answer this, Clarke-Doane points to the historical controversies in mathematics over candidate axioms for set theory, as well as over 'apparent trivialities of arithmetic such as that every natural number has a successor' (Clarke-Doane 2012, 336). Given that his debunking argument is meant to debunk our claims to knowledge of arithmetical truths about numbers, I will set aside controversies over set theoretic axioms, and consider whether Clarke-Doane's claim that the existence of disagreement over such 'apparent trivialities' as that every natural number has a successor means that it is intelligible to imagine basic arithmetic truths, such as that 1 + 1 = 2, being otherwise (while corresponding logical truths remain as they are). A major difficulty for this claim is that, arguably, *it is part of our concept of numbers as mathematical objects* that *Hume's Principle* holds of the numbers. Indeed, neo-Fregeans hold that Hume's Principle is a conceptual truth, Hume's Principle being the claim that:

> (HP) The number of Fs = the number of Gs if and only if the Fs and Gs are equinumerous.

Now given that HP, if true, would imply the existence of the natural numbers (see Appendix), nominalists have questioned whether HP should be considered a conceptual truth (can *any* concepts imply the actual existence of objects falling under those concepts, given that an empty domain seems conceptually possible?). However, arguably even if HP isn't itself a conceptual truth, nevertheless nominalists can agree that it does correctly characterize our concept of number, at least in the sense that, in order for any objects even to be *candidates* for counting as 'the natural numbers', HP would have to be true of them. But if accepting that HP would have to be true of numbers, were there any such things, is required for competency with the concept of number, then this has implications for the claim that it is intelligible to imagine the facts about numbers being different while ordinary logical truths remain the same. For, the function of HP is precisely to line up mathematical truths about numbers with logically expressible facts about collections of objects. Given HP, together with standard definitions of 0, 1, 2, and + (tacit knowledge of which is also, arguably, required for competency with the concept of number), it will be the case that if there is exactly one F then the number of Fs is 1, and if there are exactly two things that are F or G, then the number of things that are F or G is 2. Furthermore, it is a logical truth that if there is exactly one lion-behind-bush-A and exactly one lion-behind-bush-B, and nothing that is both a lion-behind-bush-A and a lion-behind-bush B, there are exactly two lions-behind-bush-A-or-B. So it will follow from this logical truth (together with Hume's Principle and appropriate definitions) that if 1 is the number of lions-behind-bush-A, and 1 is the number of lions-behind-bush-B, and there are no lions behind both bushes, then 1 + 1 = 2 (the number of lions-behind-bush-A-or-B). (See Appendix for a more detailed sketch of how these results follow from HP plus definitions.) So while it might be intelligible to imagine the basic facts about numbers being otherwise, if we accept HP and associated definitions as characterising our concept of number, then it is not intelligible to imagine the basic facts about numbers being otherwise *while logical features of the natural world remain the same*.

How does all this square with Clarke-Doane's claim that it must be intelligible to imagine the mathematical facts being different on the grounds that there is disagreement amongst those competent with the concept of number over even some of the most basic claims about the numbers, such as, to take his example, the claim that every natural number has a successor? If we assume the truth of HP on a standard impredicative understanding of the principle (so that the concepts F and G on the right hand side of HP can include concepts characterised using the 'number of' operator), then it follows from this that every natural number has a successor. This is because, for any n, once we have defined natural numbers up to n, n + 1 can be defined as the number belonging to the concept 'being 0 or 1 or 2 or … or n' (see Appendix). In this case, if impredicative HP is part of our concept of natural number (in the sense that, for any candidate objects to count as 'the natural numbers', HP would have to be true of them), then so is the claim that every natural number has a successor. How, then, could people who are competent with the concept of number disagree with this claim?

One answer would be to suggest that strict finitists who disagree with the claim that every number has a successor disagree on similar grounds that nominalists disagree with this claim (at least in the sense that, for nominalists, this claim will be counted alongside all other universally quantified claims in mathematics as only trivially true). That is, for at least some strict finitists, it is arguably the case that they *agree* that the standard Platonist concept of number requires the existence of infinitely many numbers, but question whether any system of objects satisfying the Platonist concept of number exists. Thus for example Jean Paul van Bendegem (2012) defends strict finitism on the basis of a constructivist, as opposed to Platonist, approach to mathematics. Constructivists are plausibly motivated by the thought that, given that we have no reason to believe that there are any abstract objects answering to our ordinary mathematical theories, to the extent that claims about numbers make sense at all, they make sense only as claims that can be interpreted as 'really' talking about human constructions, such as finite strings of symbols that we could in principle write down.[2] On this view, since there is an upper limit on the length of strings that it is feasible to imagine a human writing down, there will be some very large 'natural number' (understood as a string of symbols) that has no successor. But if this is the motivation for strict finitism, it looks as though strict finitists are not so much disagreeing about what is involved in the standard concept of number (according to which every number has a successor), but are rather suggesting that *this* concept is not something we have reason to think is instantiated. Instead, the constructivist strict finitist might be viewed as proposing that the standard (Platonist) conception of number should be replaced by an alternative concept (something like, 'finite string that it is humanly feasible to produce'). Such strict finitists could be (and I'm sure are!) perfectly competent with the standard Platonist concept of number (including HP), and could be able to agree on what follows from this concept. Nevertheless, like other nominalists, they doubt whether *this* particular concept is ever satisfied. The possibility of disagreement over *this* question does not show that someone could be competent with the standard Platonist concept of number and still question whether HP is true *of numbers so-conceived*.

