THE BULLETIN OF SYMBOLIC LOGIC
Volume 20, Number 3, September 2014

2012 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

LOGIC COLLOQUIUM '12

Manchester, UK
July 12–18, 2012

Logic Colloquium '12, the 2012 European Summer Meeting of the Association for Symbolic Logic, was hosted by the University of Manchester from July 12 to July 18, 2012. The lectures were held at the Chemistry Building and the Alan Turing Building on the campus. Major funding for the conference was provided by the Association for Symbolic Logic (ASL), the National Science Foundation (NSF), the London Mathematical Society, the British Logic Colloquium, and Manchester Institute of Mathematical Sciences.

The success of the meeting was due largely to the hard work of the Local Organizing Committee under the leadership of its Chair, Alex Wilkie (University of Manchester). The other members were Alexander Borovik, Mark Kambites, Jeff Paris, Mike Prest, Harold Simmons, Marcus Tressl, Alena Vencovska, and George Wilmers. The Program Committee consisted of Paola D'Aquino (Seconda Università di Napoli, Chair), Uri Abraham (Ben Gurion University of the Negev), Lev Beklemishev (Steklov Mathematical Institute), Barry Cooper (University of Leeds), Anuj Dawar (University of Cambridge), Goran Sundholm (Universitiet Leiden), Marcus Tressl (University of Manchester), and Frank Wolter (University of Liverpool).

The main conference topics were: *Computability Theory, Foundations of Mathematics and Philosophy of Logic and Mathematics, Homotopy Type Theory, Model Theory, Proof Theory and Constructive Mathematics, and Set Theory*. The program included the Turing lecture and the Goodstein lecture, three tutorial courses, thirteen invited plenary lectures, and twenty-four invited lectures in six special sessions. There were 74 contributed papers of which 14 were by title only, and 168 participants from many different countries. Twenty-nine students and recent Ph.D's were awarded grants, of whom 24 obtained awards from ASL and 5 from NSF.

The Turing lecture was delivered by Angus Macintyre (Queen Mary, University of London), with the title: *Turing meets Schanuel*. The Goodstein lecture was delivered by Michael Rathjen (University of Leeds), with the title: *From Goodstein sequences to graph theory and weak consistency*.

The following tutorial courses were given:

Ilijas Farah (York University, Toronto), *Elliott's program and descriptive set theory*.

Antonio Montalbán (University of Chicago), *Reverse mathematics*.

Boris Zilber (University of Oxford), *Pseudo-analytic structures: model theory and algebraic geometry*.

The following invited plenary lectures were presented:

Jeremy Avigad (Carnegie Mellon University), *Computability and convergence*.

Andreas R. Blass (University of Michigan, Ann Arbor), *Symbioses between mathematical logic and computer science*.

Gareth Jones (University of Manchester), *Counting rational points on definable sets*.

Péter Komjáth (Eötvös University, Budapest), *Some results on infinite graphs*.

Leonid A. Levin (Boston University), *Computers: who are they and what is beyond them*.

1079-8986/14/2003-0008/$3.40
DOI:10.1017/bsl.2014.17

Giuseppe Longo (Ecole Normal Supérieure), *Schroedinger (1944) and Turing (1952) on the logic of life: from the "coding" to the "genesis" of organization and forms*.

Menachem Magidor (Hebrew University, Jerusalem), *Getting forcing axioms by finite support iteration*.

Zlil Sela (Hebrew University, Jerusalem), *The elementary theory of free products of groups*.

V. Yu. Shavrukov, *Nonstandard elements of r.e. sets*.

Alexandra Shlapentokh (East Carolina University, Greenville), *First-order and existential definability and decidability in positive characteristic*.

Mariya I. Soskova (Sofia University), *The Turing universe in the context of enumeration reducibility*.

The proceedings of Logic Colloquium '12 will be published in a special issue of the Annals of Pure and Applied Logic.

More information about the meeting can be found at the conference webpage,
`http://www.mims.manchester.ac.uk/events/workshops/LC2012/index.php.`
Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
PAOLA D'AQUINO

Abstract of the invited Goodstein Lecture

▸ MICHAEL RATHJEN, *From Goodstein sequences to graph theory and weak consistency*.
Department of Mathematics, University of Leeds, Leeds, United Kingdom.
E-mail: `michrathjen@snafu.de`.

Termination of Goodstein sequences is one of the earliest examples of a statement that, while true, is not provable PA. In his 1944 paper though, rather than with unprovability, Goodstein was concerned with finitism. In the talk I intend to reflect on Goodstein's motives. This will be followed by relating some of the highlights of research on unprovable (in various theories) statements. In the second half I plan to talk about the inverses of Goodstein's fast growing functions and how they give rise to unprovable statements that are still weaker than consistency.

Abstract of the invited Turing Lecture

▸ ANGUS MACINTYRE, *Turing meets Schanuel*.
School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, London E1 4NS, UK.
E-mail: `a.macintyre@qmul.ac.uk`.

For various first-order theories of exponential fields, such as the reals, the complexes, and Zilber's fields, we consider the issue of constructing computable models (and, very often, prime models). Assuming Schanuel's Conjecture there are significant results, for example that of Macintyre and Wilkie that the prime model of real exponentiation is computable (and contains only computable reals, but not π). Without Schanuel, our ignorance is essentially complete. But even with it, our knowledge for the complex and Zilber cases is very limited, and very much connected to deep issues in algebraic geometry. We construct some computable structures with real trigonometric functions, but these are not models of the theory of the real trigonometric functions. We make some progress in making computable Zilber's constructions, using work on Grobner bases, and we discuss issues of how to test consistency with Schanuel's Conjecture.

Abstracts of invited plenary talks

▸ JEREMY AVIGAD, *Computability and convergence*.
Departments of Philosophy and Mathematical Sciences, Carnegie Mellon University, Pittsburgh, Pennyslvania, USA.
E-mail: avigad@cmu.edu.

Countless theorems of analysis assert the convergence of sequences of numbers, functions, or elements of an abstract space. Classical proofs often establish such results without providing explicit rates of convergence, and, in fact, it is often impossible to compute the limiting object or a rate of convergence from the given data. This results in the curious situation that a theorem may tell us that a sequence converges, but we have no way of knowing how fast it converges, or what it converges to.

On the positive side, it is often possible to "mine" quantitative and computational information from a convergence theorem, even when a rate of convergence is generally unavailable. In this talk, I will discuss examples that illustrate the kinds of information that can and cannot be obtained.

▸ ANDREAS BLASS, *Symbioses between mathematical logic and computer science*.
Department of Mathematics, East Hall, 530 Church Street, Ann Arbor, MI 48109, USA.
E-mail: ablass@umich.edu.

Computer science has strong connections with numerous aspects of mathematical logic, but those aspects are sometimes different from those traditionally studied for pure mathematical purposes. I plan to give a survey talk describing several situations where the applied and pure aspects turned out to be more closely connected than one might at first think. As a result, ideas and techniques from each side contributed to the other, or the two sides combined to suggest new and fruitful ideas. In addition to describing some of these ideas, I shall mention some open problems arising from them.

▸ SAM BUSS, *Complexity lower bounds with alternation trading proofs*.
University of California, San Diego, La Jolla, CA 92092-0112, USA.
E-mail: sbuss@math.ucsd.edu.

We discuss alternation-trading proofs and lower bounds on the time and space complexity of algorithms for satisfiability and other NP-complete problems. Alternation trading proofs achieve complexity lower bounds by speedup assumptions with classic results of Nepomnjascii on trade alternation for time complexity. Williams has proved an n^c time lower bound with $c = 2\cos(\pi/7)$ for $n^{o(1)}$ space algorithms. Although it had been conjectured that $c = 2$ was the best that present-day techniques can achieve, we can now prove that $c = 2\cos(\pi/7)$ is optimal for present methods of alternation trading proofs. To prove this, we formalize a proof system for alternation trading proofs and characterize its consequences. Related results give time space lower bounds for a wider range of parameters.

This is joint work with Ryan Williams.

This talk was not given; it is present by title.

▸ GARETH JONES, *Counting rational points on definable sets*.
School of Mathematics, The Alan Turing Building, The University of Manchester, Manchester M13 9PL, UK.
E-mail: gareth.jones-3@manchester.ac.uk.

The Pila–Wilkie theorem gives a bound for the number of rational points of bounded height lying on the transcendental part of a set definable in an o-minimal expansion of the real field. I shall explain this theorem and the better bound conjectured by Wilkie for sets definable in the real exponential field. I shall then discuss some instances of this conjecture for curves and surfaces, and some applications.

▸ PÉTER KOMJÀTH, *Some results on infinite graphs*.
Department of Computer Science, Eőtvős University, Budapest, P.O. Box 120 1518, Hungary.
E-mail: kope@cs.elte.hu.

We survey some recent results on the chromatic number of infinite graphs.

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

► LEONID A. LEVIN, *Computers: who are they and what is beyond them*.
Computer Science, Boston University, 111 Cummington Street, Boston, MA 02215, USA.
URL Address: http://www.cs.bu.edu/~lnd.

As today's computing environment is Internet, which we cannot model, I discuss an invariant-based, not model-based, notion of computability. The technical statements are simple, so will be fully explained. However, the proofs are too tricky to make sense in a talk, so I will give pointers to articles instead.

► GIUSEPPE LONGO, *Schroedinger (1944) and Turing (1952) on the logic of life: from the "coding" to the "genesis" of organization and forms*.
CNRS, CREA, Ecole Polytechnique et CIRPHLES, Ecole Normale Supérieure, Paris, France.
E-mail: longo@ens.fr.
URL Address: http://www.di.ens.fr/users/longo.

Schroedinger's and Turing's analyses of life phenomena have a twofold aspects. They both follow, first, a "coding paradigm", of embryogenesis or of human computations and deductions respectively, and then move toward a more "dynamicist" approach. Schroedinger, in the second part of his 1944 book, hints to biological organization as negentropy—a variant of Gibbs dynamical analysis of energy—that we revitalized as anti-entropy, see references. Turing, after stressing that "the nervous system is surely not a Discrete State machine" (1950), invents the mathematics for an action/reaction/diffusion process, a "continuous system" (1952), where chemical matter (a hardware with no software) organizes itself along morphogenesis.

We will hint to the paths for thought opened along the lines of Turing's dynamics by continuous deformations at the core of Turing's pioneering paper of 1952, where symmetry breakings are a key component of the dynamics.

[1] FRANCIS BAILLY and GIUSEPPE LONGO, ***Mathematics and natural sciences: the physical singularity of life***, Imperial College Press, London, 2011.

[2] ———, *Biological organization and anti-entropy*. ***Journal of Biological Systems***, vol. 17 (2009), no. 1, pp. 63–96.

[3] GIUSEPPE LONGO and MA'L MONTÉVIL, *From physics to biology by extending criticality and symmetry breakings*. ***Progress in Biophysics and Molecular Biology***, vol. 106(2011), no. 2, pp. 340–347.

[4] ERWIN SCHROEDINGER, ***What is life?*** Cambridge University Press, Cambridge, UK, 1944.

[5] ALAN M. TURING, *The chemical basis of morphogenesis*. ***Philosophical Transactions of the Royal Society***, vol. B237 (1952), pp. 37–72.

► MENACHEM MAGIDOR, *Getting forcing axioms by finite support iteration*.
Institute of Mathematics, Hebrew University of Jerusalem, Jerusalem 91904, Israel.
E-mail: mensara@savion.huji.ac.il.

Forcing axioms (like Martin's Axiom-**MA**, The Proper Forcing Axiom-**PFA** or Martin's Maximum-**MM**) provide some of the most appealing and interesting directions in which one can extend **ZFC** and decide important statement which are otherwise independent. At accepted forcing axioms provide a lot of information about sets of size $\aleph_1$. Thus the stronger forcing axioms like **PFA** and **MM** decide the value of the continuum as well as many statements about the structure of H_{ω_2}. It seems very interesting to try and get generalizations of these axioms that will have deep impact of the structure of larger sets (e.g., sets of size $\aleph_2$).

The main obstacle for getting such generalizations is the available techniques for iterating forcings. The techniques used for proving the consistency of axioms like **PFA** or **MM** is either countable support of revised countable support iteration. There are serious obstacles for generalizing it to larger cardinals.

Recently Neeman (following Mitchell and Friedman) introduced a version of finite support iteration of proper forcings which allowed him to get a model of **PFA** by different techniques. His technique can be generalized to higher cardinals and so he can get the consistency of higher versions of **PFA**.

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

Following Neeman, Gitik and the author generalized these techniques to semiproper forcings and so get a different proof of the consistency of **MM**. These techniques allow one to get higher versions of **MM**.

In this talk we shall survey these results and state some open problems. We shall try to make the talk accessible to a general Logic audience. (At least the first half of it.)

► ZLIL SELA, *The elementary theory of free products of groups*.
Department of Mathematics, Hebrew University, Jerusalem 91904, Israel.
E-mail: zlil@math.huji.ac.il.

Around 1956 R. Vaught asked the following natural question. Let A, B, C, D be arbitrary groups. Suppose that A and B have the same first order theory (such groups are called elementarily equivalent), and so do C and D. Do A*C and B*D have the same first order theory? (i.e., Is elementary equivalence preserved under free products of groups?)

A similar question for (generalized) direct products (of general structures) was answered affirmatively by Mostowski in 1952, and later generalized by Feferman and Vaught in 1959. On the other hand Olin proved in 1974 that the answer to Vaught's question is negative if we replace groups by semigroups.

We develop a geometric structure theory, that is based on the tools that were developed to solve Tarski's problem on the first order theory of a free group, to answer Vaught's problem affirmatively. This structure theory suggests a generalization of Tarski's problem to free products of arbitrary groups, as well as other (somewhat surprising) results in model theory over groups. It suggests open questions and will probably have generalizations in quite a few directions.

► V. YU. SHAVRUKOV, *Nonstandard elements of r.e. sets*.
Nijenburg 24, 1081GG Amsterdam, Netherlands.
E-mail: v.yu.shavrukov@gmail.com.

We consider the Priestley dual $(\mathcal{E}^*)^*$ of the lattice $\mathcal{E}^*$ of r.e. sets mod finite. Connections with nonstandard elements of r.e. sets in models of 1st order true arithmetic as well as with dynamic properties of r.e. sets are pointed out. Illustrations include the Harrington–Soare dynamic characterization of small subsets, a model-theoretic characterization of promptly simple sets, and relations between the inclusion ordering of prime filters on $\mathcal{E}^*$ (aka points of $(\mathcal{E}^*)^*$) and the complexity of their index sets.

The approach to $\mathcal{E}^*$ via its dual is prompted by the study of the E-tree, the dual of the lattice of Σ_1 sentences modulo provability in an appropriate r.e. theory T. Along the way we note both similarities and differences between (the duals of) the two lattices.

► ALEXANDRA SHLAPENTOKH, *First-order and existential definability and decidability in positive characteristic*.
Department of Mathematics, East Carolina University, Greenville, NC 27858, USA.
E-mail: shlapentokha@ecu.edu.
URL Address: www.personal.ecu.edu/shlapentokha.

We prove that the existential theory of any function field K of characteristic $p > 0$ is undecidable in the language of rings provided the constant field does not contain the algebraic closure of a finite field. (In the case K is uncountable we consider equations with coefficients in a finitely generated subfield.) We also complete the proof of the characteristic 2 higher transcendence degree case left out from the main theorem of [1] to show that the first-order theory of **any** function field of positive characteristic is undecidable in the language of rings without parameters.

[1] KIRSTEN EISENTRÄGER and ALEXANDRA SHLAPENTOKH, *Undecidability in function fields of positive characteristic*. ***International Mathematics Research Notices***, vol. 21 (2009), pp. 4051–4086.

► MARIYA I. SOSKOVA, *The Turing universe in the context of enumeration reducibility*.
Faculty of Mathematics and Informatics, Sofia University, Bulgaria.
E-mail: msoskova@fmi.uni-sofia.bg.

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

In the mathematical analysis of the notion of definability we wish to understand how one object can be used to specify another. Depending on the mathematical nature of the objects in question and the method for the relative specification one can distinguish between many different approaches. In every case the approach gives rise to a reducibility between the objects, with a natural structural representation as a partial order, its degree structure, a model of relative definability.

The most studied model of relative definability between sets of natural numbers is that of the Turing degrees based on the notion of Turing reducibility. A project by Ganchev, Soskov, and Soskova is to examine the standard Turing model in a wider context, the structure of the enumeration degrees. The second structure is based on a weaker form of relative computability between sets of natural numbers, enumeration reducibility. We will describe this work, comparing results that have been obtained in each structure.

Abstracts of invited tutorials

► ILIJAS FARAH, *Elliott's program and descriptive set theory*.
Department of Mathematics and Statistics, York University, 4700 Keele Street, North York, Ontario, Canada, M3J 1P3.
Matematicki Institut, Kneza Mihaila 34, Belgrade, Serbia.
E-mail: ifarah@yorku.ca.
URL Address: http://www.math.yorku.ca/~ifarah.

After his success in classifying approximately finite-dimensional C*-algebras by K-theory more than three decades ago, George Elliott proposed that more general nuclear C*-algebras might be classifiable by K-theoretic invariants. The ensuing Elliott classification program has enjoyed tremendous success and achieved a number of spectacular results ([7]). However, counterexamples constructed by Rørdam and Toms showed that the program in its original formulation needs to be revised ([1]). This was followed by further spectacular results ([8]). In the first lecture I will present the current state of the art in classification of nuclear C*-algebras. No previous acquaintance with C*-algebras will be assumed.

In the second lecture I will introduce the emerging theory of descriptive set-theoretic analysis ([5]) of Elliott's program ([4], [3]). C*-algebra ultrapowers (e.g., [6]) will be the theme of the third lecture ([2]).

[1] G. A. Elliott and A. S. Toms, *Regularity properties in the classification program for separable amenable C^*-algebras*. ***Bulletin of the American Mathematical Society. New Series***, vol. 45 (2008), pp. 229–245.

[2] I. Farah, B. Hart, and D. Sherman, *Model theory of operator algebras I: Stability*. ***Bulletin of the London Mathematical Society***, (to appear).

[3] I. Farah, A. Toms, and A. Törnquist, *The descriptive set theory of C^*-algebra invariants*. ***International Mathematics Research Notices***, (to appear), appendix with Caleb Eckhardt.

[4] ———, *Turbulence, orbit equivalence, and the classification of nuclear C^*-algebras*. ***Journal für die Reine und Angewandte Mathematik***, (to appear).

[5] G. Hjorth, *Borel equivalence relations*, ***Handbook of set theory***, 2010.

[6] E. Kirchberg, *Central sequences in C^*-algebras and strongly purely infinite algebras*, ***Operator algebras: The Abel Symposium 2004***, vol. 1, Springer, Berlin, 2006, pp. 175–231.

[7] M. Rørdam, ***Classification of nuclear C^*-algebras***, Encyclopaedia of mathematical sciences, vol. 126, Springer-Verlag, Berlin, 2002.

[8] W. Winter, *Nuclear dimension and $\mathcal{Z}$-stability of pure C^*-algebras*. ***Inventiones Mathematicae***, vol. 187 (2012), no. 2, pp. 259–342.

► ANTONIO MONTALBÁN, *Reverse mathematics*.
Department of Mathematics, University of Chicago, Chicago, IL 60657, USA.
E-mail: antonio@math.uchicago.edu.

