Macroeconomic Dynamics, **1**, 1997, 615–639. Printed in the United States of America.

FISCAL POLICY IN A GROWING ECONOMY WITH PUBLIC CAPITAL

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Public capital subject to congestion is introduced into an endogenous growth model and the transitional dynamic paths under alternative fiscal policies are characterized. Several new insights are obtained from this more general framework. During the transition, the two capital stocks always approach their common equilibrium growth rate from opposite directions. Government policy induces the more volatile response in the capital stock upon which it impinges most directly: private capital in the case of a tax, public capital in the case of expenditure. Finally, we characterize a time-varying income tax that enables the decentralized economy to replicate both the first-best transitional dynamics and steady-state equilibrium of a centrally planned economy. The steady-state component corrects for externalities that arise when government expenditure deviates from its social optimum, and the effects of congestion. The transitional component corrects for myopic behavior by the representative agent along the adjustment path.

Keywords: Fiscal Policy, Public Capital, Economic Growth

1. INTRODUCTION

The role of public investment is emerging as a major policy issue as governments around the world downsize and reassess what functions they should perform. Empirical research into the impact of public investment was stimulated by Aschauer (1989a,b) who suggested that public capital has a powerful impact on the productivity of private capital. Aschauer's results were controversial and have generated substantial empirical research directed at determining the robustness of his findings. Although the evidence regarding the productivity of public capital is mixed, there seems to be a consensus generally supporting the productivity of public investment, although suggesting that its impact is somewhat weaker than that originally argued by Aschauer.[1]

The theoretical analysis of the productivity of public investment has focused on its impact on the accumulation of private capital and output in the economy. Most of this work is based on Ramsey-type models having the characteristic that the economy converges either to a stationary state, in which all real variables including the capital stock remain constant, or to a balanced growth path along which the economy grows at some exogenously given rate. Government expenditure is

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https://doi.org/10.1017/S1365100597004045 Published online by Cambridge University Press

introduced as an argument in the production function to reflect, among other reasons, an externality in production. Two formulations can be identified. Most of the existing literature treats the current *flow* of government expenditure as the source of contribution to productive capacity; see, e.g., Aschauer (1988), Barro (1989), and Turnovsky and Fisher (1995). Although this specification has the virtue of tractability, it is open to the criticism that insofar as productive government expenditures are intended to represent public infrastructure, it is the accumulated *stock*, rather than the current flow, that is relevant.

Despite this criticism, few authors have adopted the alternative approach of specifying productive government expenditure as a stock. Arrow and Kurz (1970) were the first authors to formulate government expenditure as a form of investment. More recently, Baxter and King (1993) study the macroeconomic implications of increases in the stocks of public goods. They derive the transitional dynamic responses of output, investment, consumption, employment, and interest rates to such policies by calibrating a real business-cycle model.

The impact of fiscal policy, including the role of government capital, on long-term economic growth is an important policy issue. However, the Ramsey model, with its steady-state growth rate being determined by long-term demographic and technological factors, and therefore independent of the usual macroeconomic policy instruments, does not provide an appropriate framework for addressing this question. By contrast, the recent endogenous-growth literature places particular emphasis on fiscal policy as a determinant of long-run national growth rates and growth differentials.[2] As in the Ramsey model, most authors have introduced productive government expenditure as a flow; see, e.g., Barro (1990), Turnovsky (1996a,b). These studies therefore are subject to the limitations noted above.

A recent exception is a paper by Futagami et al., (1993), which introduces government capital as a pure public good, along with private capital, in an otherwise standard endogenous-growth model.[3] They show how this yields transitional dynamics, in contrast to models in which government expenditure impacts as a flow, when the economy is always on its balanced growth path. The authors employ their model to analyze the transitional dynamic effects of a change in the income tax rate, as well as its long-run consequences.