This solution, while perhaps plausible when considering strict finitists who wish to replace standard Platonism with a constructivist picture of the numbers, does not suffice to deal with all strict finitist attempts to question the claim that every number has a successor. Not all strict finitists are motivated by constructivism.[3] One prominent example is the mathematician Doron Zeilberger, who writes:

> I am a platonist, and I believe that *finite* integers, *finite* sets of *finite* integers, and all *finite* combinatorial structures have an existence of their own, regardless of humans (or computers). I also believe that *symbols* have an independent existence. What is completely meaningless is any kind of *infinite*, actual or potential. So I deny even the existence of the Peano axiom that every integer has a successor. (Zeilberger 2004, 32–33)

Zeilberger's finitism is motivated by suspicion of infinities quite generally (including in the mathematical realm), and not by a more general anti-Platonism. As such, it appears that one might be competent with our usual Platonistic concept of number, accept that that concept is indeed satisfied by a realm of abstract mathematical objects, and yet still question whether every number has a successor. Does this mean that HP is not, after all, a conceptual truth about the numbers, since it implies the infinitude of the numbers?

Fortunately for our purposes, we need not try to adjudicate on whether Zeilberger's finitist Platonism shows competence with the standard Platonist's concept of number. For there is a weaker version of HP, *predicative* HP, which does not imply the infinitude of the numbers, and which suffices for the purpose of our argument. If we accept predicative uses of HP (i.e. applications of HP only to concepts F and G on the right hand side that can be characterized without making use of the 'the number of' operator), then this is enough for our purposes. For given that predicative HP will produce a number n for every concept applying to exactly n things, we will be able to use it to line up the relevant logical truths concerning finite numerosities to which we have plausibly evolved to respond with corresponding facts about numbers. And given this, so long as at least *predicative* HP is characteristic of our concept of number, then we will have enough of a correspondence between facts about numerosities and facts about numbers to make it unintelligible to imagine facts of basic arithmetic being otherwise without a corresponding change in logical truths.

https://doi.org/10.1080/00455091.2019.1584936 Published online by Cambridge University Press

I conclude, then, that Clarke-Doane has not shown what he needs to show in order to raise a challenge to the sensitivity of our arithmetical beliefs to the arithmetical facts, platonistically construed. Even if it is in some sense intelligible to imagine the arithmetical facts being different than they are, competency with our ordinary Platonist concept of number (and particularly predicative Hume's Principle) ensures that it is not intelligible to imagine basic arithmetical facts about numbers being different *without corresponding differences in what we take to be logically true claims about the natural world*. So (to the extent that it makes sense to imagine this at all), *had* it been true of numbers construed platonistically that 1 + 1 = 0, it would also have been true that one lion and another lion make zero lions, and so it would not have been evolutionary advantageous to believe instead that 1 + 1 = 2.

The moral case

What is the upshot of this discussion for the debunking argument in its original form, as a challenge to belief in independent moral truths? Here it may look as though the debunker is on stronger ground, since there is no moral analogue of Hume's Principle, a neat biconditional that serves to line up – as a conceptual matter – moral truths with natural ones. In this case the existence of moral disagreement does seem relevant: people can be fully competent with moral concepts, and still disagree over any particular biconditional 'x if morally good if and only if …', where the right hand side is replaced with some purely descriptive claim. That our 'bare normative concepts', as Street (2008) puts it, don't clearly vindicate any one neat way of lining up the moral facts (on the assumption that there are such facts) with the natural facts is a clear disanalogy between the moral and mathematical cases.

However, it would be too quick to conclude that the above discussion shows only that our arithmetic beliefs, platonistically construed, do not face a debunking challenge on grounds of their failure to track the relevant facts about numbers, while placing no barriers in the way of the original debunking challenge for moral realism. Clarke-Doane and Street (at least as Clarke-Doane interprets her) hold that, to make the challenge, they have to be able to show that it is intelligible to imagine the relevant (moral/mathematical) facts being otherwise. But in the above discussion, our objection was *not* to the claim that it may be intelligible to imagine the mathematical facts concerning numbers as being otherwise, but rather, to the claim that it is intelligible to imagine the mathematical facts concerning numbers being otherwise *while the evolutionarily relevant natural facts* (including in this case logical truths about physical objects in our evolutionary environment) *remained the same*. To make the case for the insensitivity of our evolved beliefs to the mathematical facts, Clarke-Doane has to argue *both* that it is intelligible to imagine the relevant facts about numbers being otherwise, *and* that, in the situation we are being asked to imagine, nothing relevant would have changed in our evolutionary environment, so that it would still have been advantageous to believe that 1 + 1 = 2. The status of (predicative) HP as a truth about our concept of number kicks in here, making it impossible for us to imagine the facts about numbers (conceived of as satisfying HP) being different while corresponding logical truths remain the same. This is what blocks the challenge to the sensitivity of our mathematical beliefs to the facts.