In this series of three talks we will describe the program of reverse mathematics, its origins, it goals, and some of the latest results.

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

The question of which axioms are necessary to do mathematics is of great importance in Foundations of Mathematics and is the main question behind the program of Reverse Mathematics. Reverse Mathematics deals with the system of second-order-arithmetic which is rich enough to be able to express an important fragment of classical mathematics. This fragment includes number theory, calculus, countable algebra, real and complex analysis, differential equations, separable metric spaces, and combinatorics among others. Almost all of mathematics that can be modeled with, or coded by, countable objects can be done in second-order arithmetic.

The idea of Reverse Mathematics goes as follows. We start by fixing a basic system of axioms. The most commonly used basic system is called RCA_0 that essentially says that computable sets exist. Now, given a theorem of "ordinary" mathematics, the question we ask is what axioms do we need to add to the basic system to prove this theorem. It is often the case in Reverse Mathematics that we can show that certain axioms are necessary to prove a theorem by showing that the axioms follow from the theorem using the basic system. Because of this idea, this program is called Reverse Mathematics. Many different systems of axioms have been defined and studied. But a very interesting fact is that most of the theorems, whose proof-theoretic strength has been analyzed, have been proved equivalent over RCA_0 to one of five systems, that we will call the *main five systems*. Understanding why only these main five systems are so frequently equivalent to theorems of ordinary mathematics is a very intriguing question.

► BORIS ZILBER, *Pseudo-analytic structures: model theory and algebraic geometry*.
Mathematical Institute, University of Oxford, 24–29 St Giles, Oxford, OX1 3LB, UK.
E-mail: `zilber@maths.ox.ac.uk`.

This is a survey of developments around construction of pseudo-analytic structures such as the analog of the complex numbers with exponentiation. Model-theoretic ingredients are Hrushovski's construction of unusual stable structures, Shelah's theory of excellence in abstract elementary classes and the theory of Zariski structures.

The connection to algebraic geometry and number theory involves a very general form of Schanuel's conjecture as well as Diophantine conjectures around mixed Shimura varieties.

Abstracts of invited talks in the Special Session on Computability: Logical and Physical

► KLAUS AMBOS-SPIES, *On the strongly bounded Turing degrees of simple sets*.
Department of Mathematics and Computer Science, University of Heidelberg, Im Neuenheimer Feld 294, D-69120 Heidelberg, Germany.
E-mail: `ambos@math.uni-heidelberg.de`.

We study the r-degrees of simple sets under the strongly bounded Turing reducibilities $r =$ cl (computable Lipschitz reducibility) and $r =$ ibT (identity bounded Turing reducibility) which are defined in terms of Turing functionals where the use function is bounded by the identity function up to an additive constant and the identity function, respectively. We call a c.e. r-degree $\mathbf{a}$ simple if it contains a simple set and we call $\mathbf{a}$ nonsimple otherwise. As we show, the ibT-degree of a c.e. set A is simple if and only if the cl-degree of A is simple, and there are nonsimple c.e. r-degrees $> \mathbf{0}$.

Moreover, we analyze the distribution of the simple and nonsimple r-degrees in the partial ordering of the c.e. r-degrees. Among the results we obtain are the following. (i) For any c.e. r-degree $\mathbf{a}$, there is a simple r-degree $\mathbf{b}$ above $\mathbf{a}$ and, for any c.e. r-degree $\mathbf{a} > \mathbf{0}$, there is a simple r-degree $\mathbf{b}$ and a nonsimple r-degree $\hat{\mathbf{b}} > \mathbf{0}$ below $\mathbf{a}$. (ii) For any wtt-complete set A, $deg_r(A)$ is simple. So, in particular, there is a c.e. r-degree $\mathbf{a}$ with no nonsimple r-degree above it (which in turn implies that the nonsimple r-degrees are not dense in the p.o. of the c.e. r-degrees). (iii) The simple r-degrees are not dense in the p.o. of the c.e. r-degrees. (iv) For any c.e. set A which is not wtt-complete there is a nonsimple r-degree above $deg_r(A)$. So, in particular, there is a Turing-complete set A such that $deg_r(A)$ is nonsimple. (v) For any c.e.

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r-degree $\mathbf{a} > \mathbf{0}$ there are simple and nonsimple r-degrees which are incomparable with $\mathbf{a}$. (vi) Any c.e. r-degree is the join of two nonsimple c.e. r-degrees whereas the class of the nonzero c.e. r-degrees is not generated by the simple r-degrees under join.

► FAY DOWKER, *Modus ponens in physics*.
Blackett Laboratory, Imperial College, London SW7 2AZ, UK.
E-mail: `f.dowker@imperial.ac.uk`.

I will show that the classical "Boolean" rules of inference about physical events are equivalent to three assumptions: (i) something happens, (ii) modus ponens, and (iii) if an event doesn't happen then its complement (or "negation") does. Physics, specifically quantum mechanics, demands to give up the third assumption and replace it with a "principle of maximal detail". This implies that rules of inference about the physical world are dynamical.

[1] Kate Clements, Fay Dowker, and Petros Wallden, *Modus ponens in physics*, `arXiv:1201.6266v1`.

► P. D. WELCH, Σ^0_3-*Determinacy and Transfinite Turing Machine models*.
School of Mathematics, University of Bristol, Clifton, Bristol, BS8 1TW, UK.
E-mail: `philipwelch.welch@gmail.com`.

At low levels of the arithmetic hierarchy, that is below Σ^0_4, determinacy is provable in second order number theory. It is possible to characterize strategies for Σ^0_1 and Σ^0_2 games (the latter a result of Solovay) in terms of certain inductive definitions. We survey these results, and ask what can be done at Σ^0_3. There are connections with certain kinds of transfinite Turing machine models, (alternatively characterizable as certain quasi-inductive definitions), and with proof-theoretic considerations, which we explore.

Abstracts of invited talks in the Special Session on Homotopy and Type Theory

► PETER ACZEL, *The Structure Identity Principle and the Univalence Axiom*.
Schools of Mathematics and Computer Science, The University of Manchester, Oxford Road, Manchester, M13 9PL, UK.
E-mail: `petera@cs.man.ac.uk`.

The Structure Identity Principle (SIP), in a strong form, states that isomorphic structures are identical. This principle is false in axiomatic set theory, but can be true in other foundational formal systems.

In my talk I will discuss SIP in its standard set-theoretically acceptable form, which states that isomorphic structures are structurally identical. I will then briefly outline a foundational formal system for Homotopy Type Theory, with Voevodsky's Univalence Axiom and indicate how the axiom entails very strong forms of SIP.

► NICOLA GAMBINO, *Homotopy-initial W-types*.
Dipartimento di Matematica e Informatica, Università degli Studi di Palermo, via Archirafi 34, 90123 Palermo, Italy.
E-mail: `ngambino@math.unipa.it`.
URL Address: `http://www.math.unipa.it/~ngambino`.

Over the past few years, there has been significant progress in developing a dictionary relating type theory and homotopy theory. For example, Martin-Löf's rules for identity types have been shown to be intimately related to Bousfield's notion of a weak factorization system. One of the most interesting aspects of these advances is that they allow us to use homotopy-theoretic intuition when working within type theory, as demonstrated by the development of Voevodsky's Univalent Foundations program. I will illustrate this idea by introducing the notion of a homotopy-initial algebra within type theory and showing how it allows us to give a characterization of a weakening of the familiar rules for types of wellfounded trees, or W-types.

This is joint work with Steve Awodey and Kristina Sojakova.

► MARTIN HYLAND, *Identity and existence*.
Department of Pure Mathematics and Mathematical Statistics, University of Cambridge, Wilberforce Road, Cambridge CB3 0WB, UK.
E-mail: M.Hyland@dpmms.cam.ac.uk.

For many years it was apparent that there must be some connection between Type Theory and the theory of weak higher dimensional categories. The insights coming from Homotopy Type Theory provide a new angle on that idea. In particular Homotopy Type Theory suggests paying very careful attention to just how and in what ways we have weak structure.

One important aspect is the possibility to replace the computation rules of Type Theory by axioms in terms of identity types. It is important to understand that extreme form of weakness and consider whether there are additional principles (coherence conditions) which should come into play. Considerations related to this issue appear, for example, in [1] and [2]. This raises the question of the special status of the identity types, a modern form of the question 'Is existence a predicate?'.

In early work on weak categories coherence conditions and coherence theorems play a major role. However in more modern developments (the theory of quasicategories for example) such considerations play no role. This appears to be a move from a theory in an algebraic spirit to one informed by homotopy theory. In recent developments (not only in [1] and [2] but also for example in more recent unpublished work of Awodey Gambino and Sojakova) refined forms of coherence reappear. Again there is something to understand.

[1] Steve Awodey and Michael Warren, *Homotopy theoretic models of identity types*. ***Mathematical Proceedings of the Cambridge Philosophical Society***, vol. 146 (2009), pp. 45–55.

[2] Richard Garner, *On the strength of dependent products in the type theory of Martin-Löf*. ***Annals of Pure and Applied Logic***, vol. 160 (2009), pp. 1–12.

► PETER LEFANU LUMSDAINE, *A categorical road to intensional type theory*.
Department of Mathematics and Statistics, Dalhousie University, Halifax, Nova Scotia.
E-mail: p.l.lumsdaine@dal.ca.

I will discuss how ideas from model categories—specifically, the interplay of weak factorization systems with classical categorical constructions—can motivate the rules for connectives in Intensional Type Theory, giving a viewpoint complementary to Martin-Löf's original philosophical/syntactic explications of them.

Abstracts of invited talks in the Special Session on Model Theory

► ELÍAS BARO, *From commutators to Cartan subgroups in the o-minimal setting*.
Departamento de Álgebra, Facultad de Matemáticas, Universidad Complutense de Madrid, 28040, Madrid, Spain.
E-mail: eliasbaro@pdi.ucm.es.

Groups definable in o-minimal structures and groups of Finite Morley Rank sometimes have an orthogonal behaviour. For example, while the commutator subgroup of a group of FMR is definable, a counterexample of this fact in the o-minimal case has been recently found by A. Conversano. In groups of FMR it has been conjectured the conjugacy of Carter subgroups, i.e., nilpotent definable and definably connected subgroups of finite index in its normalizer. Again, the o-minimal analogue of this conjecture is false in general. However, groups of FMR and o-minimal groups share a lot of properties: for example, they both have a descending chain condition for definable subgroups and a definable and additive dimension. From this starting point, and following arguments with a FMR flavour, in [1] we study in the o-minimal setting in which situations is the commutator subgroup of a group definable. For example, we show that this is the case for solvable groups. In [2] the study of Carter subgroups in the o-minimal context lead us to the concept of Cartan subgroups. In this talk I will try to survey the results of these two papers.

[1] ELIAS BARO, ERIC JALIGOT, and MARGARITA OTERO, *Commutators in groups definable in o-minimal structures* ***Proceedings of the American Mathematical Society***, http://dx.doi.org/10.1090/S0002-9939-2012-11209-2, (to appear).

[2] ———, *Cartan subgroups of groups definable in o-minimal structures*, e-print, http://arxiv.org/abs/1109.4349, 2011.

► ABDEREZAK OULD HOUCINE, *Algebraic closure and ampleness in free groups*.
Institut Camile Jordan, Université Claude Bernard Lyon 1, 43 boulevard du 11 novembre 1918, 69622 Villeurbanne Cedex, France.
E-mail: ould@math.univ-lyon1.fr.

The first part deals with the algebraic and definable closure (*acl* and *dcl*) in free groups. We prove that if F is a free group of finite rank and A is a nonabelian subgroup of F such that F is freely indecomposable with respect to A, then $acl(A)$ is exactly the vertex group in the cyclic malnormal JSJ-decomposition of F with respect to A. We show that $dcl(A)$ is a free factor of $acl(A)$ and in particular they coincide in a free group of rank 2. In the general case, we show that a free group whose rank is greater than 4 contains a subgroup A such that $acl(A) \neq dcl(A)$. This answers a question of Z. Sela.

This is a joint work with D. Vallino.

The second part deals with ampleness in free groups where the notion of the algebraic closure intervenes in a fundamental way. Ampleness is a property that reflects the existence of geometric configurations behaving very much like projective space over a field. Pillay showed that the theory of the free group is 2-ample and conjectured that it not 3-ample. We show that the theory of the free group—and more generally the theory of any torsion-free hyperbolic group—is n-ample for any n.

This is a joint work with K. Tent.

[1] A. OULD HOUCINE and K. TENT, *Ampleness in the free group*, arXiv:1205.0929, 2012.

[2] A. OULD HOUCINE and D. VALLINO, *Algebraic and definable closure in free groups*, arXiv:1108.5641, 2011.

► FRANÇOISE POINT, *Definable sets in topological differential fields*.
F.R.S.-F.N.R.S., Department of Mathematics, Mons University, 20, Place du Parc, 7000 Mons, Belgium.
E-mail: Francoise.Point@umons.ac.be.

Let $\mathcal{K}$ be a topological $\mathcal{L}$-field as defined in [3] and its expansion $\langle K, D\rangle$, where D is a derivation on K, with a priori no interactions with the topology of K. Assume $\mathcal{K}$ is a model of a universal $\mathcal{L}$-theory T which has a model completion T_c. Under certain hypothesis on T, with N. Guzy, we showed that the expansion of T to the $\mathcal{L} \cup \{D\}$-theory T_D consisting of T together with the axioms expressing that D is a derivation, admits a model-completion $T^*_{c,D}$ which we axiomatized ([3]). Namely, to the theory $T_D \cup T_c$, we added a scheme of axioms (DL), which expresses that each differential polynomial has a zero close to a zero of its associated algebraic polynomial. This scheme (DL) generalizes the axiomatization (CODF) of the theory of closed ordered differential fields ([7]) and is related to the axiom scheme (UC) introduced by M. Tressl in the framework of large fields ([8]).

In this talk, I will review the above setting and basic properties of $T^*_{c,D}$. For instance, whenever T_c has NIP, the nonindependence property, then $T^*_{c,D}$ has NIP ([3]). With N. Guzy ([4]), using results of L. van den Dries ([2]), we showed the existence of a fibered dimension function for definable subsets in models of $T^*_{c,D}$.

Then, I will indicate how to use former results of L. Mathews ([5]) in order to get further information on definable subsets in models of $T^*_{c,D}$. This applies in particular for $T_c = RCF$, or $T_c = pCF$. In the case of $T_c = RCF$, $CODF$ has o-minimal open core (using [1]) and elimination of imaginaries ([6]).

[1] A. DOLICH, C. MILLER, and C. STEINHORN, *Structures having o-minimal open core*. ***Transactions of American Mathematical Society***, vol. 362 (2010), no. 3, pp. 1371–1411.

[2] L. VAN DEN DRIES, *Dimension of definable sets, algebraic boundedness and henselian fields*. ***Annals of Pure and Applied Logic***, vol. 45 (1989), no. 2, pp. 189–209.

[3] N. Guzy and F. Point, *Topological differential fields*. ***Annals of Pure and Applied Logic***, vol. 161 (2010), no. 4, pp. 570–598.

[4] ———, *Topological differential fields and dimension functions*. ***The Journal of Symbolic Logic***, (to appear).

[5] L. Mathews, *Cell decomposition and dimension functions in first-order topological structures*, ***Proceedings of London Mathematical Society. Series 3***, vol. 70 (1995), no. 1, pp. 1–32.

[6] F. Point, *Ensembles définissables dans les corps ordonnés différentiellement clos*, ***Comptes Rendus Mathématique. Académie des Sciences. Paris, Series I***, vol. 349 (2011), pp. 929–933.

[7] M. Singer, *The model theory of ordered differential fields*. ***The Journal of Symbolic Logic***. vol. 43 (1978), no. 1, pp. 82–91.

[8] M. Tressl, *A uniform companion for large differential fields of characteristic* 0. ***Transactions of American Mathematical Society***, vol. 357 (2005), no. 10, pp. 3933–3951.

► PIERRE SIMON, *Type decompositions in NIP theories*.
DMA, École Normale Supérieure, 45 rue d'Ulm 75005 Paris, France.
E-mail: pierre.simon.05@normalesup.org.

In his recent paper [1], Shelah proves a new characterization of NIP (or dependent) theories: a theory is NIP if and only if there are few types up to automorphisms, over saturated models. The proof goes through a fine analysis of types involving a decomposition in terms of finitely satisfiable types and some *directed* part.

In this talk, I will present the main steps of the proof and some applications.

[1] Saharon Shelah, *Dependent dreams: recounting types*, paper 950 in Shelah's archive.

Abstracts of invited talks in the Special Session on Philosophy of Mathematics and Computer Science

► LEON HORSTEN, *Human effective computability*.
Department of Philosophy, University of Bristol, 43 Woodland Road, BS81UU Bristol, UK.
E-mail: Leon.Horsten@bristol.ac.uk.

Kreisel differentiated between two forms of computability: machine-effective computability and human-effective computability (Kreisel, 1972). In his view, it has been established that machine-effective computability can be analysed in terms of Turing machines, so that the Church–Turing thesis holds for machine-effective computability. Kreisel believes that it is difficult to give a precise characterization of the notion of human-effective computability because it is unclear what the right idealisations are for this notion. However, he does say that the notion of human-effective calculability is "analogous" to the notion of informal provability.

In this talk, I propose that something like Kreisel's notion of human-effective computability can be and has been formally investigated in the framework of Epistemic Arithmetic (Shapiro, 1985). In this framework, an informal provability operator ($\Box$) is added to the first-order language of arithmetic. The operator $\Box$ is intended to express the notion of a priori knowability and is governed by the laws of $S4$ modal logic. Then the proposed formalisation of the human-effective computability of a function expressed by a formula $\phi(x, y)$ in the language of Epistemic arithmetic is:

$$\Box\forall x\exists y\Box\phi(x, y).$$

This then gives rise to Church's Thesis for human-effective computability, which is in the literature known as ECT:

$$\Box\forall x\exists y\Box\phi(x, y) \to \text{“}\phi \text{ is Turing-computable”}.$$

The principle ECT has certain interesting consequences. For instance, it entails that there are absolutely undecidable propositions of low arithmetical complexity. Thus we obtain an analogue of Gödel's disjunctive thesis (either the human mathematical mind is not a Turing machine or there are absolutely undecidable propositions):

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

Either *ECT* is false, or there are absolutely undecidable propositions.

Thus it would be of philosophical interest to know whether *ECT* is true. I will argue that, unfortunately, we have at present no convincing evidence for or against the truth of *ECT*, so in this respect we are at present no better off than with Gödel's disjunction.

[1] G. Kreisel, *Which number theoretic problems can be solved in recursive progressions on Π^1_1-paths through* **O**?. ***The Journal of Symbolic Logic***, vol. 37 (1972), pp. 311–334.

[2] S. Shapiro, *Epistemic and intuitionistic arithmetic*, ***Intensional mathematics*** (S. Shapiro, editor), North-Holland, Oxford, 1985, pp. 11–46.

► GIOVANNI SAMBIN, *Two principles of dynamic constructivism*.
Department of Mathematics, University of Padova, Via Trieste 63, 35121 Padova, Italy.
E-mail: sambin@math.unipd.it.