Although the Futagami–Morita–Shibata (FMS) model represents an important step forward, it is limited in certain key respects. First, by treating government capital as a pure public good, it fails to take account of the congestion typically associated with public capital. As Barro and Sala-i-Martin (1995) and others have argued, almost all public services are characterized by some degree of congestion. Even national defense, sometimes cited as the purest of public goods, is not congestion-free. These considerations suggest that the incorporation of congestion is an important consideration in assessing the effect of public capital on economic growth. Second, although the FMS analysis is based on a decentralized economy, its specification of fiscal policy is restrictive. The assumption of a continuously balanced budget in which the only tax is an income tax, the (endogenous) revenue of which is spent on productive capital, makes the economy behave essentially like

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a centrally planned economy. In effect, the income tax provides the mechanism whereby the central planner appropriates the resources from the private sector. Thus, insofar as the resources withdrawn from the private sector are reinvested productively by the government, raising the income tax has both a contractionary effect and a stimulating effect. The fact that this restrictive form of fiscal policy gives rise to both a growth-maximizing the welfare-maximizing tax rate is a reflection of the dual role played by the income tax rate.

The objective of the present paper is to redress these two shortcomings, by introducing congestion and a more complete array of fiscal instruments; in addition to an income tax, the government may impose a consumption tax, as well as issuing debt.[4] In contrast to the usual specification of congestion in macro growth models, which is typically to normalize aggregate government expenditure by the size of the economy, we allow for a more general parameterization of the degree of congestion, using a form of congestion function from the public goods literature. This is important because the degree of congestion turns out to be an important determinant of optimal tax policy. The introduction of government debt enables the tax and expenditure effects to be decoupled, thereby clarifying the respective roles played by each in the growth process.

The paper begins by deriving the equilibrium in a centrally planned economy, briefly characterizing its steady-state and dynamic properties. The main purpose is to provide a benchmark against which the decentralized economy can be assessed. The effects of both permanent and temporary tax shocks in such an economy are analyzed.

An important aspect of our analysis concerns the design of an optimal tax policy in an economy with gradually accumulating public capital. With the economy now following a transitional dynamic path, this requires the introduction of a more flexible tax scheme than if the economy is always on its balanced growth path, when, for example, a *fixed* income tax in conjunction with a *fixed* consumption tax—the latter essentially acting as a lump-sum tax—can replicate the first-best optimum; see, e.g., Turnovsky (1996a). To attain the first-best optimum, the decentralized economy must replicate both the steady-state equilibrium and the transitional adjustment path followed by the former. This requires the optimal income tax to consist of two components: one constant and permanent, the other time-varying and transitory. The former is required to correct for two potential permanent sources of externalities that arise from: the deviation of the actual stock of public capital from its optimum, and the degree of congestion. The latter is required to induce the right mix between private and public capital so as to ensure that the optimum transitional path is followed.

2. ANALYTICAL FRAMEWORK

We consider an economy populated by identical representative agents who consume a private consumption good C, deriving intertemporal utility represented by

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the isoelastic utility function

$$W \equiv \int_0^\infty \frac{1}{\gamma} C^\gamma e^{-\rho t}\, dt \qquad -\infty < \gamma < 1, \tag{1a}$$

where the exponent γ is related to the intertemporal elasticity of substitution, $s = 1/(1-\gamma)$, with $\gamma = 0$ corresponding to the logarithmic utility function.

Output, y, of the representative firm takes place by means of a constant returns-to-scale production function specified in the form

$$y = \alpha \left(\frac{K_g^s}{k} \right)^\beta k \qquad \alpha > 0; \quad 0 < \beta < 1, \tag{1b}$$

where k denotes the representative firm's capital stock of private capital and K_g^s denotes the services derived by the firm from its use of public capital. Equation (1b) embodies the assumption that the services of public capital enhance the productivity of private capital, though at a diminishing rate. Neither form of capital is subject to depreciation. The model abstracts from labor so that private capital should be interpreted broadly to include human, as well as physical, capital; see Rebelo (1991).

The productive services derived by the agent from government expenditure are represented by

$$K_g^s = K_g (k/K)^{1-\sigma} \qquad 0 \le \sigma \le 1, \tag{1c}$$

where K_g denotes aggregate public capital and K denotes the aggregate private capital stock. Equation (1c) incorporates the possibility that the public capital may be associated with congestion.[5] The specification in (1c) characterizes what one can call relative congestion, in that the productive services derived by an individual agent from a given stock of public capital is enhanced as his/her individual capital stock increases relative to the aggregate.[6] This encourages the use of private capital and is important in the determination of the optimal tax rate.[7] In particular, (1c) implies that, for the level of public capital services, K_g^s, available to the individual firm to remain constant over time, given its individual capital stock, k, the growth rate of K_g must be related to that of K in accordance with $\dot{K}_g/K_g = (1-\sigma)\dot{K}/K$ so that σ parameterizes the degree of (relative) congestion associated with the public good.