So for the sensitivity challenge to work in the moral case, Clarke-Doane's debunker does not just need to show that it is intelligible to imagine the moral facts being otherwise. They also need to show that, in the situation we are being asked to imagine, we should expect that there would be no *corresponding change in the evolutionarily relevant natural facts*. And this is a much bigger task. For, even if no *particular* bridge principle lining up moral facts with natural facts can claim the status of a conceptual truth, the supervenience of the moral on the natural is widely regarded as a conceptual truth. Thus, for example, Michael Smith claims that

> Everyone agrees that the moral features of things *supervene* on their natural features … That is, everyone agrees that two possible worlds that are alike in all of their natural features must also be alike in their moral features; that the moral features of things cannot float free of their natural features. Moreover, everyone agrees that this is a platitude; that it is an *a priori* truth. For recognition of the way in which the moral supervenes on the natural is a constraint on the proper use of moral concepts. (Smith 1994, 21–22)

And, while his ultimate purpose is to challenge the orthodoxy concerning the supervenience thesis, Rosen (2014) calls the supervenience of the normative on the natural 'the least controversial thesis in metaethics'.[4] But if grasp of our moral concepts does involve (perhaps, for many of us, tacit) acceptance of the supervenience of the moral on the natural, then this causes problems for the sensitivity challenge *even if our moral concepts do not rule out the possibility of the moral facts being otherwise*.

To see this, consider Smith's formulation of supervenience, that it is 'a constraint on the proper use of moral concepts' that 'two possible worlds that are alike in all of their natural features must be alike in their moral features'. Suppose now that we are asked to imagine – as is conceptually possible – the moral facts being other than they in fact are. Then, on Smith's characterization of the supervenience of the moral on the natural as a conceptual truth, *the proper use of our moral concepts requires* that the world we are imagining is one in which the natural facts are also other than they in fact are (since it is 'a constraint on the proper use of moral concepts' that there can be no difference in moral features without a difference in natural features). But then we cannot presume that in *this* world that we are now imagining, whose natural features are different from those of the actual world, our counterparts would have been subject to the very same evolutionary constraints and have evolved the very same moral beliefs, so we cannot presume that *had* the moral facts been different, our evolved moral beliefs would nevertheless have been the same.

One might worry that something has gone wrong in this argument, especially given the phenomenon Clarke-Doane points to of reasonable moral disagreement in establishing that it is intelligible to imagine the moral facts being different while the descriptive facts remain the same. When we disagree over the moral facts, these disagreements can happen even despite full agreement over all the relevant non-moral facts. For many moral disagreements are at root disagreements over what the correct bridge principles are that link moral facts to natural facts, and in such cases, disagreement concerning the moral facts will arise even in cases of full agreement in non-moral matters. Surely the intelligibility of these disagreements means this that we *can* conceive of the moral facts being other than they are while the descriptive/natural facts remain the same? After all, even if, in such disagreements, I believe that I am in fact right in my moral assessments, it would be intellectual arrogance in the extreme not to concede that it is at least *conceivable* that it is my opponent who has things right, and hence that the moral facts *could* be otherwise while the natural facts remain the same.

This reasoning, though, is fallacious. What these disagreements show is that it is perfectly intelligible to imagine that the moral facts may be *different from how we take them to be*, while the natural facts remain the same. Recognising that one's opponent *may* have things right is simply recognising that our own beliefs concerning the moral facts might be wrong (and wrong not in virtue of some mistake about the natural facts, but in virtue of some mistake about how moral matters relate to natural matters). But to undermine our moral beliefs on the grounds that they are the products of forces that are insensitive to the actual moral facts (whatever they may be), one must argue, not for the claim

(a) Had (as is intelligible to imagine) the moral facts been different from *how we in fact take them to be*, our moral beliefs would still have been the same,

but rather for the claim,

(b) Had (as is intelligible to imagine) the moral facts been different from *how they in fact are*, our moral beliefs would still have been the same, for it is only this latter claim that would show that our beliefs do not vary along with variations of the moral facts (whatever these may be). The evolutionary challenge to the realist from sensitivity aims to establish that the realist's moral beliefs are the result of evolutionary forces that are not truth-tracking. To make this challenge, the debunker needs to show that the realist's beliefs would not vary with variations in the actual moral facts. Thus, Clarke-Doane points out, the realist's counter claim that 'we were selected to have true moral beliefs'

> does not merely mean that we were selected to have certain moral beliefs, and those beliefs are (actually) true. The latter claim could be true even if evolutionary forces were "indifferent" to the moral truths but "just happened to land" us on them "by chance." The claim that we were selected to have true moral beliefs has counterfactual force. It implies that had the moral truths been very different, our moral beliefs would have been correspondingly different – that it would have benefited our ancestors to have correspondingly different moral beliefs. (Clarke-Doane 2012, 319)

The debunking challenge, insofar as it concerns consideration of how our beliefs would vary with variations in the moral facts, functions as a challenge to the moral realist *even if* as a matter of fact the realist's moral beliefs are true. It provides a sense in which, even if those beliefs were true, they would be so only coincidentally, since the very same evolutionary forces would result in the very same moral beliefs in worlds where the moral facts were very different.[5]

In his (2014) paper, 'No Coincidence', Matthew Bedke challenges this interpretation of the sensitivity considerations at work in debunking arguments, arguing that what the debunker needs to establish is that our moral beliefs are *oblivious* to the moral facts where

> The belief that P based on justification J is *oblivious* to the target fact when it meets these conditions – when, were P not the case (i) one would believe that P, (ii) one would have the same justification J for believing that P, and (iii) the same causal explanations for why one believes that P and why one has justification J would hold. (Bedke 2014, 114)

Importantly, for Bedke, the relevant subjunctive here is concerned with

> *allodoxic* possibilities, not counterfactuals. Allodoxic possibilities are false belief possibilities – they are those we can assume to obtain contrary to what we actually believe and our justifying bases for believing it. (Bedke 2014, 119)

If this is right, then when the debunker challenges the realist that evolutionary forces would still have led them to the same moral beliefs even had those beliefs were false, the subjunctive need not force us to consider what would be the case in another possible world (with different moral facts), but only what – for all we know – might in fact be the case in our world. And if this is right then we cannot rely on supervenience to suggest that, had the moral facts been different the evolutionary circumstances might have been relevantly different too, since we cannot assume that the closest world in which the moral facts are other than we take them to be isn't in fact our own world.