In an evolutionary view, all of mathematics is produced by humanity through a dynamic process of abstraction [2]. Then, instead of reducing all notions to one or just a few, one needs a foundational theory which is maximal in its capacity of conceptual distinctions, and thus minimalist in its assumptions [1]. The relevance for mathematics of our minimalist foundation has been illustrated in [3, 4].

Since our foundation can keep real and ideal aspects of mathematics distinct, they should obey two principles: on one hand real aspects have to always admit a computational interpretation, and on the other hand ideal aspects have to be conservative over the real ones [5]. We will discuss the impact of these two methodological principles on philosophy of mathematics and of computing science.

[1] M. E. Maietti and G. Sambin, *Toward a miminalist foundation for constructive mathematics*, ***From sets and types to topology and analysis. Towards practicable foundations for constructive mathematics*** (L. Crosilla and P. Schuster, editors), Oxford Logic Guides, vol. 48, Clarendon Press, Oxford, 2005, pp. 91–114.

[2] G. Sambin, *Steps towards a dynamic constructivism*, ***In the scope of logic, methodology and philosophy of science. XI International Congress of Logic, Methodology and Philosophy of Science, Cracow, August 1999, vol. 1***, (P. Gärdenfors, J. Wolenski, and K. Kijania-Placek, editors), Kluwer 2002, pp. 261–284.

[3] ———, *Two applications of dynamic constructivism: Brouwer's continuity principle and choice sequences in formal topology*, ***One Hundred years of Intuitionism*** (***1907–2007***) (The Cerisy Conference), (M. van Atten, P. Boldini, M. Bourdeau, and G. Heinzmann, editors), Birkhäuser 2008, pp. 301–315.

[4] ———, *A minimalist foundation at work*, ***Logic, Mathematics, Philosophy, Vintage Enthusiasms. Essays in Honour of John L. Bell*** (D. DeVidi, M. Hallett, and P. Clark, editors), The Western Ontario Series in Philosophy of Science, vol. 75, NewYork/Dordrecht, Springer 2011, pp. 69–96.

[5] ———, *Real and ideal in constructive mathematics*, ***Epistemology versus Ontology, Essays on the Philosophy and Foundations of Mathematics in honour of Per Martin-Löf***, (P. Dybjer, S. Lindström, E. Palmgren, and G. Sundholm, editors), Logic, Epistemology and the Unity of Science 27, Springer, Berlin 2012, pp. 69–85.

► JOHAN VAN BENTHEM, *Computation as social agency: What and how?*
ILLC, Science Park 904, University of Amsterdam, 1090 GE Amsterdam, The Netherlands.
Department of Philosophy, Stanford University, Building 90, 450 Serra Mall, Stanford, CA 94305-2155, USA.
E-mail: J.vanBenthem@uva.nl.

Starting from a brief history of different models for computation, we will explore the view of computation as social interaction. We will discuss some new general topics that emerge in this perspective, including systematic "epistemization" of existing algorithms, different notions of information that may be involved, and perhaps even the computing power of conversation.

Abstracts of invited talks in the Special Session on Proof Theory

▸ ARNOLD BECKMANN, *Feasible computation on general sets*.
Department of Computer Science, Swansea University, Singleton Park, Swansea SA2 8PP, UK.
E-mail: a.beckmann@swansea.ac.uk.

Polynomial time computation on finite strings is a central notion in complexity theory. Polynomial time in more general settings has been considered by several authors. In this talk we will discuss a proposal to define feasible computation on general sets. Our approach is based on the Bellantoni–Cook scheme characterizing polynomial time on finite strings in terms of "safe recursion" [1]—we denote our class as safe recursive set functions (SRSF). We establish an exact characterization of the functions that can be computed by SRSF functions on hereditarily finite sets. Namely, using a natural interpretation of finite strings as sets, we prove that the problems decided by safe recursive set functions are exactly those computed by an alternating exponential time Turing machine with polynomially many alternations. This complexity class has been considered before and is known to exactly characterize the complexity of validity in the theory of the real numbers as an ordered additive group by Berman [2]. We also give characterizations of the safe recursive functions acting on arbitrary sets using Gödel's L-hierarchy of constructible sets and refinements of it. As a corollary, we prove that the safe recursive set functions on binary omega-sequences are identical to those defined to be computable in "polynomial time" by Schindler [3].

This is joint work with Samuel R. Buss and Sy-David Friedman.

[1] Stephen Bellantoni and Stephen Cook, *A new recursion-theoretic characterization of the polytime functions*. ***Computational Complexity***, vol. 2 (1992), no. 2, pp. 97–110.

[2] Leonard Berman, *The complexity of logical theories*. ***Theoretical Computer Science***, vol. 11 (1980), pp. 71–77.

[3] Ralf Schindler, P $\neq$ NP *for infinite time Turing machines*. ***Monatshefte für Mathematik***, vol. 139 (2003), no. 4, pp. 335–340.

▸ ELLIOTT SPOORS AND STAN WAINER, *A hierarchy of ramified theories "around" PRA*.
School of Mathematics, University of Leeds, Leeds LS2 9JT, UK.
E-mail: s.s.wainer@leeds.ac.uk.

A two-sorted arithmetic EA(I;O) of elementary recursive strength, based on the Bellantoni–Cook variable separation, is first enriched by addition of quantifiers over "input" or "normal" variables, and then extended, by ramifying higher levels of inputs, to a hierarchy of theories whose provably computable functions coincide with the levels of the Grzegorczyk hierarchy. This may be further extended by introducing numerical inputs of transfinite rank so that, for example, the Ackermann function is obtained at level ω. The methods are those of predicative proof theory, but here they appear "in miniature", controlled by the "slow-growing" bounding functions rather than the "fast-growing" ones.

▸ THOMAS STRAHM, *Unfolding schematic formal systems: From nonfinitist to feasible arithmetic*.
Institut für Informatik und angewandte Mathematik, Universität Bern, Neubrückstr. 10, CH-3012 Bern, Switzerland.
E-mail: strahm@iam.unibe.ch.
URL Address: http://www.iam.unibe.ch/~strahm.

The notion of *unfolding a schematic formal system* was introduced in Feferman [4] in order to answer the following question:

> *Given a schematic system* S*, which operations and predicates, and which principles concerning them, ought to be accepted if one has accepted* S*?*

A paradigmatic example of a schematic system S is the basic system NFA of nonfinitist arithmetic. In Feferman and Strahm [5], three unfolding systems for NFA of increasing strength have been analyzed and characterized in more familiar proof-theoretic terms; in particular, it was shown that the full unfolding of NFA, $\mathcal{U}(\mathsf{NFA})$, is proof-theoretically equivalent to predicative analysis.

More recently, the unfolding notions for a basic schematic system of finitist arithmetic, FA, and for an extension of that by a form BR of the so-called bar rule have been worked out in Feferman and Strahm [6]. It is shown that $\mathcal{U}(\mathsf{FA})$ and $\mathcal{U}(\mathsf{FA}+\mathsf{BR})$ are proof-theoretically equivalent, respectively, to primitive recursive arithmetic, PRA, and to Peano arithmetic, PA.

The most recent application of the unfolding procedure is in the context of a natural schematic system FEA for *feasible arithmetic* in Eberhard and Strahm [3]. The main results obtained are that the provably convergent operations on binary words for the operational as well as the full predicate unfolding $\mathcal{U}(\mathsf{FEA})$ are precisely those being computable in polynomial time. The upper bound computations make essential use of a specific theory of truth TPT over combinatory logic, which has recently been introduced in Eberhard and Strahm [2] and Eberhard [1] and whose proof-theoretic analysis is due to Eberhard [1].

In this talk we will survey the unfolding procedure and its application to the various arithmetical systems, with some emphasis on the unfolding of feasible arithmetic.

[1] SEBASTIAN EBERHARD, *A feasible theory of truth over combinatory logic*, preprint, 2011.

[2] SEBASTIAN EBERHARD and THOMAS STRAHM, *Weak theories of truth and explicit mathematics*. ***Festschrift for Helmut Schwichtenberg***, Ontos Verlag, Heusenstamm (to appear).

[3] ———, *Unfolding feasible arithmetic and weak truth*, submitted.

[4] SOLOMON FEFERMAN, *Gödel's program for new axioms: Why, where, how and what?*, ***Gödel '96*** (***Brno, Czech Republic***), (Petr Hájek, editor), Lecture Notes in Logic vol. 6, Springer, Berlin 1996, pp. 3–22.

[5] SOLOMON FEFERMAN and THOMAS STRAHM, *The unfolding of nonfinitist arithmetic*. ***Annals of Pure and Applied Logic***, vol. 104 (2000), no. 1–3, pp. 75–96.

[6] ———, *Unfolding finitist arithmetic*. ***The Review of Symbolic Logic***, vol. 3 (2010), no. 4, pp. 665–689.

► ALBERT VISSER, *Provability logic and the arithmetics of a theory*.
Department of Philosophy, Utrecht University, Janskerkhof 13A, 3512BL Utrecht, The Netherlands.
E-mail: albert.visser@phil.uu.nl.

We propose a particular way of viewing theories. We look at theories as a class of interpretations of a given weak arithmetical theory (like S^1_2). Consider a theory U. We view the interpretations of the given weak arithmetical theory in U as 'occurrences' of that given theory in U.

We will call an interpretation N of the given weak theory in U *an arithmetic of* U. The arithmetics of U have a natural ordering, the (definable) initial embedding ordering $\preceq$.

From the perspective of theories as containers of (possibly) lots of arithmetics, we study the provability logics of theories. We fully characterize the propositional modal principles for provability that hold in all arithmetics in a given theory U. The only assumption being a constraint on the complexity of the set of axioms of U. The comparatively easy success of this characterization contrasts with the remaining great open questions of provability logic concerning the provability logics of theories like S^1_2 or $\mathrm{I}\Delta_0+\Omega_1$ (for a fixed arithmetic given by the identical embedding interpretation).

We provide an example of a theory U where the provability logic of U is not assumed at any arithmetic N in U. The idea of the example is very simple, but to verify its correctness requires some work and some theory. We need a sharpened version of a theorem independently due to Harvey Friedman and to Jan Jan Krajíček. Very roughly, this theorem says that $\preceq$-below any arithmetic N of a finitely axiomatized sequential theory, there is an arithmetic M that is Σ_1-sound. We explain the formulation of the theorem and the methods needed to prove it.

Abstracts of invited talks in the Special Session on Set Theory

▶ DAVID ASPERÓ, *Ω-completeness and forcing axioms*.
Institute of Discrete Mathematics and Geometry, Technische Universität Wien, Wiedner Hauptstrasse 8-10/104, 1040 Wien, Austria.
E-mail: david.aspero@tuwien.ac.at.

Forcing is the most powerful method for proving independence from the usual axioms of set theory. I will talk about extensions of ZFC intended to neutralise the effects of forcing. I will focus on axioms for $H(\omega_2)$, but will most likely mention also results concerning forcing axioms for higher fragments of the universe.

[1] David Asperó and Miguel Angel Mota, *A generalization of Martin's Axiom*, in preparation, 2012.

[2] David Asperó and Ralf Schindler, *Martin's Maximum^{++} and Woodin's Axiom* $(*)$, in preparation, 2012.

▶ JAMES CUMMINGS, *Infinitary methods in finite combinatorics*.
Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, USA.
E-mail: jcumming@andrew.cmu.edu.

I will give an overview of some recent developments in the application of infinitary methods in finite combinatorics, notably the theories of *graph limits* [1] and *flag algebras* [2]. I will discuss some recent joint work with Michael Young and Ashwini Aroskar in this general area.

[1] L. Lovász and B. Szegedy, *Limits of dense graph sequences*. ***Journal of Combinatorial Theory. Series B***, vol. 96 (2006), pp. 933–957.

[2] A. Razborov, *Flag algebras*. ***The Journal of Symbolic Logic***, vol. 72 (2007), no. 4, pp. 1239–1282.

▶ MIRNA DŽAMONJA, *Forcing Axioms*.
University of East Anglia, Norwich, NR4 7TJ, United Kingdom.
E-mail: M.Dzamonja@uea.ac.uk.

We present several forcing axioms and iterations to obtain them, whose novelty is either the way they are obtained or the cardinal they apply to, or both. The work presented is part of work in progress or published with various groups of authors, as will be explained in the talk.

▶ DILIP RAGHAVAN, *Bounding, splitting, and almost disjointness*.
Graduate School of System Informatics, Kobe University, Rokko-dai 1-1, Nada, Kobe 657-8501, Japan.
E-mail: raghavan@math.toronto.edu.

A famous (still) open problem in the theory of cardinal invariants of the continuum asks whether $\mathfrak{d} = \aleph_1$ implies $\mathfrak{a} = \aleph_1$. A variation of this question is "does $\mathfrak{b} = \mathfrak{s} = \aleph_1$ already imply that $\mathfrak{a} = \aleph_1$?". We will shed some light on this question by examining when it is possible to destroy a MAD family without increasing either $\mathfrak{b}$ or $\mathfrak{s}$.

This is joint work with J. Brendle.

The talk was given by James Cummings.

Abstracts of contributed talks

▶ M. A. NAIT ABDALLAH, *On an application of Curry–Howard correspondence to quantum mechanics*.
Department of Computer Science, UWO, London, Canada; INRIA, Rocquencourt, France.
E-mail: areski@yquem.inria.fr.

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

We present an application of Curry–Howard correspondence to the formalization of physical processes in quantum mechanics. Quantum mechanics' assignment of complex amplitudes to events and its use of superposition rules are puzzling from a logical point of view.

We provide a Curry–Howard isomorphism based logical analysis of the photon interference problem [2], and develop an account in that framework. This analysis uses the logic of partial information [1] and requires, in addition to the resolution of systems of algebraic constraints over sets of λ-terms, the introduction of a new type of λ-terms, called *phase λ-terms*. The numerical interpretation of the λ-terms thus calculated matches the expected results from a quantum mechanics point of view, illustrating the adequacy of our account, and thus contributing to bridging the gap between constructive logic and quantum mechanics.

The application of this approach to a photon traversing a Mach–Zehnder interferometer [2], which is formalized by context

$$\begin{aligned}\Gamma = \{x: s,\ \langle P, \pi\rangle : s \to a^\star \to a,\ Q: s \to b^\star \to b,\\ \langle J, \pi\rangle : a \to a',\ \langle J', \pi\rangle : b \to b',\ \langle P', \pi\rangle : b' \to c^\star \to c,\ Q': b' \to d^\star \to d,\\ P'': a' \to d'^\star \to d',\ Q'': a' \to c'^\star \to c',\\ R: c \vee c' \to C,\ S: d \vee d' \to D,\\ \xi_1: a^\star,\ \xi_2: b^\star,\ \xi_1': c^\star,\ \xi_2': d^\star,\ \xi_1'': d'^\star,\ \xi_2'': c'^\star\},\end{aligned}$$

yields inhabitation claims:

$$R(\mathrm{in}_1(Q''(J(Px\xi_1))\xi_2'')) + R(\mathrm{in}_2(P'(J'(Qx\xi_2))\xi_1')): C$$

$$S(\mathrm{in}_1(P''(J(Px\xi_1))\xi_1'')) + \langle S(\mathrm{in}_2(Q'(J'(Qx\xi_2))\xi_2')), \pi\rangle : D$$

which are the symbolic counterpart of the probability amplitudes used in the standard quantum mechanics formalization of the interferometer, with constructive (respectively destructive) interference at C (resp. D).

[1] M. A. Nait Abdallah, ***The logic of partial information***, EATCS Research Monographs in Theoretical Computer Science, Springer, Berlin, 1995.

[2] P. Grangier, G. Roger, and A. Aspect, *Experimental evidence for a photon anticorrelation effect on a beam splitter: a new light on single-photon interference.* ***Europhysics Letters***, vol. 1 (1986), pp. 173–179.

► TOSHIYASU ARAI, *Proof theoretic bounds of set theories*.
Graduate School of Science, Chiba University, 1-33, Yayoi-cho, Inage-ku, Chiba, 263-8522, Japan.
E-mail: tosarai@faculty.chiba-u.jp.

I will explain how to describe bounds on provability in set theories.

► SERIKZHAN BADAEV, MANAT MUSTAFA, AND ANDREA SORBI, *A note on computable Friedberg numberings in the Ershov hierarchy*.
Department of Mechanics and Mathematics, Al-Farabi Kazakh National University, Al-Farabi avenue, 71, Almaty, 050038, Kazakhstan.
E-mail: serikzhan.badaev@kaznu.kz.
E-mail: manat.mustafa@kaznu.kz.
Dipartimento di Scienze Matematiche ed Informatiche "Roberto Magari", Università di Siena, 53100 Siena, Italy.
E-mail: sorbi@unisi.it.

Minimal numberings became a fashionable research topic in the classical theory of numberings at the end of the sixties. One of the main questions on minimal numberings, that is the problem of finding, up to equivalence of numberings, the possible number of minimal numberings, was settled by Yu. L. Ershov. A Friedberg numbering is a special but very important case of minimal numbering. The theory of minimal numberings, and in particular Friedberg numberings, has many successful applications in classical recursion theory, recursive model theory ([2], [5]). The powerful method for constructing families of c.e. sets with a finite number of Friedberg numberings, due to Goncharov [3], is the starting point for important researches on algorithmic dimension of computable models. Another application of this result was found by Kummer ([6]).

In [4], S. S. Goncharov showed that there exist classes of recursively enumerable sets admitting up to equivalence exactly one Friedberg numbering which does not induce the least element in the corresponding Rogers semilattice. Later, a simple example of such a class was found by M. Kummer: This example appears in the paper of S. A. Badaev and S. S. Goncharov ([1]). We generalize this result to all finite levels of the Ershov hierarchy, by showing that for every $n \geq 1$, there exists a Σ_n^{-1}-computable family of sets, with only one Friedberg numbering up to equivalence, that does not induce the least element in the Rogers semilattice of the family.

[1] S. A. Badaev and S. S. Goncharov, *On computable minimal enumerations*, ***Algebra. Proceedings of the Third International Conference on Algebra, Dedicated to the Memory of M. I. Kargopolov***, Krasnoyarsk, August 23–28, 1993, Walter de Gruyter, Berlin–New York, 1995, pp. 21–32.

[2] Yu. L. Ershov and S. S. Goncharov, ***Constructive models***, translated from the Russian, (English) (Siberian School of Algebra and Logic) Siberian School of Algebra and Logic, Consultants Bureau, New York, NY, xii, 293 pp. (2000) 1980.

[3] S. S. Goncharov, *Computable single-valued numerations*, ***Algebra and Logic***, vol. 19 (1980), no. 5, pp. 325–356.

[4] ———, *The family with unique univalent but not the smallest enumeration*, ***Trudy Inst. Matem. SO AN SSSR***, vol. 8 (1988), pp. 42–48, Nauka, Novosibirsk, (Russian).

[5] ———, *Problem of the number of nonself-equivalent constructivizations*, ***Algebra and Logic***, vol. 19 (1980), no. 6, pp. 401–414.

[6] M. Kummer, *Some applications of computable one-one numberings*, ***Archive for Mathematical Logic***, vol. 30 (1990), no. 4, pp. 219–230.

► YERZHAN BAISSALOV, *On linearly minimal Lie and Jordan algebras*.
Department of Mechanics and Mathematics, Eurasian National University, Astana, Munaitpasov 5, Kazakhstan.
E-mail: baisalov_er@enu.kz.

Let $\mathfrak{A} = \langle A; +, \cdot \rangle$ be an infinite algebra over field Φ and $\mathfrak{T}(\alpha)$ be its multiplication algebra [2].

The algebra $\mathfrak{A}$ is called *linearly minimal*, if each nonzero element $\mathfrak{t}$ of $\mathfrak{T}(\alpha)$ is a surjective linear transformation of $\mathfrak{A}$ (i.e., $\mathfrak{t}\colon A \to A$ is an onto map) with finite kernel. The property of linear minimality is much weaker than one of definable minimality when we demand each definable (by a formula of first-order logic) set of the algebra to be finite or cofinite.

The notion of linear minimality can be also defined for rings. It is shown in [1] that the classes of linearly minimal rings and algebras coincide, and there are two main possibilities for a (nontrivial) linearly minimal algebra $\mathfrak{A}$: either $\mathfrak{A}$ is field or $\mathfrak{A}$ is an infinite-dimensional central algebra over finite field Φ. Note also that when the linearly minimal algebra $\mathfrak{A}$ has trivial multiplication (i.e., $a \cdot b = 0$ for all $a, b \in A$) the study of the algebra is reduced to the study of the definably minimal Abelian group $\langle A; + \rangle$ which is done, for example, in [4].

Theorem 1. *Any nontrivial linearly minimal Lie algebra has an infinite locally finite subalgebra, which is linearly minimal too.*

Theorem 2. (1) *Any nontrivial linearly minimal Jordan algebra over field Φ with char$\Phi \neq 2$ is unital.*

(2) *Any linearly minimal unital Jordan algebra is a division algebra. If it is not associative then any element generates a finite subalgebra.*

Remark. When *char*$\Phi = 2$ the notion of linear minimality can be easily adapted for quadratic Jordan algebras [3].

Conjecture. Any nonassociative linearly minimal Jordan algebra has an infinite locally finite subalgebra, which is linearly minimal too.

The following results (obtained jointly with R. Bibazarov, B. Duzban, A. Syzdykova, and B. Tuktibaeva) are the first steps toward the classification of linearly minimal algebras: any nontrivial linearly minimal alternative algebra is a field, and so is any nontrivial linearly minimal Novikov algebra.

I think that the situation is not so easy and simple in the case of linearly minimal Lie or Jordan algebras.

[1] Yerzhan Baisalov, *On linearly minimal rings and algebras*. ***Algebra i Logika***, vol. 51, (2012) no. 2, pp. 159–167 (in Russian).

[2] Nathan Jacobson, ***Lie algebras***, Dover Books on Mathematics, Dover, 1979.

[3] Kevin McCrimmon, ***A taste of Jordan algebras***, Universitext, Springer, Berlin, 2004.

[4] Bruno Poizat, ***Groups stables***, Nur Al-Mantiq Wal-Ma'rifah, Launey, 1987.

▸ CAROLINA BLASIO AND JOÃO MARCOS, *Logics for discussion, and for agreement*.
IFCH / UNICAMP, Campinas–SP, Brazil.
E-mail: `carolblasio@gmail.com`.
DIMAp / UFRN, Campus Universitário, Natal–RN, Brazil.
E-mail: `jmarcos@dimap.ufrn.br`.

For a given society of reasoning agents, we will entertain situations in which they are consulted upon their opinion about the informational content of logical expressions. In the simplest case, the agents are mere sources of unstructured sentences that may be used to *assert* or to *deny* certain facts. However, invoking a judgmental attitude on the part of the agent, we will assume instead that such sentences represent information that is either *accepted* or *rejected* by the agent. For a source immersed in a classic-like environment, for instance, acceptance may be taken as dual to rejection and these may be reduced to checking satisfiability of an atom by a given assignment.

When collecting and processing the opinions of given agents, one may adopt several different strategies in defining the underlying logic of their society. A *bold* strategy, for instance, would be one in which a given sentence is accepted by the society when at least one of the involved agents sees reason to accept it. The idea of processing the information received from agents involved in a discussion appears, e.g., in some of the oldest papers on paraconsistent logic: inconsistent opinions should be somehow accommodated when boldly collected. In the present contribution we shall show that the correct way of dualizing the latter approach in terms of a *cautious* collecting strategy would be one in which a given sentence is accepted when none of the involved agents sees reason to reject it. This will allow us to smoothly accommodate undeterminedness phenomena, typical of paracomplete logics. For some interesting illustrations we will concentrate on cases in which agents are classic-like and sentences are structured. As we shall prove, the natural broadly truth-functional semantics behind such approaches have nondeterministic features, yet is computationally well-behaved.

▸ QUENTIN BROUETTE, *A nullstellensatz and a positivstellensatz for ordered differential fields*.
Université de Mons, 20 Place du Parc, 7000 Mons, Belgique.
E-mail: `quentin.brouette@gmail.com`.

We consider ordered differential fields endowed with m commuting derivations $\delta_1, \dots, \delta_m$. Their theory has a model completion called m-CODF. An axiomatization of m-CODF was given by M. Singer [3] (in case $m = 1$) and later by M. Tressl [4] and C. Rivière [2] (in the general case).

We define the differential real radical of a differential ideal I (denoted by $\mathcal{R}^{\omega}(I)$ below) and note that it is the smallest differential real ideal containing I.

Throughout K is a model of m-CODF.

We first obtain the analogue in this context of Dubois' nullstellensatz for real closed fields (see [1]).

Theorem 1 (Nullstellensatz). *Let I be a differential ideal of $K\{X_1, \dots, X_n\}$,*

$$\mathcal{I}(\mathcal{V}(I)) = \mathcal{R}^{\omega}(I).$$

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

For any differential polynomial f, let f^* be the ordinary polynomial obtained by substituting for each $\delta_1^{e_1}\dots\delta_m^{e_m}X_i$ a new variable Y_k.

Let $g_1,\dots,g_s \in K\{X_1,\dots X_n\}$, $W := \{\bar{x} \in K^n : g_1(\bar{x}) \geq 0,\dots,g_s(\bar{x}) \geq 0\}$, and $W^* := \{\bar{x} \in K^d : g_1^*(\bar{x}) \geq 0,\dots,g_s^*(\bar{x}) \geq 0\}$ where d is such that for all $i = 1,\dots,s$, $g_i^* \in K[Y_1,\dots,Y_d]$.

Using a result of density of differential tuples, we obtain a differential version of Stengle's positivstellensatz (see [1]).

THEOREM 2 (Positivstellensatz). *Let $f \in K\{X_1,\dots,X_n\}$ and P be the cone of $K\{X_1,\dots,X_n\}$ generated by $g_1,\dots,g_s$.*

Suppose that there exists an open set O such that $O \subset W^* \subset \mathrm{cl}(O)$.

$$\forall \bar{x} \in W, f(\bar{x}) \geq 0 \Leftrightarrow \exists m \in \mathbb{N}, g, h \in P, \forall \bar{x} \in W : f(\bar{x}).g(\bar{x}) = f^{2m}(\bar{x}) + h(\bar{x}).$$

[1] J. BOCHNAK, M. COSTE, and M.-F. ROY, ***Géométrie algébrique réelle***, Springer-Verlag, Berlin 1987.

[2] C. RIVIÈRE, *The theory of closed ordered differential fields with m commuting derivations*. ***Comptes Rendus de l'Académie des Sciences Paris***, ser. I, vol. 343 (2006), pp. 151–154.

[3] M. SINGER, *The model theory of ordered differential fields*. ***The Journal of Symbolic Logic***, vol. 43 (1978), no. 1, pp. 82–91.

[4] M. TRESSL, *The uniform companion for large differential fields of characteristic zero*. ***Transactions of the American Mathematical Society***, vol. 357 (2005), pp. 3933–3951.

► JAN DOBROWOLSKI, *New examples of small Polish structures*.
Instytut Matematyczny, Uniwersytet Wrocławski, Plac Grunwaldzki 2/4, 50-384 Wrocław, Poland.
E-mail: dobrowol@math.uni.wroc.pl.

We answer some questions from [1] by giving suitable examples of small Polish structures. First, we present a class of small Polish group structures without generic elements. Next, we give a first example of a small nonzero-dimensional Polish G-group.

[1] KRZYSZTOF KRUPIŃSKI, *Some model theory of Polish structures*. ***Transactions of the American Mathematical Society***, vol. 362 (2010), no. 7, pp. 3499–3533.

► PHILIP EHRLICH, *The surreal number tree*.
Department of Philosophy, Ohio University, Athens, OH 45701, USA.
E-mail: ehrlich@ohio.edu.

In his monograph *On Numbers and Games* [1], J. H. Conway introduced a real-closed field No, that is so remarkably inclusive that, subject to the proviso that numbers—construed here as members of ordered fields—be individually definable in terms of sets of NBG, it may be said to contain 'All Numbers Great and Small.' In addition to its inclusive structure as an ordered field, No has a rich algebraico-binary tree-theoretic structure, or simplicity hierarchy, that emerges from the recursive clauses in terms of which it is defined.

Among the striking simplicity-hierarchical features of No is that every surreal number can be assigned a canonical "proper name"—called its *Conway name* (or *normal form*)—that is a reflection of its characteristic simplicity-hierarchical properties.

In [2], answers are provided for the following two questions that are motivated by No's structure as an ordered binary tree: (i) Given the Conway name of a surreal number, what are the Conway names of its two immediate successors? (ii) Given a chain of surreal numbers of infinite limit length, what is the Conway name of the immediate successor of the chain?

The purpose of this talk is to provide an introduction to [2].

[1] J. H. CONWAY, ***On numbers and games***, Academic Press, Waltham, MA, 1976.

[2] PHILIP EHRLICH, *Conway names, the simplicity hierarchy and the surreal number tree*. ***The Journal of Logic and Analysis***, vol. 3 (2011), no. 1, pp. 1–26.

► THOMAS MACAULAY FERGUSON, *Ramsey's footnote and Priest's connexive logics*.
Philosophy Department, Brooklyn College, 2900 Bedford Avenue, Brooklyn, NY 11210, USA.
E-mail: tferguson@gc.cuny.edu.

The family of logics known as *connexive logics* are characterized by two theses. Using the notation of [1], these are

1: *Aristotle's Thesis* (AT) $\sim(\varphi >\sim \varphi)$, and
2: *Boethius' Thesis* (BT) $\sim((\varphi > \psi) \wedge (\varphi >\sim \psi))$.

Connexive logics are distinct in that, in verifying AT and BT, they have properly superclassical theorems. Another feature of these theses is that they have historically been appealed to often in philosophical literature, albeit subtly.

One such instance is in Frank Ramsey's footnote in [3]:

> If two people are arguing "If p will q?" and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; so that in a sense "If p, q" and "If p, $\bar{q}$" are contradictories.

The assertion that $p > q$ and $p >\sim q$ are *contradictories* yields two consequences, BT itself, and

3: *Conditional Excluded Middle* (CEM) $(\varphi > \psi) \vee (\varphi >\sim \psi)$

Logics have been generated accounting for CEM in the spirit of Ramsey such as Stalnaker's C2 (see [1]); what has *not* been noted is that BT is a consequence of the footnote. Relaxing the term "contradictories" to "contraries," Ramsey's assertion is equivalent to BT.

By following this weakening along the lines of [1], one arrives at a family of connexive logics, including Graham Priest's connexive logics described in [2]. As this semantics is developed, an alternative semantics in the spirit of Ramsey's footnote for connexive logics emerges.

[1] Donald Nute, ***Topics in conditional logic***, Philosophical Studies Series in Philosophy, D. Reidel Publishing Company, 1980.

[2] Graham Priest, *Negation as cancellation, and connexive logic*. ***Topoi***, vol. 18, no. 2, pp. 141–148.

[3] Frank Ramsey, *General propositions and causality*, ***Philosophical papers*** (D. H. Mellor, editor), Cambridge University Press, New York, 1990, pp. 145–163.

► MICHÈLE FRIEND, *Genetic proofs, reductions and rational reconstruction proofs*.
Philosophy, George Washington University, 801 22nd Street NW, Washington, DC, USA.
E-mail: Michele@gwu.edu.

Some theorems in mathematics have several proofs. Why? It is clear that proofs are not meant to only convince us of the truth of the theorem being proved. Rather, they give us explanations. I explore the philosophical implications of three conceptions of proof: the genetic conception, reductions, and the rational reconstruction conception.

The genetic conception traces a theorem back to the setting in which the mathematician thought of the proof and the theorem. We learn the historical origin of the theorem. The reduction traces the theorem back to some more primitive conceptions. The primitive conceptions are associated with a foundational project, such as constructivism, realism, or logicism. With such a proof, we learn the conceptual justification for the theorem. The 'rational reconstruction' conception of proof is one where we demonstrate that it is possible to understand a theorem in a novel setting, for example, we might give a proof in Topos theory of a theorem in geometry. We learn the spread of the theorem: in what other theories it is provable. The lessons become more interesting when the novel setting is inconsistent with the original setting.

As an example, I shall focus on Lobachevsky's solution to the problem of indefinite integrals. I shall compare Lobachevsky, Beltrami, and Rodin's constructions and re-constructions, and offer one of my own, pointing out the lessons in each case. Lobachevsky gives the genetic proof, Beltrami reduces the proof to Euclidean geometry, which was thought to be more obvious or primitive. Rodin reconstructs the proof in topos theory to give a neutral proof, which is closer to Lobachevsky's purpose. I give a reconstruction using techniques developed in the paraconsistency literature, in order to justify using what might look like contradictory methods. Each type of proof teaches us different lessons.

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

▸ JEROEN GOUDSMIT, *On the admissible rules of Gabbay–de Jongh logics*.
Department of Philosophy, Utrecht University, Janskerkhof 13a, The Netherlands.
E-mail: jeroen.goudsmit@phil.uu.nl.
URL Address: http://jeroengoudsmit.com.

The admissible rules of a logic are those rules that can be added without affecting provability. Not all admissible rules of propositional intuitionistic logic (IPC) are derivable, a property shared with numerous modal and intermediate logics. Dick de Jongh and Albert Visser conjectured that the now well-known *Visser rules* characterize admissibility of IPC, in that these rules are sufficient and necessary to derive all admissible rules of IPC. Rozière [3] and Iemhoff [2] independently proved this.

We posit the de Jongh rules, an ostensible generalization of the Visser rules. Much like the Visser rules, they can be stratified along the natural numbers. We study the strata separately, and for logics that admit the disjunction property, we can tie admissibility of these rules to a stratified version of the extension property. Furthermore, these rules characterize admissibility for the Gabbay–de Jongh logics [1].

Many of our results hold in arbitrary intermediate logics. Internally, we mostly work with sets of formulae, shying away from semantics whenever sensible. This work might smoothen some other proofs in the literature. We employ these techniques to prove that a rule is admissible for the n^{th} Gabbay–de Jongh logic if and only if it can be derived in the same logic enriched with the de Jongh rules up to the $(n+1)^{\text{th}}$ stratum. We also prove that the Gabbay–de Jongh logics have finitary unification type. This is joint work with Rosalie Iemhoff.

[1] Dov M. Gabbay and Dick H. J. de Jongh, *A sequence of decidable finitely axiomatizable intermediate logics with the disjunction property*. ***The Journal of Symbolic Logic***, vol. 39 (1974), no. 1, pp. 67–78.

[2] Rosalie Iemhoff, *On the admissible rules of intuitionistic propositional logic*. ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 1, pp. 281–294.

[3] Paul Rozière, ***Règles admissibles en calcul propositionnel intuitionniste***, Université de Paris, 1992.

▸ CHRISTOPHER HAMPSON AND AGI KURUCZ, *The modal logic of 'elsewhere' as a component in product logics*.
Department of Informatics, King's College London, Strand, London, WC2R 2LS, UK.
E-mail: christopher.hampson@kcl.ac.uk.
E-mail: agi.kurucz@kcl.ac.uk.

The finitely axiomatizable and decidable modal logic **Diff** of *elsewhere* (or *difference operator*) is known to be Kripke complete with respect to the class of symmetric, pseudo-transitive frames. These frames closely resemble **S5**-relations (i.e., equivalence relations) and it is little surprise that the validity problems for **Diff** and **S5** have the same co-NP complexity, and both logics enjoy the finite model property.

Here we turn our attention to decision and axiomatization problems of two-dimensional product logics $L_1 \times L_2$, by which we mean the multimodal logic of all product frames where the first component is a frame for L_1 and the second a frame for L_2. It is well-known that product logics of the form $L \times$ **S5** are usually decidable, whenever L is a decidable (multi)modal logic. We even have that **S5** $\times$ **S5** enjoys the exponential finite model property. Here we present some cases where the transition from $L \times$ **S5** to $L \times$ **Diff** not only increases the complexity of the validity problem, but in fact introduces undecidability and the lack of finite model property. We also show that no modal product logic of the form $L \times$ **Diff** is finitely axiomatizable, whenever L is between **K** and **S5**.

[1] D. M. Gabbay, A. Kurucz, F. Wolter, and M. Zakharyaschev, ***Many-dimensional modal logics: Theory and applications***, Studies in Logic, Elsevier, Amsterdam 2003.

▸ DANIEL HOFFMANN, *Multiplicatively iterative higher derivations*.
Instytut Matematyczny, Uniwersytet Wrocławski, Plac Grunwaldzki 2/4, 50-384 Wrocław, Poland.
E-mail: daniel.max.hoffmann@gmail.com.

For fields of characteristic $p > 0$, we will show that every derivation D such that $D^{(p)} = D$ expands to a multiplicatively iterative Hasse–Schmidt derivation. It is known in the case of the standard (i.e., additive) iterativity condition: see Theorem 27.4. in [2], where the author expands a derivation D such that $D^{(p)} = 0$ to an iterative Hasse–Schmidt derivation. Afterward we will focus on a geometric axiomatization of existentially closed Hasse–Schmidt fields with one multiplicatively iterative derivation as was done in [1] for the standard iterativity condition.

[1] Piotr Kowalski, *Geometric axioms for existentially closed Hasse fields*. ***Annals of Pure and Applied Logic***, vol. 135 (2005), no. 1–3, pp. 286–302.

[2] Hideyuki Matsumura, ***Commutative ring theory***, Cambridge Studies in Advanced Mathematics, Cambridge University Press, Cambridge, UK 1989.

▸ TOMÁŠ HOLEČEK, *On a rule of propositional function instantiation*.
Department of Philosophy, Faculty of Humanities, Charles University, U kříže 8, 15800, Praha 5, Czech Republic.
E-mail: holecek@ojrech.cz.

The proofs in the first edition of Principia Mathematica [3] include asserted instances of previously asserted propositional functions (pfs), a process somewhat similar to the use of axiom-schemata. However, it was not possible to explicitly state a general rule for this essential step, i.e., something like "If a pf φ is asserted, we can assert any pf which is an instance of φ." Two reasons made it impossible: inadmissibility of inductive definitions based on syntax of pfs and reluctance to express by general rule what we always need to apply as a particular. In reconstruction of this classical account on type theory [1, 2], we can easily define the instantiation by induction and explicate the rule in meta-language. In this talk, we will discuss the significance that the original reluctance has.

[1] Fairouz Kamareddine, Twan Laan, and Rob Nederpelt, *Types in logic and mathematics before 1940*. this Journal, vol. 8 (2002), no. 2, pp. 185–245.