If it is allodoxic possibilities that matter in the tracking argument, then one worry we may have is that this kind of 'obliviousness' to the facts comes too cheap. We take it as a conceptual truth that the moral facts supervene on the natural facts. But presumably when we are considering the *allodoxic* possibility of our having the very same moral beliefs even if those moral beliefs were wrong, we're meant to be considering a world where the supervenience base remains just as it is, but the moral facts are otherwise. But if this is right, then it would seem that it is all too easy for our beliefs to count as oblivious. As Clarke-Doane points out,

> For virtually *any* supervenient property, F, it appears that had – per impossible – the contents of our explanatorily basic F-beliefs been false, we still would have believed them. (Clarke-Doane 2016, 27) [6]

One worry about this formulation of the sensitivity requirement is that it may make it too easy for our beliefs to fail to be sensitive to the facts. As such it would only undermine the moral realist's claim to moral knowledge if it also undermined lots of other knowledge claims that we do not take to be problematic.

Whether or not this is correct, I would like to suggest that if Bedke's diagnosis of the debunking challenge is right it shows that, contrary to Clarke-Doane's presentation, it is not really a modal 'tracking' challenge at all (in the sense of a challenge to the realist to show that our D-beliefs covary with the D-facts). In Clarke-Doane's picture as I have been presenting it, the D-realist's confidence in their D-beliefs is undermined by showing the realist that, *even if those beliefs were in fact true*, this would still be in an important sense coincidental since, in worlds in which they were false we would still be subject to the very same evolutionary forces and so still have the same D-beliefs. In Bedke's picture, by contrast, although the truth of our D-beliefs is also argued to be coincidental, the source of the coincidence appears somewhat different, lying not in the failure of our D-beliefs to covary with the D-facts, but in a lack of an explanation of the source of our D-beliefs that connects the D-beliefs we actually have with the relevant D-facts.[7] But then it seems that any substantial modal aspect to the challenge disappears altogether; instead, it appears, the realist's moral beliefs are undermined simply because the realist's assumption that they are, as a matter of fact, true relies on positing an unexplained coincidence in the actual world.

I am sympathetic to the thought that this kind of coincidence is really what is at issue in evolutionary debunking arguments, and that an evolutionary debunking challenge can be raised that does not depend on showing that our moral beliefs do not vary with the actual moral facts. For example, David Enoch (2010) argues that the best version of the debunking challenge should be understood in non-modal terms as the challenge for the realist to explain the correlation that they claim to hold between their moral beliefs and the moral facts. However, if this is the form the debunking challenge takes, then arguably (again following Enoch 2010), it can be relatively easily defused by the realist. I am content in this paper to argue that, *if* the debunking challenge involves (as Clarke-Doane's (2012) presentation suggests) a 'tracking' challenge to the modal sensitivity of our moral beliefs to the moral facts, then the supervenience of the moral on the natural as a conceptual truth presents a significant obstacle to the development of this challenge. On the other hand, if the challenge involves arguing that we have no reason to believe that the moral facts are as we take them to be, on the grounds that our explanation of how we have come to these beliefs does not connect those beliefs to the facts and as such would appear to render any correlation an unexplained coincidence, I defer to Enoch's (2010) discussion of how the realist may explain the alleged correlation between our moral beliefs and the moral facts.

Let us return, then, to the sensitivity challenge as pressed by Clarke-Doane (2012). At least if we accept Smith's formulation of the supervenience thesis as a conceptual truth, then, I claim, this formulation rules out the intelligibility of imagining the moral facts being different than they in fact are, without there also being a corresponding difference in the natural facts. And this is so even though the phenomenon of moral disagreement shows that it is certainly intelligible to imagine the moral facts being different from *how we take them to be*, without imagining a corresponding difference in the natural facts. For again, according to Smith, competent use of moral concepts requires us to accept 'that two possible worlds that are alike in all of their natural features must also be alike in their moral features'. It follows from this that two possible worlds that differ in their moral features must differ in at least some of their natural features. But in this case, according to Smith's formulation, it is a constraint on the proper use of our moral concepts that if we are imagining the moral facts being different from how they in fact are (i.e. in this world), this will involve us in imagining a possible world whose *natural* features differ from those of our own world. The phenomenon of moral disagreement tells us only that we can imagine ourselves being wrong about the moral features of our world, not that we can imagine those features being *other than they actually are*, without a corresponding change in the natural features.