[2] Twan Laan, *A formalization of the ramified type theory*. ***Computing Science Report***, 94/33, Eindhoven University of Technology, Eindhoven, 1994.

[3] Alfred North Whitehead and Bertrand Russell, ***Principia Mathematica***, vol. I, II, III, Cambridge University Press, 1910, 1912, 1913.

▸ KRZYSZTOF KAPULKIN, *Fibration categories and type theory*.
Department of Mathematics, University of Pittsburgh, 139 University Place, Pittsburgh, PA 15260, USA.
E-mail: krk56@pitt.edu.

The connections between Martin-Löf Type Theory and homotopy theory are now very intensively studied (see e.g., [1, 3]), especially in the context of Vladimir Voevodsky's 'Univalence Foundations' program. We propose the framework of fibration categories (cf. [2]) for a systematic development of these connections.

We start by verifying that the classifying category of MLTT has a natural structure of a fibration category. Further, we formalize within type theory several notions and theorems about fibrations categories such as right properness and the factorization lemma. Our special interests are in the study of the loop functor Ω, spectra of types, and the homotopy limits. In particular, we internalize in type theory the construction of homotopy limits for model categories. All the formalization is done in the Coq proof assistant.

This is joint work with Jeremy Avigad (Carnegie Mellon University).

[1] Steve Awodey and Michael A. Warren, *Homotopy theoretic models of identity types*. ***Mathematical Proceedings of the Cambridge Philosophical Society***, vol. 45 (2009), no. 146, pp. 45–55.

[2] Kenneth Brown, *Abstract homotopy theory and generalized sheaf cohomology*. ***Transactions of the American Mathematical Society***, vol. 186 (1973), pp. 419–458.

[3] Nicola Gambino and Richard Garner, *The identity type weak factorisation system*. ***Theoretical Computer Science***, vol. 409 (2008), no. 1, pp. 94–109.

► LAURENCE KIRBY, *Ordinal exponentiations of sets*.
Department of Mathematics, Baruch College, City University of New York, 1 Bernard Baruch Way, New York, NY 10010, USA.
E-mail: laurence.kirby@baruch.cuny.edu.

In the 1950s Tarski generalized to all sets the addition operation on the von Neumann ordinals ([2]; see also [1]). Scott followed with definitions of multiplication and exponentiation. The "high school algebra" laws of exponentiation fail in the generalized von Neumann arithmetic. The situation can be remedied by replacing the usual ordinal arithmetic of sets with one based on the finite Zermelo ordinals, in which the successor of n is $\{n\}$. In the Zermelo arithmetic the "high school algebra" exponentiation laws hold.

Each of the two arithmetics of sets has advantages. The von Neumann arithmetic's elegance, flexibility, and straightforward extension to the infinite arise largely from the fact that the order on the ordinals is the restriction of the membership relation. The Zermelo arithmetic, as well as the advantage mentioned above, is more economical.

[1] LAURENCE KIRBY, *Addition and multiplication of sets*. ***Mathematical Logic Quarterly***, vol. 53 (2007), no. 1, pp. 52–65.

[2] ALFRED TARSKI, *The notion of rank in axiomatic set theory and some of its applications*. ***Bulletin of the American Mathematical Society***, vol. 61 (1955), p. 443, reprinted in ***Alfred Tarski, Collected papers***, vol. 3, (Steven R. Givant and Ralph N. McKenzie, editors), Birkhäuser, Basel and Boston, 1986, p. 622.

► ALEXANDER P. KREUZER, *Nonprincipal ultrafilters, program extraction and higher order reverse mathematics*.
Technische Universität Darmstadt, Faculty of Mathematics, AG Logik, Schlossgartenstrasse 7, 64289 Darmstadt, Germany.
E-mail: akreuzer@mathematik.tu-darmstadt.de.
URL Address: http://www.mathematik.tu-darmstadt.de/~akreuzer.

We investigate the strength of the existence of a nonprincipal ultrafilter over fragments of higher order arithmetic. Let $(\mathcal{U})$ be the statement that a nonprincipal ultrafilter on $\mathbb{N}$ exists and let ACA_0^ω be the higher order extension of ACA_0. We show that $\mathsf{ACA}_0^\omega + (\mathcal{U})$ is Π^1_2-conservative over ACA_0^ω and thus that $\mathsf{ACA}_0^\omega + (\mathcal{U})$ is conservative over PA.

Moreover, we provide a program extraction method and show that from a proof of a strictly Π^1_2 statement $\forall f\, \exists g\, \mathsf{A}_{qf}(f, g)$ in $\mathsf{ACA}_0^\omega + (\mathcal{U})$ a realizing term in Gödel's system T can be extracted. This means that one can extract a term $t \in T$, such that $\forall f\, \mathsf{A}_{qf}(f, t(f))$.

[1] ALEXANDER P. KREUZER, *Nonprincipal ultrafilters, program extraction and higher order reverse mathematics*. ***Journal of Mathematical Logic***, vol. 12 (2012), no. 1.

► KANAT KUDAIBERGENOV, *Generalizations of o-minimality to partial orders*.
KIMEP University, 4 Abay Avenue, Almaty 050010, Kazakhstan.
E-mail: kanat@kimep.kz.

For linearly ordered structures one has an important notion of o-minimality and several its generalizations. In this talk I will discuss some generalizations of o-minimality to partial orders.

► SORI LEE, *Sight realizability: the arithmetic in subtoposes of the effective topos*.
DPMMS, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WB, United Kingdom.
E-mail: S.Lee@dpmms.cam.ac.uk.

The internal (first-order) arithmetic of the effective topos is Kleene's realizability. This talk presents a realizability-like description for the arithmetic in *subtoposes* of the effective topos.

The effective topos has as its least (nondegenerate) subtopos the category of sets, whose internal arithmetic is of course the true arithmetic. Also well-known is the fact that the (opposite) semilattice of Turing degrees embed into the lattice of subtoposes of the effective topos, manifesting the vast size of the latter structure. The work [1] establishes an infinite family of new examples, with the technique behind being to understand entities that represent subtoposes in terms of a certain kind of well-founded trees called *sights*.

As a by-product of this, we obtain our "realizability" semantics for the arithmetic in subtoposes. If θ is a subtopos, we define a relation 'θ-realizes' between numbers and arithmetic sentences in the same inductive way as the original realizability, with only changes in the implication and universal quantifier clauses. For example, the implication clause has the following look.

> n '*θ-realizes*' $\phi \Rightarrow \psi$ if for each θ-realizer m of ϕ there is a "$(\varphi_n(m), \theta)$-dedicated" sight S such that each "$\varphi_n(m)$-value of" S does θ-realizes ψ.

The subtopos θ is secretly just a sequence of collections of natural number sets, and each notion appearing above ('sight', 'dedicated', etc.) is free of topos theory. This leaves us with plenty of models of Heyting arithmetic described in elementary terms.

In the talk we introduce and discuss this 'sight realizability'.

[1] Sori Lee, ***Subtoposes of the effective topos***, Master's Thesis, Utrecht University, `arXiv:1112.5325`, 2011.

[2] Sori Lee and Jaap van Oosten, *Basic subtoposes of the effective topos*, `arXiv:1201.2571`, 2012.

▸ LAURENŢIU LEUŞTEAN, *An application of proof mining in nonlinear analysis*.
Simion Stoilow Institute of Mathematics of the Romanian Academy, 21 Calea Griviţei, 010702, Bucharest, Romania.
E-mail: `laurentiu.leustean@imar.ro`.

Proof mining is an area of applied proof theory concerned with the extraction of hidden finitary and combinatorial content from proofs that make use of highly infinitary principles. This line of research, developed by Ulrich Kohlenbach in the 90's, has its roots in Georg Kreisel's program on *unwinding of proofs*, initiated in the 50's. A comprehensive reference for proof mining is Kohlenbach's book [3].

We present an application of proof mining to the asymptotic behavior of firmly nonexpansive mappings, a class of functions which play an important role in metric fixed point theory and optimization due to their correspondence with maximal monotone operators.

We obtain effective and highly uniform rates of asymptotic regularity for the Picard iterations of firmly nonexpansive mappings in uniformly convex W-hyperbolic spaces, a class of geodesic spaces that generalize both CAT(0) spaces and uniformly convex Banach spaces. In the case of CAT(0) spaces, the rate of asymptotic regularity is quadratic. These results, contained in a joint paper with D. Ariza-Ruiz and G. Lopez-Acedo [1], are new even for uniformly convex Banach spaces. Furthermore, they are guaranteed by general logical metatheorems proved by P. Gerhardy and U. Kohlenbach [2] for different classes of metric and normed spaces and adapted in [4] to uniformly convex W-hyperbolic spaces.

[1] D. Ariza-Ruiz, L. Leuştean, and G. Lopez-Acedo, *Firmly nonexpansive mappings in classes of geodesic spaces*, `arXiv:1203.1432v1 [math.FA]`, 2012.

[2] P. Gerhardy and U. Kohlenbach, *General logical metatheorems for functional analysis*. ***Transactions of the American Mathematical Society***, vol. 360 (2008), pp. 2615–2660.

[3] U. Kohlenbach, ***Applied proof theory: Proof interpretations and their use in mathematics***, Springer Monographs in Mathematics, Springer-Verlag, 2008.

[4] L. Leuştean, ***Proof mining in $\mathbb{R}$-trees and hyperbolic spaces***, Electronic Notes in Theoretical Computer Science, vol. 165, 2006, pp. 95–106.

▸ DAVID MILLER, *Probabilistic generalizations of deducibility*.
University of Warwick, Coventry CV4 7AL, UK.
E-mail: `dwmiller57@yahoo.com`.

The guiding idea of the *logical interpretation of probability* (von Kries, Waismann, Popper, Kneale, and others) is that the upper limit of probability is certainty or logical necessity. The relation $p(c \mid a) = 1$ (or, more accurately, $\forall b\, p(c \mid ab) = 1$) represents a *logically necessary connexion* between the sentences a and c, that is, the *deducibility* of c from a, while lesser values of p indicate that, whatever connexion there may be between a and c, it falls short of necessity. It is generally understood, in addition, that p is a measure on the ranges of its arguments, the sets of possibilities that they admit; whence the conditional probability

$p(c \mid a)$ measures the proportion (in an appropriately generalized sense) of those possibilities admitted by a that are admitted also by c, and takes its maximum value, namely 1, when they all are. As anticipated, $p(c \mid a) = 1$ when c is deducible from a. At the other extreme $p(c \mid a) = 0$ if the range of a excludes every possibility that is admitted by c, that is, if $\neg c$ is deducible from a. The function p, so construed, satisfies all the usual (finitary) axioms for probability, such as those of Kolmogorov, or those of the more general system given in [1], appendix $*$v.

Less often considered are several other functions that provide alternative, and perhaps more illuminating, generalizations of deducibility. Since c is deducible from a if & only if $\neg a$ is deducible from $\neg c$, for example, and also if & only if c is deducible from $a \vee c$, the functions $q(c \mid a) = p(\neg a \mid \neg c)$ and $d(c \mid a) = p(c \mid a \vee c)$, which are not in general equal to $p(c \mid a)$, also suggest necessary conditions for deducibility. Deducibility itself may be defined by $\forall b\, q(b \to c \mid a) = 1$ and by $\forall b\, d(cb \mid ab) = 1$. The full range of possibilities is explored, and some philosophical consequences are drawn.

[1] Karl R. Popper, ***The logic of scientific discovery***, Hutchinson, London, 1959.

► TAKAKO NEMOTO, *The proof theoretic strengths of determinacy between Σ^0_1 and Δ^0_2*.
School of Information Science, Japan Advanced Institute of Science and Technology, 1-1 Asahidai, Nomi, Ishikawa, 923-1292, Japan.
E-mail: nemototakako@gmail.com.
URL Address: http://iam.unibe.ch/~nemoto/.

[3] proved the equivalences between $\Sigma^0_1 \wedge \Pi^0_1$ determinacy and Π^1_1 comprehension, and between Δ^0_2 determinacy and Π^1_1 transfinite recursion over RCA_0. The idea of the latter proof is as follows:

1. Find a well-ordering W such that a given Δ^0_2 game can be represented as a union of $(\Sigma^0_1 \wedge \Pi^0_1)$ games along W.
2. Iterate the proof of the former equivalence.

Then, by restricting the well-ordering in 1, we can define many subclasses of Δ^0_2 games. In this talk, we show the equivalences between the determinacy of such classes and schemata of restricted transfinite recursion. We also consider the proof theoretic strengths of them by constructing β-models.

[1] Takako Nemoto, *Determinacy of Wadge classes and subsystems of second order arithmetic*. ***Mathematical Logic Quarterly***, vol. 55 (2009), pp. 154–176.

[2] S. G. Simpson, ***Subsystems of second order arithmetic***, Springer, Berlin, 1999.

[3] K. Tanaka, *Weak axioms of determinacy and subsystems of analysis I: Δ^0_2-games*. ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 36 (1990), pp. 481–491.

► VLADISLAV NENCHEV, *Dynamic relational mereotopology: First-order and modal logics for stable and unstable relations*.
Department of Mathematical Logic, Faculty of Mathematics and Informatics, Sofia University, 1164 Sofia, 5 James Bourchier Boulevard, Bulgaria.
E-mail: lucifer.dev.0@gmail.com.

This paper presents first-order and modal logics of stable and unstable versions of four mereotopological relations: *part-of*, *overlap*, *underlap*, and *contact* (denoted $\leq$, O, U, and C). These relations are formally defined with Contact algebras $(\underline{B}, \mathsf{C})$, where $\underline{B}$ is a Boolean algebra and C is the contact relation (see [3], [2]). Standard models for Contact algebras are the regular closed sets in a topological space and the topological contact. The other relations are defined: $\leq$ is the Boolean ordering, O corresponds to nonempty intersection, and U is the dual of O. *Mereotopological structures* are relational structures with the four relations $\leq$, O, U, and C (see [2]). Their stable and unstable counterparts are defined over Cartesian products of mereotopological structures. Let I be a set of moments of time and $(W_i, \leq_i, \mathsf{O}_i, \mathsf{U}_i)$ be a mereotopological structure for every $i \in I$. Then the stable and unstable contact is defined $x\ \mathsf{C}^\forall\ y$ iff $(\forall i \in I)(x_i\ \mathsf{C}_i\ y_i)$, $x\ \mathsf{C}^\exists\ y$ iff $(\exists i \in I)(x_i\ \mathsf{C}_i\ y_i)$. $\leq^\forall$, $\leq^\exists$, $\mathsf{O}^\forall$, $\mathsf{O}^\exists$, $\mathsf{U}^\forall$, and $\mathsf{U}^\exists$ are defined similarly in [1].

The current system is an extension of the one presented in [1]. Both systems are relational variants of the dynamic mereotopology from [3]. These works are developments in the area of alternative theories of space and time, started by Alfred Whitehead. They combine relations from mereotopology with simple temporal properties like *stability* and *unstability*. Whitehead used mereotopology as a base to build a new point-free theory of space. The fact that we combine spacial and temporal properties in one, rather than using different operators for space and time, corresponds to Whitehead's idea that the theory of time cannot be separated from the theory of space (see [4]).

The paper continues with axiomatization of the new system. The completeness is proved via a generalization of the Stone-like representation techniques for distributive lattices. We prove completeness for the quantifier-free fragment of the corresponding first-order logic, its decidability and show that its satisfiability problem is in NP. The full first-order logic is hereditary undecidable. We use the relational structures as semantic base of a polymodal logic for which we provide a complete axiomatization.

[1] VLADISLAV NENCHEV, *Logics for stable and unstable mereological relations*. ***Central European Journal of Mathematics***, vol. 9 (2011), no. 6, pp. 1354–1379.

[2] YAVOR NENOV and DIMITER VAKARELOV, *Modal logics for mereotopological relations*. ***Advances in Modal Logic***, vol. 7, College Publications, Joplin, MO, 2008, pp. 249–272.

[3] DIMITER VAKARELOV, *Dynamic mereotopology: A point-free theory of changing regions. I. Stable and unstable mereotopological relations*. ***Fundamenta Informaticae***, vol. 100 (2010), no. 1–4, pp. 159–180.

[4] ALFRED N. WHITEHEAD, ***Process and reality***, MacMillan, New York, 1929.

▸ JOSEPH W. NORMAN, *Hot buttered conditionals, tangled up in grue: Goodman's riddles solved by parametric probability analysis*.

Department of Internal Medicine, University of Michigan, 1500 East Medical Center Drive, Ann Arbor, MI 48109, USA.
E-mail: `jwnorman@umich.edu`.

Conditional statements, whether factual or counterfactual, make perfect sense as constraints on probabilities. Using probability networks, the computed results of probability queries are quotients of sums of products of input probabilities. With inputs $\mathrm{Pr}_0(H) = x$ and $\mathrm{Pr}_0(M \mid H) = y$ stating that the butter was heated with probability x and that the conditional probability of melting given heating is y, the query $\mathrm{Pr}(M \mid H)$ yields the result xy/x: the product $\mathrm{Pr}_0(H) \times \mathrm{Pr}_0(M \mid H)$ divided by $\mathrm{Pr}_0(H)$.

Interpreting the sentence "If that piece of butter had been heated it would have melted" as the constraint $y = 1$ and "If that piece of butter had been heated it would *not* have melted" as $y = 0$ produces the desired semantics: these constraints are clearly inconsistent; and neither constraint on y affects $\mathrm{Pr}(H)$, which is x. Note that if $x > 0$ then the output $\mathrm{Pr}(M \mid H)$ simplifies to the input y; however if $x = 0$ then $\mathrm{Pr}(M \mid H)$ yields the indeterminate form $0/0$ regardless of y. Algebra says what we mean.

Concerning "grue" we model explicitly the ambiguous correlation between the basic green/blue proposition, the composite grue/bleen proposition, and time. Parametric probability analysis demonstrates that it is always correct to make complementary predictions about future green and future grue, regardless of how this ambiguity is resolved.

[1] NELSON GOODMAN, ***Fact, fiction, and forecast***, 4th edition, Harvard, Cambridge, MA, 1983.

[2] JOSEPH W. NORMAN, *The logic of parametric probability*, preprint at `arXiv: 1201.3142v2 [math.LO]`, January, 2012.

▸ FLORIAN PELUPESSY, *Adjacent Ramsey and unprovability*.

Department of Mathematics, Ghent University, Krijgslaan 281 Gebouw S22, 9000 Ghent, Belgium.
E-mail: `pelupessy@cage.ugent.be`.

In [1] Friedman introduces adjacent Ramsey theory, including a series of theorems independent of Peano Arithmetic which, though similar to other Ramsey-like theorems, avoid its language. We examine the provability and phase transitions in Peano Arithmetic and its fragments of one of those theorems.

Following the notations from [1], we call a function $C\colon R^k \to \mathbb{N}^r$ f-limited if $\max C(x) \leq \max(f(\max x), 1)$. Let AR^k_f be the following statement:

> For every r there exists R such that for every f-limited function $C\colon R^k \to \mathbb{N}^r$ there are $x_1 < \cdots < x_{k+1} < R$ with $C(x_1, \ldots, x_k) \leq C(x_2, \ldots, x_{k+1})$.
>
> Take $f^k_\alpha(i) = \sqrt[H_\alpha^{-1}(i)]{\log^k(i)}$ and $g_\alpha(i) = \log^{H_\alpha^{-1}(i)}(i)$ where the k in the exponent at the log indicates number of iterations and H_α is the α-th function in the Hardy hierarchy.

THEOREM 1. *Take* $\omega_0 = 1$, $\omega_{n+1} = \omega^{\omega_n}$, *then*:

$\mathrm{I}\Sigma_k \nvdash \mathrm{AR}^k_{\mathrm{id}}$,
$\mathrm{PA} \vdash \forall k \mathrm{AR}^k_{g_\alpha}$ *if and only if* $\alpha < \varepsilon_0$,
$\mathrm{I}\Sigma_{k+1} \vdash \mathrm{AR}^{k+1}_{f^k_\alpha}$ *if and only if* $\alpha < \omega_{k+2}$.

[1] HARVEY FRIEDMAN, *Adjacent Ramsey theory*, `http://www.math.osu.edu/~friedman.8/pdf/PA%20incomp082910.pdf`

▸ MIKHAIL G. PERETYAT'KIN, *On model-theoretic properties that are not preserved on the pairs of mutually interpretable theories*.
Institute of Mathematics, 125 Pushkin Street, 050010 Almaty, Kazakhstan.
E-mail: `m.g.peretyatkin@predicate-logic.org`.

We consider theories in first-order predicate logic with equality. *Incomplete theories* are normally studied. An infinite model $\mathfrak{M}$ is said to be *minimal* (synonym: a *Jónsson model*), if for any model $\mathfrak{N}$, $\mathfrak{N} \preccurlyeq \mathfrak{M} \wedge \mathrm{Card}(\mathfrak{N}) = \mathrm{Card}(\mathfrak{M})$ implies $|\mathfrak{N}| = |\mathfrak{M}|$. An interpretation I of theory T in domain $U(x)$ of theory H is called $\exists \cap \forall$-*presentable*, if the domain $U(x)$ of the interpretation and I-image of each predicate of T is presentable in H by an $\exists$-formula, and simultaneously, by a $\forall$-formula. Theories T and H are called *mutually* $\exists \cap \forall$-*definably interpretable in each other*, if there is an $\exists\cap\forall$-definable interpretation I of T in H and an $\exists\cap\forall$-definable interpretation J of H in T such that, for any sentence Φ of T and any sentence Ψ of H, we have $T \vdash \Phi \leftrightarrow J(I(\Phi))$ and $H \vdash \Psi \leftrightarrow I(J(\Psi))$ ensuring an isomorphism of the Tarski–Lindenbaum algebras of these theories.

THEOREM 1. *There are complete decidable theories T_i and H_i, $i = 0, 1$, of finite pure predicate signatures without finite models, such that T_i and H_i are mutually $\exists \cap \forall$-definably interpretable in each other for $i = 0, 1$; moreover, the following properties are satisfied*:

(a) *T_0 is model-complete, while H_0 is not model-complete*;
(b) *T_1 is finitely axiomatizable, while H_1 is not finitely axiomatizable*;
(c) *T_1 has a minimal model, while H_1 does not have such a model*;
(d) *T_1 has a model with first-order definable elements, while H_1 does not have such a model; furthermore, H_1 does not even have a model with almost first-order definable* (*algebraic*) *elements*.

By $\mathcal{L}^{ea}$, we denote the collection of those model-theoretic properties $\mathfrak{p}$ which are preserved on each pair of mutually $\exists \cap \forall$-definably interpretable computably axiomatizable theories of finite signatures. Theorem 1 shows that the semantic layer $\mathcal{L}^{ea}$ covers neither Cartesian, nor Cartesian-quotient, nor even model quasiexact layer of model-theoretic properties. This gives a negative answer to Question 7 posed in [1].

[1] M. G. PERETYAT'KIN, *Finitely axiomatizable theories and similarity relations*. ***American Mathematical Society Translations, Series 2***, vol. 195 (1999), pp. 309–346.

▸ PAULA QUINON, *Numerals and numbers. Problems of encodings and denotations*.
Department of Philosophy, Lund University, Kungshuset, 222 22 Lund, Sweden.
E-mail: `paula.quinon@fil.lu.se`.

This talk proposes a study of so-called "deviations" which are claimed to occur as consequences of accepting the formal definition of the concept of computability that assumes of human intuitions about computation that they concern operations on strings (as captured

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

by Turing's thesis) rather than abstract knowledge of functions defined on natural numbers (Church's thesis) [3].

The study involves specification of the relationship between the syntactic (numerals) and the semantic (numbers) levels of the language of number theory, and the denotation functions acting between those two. The "deviations"—resulting in "computability" of some uncomputable functions (the halting problem is the most commonly quoted example)—have been claimed to occur on both of those levels ([1], [2], [3]). Certain constraints on the properties of the denotation functions have been also proposed. These constraints aim to single out the class of dentation functions acceptable for number-theoretical purpose ([4]).

The central claim of this talk is that the harmful aspect of these "deviations" can be avoided by detailed insight into the dichotomy between syntax and semantics. Additionally, some remarks on denotation functions are formulated. I claim that the presented results shed some light on the number-concept as investigated by cognitive scientists.

[1] Paul Benacerraf, *Recantation, or: any old ω-sequence would do after all*. ***Philosophia Mathematica***, vol. 4 (1996), no. 3, pp. 184–189.

[2] B. Jack Copeland and Diane Proudfoot, *Deviant encodings and Turing's analysis of computability*. ***Studies in History and Philosophy of Sciences***, vol. 41 (2010), no. 3, pp. 247–252.

[3] Michael Recorla, *Church's thesis and the conceptual analysis of computability*. ***Notre Dame Journal of Formal Logic***, vol. 48 (2007), pp. 253–280.

[4] Stewart Shapiro, *Acceptable notation*. ***Notre Dame Journal of Formal Logic***, vol. 23 (1982), no. 1, pp. 14–20.

► JASON RUTE, *Martingale convergence and algorithmic randomness*.
Department of Mathematical Sciences, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213, USA.
E-mail: `jrute@cmu.edu`.
URL Address: `www.math.cmu.edu/~jrute`.

Recently there has been a good deal of interest in the interaction between algorithmic randomness and computable analysis, especially a.e. convergence theorems. In this talk I will show a fruitful relationship between martingale convergence and randomness.

Martingales, which are a formalization of the notion of betting strategy, have historically been studied in two contexts. On the computability side, they have become a useful tool for information theory and algorithmic randomness; while on the analysis side, martingales have also become the foundation of modern probability theory and finance, with a variety of applications to analysis. Traditionally, algorithmic randomness has been concerned with at which points a nonnegative dyadic martingale succeeds (wins arbitrarily large amounts of money), while probability theory has been concerned with whether more general classes of martingales converge pointwise a.e.

I will present a variety of martingale convergence theorems, and I will show how they relate to Schnorr, computable, Martin-Löf, and weak 2-randomness. These martingale convergence theorems imply facts about differentiability, the law of large numbers, and de Finetti's theorem. They also are closely related to the ergodic theorems.

Further, the tools used to study randomness and martingales have close connections to constructive and computable analysis, reverse mathematics, proof theory, and hard/quantitative/numerical analysis.

► LUCA SAN MAURO, *Aspects of the theory of computable enumerable equivalence relations*.
Scuola Normale Superiore, Piazza dei Cavalieri 7, Pisa, Italy.
E-mail: `luca.sanmauro@sns.it`.

This talk is about computable enumerable equivalence relations (ceers). We study them under the following reducibility: if R, S are equivalence relations on ω, we say that R is *reducible* to S ($R \leq S$) if there exists a computable function f such that, for every x, y, $xRy \Leftrightarrow f(x)Sf(y)$.

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The reducibility was introduced by Ershov [2], with respect to the theory of numberings, and later developed in Bernardi and Sorbi [1], and Lachlan [4].

Recently, new motivations occurred while considering a computable analog of the so-called Borel reducibility, as in Gao and Gerdes [3].

In this talk, we focus on two of the main aspects of the topic. Firstly, we approach the degree structure generated by the reducibility. We show that the structure form a bounded poset which is neither a lower semilattice, nor an upper semilattice. In addition, we prove that its first order theory is undecidable.

Secondly, we turn our attention to universal ceers. We review classical definitions from the existing literature and we show that: the uniformly effectively inseparable ceers are universal, while there are effectively inseparable ceers that are not universal.

This is joint work with Uri Andrews, Steffen Lempp, Joseph S. Miller, Keng Meng Ng, and Andrea Sorbi.

[1] C. Bernardi and A. Sorbi, *Classifying positive equivalence relations*, ***The Journal Symbolic Logic***, vol. 48 (1983), no. 3, pp. 529–538.

[2] Yu. L. Ershov, ***Theory of numberings***, Nauka, Moskow, 1977.

[3] S. Gao and P. Gerdes, *Computably enumerable equivalence relations*. ***Studia Logica***, vol. 67 (2001), no. 1, pp. 27–59.

[4] A. H. Lachlan, *A note on positive equivalence relations*. ***Mathematical Logic Quarterly***, vol. 33 (1987), no. 1, pp. 43–46.

▸ SAM SANDERS, *Reuniting the antipodes: Bringing together Constructive and Nonstandard Analysis*.

Department of Mathematics, Ghent University, Krijgslaan 281, 9000 Gent, Belgium.
E-mail: sasander@cage.ugent.be.
URL Address: http://cage.ugent.be/~sasander.

Constructive Analysis was introduced by Errett Bishop to identify the *computational meaning* of mathematics. In the spirit of intuitionistic mathematics, notions like *algorithm*, *explicit computation*, and *finite procedure* are central. The exact meaning of these vague terms was left open, to ensure the compatibility of Constructive Analysis with several traditions (classical, intuitionistic and recursive) in mathematics. Constructive Reverse Mathematics (CRM) is a spin-off of Harvey Friedman's famous *Reverse Mathematics* program, based on Constructive Analysis. Bishop famously derided Nonstandard Analysis for its lack of computational meaning. In this talk, we introduce "Ω-invariance: a simple and elegant definition of *finite procedure* in (classical) Nonstandard Analysis. Using an intuitive interpretation, we obtain many results from CRM, thus showing that Ω-invariance is quite close to Bishop's notion of *finite procedure* and algorithm. We briefly discuss philosophical implications and future work with regard to Per Martin-Löf's *Type Theory*, which is intended as a foundation for Constructive Analysis.

This research is generously sponsored by the John Templeton Foundation.

▸ ANDREY SARIEV, *The ω-Turing degrees*.

Faculty of Mathematics and Computer Science, Sofia University, 5 James Bourchier Boulevard, 1164 Sofia, Bulgaria.
E-mail: acsariev@gmail.com.

In this paper the study of the partial ordering of the ω-Turing degrees is initiated. Informally, the considered structure is derived from the structure of ω-enumeration degrees described by Soskov [1] by replacing the usage of the enumeration reducibility and the enumeration jump in the definitions with Turing reducibility and Turing jump respectively. The main results include a jump inversion theorem, existence of minimal elements and minimal pairs.

[1] I. N. Soskov, *The ω-enumeration degrees*. ***Journal of Logic and Computation***, (to appear).

[2] I. N. Soskov and H. Ganchev, *The jump operator on the ω-enumeration degrees*. ***Annals of Pure and Applied Logic***, (to appear).

▸ ANTON SETZER, *How to reason coinductively informally*.
Department of Computer Science, Swansea University, Singleton Park, Swansea SA2 8PP, UK.
E-mail: a.g.setzer@swan.ac.uk.
URL Address: http://www.cs.swan.ac.uk/~csetzer/.

This research was supported by EPSRC grant EP/G033374/1 "Theory and application of induction-recursion", and written as a visiting fellow of the Newton Institute, Cambridge University.

Whereas formally an inductively defined set is defined as a least fixed point, one rarely argues directly using this definition. Instead we use usually the induction rules derived from this principle. In fact we have developed a culture of informally arguing inductively by referring to the induction hypothesis, and often use extended induction principles such as course of value induction. Coinductively defined sets are greatest fixed points, however proofs about coinductively defined sets are usually either carried out by referring directly to the definition, or using game theoretic approaches. Therefore, coinductive proofs appear to be quite complicated and are usually not taught in the early parts of a mathematics or computer science curriculum.

In the interactive theorem prover Agda, proofs by induction are given as recursive functions which pass a termination checker. The termination checker verifies that the induction hypothesis is used correctly. In a similar way coinductive proofs are given as well as recursive functions, passing the termination checker. The termination checker checks whether the recursive call, which we call the *coinduction hypothesis* is used correctly.

In this talk we will develop rules for coinduction in the same way as it is done for induction. We show how informal proofs by coinduction can be carried out by referring to the coinduction hypothesis in an appropriate way. As when referring to the induction hypothesis for inductive proofs the same care has to be applied when referring to the coinduction hypothesis in coinductive proofs.

We will illustrate this by showing how to carry out informal proofs by bisimulation.

▸ ANDREW SWAN, *The failure of the existence property for CZF*.
School of Mathematics, University of Leeds, Leeds, LS2 9JT, UK.
E-mail: aws@maths.leeds.ac.uk.

A theory, T is said to have the *existence property* (sometimes called the *set existence property*) if for each formula $\phi(x)$ such that $T \vdash (\exists x)\phi(x)$, there is another formula, $\chi(x)$ such that $T \vdash (\exists! x)(\phi(x) \wedge \chi(x))$. The existence property is sometimes expected for constructive theories on the basis that the existence of mathematical objects should only asserted if they can be "mentally constructed." However, the existence property fails for some set theories regarded as constructive. In this talk we will show the new result that in fact the existence property fails for what is today the most widely studied constructive set theory, CZF.

The cause of this failure is the subset collection axiom schema. Subset collection can be regarded as a strengthened version of the exponentiation axiom that is validated by Peter Aczel's interpretation of set theory into Martin-Löf type theory. Because of this interpretation it can be regarded as predicative, as opposed to the much stronger power set axiom. We show that subset collection asserts the existence of a particular set of multivalued functions from Baire space to the naturals that cannot be defined from within CZF.

To prove this we define the notion of embedding one realizability interpretation into another. We will show that there are two realizability interpretations with essentially different witnesses of subset collection that can both be embedded into the standard McCarty style realizability interpretation of IZF.

▸ TINKO TINCHEV, *Modal approach to region-based theories of space: canonicity*.
Faculty of Mathematics and Informatics, Sofia University, 1164 Sofia, 5 James Bourchier, Bulgaria.
E-mail: tinko@fmi.uni-sofia.bg.

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

Region-based theories of space study properties of the regions—formal analog of the "bodies"—instead of abstract notions like points with respect to axiomatization, complexity of satisfiability problem, etc. Usually the regions are taken to be the regular closed sets (or regular open sets) in a given topological space $\mathbb{T} = (T, \tau)$ from a class $\mathfrak{T}$ of spaces. Typical properties are contact, n-contact, internal (strong) contact, connectedness, n-connectedness, boundedness, convexity, etc. For example, n regions $A_1, \dots, A_n$ are in n-contact if they have a common point, $A_1 \cap \dots \cap A_n \neq \varnothing$. On the other hand, regular closed sets form a Boolean algebra under inclusion with bottom $0 = \varnothing$ and top $1 = T$. Normally, the first-order theories of this kind for the spaces (classes of spaces) which take attention are very complex. In contrast their universal fragments often allow formal handling and for practical spatial reasoning are good enough.

In the present talk we propose a sufficiently general condition for completeness with respect to the canonical model whenever the above mentioned universal fragments are treated as fragments of appropriate modal language.

► J. A. TUSSUPOV, *Categoricity and complexity relations over structures with two equivalences*.
Information Systems, Eurasian National University, Astana, Munaitpasova 5, Kazakhstan.
E-mail: tussupov@mail.ru.

In paper [1] authors showed that for each computable ordinal α there is a structure that is Δ^0_α categorical but not relatively Δ^0_α categorical. This structure of the countable relational language. S. S. Goncharov [2] suggested the method of definability structure with countable computable set of predicates where arity of them bounded by finite number to the oriented graph. J. A. Tussupov [3] suggested the method of definability oriented graph to the bipartite graph and to the structure with two equivalence. Let $\sigma_0 = \langle P^2(x, y)\rangle$ is signature with the bipartition binary predicates $P(x, y)$ and $\sigma_1 = \langle E^2_0(x, y), E^2_1(x, y)\rangle$ is signature with two equivalences $E^2_0(x, y), E^2_1(x, y)$.

Let $\mathcal{A}$ structure of signature σ_i, where $i = 0, 1$.

THEOREM 1. *For each computable ordinal α there is a computable structure $\mathcal{A}$ of signature σ_i that is Δ^0_α categorical but not relatively Δ^0_α (and without formally Σ^0_α Scott family).*

THEOREM 2. *For each computable ordinal α there is a computable structure $\mathcal{A}$ of signature σ_i with additional relation R that is intrinsically Σ^0_α but not relatively intrinsically Σ^0_α* on $\mathcal{A}$.

[1] J. CHISHOLM, E. B. FOKINA, S. S. GONCHAROV, V. S. HARIZANOV, J. F. KNIGHT, and S. MILLER, *Intrinsic bounds on complexity and definability at limit levels*. ***The Journal of Symbolic Logic***, vol. 74 (2009), no. 3, pp. 1047–1060.

[2] S. S. GONCHAROV, *Isomorphisms and definable relations on computable models*. ***Proceeding of the Logic Colloquium 2005, Athens***, pp. 26–45.

[3] J. A. TUSSUPOV, *Isomorphisms and algorithmic properties structures with two equivalences*. ***Abstracts of Logic Colloquium 2011***, Barcelona, Spain, July 11–16, pp. 107–109.

► JEROEN VAN DER MEEREN, *Well-partial-orderings and recursively defined trees*.
Department of Mathematics, Ghent University, Krijgslaan 281 S22, B 9000 Gent, Belgium.
E-mail: jvdm@cage.ugent.be.

Well-partial-orderings play an important role in for example logic, mathematics, and computer science [1]. They are the essential ingredient of famous theorems like Higman's lemma and Kruskal's theorem. The maximal order type of a well-partial-ordering is most of the time also the proof-theoretical ordinal of a specific theory T. There exists a general principle for computing the maximal order type of well-partial-orderings of recursively defined trees [3]. In this talk, I will introduce those recursively defined trees and discuss recent results of their maximal order types. These recursively defined trees are introduced for studying trees with a Friedman-style gap-condition [2].

[1] T. BECKER and V. WEISPFENNING, in cooperation with H. Kredel, ***Gröbner bases: A computational approach to commutative algebra***, Graduate Texts in Mathematics, vol. 141, Springer-Verlag, Berlin, 1993.

[2] S. G. SIMPSON, *Nonprovability of certain combinatorial properties of finite trees*, ***Harvey Friedman's research on the foundations of mathematics***, Studies in Logic and the Foundation

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

of Mathematics, (L. A. Harrington, M. D. Morley, A. Scedrov, and S. G. Simpson, editors), Elsevier Science Publishers B.V., Amsterdam, The Netherlands, 1985, pp. 87–117.

[3] A. WEIERMANN, *A computation of the maximal order type of the term ordering on finite multisets*, ***Mathematical theory and computational practice*** (***5th Conference on Computability in Europe, Heidelberg, Germany, July 19–24, 2009***), (K. Ambos-Spies, B. Löwe, and W. Merkle, editors), vol. 5635/2009, Lecture Notes in Computer Science, Springer Berlin / Heidelberg, 2009, pp. 488–498.