The supervenience claim, at least as characterized by Smith, rules out the intelligibility of imagining a world that is in natural respects just like ours, but that differs in its moral features. And as such, it presents an obstacle in the way of the debunker who wishes to claim that 'had (as it is intelligible to imagine) the moral facts been otherwise, we would have still evolved the very same moral beliefs', since the debunker cannot simply assume that a world with different moral features from our own would have been naturally similar enough to our own such so as to put us under the very same evolutionary pressures to form the very same moral beliefs. There is, though, a respect in which the obstacle presented by the supervenience claim to the debunking argument in ethics is less substantial than the obstacle presented by Hume's Principle in the analogous argument in the philosophy of mathematics. In the philosophy of mathematics case, we can say precisely what we would have to conceive of as being different in the world where we imagine $1 + 1 = 0$ being true. In particular, the logical truth 'if there is exactly one lion-behind-bush-A and exactly one lion-behind-bush -B, and nothing that is both a lion-behind-bush-A and a lion-behind-bush B, there are exactly two lions-behind-bush-A-or-B' would have (*per impossibile*) to be false. Either it is not intelligible to imagine such a world, or at the very least, imagining such a world would also involve imagining significant changes in our evolutionary environment.

In the ethical case matters are somewhat different. While we cannot conceive of a world in which the moral facts are different without a corresponding difference in the natural facts, the supervenience thesis itself does not allow us to say precisely *what* in the supervenience base would have to be different for us to be in a world where, to use Street's example, 'our children's lives [were] worthless' (Street 2008, 208). For all the bare supervenience thesis tells us, the difference between a world in which our children's lives matter and a world in which they are worthless might be a small matter of no evolutionary relevance – a difference in the spin of some distant elementary particle, for example.[8] If it is even conceivable that differences in the moral facts could be grounded in differences in underlying natural facts that are *entirely irrelevant* to our evolutionary context, then even notwithstanding the supervenience of the moral on the natural, the debunker would still be able to argue that it is intelligible to imagine the moral facts being otherwise, while all the evolutionary relevant natural facts that guide the formation of our moral beliefs remained the same.

While the supervenience of the moral on the natural does not, then, rule out the evolutionary debunker's strategy outright, notice that it does at the very least raise the bar for the evolutionary debunker in ethics. For, to raise a plausible sensitivity challenge to our moral beliefs, the debunker would still have to show that the closest possible worlds in which the moral facts are significantly different are worlds in which there are no significant differences in evolutionarily relevant matters. And, at least to the extent that our moral theorizing involves us in the development of a conception of the kind of thing that has moral relevance/moral worth, it is not at all clear that, 'for all our bare normative concepts tell us' (Street 2008, 208), the closest worlds to our own in which our children's lives don't matter *could* differ only in recherché, evolutionarily irrelevant matters. A world in which our children's lives don't matter would, a realist would presumably counter, have to be a world in which our children have no, or at least only a negligible, moral status. But to the extent to which an entity's having a significant moral status depends in part on such things as its capacity for agency or suffering (to take just two plausible examples), it is hard to see how a world in which our children's lives didn't matter could be anything other than wildly different from our own world. It may not be a world in which one thing and another thing makes zero things, but at least to a realist who believes that the moral facts are grounded in *morally relevant* natural facts, conceiving of what things would have to be like *naturally* in a world where our children's lives didn't matter might be almost as difficult as conceiving of a world where $1 + 1 = 0$. At any rate, the option is certainly open to the realist to respond to the debunker by countering that a world in which our children's lives did not matter would have to be so different from our own world as to make it implausible that our counterparts would have been subjected to the very same evolutionary forces as we have.

Conclusion

I conclude, then, that to the extent that the debunking challenge is a challenge to the modal sensitivity of our mathematical or moral beliefs to the mathematical or moral facts, construed realistically,[9] the challenge cannot straightforwardly be made either in the mathematical or the moral case. In both cases, the challenge encounters difficulties not because we cannot intelligibly imagine the mathematical or moral facts being different, but rather because, at least if we accept some widely accepted principles as governing the realist's mathematical and moral concepts respectively, we cannot intelligibly imagine the mathematical or moral facts being different *while the natural facts that provide the backdrop for our evolved mathematical or moral beliefs remain fixed*. In the mathematical case, it is the status of Hume's Principle as characterizing what would have to be true of any objects for them to count as 'natural numbers' that ensures, at least in the case of basic arithmetic knowledge, it is not intelligible to imagine the facts about numbers, construed realistically, being different without there being corresponding differences in the natural world. In the moral case, while there is no easy analogue to Hume's Principle, lining up particular moral truths with particular natural facts, nevertheless, to the extent that the supervenience of the moral on the natural is accepted *as a conceptual matter*, it is likewise not intelligible to imagine the moral facts being different than they are without any changes in the natural world. In the moral case, the option remains open for the debunker to argue that, although a world in which the moral facts were different would also have to be a world in which the natural facts would differ, these differences in the natural facts would not be evolutionarily relevant. But it is not at all clear that a realist who believes that the supervenience of the moral on the natural involves a supervenience of the moral on *morally relevant* natural features should accept this. At the very least, considerations of how natural facts would have to vary with variations in the moral facts present a significant obstacle for an easy debunking argument against moral realism.