▸ MITKO YANCHEV, *Part restrictions in Description Logics: reasoning in polynomial time complexity*.
Faculty of Mathematics and Informatics, Sofia University, 5 James Bourchier Boulevard, 1164 Sofia, Bulgaria.
E-mail: yanchev@fmi.uni-sofia.bg.

Description Logics (DLs) are logical formalism widely used in knowledge-based systems for both explicit knowledge representation in the form of taxonomy, and inferring new knowledge out of the presented structure by means of a specialized inference engine ([1]). The representation language, called *concept language*, comprises expressions with only unary and binary predicates, called *concepts* and *roles*. In the semantics these are interpreted as subsets and binary relations respectively. With their syntax and interpretation various DLs can be viewed as syntactical variants or restricted fragments of some modal logics. Concept languages differ mainly in the constructors adopted for building complex concepts and roles, and they are compared with respect to their expressiveness, as well as with respect to the complexity of reasoning in them. The language $\mathcal{AL}$ is usually considered as a "core" one, having the basic set of constructors: $\neg A$ (atomic negation), $C \sqcap D$ (intersection), $\forall P.C$ (universal role quantification), and $\exists P.\top$ (restricted existential role quantification).

In the present talk we introduce new concept constructors, called *part restrictions*, capable to distinguish a part of a set of successors. These are $MrP.C$ and (the dual) $WrP.C$, where r is an arbitrary rational number in $(0, 1)$, P is a role, and C is a concept. The concept $MrP.C$ is interpreted by the set of all objects x such that $\left|\{y \mid (x, y) \in P^{\mathcal{I}} \;\;\&\;\; y \in C^{\mathcal{I}}\}\right| > r\left|\{y \mid (x, y) \in P^{\mathcal{I}}\}\right|$. Part restrictions essentially enrich the expressive capabilities of Description Logics, and, as we show for a particular language, they do that with no extra cost of complexity.

We consider the language $\mathcal{ALP}^{\varepsilon}$ extending $\mathcal{AL}$ with limited part restrictions adopting only atomic and negated atomic concepts. We show that this language, while extending the expressive power of $\mathcal{AL}$, keeps the same P-time upper bound for the complexity of the main reasoning task in DLs—checking the subsumption between concepts. For, a completion calculus based on tableau technique is used.

[1] F. Baader, D. Calvanese, D. McGuinness, D. Nardi, and P. Patel-Schneider, editors, ***The description logic handbook: Theory, implementation, and applications***, Cambridge University Press, New York, 2003.

▸ FAN YANG, *Implications in dependence and independence logic*.
Department of Mathematics and Statistics, PO Box 68, FIN-00014 Helsinki, Finland.
E-mail: fan.yang@helsinki.fi.

Dependence logic (**D**) [Väänänen, 2007] and independence logic (**Ind**) [Grädel, Väänänen, 2011] are new logics incorporating the concept of dependence and independence into first-order logic. The compositional semantics of these logics are defined with respect to sets of assignments, called *teams*. Team semantics was originally introduced by Hodges [1997] for independence friendly logic [Hintikka, Sandu 1989]. Both dependence logic and independence logic have the same expressive power as existential second order logic [Galliani, Grädel, Kontinen, Väänänen].

The negation of neither of these two logics is classical; this fact therefore raises the question of how to define implications in these two logics. Basing on team semantics and the downward closure property of dependence logic, Abramsky and Väänänen [2009] introduced intuitionistic implication ($\to$) and linear implication ($\multimap$) for dependence logic. In this talk, we show that on sentence level, dependence logic extended with these two implications

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

($\mathbf{D}^{[\rightarrow,\multimap]}$) have the same expressive power as the full second order logic [Yang 2010], while on the formula level, $\mathbf{D}^{[\rightarrow,\multimap]}$ characterizes exactly second order downward monotone properties.

On the other hand, dependence logic is contained in independence logic [Grädel, Väänänen 2011], and independence logic can be, in certain sense, broken into two logics, namely inclusion logic ($\mathbf{I}$) and exclusion logic ($\mathbf{E}$) [Galliani 2011]. As independence logic does not have the downward closure property, the intuitionistic implication or linear implication does not do the same job in independence logic as in dependence logic. In this talk, we introduce a maximal implication ($\hookrightarrow$) in the context of independence logic and show that on the sentence level, independence logic extended with maximal implication ($\mathbf{Ind}^{\hookrightarrow}$) has the same expressive power as the full second order logic (thus on the sentence level, $\mathbf{Ind}^{\hookrightarrow} = \mathbf{D}^{[\rightarrow,\multimap]}$). The same hold for $\mathbf{I}$ extended with $\hookrightarrow$ ($\mathbf{I}^{\hookrightarrow}$) and $\mathbf{E}$ extended with $\hookrightarrow$ ($\mathbf{E}^{\hookrightarrow}$) as well, namely on the sentence level

$$\mathbf{D}^{[\rightarrow,\multimap]} = \mathbf{Ind}^{\hookrightarrow} = \mathbf{I}^{\hookrightarrow} = \mathbf{E}^{\hookrightarrow}.$$

In addition, on the formula level, both $\mathbf{Ind}^{\hookrightarrow}$ and $\mathbf{I}^{\hookrightarrow}$ characterize exactly second order properties.

Abstracts of talks presented by title

▶ ANATOLY P. BELTIUKOV, *Deductive synthesis of polynomial algorithms on finite partially ordered models of second-order logic*.
Udmurt State University, Universitetskaya 1, Izhevsk, Russia.
E-mail: belt@uni.udm.ru.

A method of constructing formal intuitionistic theories, destined for deductive synthesis of polynomial algorithms, working on finite realizational models of the second-order predicate logic is proposed. Built theories suppose some relation of partial order defined on a model.

Formal theories are built in the form of sequent calculi, focused on inverse search of inference. Special rules are constructed to deal with partial orderings. Cyclic and recursive algorithms are extracted from the applications of these rules.

Estimating computational complexity of algorithms we consider second-order algorithms, that may have at entry also algorithms, but working only with data. In addition, if entrance algorithms are time polynomial then the resulting algorithm is also time polynomial. Degree of this polynom is also limited with a polynom from degrees of the initial polynoms.

The theories can be applied to programming of information systems. Proposed methods of extracting algorithms are suitable for direct construction of programs on such actual programming languages as JavaScript.

This work is continuing the works [1], [2], [3].

[1] A. P. Beltiukov, *Intuitionistic formal theories with realizability in subrecursive classes*. ***Annals of Pure and Applied Logic***, vol. 89 (1997), pp. 3–15.

[2] ———, *A strong induction scheme that leads to polynomially computable realizations*. ***Theoretical Computer Science***, vol. 322 (2004), pp. 17–39.

[3] ———, *A Polynomial Programming Language*. ***Transactions of the Institute for Informatics and Automation Problems of the National Academy of Sciences of Armenia, Mathematical Problems of Computer Science***, vol. 27 (2006), pp. 11–19.

▶ MARIJA BORIČIĆ, *Hypothetical syllogism rule probabilized*.
Faculty of Organizational Sciences, University of Belgrade, Jove Ilića 154, 11000 Beograd, Serbia.
E-mail: marija.boricic@fon.bg.ac.rs.

Let A, B, and C be propositional formulae with the following probabilities of their truthfulness $P(A) = a$, $P(B) = b$, and $P(C) = c$. Then, the probabilistic versions of the hypothetical syllogism inference rule can be given as follows:

$$\frac{P(A \rightarrow B) = r \quad P(B \rightarrow C) = s}{\max\{r - a, r + s - 1\} \leq P(A \rightarrow C) \leq \min\{s + 1 - a, r + c\}}$$

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

in Hailperin-style, and

$$\frac{P(A \to B) \geq 1-\varepsilon \quad P(B \to C) \geq 1-\varepsilon}{P(A \to C) \geq 1-2\varepsilon},$$

for each $0 \leq \varepsilon \leq \frac{1}{2}$, in Suppes-style. These rules contain the probabilistic versions of both *modus ponens* (see [1] and [2]), for $a = 1$, and *modus tollens* rule (see [3]), for $c = 0$. We can show that a complete proof-theoretical treatment of probability operators, considered as a part of a polymodal language containing formulae of the form A^{α}, with the intended meaning that $P(A) \in \alpha$, where α is an element of a finite algebra of subsets of [0, 1], can be based on this approach. On the other side, in case when implication $A \to B$ is interpreted as conditional probability $P(B|A)$, although the probabilistic versions of *modus ponens* and *modus tollens* are quite natural (see [1], [2] or [3]), there are arguments that probabilistic versions of the hypothetical syllogism inference rule lose their usual logical sense.

[1] T. Hailperin, *Probability logic*. ***Notre Dame Journal of Formal Logic***, vol. 25 (1984), pp. 198–212.

[2] P. Suppes, *Probabilistic inference and the concept of total evidence*, ***Aspects of inductive inference***, (J. Hintikka and P. Suppes, editors), North-Holland, Amsterdam, 1966, pp. 49–55.

[3] C. G. Wagner, *Modus tollens probabilized*, ***British Journal for the Philosophy of Science***, vol. 54 (2004), no. 4, pp. 747–753.

▸ JOHN CORCORAN, *Tarski's extensional functions*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

"Extensional functions" appear twice in [1]—in different senses. In his 1923 dissertation [1, pp. 1–23], "functions" are interpretable as type-theoretic mathematical "mappings": not linguistic expressions [1, pp. 5, 23]. In Tarski's 1935 truth-definition paper [1, pp. 152–278], "functions" are expressions containing free variables. Below *mapping* and *function* have contrasting senses: *mapping* always has the mathematical sense, *function* always the linguistic sense. Mappings never contain variables, functions always contain variables.

Arithmetic languages have the three-character absolute-value *function* '$|x|$' associated with the absolute-value mapping carrying numbers to their absolute values. Set-theoretic languages have the three-character singleton function '$\{x\}$' associated with the singleton mapping carrying sets to their singletons. Functions, which are not names, convert into names by replacing variables with constants: '$|-2|$' names two; '$\{\omega\}$' names ω's singleton.

The respective metalanguages have the three-character *quotation function* ' 'x' ' convertible to expression names: ' 'a' ' *names* the first letter—the one-character expression 'a'. As Tarski knew, the *expression* ' 'x' ' is ambiguous: in one sense it is a *function*; in another sense it is a name of the 24th letter, ecks. The three-character expression ' 'x' ' is named with a *five-character* expression ' ' 'x' ' '—applying two-character single-quotation twice.

The singleton function '$\{x\}$' and the absolute-value function '$|x|$' are both *extensional* in Tarski's implicit sense [1, p. 161]—substituting coextensive names for the variable produces coextensive names: $|2| = |1 + 1|$ and $\{\omega\} = \{\omega - 1\}$. However, as Tarski implied, the quotation function ' 'x' ' is nonextensional: '2' $\neq$ '1+1' and 'ω' $\neq$ '$\omega - 1$'—no one-character expression is a three-character expression.

[1] Alfred Tarski, ***Logic, semantics, metamathematics***, Hackett, Indianapolis, 1983.

▸ JOHN CORCORAN AND HASSAN MASOUD, *Existential-import sentence schemas: classical and relativized*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

The variable-enhanced-English sentence schema 'for every integer x P(x)' translates the first-order schema '$\forall x$ P(x)' interpreted in the integers: "for every integer x' translates '$\forall x$' applied to integers.

The *Classical Existential-Import Schema*, *CEIS*, has as instances every conditional whose antecedent is a universal sentence and whose consequent is *the* corresponding existential—replacing the initial universal quantifier by the existential.

If for *every* integer x $\mathrm{P}(x)$, then for *some* integer x $\mathrm{P}(x)$.

$$[\forall x\, \mathrm{P}(x) \rightarrow \exists x\, \mathrm{P}(x)].$$

Obviously every CEIS instance is tautological [logically true].

The *Relativized Existential-Import Schema*, *REIS*, has as instances every conditional whose antecedent is a universalized conditional and whose consequent is *the* corresponding existentialized conjunction—replacing the initial universal quantifier by the existential *and* replacing the conditional connective by conjunction.

If for every integer x [if $\mathrm{A}(x)$, *then* $\mathrm{C}(x)$], then for *some* integer x [$\mathrm{A}(x)$ *and* $\mathrm{C}(x)$].

$$[\forall x\, (\mathrm{A}(x) \rightarrow \mathrm{C}(x)) \rightarrow \exists x\, (\mathrm{A}(x)\ \ \&\ \ \mathrm{C}(x))].$$

Nontautological REIS instances are familiar. But, contrary to textbook impressions, *certain* instances of REIS *are* tautological. A necessary and sufficient condition for REIS instances to be tautological follows.

THEOREM. *A REIS instance is tautological iff the existentialization* $\exists x\, \mathrm{A}(x)$ *of the antecedent condition* $\mathrm{A}(x)$ *is tautological.*

$[\forall x\, (\mathrm{A}(x) \rightarrow \mathrm{C}(x)) \rightarrow \exists x\, (\mathrm{A}(x)\ \ \&\ \ \mathrm{C}(x))]$ *is tautological*

if and only if

$\exists x\, \mathrm{A}(x)$ *is tautological.*

"If" is obvious. The four key ideas in our "only-if" proof are:

(1) $\exists x\, \mathrm{A}(x)$ *is tautological if* $\sim \exists x\, \mathrm{A}(x)$ *implies* $\exists x\, \mathrm{A}(x)$.
(2) $\sim \exists x\, \mathrm{A}(x)$ *implies* $\forall x\, (\mathrm{A}(x) \rightarrow \mathrm{C}(x))$.
(3) $\forall x\, (\mathrm{A}(x) \rightarrow \mathrm{C}(x))$ *implies* $\exists x\, (\mathrm{A}(x)\ \ \&\ \ \mathrm{C}(x))$, *by the hypothesis.*
(4) $\exists x\, (\mathrm{A}(x)\ \ \&\ \ \mathrm{C}(x))$ *implies* $\exists x\, \mathrm{A}(x)$.

This lecture complements this BULLETIN vol. 11 (2005), p. 460; vol. 11 (2005), pp. 554–555; vol. 12 (2006) pp. 219–240 and vol. 13 (2007) pp. 143–144; and vol. 17 (2011), pp. 324–325.

▸ JOHN CORCORAN AND JOAQUIN MILLER, *Meanings of show*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

We study uses of *show* in logic: literal, metaphorical, elliptical, etc. Some are confused—through not distinguishing literal from figurative uses.

A teacher *shows* students that not every integer is either positive or negative.
An easy proof *shows* that not every integer is either positive or negative.
Euclid's construction *shows* how to construct equilateral triangles.

Zero *shows* that not every integer is either positive or negative.

Zero is a counterexample for "every integer is either positive or negative". A counterexample for a given proposition *shows* the proposition false. Thus, zero *shows* "every integer is either positive or negative" to be false. Compare [1].

The fact that zero is neither positive nor negative *shows* that not every integer is either positive or negative.

The theorem that zero is an integer which is neither positive nor negative *shows* that not every integer is either positive or negative.

The proposition "zero is zero" does not say that it—the proposition "zero is zero"—is tautological; however, according to some, it does *show* that it is. In fact, some logicians say *every tautology itself shows that it is a tautology* ([2], 6.127).

The grammatical categories of the verb *show* are diverse. It occurs as a two-place verb completed by both subject and direct object, but it also occurs as a three-place verb requiring

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for its completion an indirect object as well. Moreover, sometimes it requires a human subject; it is an action verb—like *teach* and *infer*. Other times it requires an inert nonhuman subject; it is a timeless relation verb—like *equal* and *imply*.

[1] JOHN CORCORAN, *Counterexamples and proexamples*, this BULLETIN, vol. 11 (2005), p. 460.

[2] LUDWIG WITTGENSTEIN, ***Tractatus Logico-Philosophicus***, Kegan Paul, London, 1921.

► JOHN CORCORAN AND SRIRAM NAMBIAR, *Conversely: extrapropositional and prosentential*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

This self-contained lecture examines uses and misuses of the adverb *conversely* with special attention to logic and logic-related fields. Sometimes adding *conversely* after a conjunction such as *and* signals redundantly that a converse of what preceded will follow.

Tarski read Church and, conversely, Church read Tarski. (1)

In such cases, *conversely* serves as an *extrapropositional* constituent of the sentence in which it occurs: deleting *conversely* doesn't change the proposition expressed. Nevertheless it does introduce new implicatures: a speaker would implicate belief that the second sentence expresses a converse of what the first expresses.

Perhaps because such usage is familiar, the word *conversely* can be used as "sentential pronoun"—or *prosentence*—representing a sentence expressing a converse of what the preceding sentence expresses.

Tarski read Church and conversely. (2)

This would be understood as expressing the proposition expressed by 1.

Prosentential usage introduces ambiguity when the initial proposition has more than one converse. Confusion can occur if the initial proposition has nonequivalent converses.

Every proposition that is the negation of a false proposition is true and conversely.

One sense implies that every proposition that is the negation of a true proposition is false, which is true of course. But another sense, probably more likely, implies that every proposition that is true is the negation of a false proposition, which is false: the proposition that one precedes two is not a negation and thus is not the negation of a false proposition.

The above also applies to synonyms of *conversely* such as *vice versa*. Although *prosentence* has no synonym, extrapropositional constituents are sometimes called *redundant rhetoric*, *filler*, or *expletive*.

Authors discussed include Aristotle, Boole, De Morgan, Peirce, Frege, Russell, Tarski, and Church.

► JOHN CORCORAN AND DANIEL NOVOTNÝ, *Formalizing Euclid's first axiom*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

Euclid's ***Elements*** divides its ten premises into two groups of five.

The first five (*postulates*)—applying in geometry but nowhere else—are *specifically* geometrical. The first: "to draw a line from any point to any point"; the last: the parallel postulate.

The second five (*axioms*) apply in geometry *and* elsewhere. They are nonlogical principles governing *magnitude types* both geometrical (e.g., lengths, areas) and nongeometrical (e.g., durations, weights). Euclid called axioms *koinai ennoia*: *koinai* ("shared", "communal", etc.), ennoia ("designs", "thoughts", etc.). The first axiom is:

Ta toi autoi isa kai allelois estin isa.

Things that equal the same thing equal one another.

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One first-order translation in variable-enhanced English (cf. [2, p. 121] is:

$$\text{Given two things } x, y, \textit{if} \text{ for something } z, x \text{ and } y \text{ equal } z, \quad (1)$$
$$\textit{then } x \text{ equals } y.$$

Translation (1) overlooks Euclid's plural construction not limited to two. Second-order translations avoid that objection.

$$\text{For any set } S, \textit{if} \text{ for something } z, \text{ everything } x \text{ in } S \text{ equals } z, \quad (2)$$
$$\textit{then} \text{ anything } x \text{ in } S \text{ equals anything } y \text{ in } S.$$

Translations (1) and (2) are "too broad": they cover all magnitude types but by amalgamating them into a hodgepodge universe containing all magnitude types—a universe violating category restrictions and not itself a magnitude type.