Notes

1. In the mathematical case, this dialectic is borne out in debates over so-called 'makes no difference' arguments for nominalism. Nominalists argue that, given the supposed acausal nature of mathematical objects, their existence or otherwise would make no difference to spatiotemporal going on, so that 'if all the objects in the mathematical realm suddenly *disappeared*, nothing would change in the physical world' (Balaguer, 1998, 132). On the other hand, Platonists question the intelligibility of such thought experiments (e.g. Baker 2003), holding that on their picture of the relation of mathematical to physical objects, a world with no mathematical objects (even if we can conceive of such a thing) would have to be very different indeed from our world, so much so that it may not even be intelligible to imagine *our* world being maths-free.
2. Compare with Arend Heyting's character of the intuitionist (a constructivist, though admittedly not a strict finitist) in his *Disputation*: 'We have no objection against a mathematician privately admitting any metaphysical theory he likes, but Brouwer's program entails that we study mathematics as something simpler, more immediate than metaphysics. In the study of mental mathematical constructions "to exist" must be synonymous with "to be constructed".' (Heyting 1956, 2).
3. I am grateful to anonymous referees for this journal for drawing my attention to some examples.
4. It is worth noting that, while the supervenience of the moral on the natural is certainly widely accepted in some form or other, assent is not universal. Aside from the challenge raised by Rosen (2014), other challenges to ethical supervenience are found in, e.g. Raz (2000), Sturgeon (2009), and Roberts (2017).
5. I should note that the sensitivity of our D-beliefs to the D-facts involves more than the simple *variation* of D-beliefs with variation in the D-facts, since our beliefs could vary with variations in the facts while still failing to get things right about those facts. However, as the debunking argument that we are concerned with here involves the claim that, had the moral facts been different our evolved beliefs would still have been the same (and thus a claim

concerning a lack of variation), we can set aside this additional complexity. I am grateful though, to an anonymous referee for raising this possibility, which could potentially be used to launch a further debunking argument even if it is accepted that changes in D-facts are likely to lead to a change in the evolutionary environment and thus a corresponding change in our evolved D-beliefs.

6. It's perhaps worth noting that, although Clarke-Doane (2012) is aimed at arguing that, to the extent that there is a sensitivity challenge to moral realism there is an analogous challenge to mathematical realism, the paper from which this quote is taken, Clarke-Doane (2016), aims to show that we cannot in fact launch a compelling sensitivity challenge to moral realism.
7. Thus, for example, Bedke (2014, 119) argues that our belief that water is H_2O is not oblivious to the facts since if we consider the allodoxic possibility that water is not H_2O, 'we would have a lot of explaining to do for why we have the false belief and the misleading evidence we have'. On the other hand, considering the moral claim that pain is bad, Bedke suggests that if we consider the allodoxic possibility that pain is not bad, we are not similarly at a loss for an explanation of why we have the belief that it is bad, since the badness of pain does not feature in the explanation of our belief that pain is bad.
8. I am grateful to an anonymous referee for pushing me to clarify this point, and to an editor for CJP for suggesting this example of an apparently evolutionarily irrelevant change in the supervenience base.
9. What I haven't here argued is that the debunking challenge *is* best understood as taking this form. I have followed Clarke-Doane's (2012) reconstruction of Street's challenge in order to consider whether, *if* the challenge takes this form, it is a genuine one, either for moral or mathematical realists. As noted above though, there are alternative construals of the debunking argument in metaethics (including Enoch (2010), which presents the challenge as a metaethical analogue of Benacerraf's original epistemological challenge to Platonism) that do not require that we make sense of the conceptual possibility of the moral facts being otherwise while the evolutionary environment remains the same. This paper in no way speaks to those alternative formulations of the debunking considerations.
10. The formulation of the account of number presented here is borrowed from Edward Zalta's (2017) and (2018) presentation of Frege's Theorem in the Stanford Encyclopedia of Philosophy.
11. These conditions are notational variants of those presented in Zalta (2018).
12. See Zalta (2018) for details.

Acknowledgments

I am grateful to members of the White Rose Early Career Ethics Researchers (WRECERS) group (including Richard Yetter Chappell, Daniel Elstein, Johan Gustafsson, and Jack Woods) and of the Practical Philosophy group at York (including Rebecca Davis, Christopher Jay, and Christian Piller) for discussions of earlier versions of this paper. I am grateful also to Debbie Roberts and Alan Thomas for comments on earlier drafts. I received insightful and challenging comments from two anonymous referees and an editor for Canadian Journal of Philosophy and I thank them for their time and efforts in helping bring this paper to its published form. This research was originally carried out during a period of leave that was made possible by the Leverhulme Trust under their Research Fellowship scheme (award number RF-2016-245). I am extremely grateful to the Leverhulme Trust for supporting this research.

Funding

This research was funded by the Leverhulme Trust under their Research Fellowship scheme (grant number RF-2016-245).

Notes on contributor

Mary Leng completed her PhD at the University of Toronto in 2002 before holding a Research Fellowship at St John's College, Cambridge. She lectured at the University of Liverpool (2006–2011) before moving to the University of York in 2012, where she remains. Her research interests are in the philosophy of mathematics and science, and in particular, realism/anti-realism debates. She is currently working on analogies between debates over realism/anti-realism in the philosophy of mathematics and in metaethics.