Translation (3) is a *second-order axiom schema* (cf. [1]) having one instance for each magnitude type. 'MAG' is placeholder for magnitude words such as *length*, *area*, etc.

$$\text{For any set } S, \textit{if} \text{ for some MAG } z, \text{ every MAG } x \text{ in } S \text{ equals } z, \quad (3)$$
$$\textit{then} \text{ any MAG } x \text{ in } S \text{ equals any MAG } y \text{ in } S.$$

We treat several other translations and formalizations.

[1] John Corcoran, *Schemata*. this Journal, vol. 12 (2006), pp. 219–240.
[2] Alfred Tarski, ***Introduction to logic***, Dover, New York, 1995.

► JOHN CORCORAN AND JOSÉ MIGUEL SAGÜILLO, *Euclid's weak first axiom*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

"Things that equal the same thing equal one another" [TES] is Euclid's first axiom in ***Elements***—written about 300 BCE, some 50 years after Aristotle's ***Analytics***. There is no trace of TES before Euclid, but afterward it was often repeated verbatim or nearly verbatim. For example, over 400 years later, Galen stated it at least three times [1, pp. 430–442]; over 700 years later Proclus stated it several times. It still influences logic. After distinguishing geometrical equality ["has-the-same-size-as"] from logical identity ["is-the-same-entity-as"], Tarski adapted TES for line-segment geometry where congruence and equality are coextensive—"Two segments congruent to the same segment are congruent to each other" [3, p. 121].

TES is the only ***Elements*** axiom governing equality alone: three others govern equality with addition, subtraction, and coincidence, respectively. A closelyrelated proposition (*the first axiom's twin*)—"Things that the same thing equals equal one another" [TSE]—has similar practical applications. After all, things that equal the same thing are things that the same thing equals [TES–TSE], and conversely [TSE–TES]; both are consequences of equality's symmetry.

Euclid's first axiom seems poorly chosen if characterizing equality were a goal. It is particularly weak. As a measure of its weakness, note that it doesn't imply any of the following: reflexivity, symmetry, transitivity, TSE (its twin), TES–TSE, and TSE–TES.

The independence results use the counterargument method [2, pp. 32ff] from Aristotle's ***Analytics*** and available to Euclid—after first-order translations following Tarski's examples [3, pp. 120–125].

This lecture treats the history, philosophy, and logic of Euclid's first axiom.

[1] Jonathan Barnes, ***Truth, etc.***, Oxford University Press, Oxford, 2007.
[2] John Corcoran, *Argumentations and logic*. ***Argumentation***, vol. 3 (1989), pp. 17–43.
[3] Alfred Tarski, ***Introduction to logic***, Dover, New York, 1995.

► FREDRIK ENGSTRÖM, *Generalized quantifiers in dependence logic*.
Department of Philosophy, Linguistics and Theory of Science, University of Gothenburg, Box 200, 405 30 Gothenburg, Sweden.
E-mail: fredrik.engstrom@gu.se.
URL Address: http://engstrom.morot.org.

Partially ordered quantifier prefixes, or branching quantifiers, are needed when formalizing natural languages. The case of partially ordered existential and universal quantifiers has been studied in length, leading to systems like Hintikka and Sandu's IF-logic and Väänänen's dependence logic. For a compositional semantical analysis of these systems the framework, invented by Hodges, using sets of assignments instead of single assignments, is needed.

Branching of generalized quantifiers, however, is yet to be analyzed compositionally. We will in this talk present a compositional account of partially ordered monotone generalized quantifiers. It is based on dependence logic but with a modified dependence atom. Instead of using functional dependence we are forced to use multivalued dependence: A set of assignments X satisfies the multivalued dependence $[\bar{x} \twoheadrightarrow y]$ if

$$\forall s, s' \in X \left(s(\bar{x}) = s'(\bar{x}) \to \exists s_0 \in X \left(s_0(\bar{x}, \bar{y}) = s(\bar{x}, \bar{y}) \wedge s_0(\bar{z}) = s'(\bar{z}) \right) \right),$$

where $\bar{z}$ are the variables in X which are not in $\bar{x}$ or $\bar{y}$. Galliani proved in [3] that this atom if definably equivalent to the *independence atom* recently introduced by Väänänen and Grädel.

We will end by characterizing the expressive power of these extensions of dependence logic by monotone generalized quantifiers in terms of quantifier extensions of existential second-order logic.

[1] Fredrik Engström, *Generalized quantifiers in dependence logic*. ***Journal of Logic, Language and Information***, vol. 21 (2012), no. 3, pp. 299–324.

[2] Fredrik Engström and Juha Kontinen, *Characterizing quantifier extensions of dependence logic*. ***The Journal of Symbolic Logic***, (to appear).

[3] Pietro Galliani, *Inclusion and exclusion dependencies in team semantics—on some logics of imperfect information*. ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 1, pp. 68–84.

▸ HADI FARAHANI AND HIROAKIRA ONO, *Substructural view of Glivenko theorems and negative translations*.

Department of Computer Sciences, Shahid Beheshti University, Evin, Tehran, Iran.
E-mail: hadimathematics@gmail.com.
Research Center for Integrated Science, Japan Advanced Institute of Science and Technology, Nomi, Ishikawa, 923-1292, Japan.
E-mail: ono@jaist.ac.jp.

In [3], the second author has developed a proof-theoretic approach to Glivenko theorems for substructural propositional logics. In the present talk, by using the same techniques, we will extend them for substructural predicate logics relative not only to classical predicate logic but also to an arbitrary involutive substructural predicate logic over QFLe. It will be pointed out that in this extension, the following *double negation shift* scheme (DNS) plays an essential role.

$$(DNS)\colon \forall x\, \neg\neg\varphi(x) \to \neg\neg\forall x\, \varphi(x).$$

Among others, it is shown that the Glivenko theorem holds for QFLe† + (DNS) relative to classical predicate logic. Moreover, this logic is the weakest one among predicate logics over QFLe for which the Glivenko theorem holds relative to classical predicate logic.

Then we will study negative translations of substructural predicate logics by using the same approach. Our substructural analysis of Glivenko theorems will induce negative translation results of involutive substructural predicate logics over QFLe in a natural way. We introduce a negative translation, called extended Kuroda translation and the existence of the weakest logic is proved among such logics for which the extended Kuroda translation works. Thus we give a clearer unified understanding of negative translations by substructural point of view.

[1] J. Avigad, *A variant of the double-negation translation*, ***Carnegie Mellon Technical Report CMU-PHIL***, vol. 179, 2006.

[2] G. Ferreira and P. Oliva, *On various negative translations*, ***Third International Workshop on Classical Logic and Computation*** (Brno, Czech Republic), (Steffen van Bakel, Stefano Berardi, and Ulrich Berger), vol. 47, Electronic Proceedings in Theoretical Computer Science, 2011, pp. 21–33.

[3] H. Ono, *Glivenko theorems revisited*. ***Annals of Pure and Applied Logic***, vol. 161 (2009), pp. 246–250.

▸ VOLKER HALBACH, *Self-reference*.
University of Oxford, New College, OX1 3BN, England.
E-mail: volker.halbach@new.ox.ac.uk.
URL Address: http://users.ox.ac.uk/~sfop0114/.

A Gödel sentence is a sentence that "says about itself" that it's not provable; a Henkin sentence is a sentence that "says about itself" that it's provable; a Σ_1-truth teller is a sentence that 'says about itself' that it is Σ_1-true.

I will try to look more closely at the way such self-referential statements are constructed. It is well known that the properties of self-referential sentences may depend on the chosen Gödel coding and on the formula representing the property in question. It is less understood that the properties of self-referential statements depend also on the way, self-reference is obtained once the representing formula and the Gödel coding have been fixed.

Kreisel [1] constructed a refutable Henkin sentence. To this end he employed a noncanonical provability predicate but also a noncanonical construction to obtain self-reference. I will discuss some applications of Kreisel's basic technique and related observations by Albert Visser.

[1] Georg Kreisel, *On a problem of Henkin's*. ***Indagationes Mathematicae***, vol. 15 (1953), pp. 405–406.

▸ BEIBUT KULPESHOV, *On self-definable subsets in weakly o-minimal structures*.
Department of Information Systems and Mathematical Modelling, International Information Technology University, 8 A Zhandosov str., Almaty, Kazakhstan.
E-mail: kulpesh@mail.ru.

We continue studying the notion of *weak o-minimality* originally studied by D. Macpherson, D. Marker, and C. Steinhorn in [2]. A subset A of a linearly ordered structure $M = \langle M, =, <, \dots \rangle$ is *convex* if for any $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. *A weakly o-minimal structure* is a linearly ordered structure M such that any definable (with parameters) subset of M is a finite union of convex sets in M. Real closed fields with a proper convex valuation ring provide an important example of weakly o-minimal structures.

If M is a structure and $A \subset M$, we say that A is *self-definable* if A is definable in M with parameters which are elements of A. A self-definable subset A of an $\aleph_0$-categorical structure M is *good* if for all $n < \omega$ every n-type over A realized in M is isolated. Self-definable sets were considered in [3] concerning the notion of *Jordan set*.

Here we discuss some equivalent conditions for goodness of self-definable subsets in an $\aleph_0$-categorical weakly o-minimal theory, and for this we use some results obtained in [1].

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[2] H. D. Macpherson, D. Marker, and C. Steinhorn, *Weakly o-minimal structures and real closed fields*. ***Transactions of the American Mathematical Society***, vol. 352 (2000), pp. 5435–5483.

[3] H. D. Macpherson and C. Steinhorn, *On variants of o-minimality*. ***Annals of Pure and Applied Logic***, vol. 79 (1996), pp. 165–209.

▸ GRAHAM E. LEIGH, *Theories of truth over intuitionistic logic*.
Faculty of Philosophy, University of Oxford, 10 Merton Street, Oxford OX1 4JJ, UK.
E-mail: graham.leigh@philosophy.ox.ac.uk.

We investigate the role classical principles play in restricting the freedom to add semantic concepts such as truth to the language of arithmetic. In particular we consider two collections of natural principles of truth both of which are consistent over Heyting arithmetic, but inconsistent over classical Peano arithmetic. We show that the two intuitionistic theories of truth have the same Π^0_2 consequences as their consistent classical counterparts and argue that in the analysis of formal theories of truth, intuitionistic logic can play an intermediary

https://doi.org/10.1017/bsl.2014.17 Published online by Cambridge University Press

role between full classical logic in which paradoxes abound and much weaker logics such as partial or para-consistent logics that are mathematically not well understood.

▸ BENJAMIN RIN, *The computational strengths of α-length ITTMs*.
Logic and Philosophy of Science, University of California, Irvine, CA 92697, USA.
E-mail: `brin@uci.edu`.

In [2], open questions are raised regarding the computational strengths of so-called ∞-α-Turing machines, a family of models of computation resembling the infinite-time Turing machine (ITTM) model of [1], except with α-length tape (for any $\alpha \geq \omega$). Let T_α refer to the model of length α. So T_ω is just the ITTM model. Let $\succ$ stand for "is computationally stronger than". In attempting to address the open questions, I present the following results: (1) $T_{\omega_1} \succ T\omega$. (2) There exists a countable α such that $T_\alpha \succ T_\omega$. In fact, there is a hierarchy of countable machines of increasing strength, corresponding to the (weak) transfinite Turing-jump operator $^\nabla$. (3) There is a countable ordinal μ' such that for every countable $\mu \geq \mu'$, neither $T_\mu \succ T_{\omega_1}$ nor $T_{\omega_1} \succ T_\mu$—that is, the machines T_{ω_1} and T_μ are computation-strength incommensurable. The same holds true for any machine of length greater than T_{ω_1}.

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[2] Peter Koepke, *Ordinal Computability*, ***Lecture Notes in Computer Science***, vol. 5365 (2009), pp. 280–289.

▸ GEMMA ROBLES, *Depth relevance and the contraction axiom*.
Departamento de Psicología, Sociología y Filosofía, Universidad de León, Campus Vegazana, s/n, 24071, León, Spain.
E-mail: `gemmarobles@gmail.com`.
URL Address: `http://grobv.unileon.es`.

A propositional logic has the *depth relevance property* (drp) if in all its theorems of the form $A \rightarrow B$, A and B share a propositional variable at the same depth (see [1]). In [1], a particular logic, DR, is defined by restricting with the drp the class of logics verified by Meyer's six-valued Crystal matrix. DR is motivated by its rejection of the contraction axiom $(A \rightarrow .A \rightarrow B) \rightarrow .A \rightarrow B$ used in the derivation of Curry Paradox in naive set theory.

The aim of this paper is to generalize Brady's strategy by defining a class of general model structures built upon what we label weak relevant matrices. The contraction axiom, together with a number of related theses, is falsified in any of these model structures.

Acknowledgments. Work supported by research project FFI2011-28494, financed by the Spanish Ministry of Economy and Competitiveness. G. Robles is supported by Program Ramón y Cajal of the Spanish Ministry of Economy and Competitiveness.

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▸ FRANCISCO SALTO, GEMMA ROBLES, AND JOSÉ M. MÉNDEZ, *Strong relevant matrices*.
Departamento de Psicología, Sociología y Filosofía, Universidad de León, Campus Vegazana, s/n, 24071, León, Spain.
E-mail: `francisco.salto@unileon.es`.
URL Address: `http://www3unileon.es/personal/wwdfcfsa/web/html`.
Departamento de Psicología, Sociología y Filosofía, Universidad de León, Campus Vegazana, s/n, 24071, León, Spain.
E-mail: `gemmarobles@gmail.com`.
URL Address: `http://grobv.unileon.es`.
Universidad de Salamanca, Edificio FES, Campus Unamuno, 37007, Salamanca, Spain.
E-mail: `sefus@usal.es`.
URL Address: `http://web.usal.es/~sefus`.

The aim of this paper is to define a general class of logical matrices called "Strong relevant matrices" (srm). Any logic S verified by a srm has the following properties.

1. (Strong variable-sharing property). If $A \to B$ is a theorem of S, then some variable occurs as an antecedent part (ap) or else as a consequent part (cp) of both A and B.
2. (No loose pieces property). If A is a theorem of S and A contains no conjunctions as aps and no disjunctions as cps, every variable in A occurs once as ap and once as cp.

Our result generalizes that of Anderson and Belnap for the logics E and R (see [1], § 22.1.2).

Acknowledgments. Work supported by research project FFI2011-28494, financed by the Spanish Ministry of Economy and Competitiveness. G. Robles is supported by Program Ramón y Cajal of the Spanish Ministry of Economy and Competitiveness.

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▸ DENIS I. SAVELIEV, *On Zariski topologies on Abelian groups with operations*.
Department of Mathematical Logic and Theory of Algorithms, Faculty of Mechanics and Mathematics, M. V. Lomonosov Moscow State University, Vorobievy Gory, GSP-1, Main Building, Moscow, 119991, Russia.
E-mail: `d.i.saveliev@gmail.com`.

We consider universal algebras consisting of an Abelian group endowed with operations (of arbitrary arity) satisfying the generalized distributivity law, i.e., such that the unary operations obtaining from them by fixing all but one arguments are endomorphisms of the group. Instances of such algebras include rings, modules, linear algebras, differential rings, etc. Given such an algebra K, a closed basis of the Zariski topology on its Cartesian product K^n consists of finite unions of sets of solutions of equations $t(x_1, \dots, x_n) = 0$ for all terms t of n variables over K; it is the least T_1 topology in which all operations are continuous. We prove that for every such infinite K and any n, the space K^n is nowhere dense in the space K^{n+1}. A fortiori, all such K are nondiscrete (this fact was previously established for commutative associative rings by Arnautov [1]). Our proof uses a multidimensional generalization of Hindman's Finite Sums Theorem, a strong Ramsey-theoretic result obtained via algebra of ultrafilters [2].

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▸ DAN E. WILLARD, *On the allowed limited permissible extents in which various different self-justifying logics can formally recognize their own consistency*.
Computer Science & Mathematics Departments, University at Albany, NY 12222, USA.
E-mail: `dew@cs.albany.edu`.

We have published a series of articles during 2001–2009 about generalizations and boundary-case exceptions to the Second Incompleteness Theorem, including six papers in the ***JSL*** and ***APAL***. (Citations to these articles and a formalism that both unifies and extends their techniques can be found in [2].) Our goal in this talk is to summarize the significance of the latter's unification formalism.

Our partial evasions of the Second Incompleteness can obviously elude the force of Second Incompleteness Theorem under only unusual extremal circumstances because the combined work of Pudlák, Solovay, Nelson, and Wilkie–Paris implies that essentially all natural axiom systems, that merely recognize Successor as a total function, are unable to recognize their own consistency, when a Hilbert-style method of deduction is used. Our six journal articles and [2]'s unification formalism do show that boundary-case exceptions to the Second Incompleteness Theorem do exist when either:

(1) The assumption that successor is a total function is dropped, in a context where Addition and Multiplication are treated as two 3-way relations (e.g., see [1])

(2) Or when Addition is treated as a total function (e.g., as by "$\forall x\, \forall y\, \exists z\, \mathrm{Add}(x, y, z)$"), and the self-justifying system can recognize its consistency under a deduction method that lacks a Modus Ponens Rule, such as semantic tableaux,

It is obvious that Items (1) and (2) amount to being no more than being *Boundary-Case Exceptions* to the Second Incompleteness Theorem, in light of the aforementioned power of the Second Incompleteness Theorem. Our papers have diligently used the preceding italicized phrase, so as to avoid any possible confusion.

Our report [2] illustrates how such results are of epistemological interest because they explain how a Thinking Being can maintains at least some *instinctive* (partial) faith in its own consistency, despite the formidable barriers imposed by the Second Incompleteness Theorem.

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[2] ———, *A detailed examination of methods for unifying, simplifying and extending several results about self-justifying logics*, `http://arxiv.org/abs/1108.6330`.

▸ XUNWEI ZHOU, *Meaningfulness—meaninglessness duality for distinguished sets*.
Institute of Information Technology, Beijing Union University, 97 Beisihuandong Road, Beijing 100101, China.
E-mail: `zhouxunwei@263.net`.

An empty set has nothing in it. A universal set represents the universe, it is ubiquitous. In mutually inversistic set theory, an empty set and a universal set are called by a joint name distinguished sets. But in abstract set operations, distinguished sets are bound to yield, to be operated on. For example, the intersection of P and ∼P is the empty set $\emptyset$. In mutually inversistic set theory, $\mathrm{P} \cap \sim \mathrm{P} = \emptyset$ is called a quasi-set connection proposition. In mutually inversistic set theory, there is the meaningfulness—meaninglessness duality for distinguished sets: distinguished sets occurring in quasi-set connection propositions and power sets are meaningful, occurring elsewhere are meaningless. Meaningless distinguished sets correspond to proper classes in axiomatic set theory. Mutually inversistic set theory is logical-mathematical paradox-free. It is free from Russell's paradox, because $\mathrm{x} \notin \mathrm{x}$ is a universal set, a meaningless distinguished set. It is free from the greatest ordinal number paradox, because the set of all ordinal numbers is a universal set, a meaningless distinguished set. It is free from the greatest cardinal number paradox, because the set of all sets is a universal set, a meaningless distinguished set. There are logical-mathematical paradoxes in naïve set theory. Axiomatic set theory is logical-mathematical paradox-free, but is complex. It has to construct sets and classes in parallel. Mutually inversistic set theory is logical-mathematical paradox-free, and is simpler than axiomatic set theory. It need not introduce classes.