ORCID

Mary Leng http://orcid.org/0000-0001-9936-5453

References

Baker, A. 2003. "Does the Existence of Mathematical Objects Make a Difference?" *Australasian Journal of Philosophy* 81 (2): 246–264. doi:10.1080/713659635.
Balaguer, M. 1998. *Platonism and Anti-Platonism in Mathematics*. Oxford: OUP.
Bedke, M. 2014. "No Coincidence." In *Oxford Studies in Metaethics*, edited by R. Shafer Landau, 9: 102–125. Oxford: OUP.
Clarke-Doane, J. 2012. "Morality and Mathematics: The Evolutionary Challenge." *Ethics* 122 (2): 313–340. doi:10.1086/663231.
Clarke-Doane, J. 2016. "Debunking and Dispensability." In *Explanation in Ethics and Mathematics*, edited by U. D. Leibowitz and N. Sinclair, 23–36. Oxford: OUP.
Enoch, D. 2010. "The Epistemological Challenge to Metanormative Realism: How Best to Understand It, and How to Cope with It." *Philosophical Studies* 148 (3): 413–438. doi:10.1007/s11098-009-9333-6.
Heyting, A. 1956. *Intuitionism: An Introduction*. Amsterdam: North-Holland.
Raz, J. 2000. "The Truth in Particularism." In *Moral Particularism*, edited by B. Hooker and M. O. Little, 48–78. Oxford: OUP.
Roberts, D. 2017. "Why Believe in Normative Supervenience?" In *Oxford Studies in Metaethics*, edited by R. Shafer-Landau, 2–21. Vol. 13. Oxford: OUP.
Rosen, G. 2014. "What Is Normative Necessity?" Accessed October 2017. https://www.academia.edu/9159728/Normative_Necessity
Smith, M. 1994. *The Moral Problem*. Oxford: Blackwell.
Street, S. 2006. "A Darwinian Dilemma for Realist Theories of Value." *Philosophical Studies* 127 (1): 109–166. doi:10.1007/s11098-005-1726-6.

Street, S. 2008. "Reply to Copp: Naturalism, Normativity, and the Varieties of Realism Worth Worrying About." *Philosophical Issues* 18 (1): 207–228. doi:10.1111/j.1533-6077.2008.00145.x.

Sturgeon, N. 2009. "Doubts about the Supervenience of the Evaluative." In *Oxford Studies in Metaethics*, edited by R. Shafer-Landau, 53–92. Vol. 4. Oxford: OUP.

van Bendegem, J. P. 2012. "A Defence of Strict Finitism." *Constructive Foundations* 7 (2): 141–149.

Zalta, E. N. 2017. "Frege's Theorem and Foundations for Arithmetic." In *The Stanford Encyclopedia of Philosophy (Summer 2017 Edition)*, edited by E. N. Zalta. Accessed 23 May 2018. https://plato.stanford.edu/archives/sum2017/entries/frege-theorem/

Zalta, E. N. 2018. "Gottlob Frege." In *The Stanford Encyclopedia of Philosophy (Spring 2018 Edition)*, edited by E. N. Zalta. Accessed 23 May 2018. https://plato.stanford.edu/archives/spr2018/entries/frege/

Zeilberger, D. 2004. ""Real" Analysis Is a Degenerate Case of Discrete Analysis." In *New Progress in Difference Equations (Proc. ICDEA 2001)*, edited by B. Aulbach, S. Elaydi, and G. Ladas. 25–34. Taylor & Francis: London.

Appendix

How does Hume's Principle together with definitions serve to line up logical truths about lions with the mathematical facts concerning addition? The following is just a sketch (without proofs) of the key elements needed to line up the logical truth that, if there is exactly one lion behind bush A and exactly one lion behind bush B and no lion behind both bushes, then there are exactly two lions behind bush A or B with the mathematical truth that 1 + 1 = 2.[10]

Let $L_A x$ = x is a lion behind bush A, and let $L_B x$ = x is a lion behind bush B. Assume that there is exactly one lion behind bush A, exactly one lion behind bush B, and no lions behind both bushes, i.e.

(i) $\exists x(L_A x \ \& \ \exists y(L_A y \supset y = x))$
(ii) $\exists x(L_B x \ \& \ \exists y(L_B y \supset y = x))$
(iii) $\sim\exists x(L_A x \ \& \ L_B x)$

From (i)-(iii), it can be derived that there are exactly two lions behind bush A or B:

(iv) $\exists x \exists y((((L_A x \vee L_B x) \ \& \ (L_A y \vee L_B y)) \ \& \ x \neq y) \ \& \ \exists z \ ((L_A z \vee L_B z) \supset z = x \vee z = y))$

To connect these facts up with statements about numbers, we need to introduce Hume's Principle plus some definitions, as follows.

$$\text{HP}: \ \#F = \#G \text{ if and only if } F \approx G$$

(where here the operator '#' takes concepts to objects, and the relation '≈' of equinumerosity holds between concepts F and G if and only if the Fs and the Gs can be put in one-one correspondence).

We note now that the following conditions on concepts pick out equinumerous concepts (in the sense that, if concepts F and G both satisfy condition (n) for some n, then F and G are equinumerous):

Condition (0):	Nothing falls under F $\sim\exists xFx$
Condition (1):	Exactly one thing falls under F $\exists x(Fx \,\&\, \exists y(Fy \supset y = x))$
Condition (2):	Exactly two things fall under F. $\exists x\exists y(((Fx \,\&\, Fy) \,\&\, x \neq y) \,\&\, \exists z(Fz \supset z = x \vee z = y))$
Condition (3):	Exactly three things fall under F. $\exists x\exists y\exists z((((Fx \,\&\, Fy) \,\&\, Fz) \,\&\, ((x \neq y \,\&\, x \neq z) \,\&\, y \neq z)) \,\&\, \exists w(Fw \supset ((w = x \vee w = y) \vee w = z)))$

(etc).[11]

Supposing, then, that there is a concept satisfying condition 0, the number belonging to this concept will also belong to every other concept satisfying condition 0 (i.e. to every concept that applies to exactly zero objects).

In fact, we can easily find a concept satisfying condition 0 – that being the concept of being non-self-identical. It will be useful here to introduce the λ-notation for concepts, where '[λx: ϕ(x)]' stands for the concept of 'being an x such that ϕ(x)'. In this notation, the concept of being non-self-identical can be written as [λx: x ≠ x], and we can define 0 as follows:

$$0 =_{df} \#[\lambda x: \; x \neq x]$$

(The existence and uniqueness of 0, so-defined, is a consequence of Hume's principle.)

How about condition 1? Once we have proved the existence and uniqueness of 0, a neat way of finding a concept satisfying condition 1 is to pick the concept 'identical with 0' (since this applies to one and only one thing, namely 0). If we did this we could define $1 =_{df} \#[\lambda x: x = 0]$. (From HP it would then follow that 1 exists, is unique, and differs from 0.) Similarly, we could define $2 =_{df} \#[\lambda x: x = 0 \vee x = 1]$, and in general, $n + 1 =_{df} \#[\lambda x: x = 0 \vee x = 1 \vee \ldots \vee x = n]$. In this way, the infinitude of the natural numbers easily falls out of Hume's principle.

However, this reasoning uses an impredicative form of Hume's principle (since, once we've used HP to introduce 0, we've assumed that the concept of being identical with 0 is one to which HP can now apply). We've conceded that the conceptual possibility of strict finitism could speak against taking *impredicative* HP as definitive of the concept of number, so with this concession in mind we cannot make use of the standard definition of number to show that HP as a conceptual truth lines up the logical truths about small finite numerosities with facts about numbers. Instead, for each number n that we wish to assert to exist, we must find some concept that applies to exactly n objects. Suppose that there are (as the strict finitist claims) only finitely many objects. Then a finite language can contain a name for each distinct such object – call the objects that exist $a_1, \ldots, a_k$, with $a_i = a_j$ if and only if $i = j$. From these names we can define concepts that allow us to define all the natural numbers up to k as follows: $1 =_{df} \#[\lambda x: x = a_1]$; $2 =_{df} \#[\lambda x: x = a_1 \vee x = a_2]$, … and $k =_{df} \#[\lambda x: x = a_1 \vee x = a_2 \vee \ldots \vee x = a_k]$. (On the other hand, if we adopt, as Russell did, an axiom of infinity, we will be able to find concepts that enable us define all the natural numbers while still only using predicative HP.)

Now, our definitions of the numbers 1 and 2, together with the fact that conditions (1) and (2) pick out equinumerous concepts (applying to all concepts that pick out one or two objects respectively), will allow us to conclude from (i)-(iv) (using HP) that $\#L_A = 1$, $\#L_B = 1$, and $\#(L_A \vee L_A) = 2$. Finally, to tie this to the mathematical fact that $1 + 1 = 2$ we need to introduce the definitions of successor and of addition.

First, to define successor, we will follow Frege in defining the concept 'x immediately precedes y' in terms of x being the number belonging to some concept that holds of all of the Fs bar one. For example, if w is one of the Fs, we can define the concept of being an F but not being equal to w: $[\lambda z: Fz \& z \neq w]$, and say that the number belonging to this concept immediately precedes the number belonging to the concept F. Our definition of 'Precedes' is then as follows:

$$\textit{Precedes}(x,\ y) =_{df} \exists F \exists w((Fw \& y = \#F)x = \#[\lambda z:\ Fz \& z \neq w])$$

We can see from our definitions above that 0 Precedes 1, 1 precedes 2, 2 Precedes 3, and so on. By HP we can show that the 'Precedes' relation is functional (i.e. if Precedes(x, y) and Precedes(x, z), then y = z, and 1–1 (i.e. if Precedes(x, y) and Precedes(z, y), then x = z. Since the relation is functional, if m Precedes n we can call n *the* successor of m, or s(m). We can define the notion of a natural number using the ancestral, Precedes+, of the Precedes relation, so that the natural numbers are the successors of the successors of … of the successors of 0.

$$x \text{ is a natural number} =_{df} \text{Precedes}^{+}(0,\ n).$$ [12]

It follows from this that if n is a natural number and $n \neq 0$, then there is a natural number m such that m Precedes n (so that $n = s(m)$).

With our successor notation in place, addition can now be defined recursively on the natural numbers as follows:

$$m + 0 = m$$

$$m + s(n) = s(m + n)$$

(For the strict finitist, we should qualify the second clause with 'if $s(m + n)$ exists', since for strict finitists, not every number will have a successor.)

The following lemma (provable by induction on #G) allows us to line up logical truths with facts about addition in general.

$$\text{If } \#F = m \text{ and } \#G = n \text{ and } \#[\lambda x:\ Fx \& Gx] = 0,\ \text{then} \#[\lambda x:\ Fx \vee Gx] = m + n.$$

With these ingredients, we get $\#(L_A \vee L_B) = \#L_A + \#L_B = 1 + 1$. But we also have shown that $\#(L_A \vee L_B) = 2$. So (predicative) HP plus definitions tells us that if (i)-(iii) all hold, then $1 + 1 = 2$ must also hold, as required.