The case $\sigma = 1$ corresponds to a nonrival, non-excludable public capital good that is available equally to each firm, independent of the size of the economy; there is no congestion. There are few examples of such pure public goods, so that this case should be treated largely as a benchmark. At the other extreme, if $\sigma = 0$, then only if K_g increases in direct proportion to the aggregate capital stock, K, does the level of the public service available to the individual firm remain fixed. We refer to this case as being one of *proportional* congestion, meaning that the congestion grows in direct proportion to the size of the economy.[8] Road services and infrastructure that play a productive role in facilitating the distribution of the firm's output may serve as examples of public goods subject to this type of

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congestion. In between, $0 < \sigma < 1$ describes partial congestion, where K_g can increase at a slower rate than does K and still maintain a fixed level of public services to the firm.[9] This intermediate case also can be thought of as being some aggregate composite of the types of public capital we have noted.

Substituting (1c) into (1b), the firm's production function can be expressed as

$$y = \alpha \left[\frac{K_g}{K} \left(\frac{K}{k} \right)^{\sigma} \right]^{\beta} k. \tag{1b'}$$

Provided $\sigma \neq 1$, so that the public good is associated with some congestion, aggregate private capital is introduced into the production function of the individual firm in a way analogous to that of Romer (1986).

With all agents being identical, the aggregate and individual capital stocks are related by $K = Nk$, where N is the number of representative agents (firms). Thus, in equilibrium, the individual output y and aggregate output $Y = Ny$ may be expressed as

$$y = \alpha \left(\frac{K_g}{K} N^{\sigma} \right)^{\beta} k; \qquad Y = \alpha \left(\frac{K_g}{K} N^{\sigma} \right)^{\beta} K. \tag{1b''}$$

The critical difference between the perception of the world as seen by the representative firm and as seen by the central planner is as follows. The representative firm treats the aggregate capital stock K as given, with the relationship $K = Nk$, as employed in (1b″) holding as an equilibrium one. The central planner, on the other hand, takes this relationship into account when determining his/her decisions.

For expositional convenience, we set the number of agents $N = 1$, enabling us to drop the distinction between aggregate and individual quantities in equilibrium. Although this normalization is not innocuous, it suffices for our purposes. In the absence of this normalization, the growth equilibrium would remain identical to the one that we derive below, with the exception that the productivity parameter, α, is scaled to αN^{σ}. Thus, the tax and expenditure effects upon which we are focusing remain unchanged. However, a change in σ would lead to a change in αN^{σ} (for $N \neq 1$) and this would need to be considered in assessing the impact of a change in the degree of congestion on the equilibrium.[10]

We assume that new output can be transformed costlessly either to consumption, to new private capital, or to new public capital, subject to the economywide resource constraint

$$Y = C + \dot{K} + \dot{K}_g. \tag{2a}$$

With costless adjustment and no depreciation, private capital and private investment, I, are related by

$$\dot{K} = I. \tag{2b}$$

For an equilibrium with steady ongoing growth to be sustained, the current flow of government expenditure, G, must itself be tied to some index of growth in the economy. Whereas several such measures are plausible, a natural case to

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consider is that the central planner sets gross public investment as proportional of output[11]:

$$\dot{K}_g = G = gY \qquad 0 < g < 1. \tag{2c}$$

In analyzing the centrally planned economy, it is instructive to proceed in two stages. In the first, we assume that g is set arbitrarily in (2c), whereas in the second, g is set optimally along with C and K. This latter case is equivalent to optimizing directly with respect to G (along with C and K) and the outcome is independent of the specific constraint $G = gY$. One advantage of this two-stage approach is that it preserves comparability of our analysis with much of the existing literature which, by tying expenditures to tax revenues for a given tax rate is essentially assuming that g is set arbitrarily.[12] More important, this approach enhances our understanding of the optimal tax structure which, as we show in equation (20), reflects two potential sources of externalities for the representative agent: one due to deviations in government expenditure from its social optimum; and another due to the degree of congestion, σ, associated with the public good.

3. CENTRALLY PLANNED ECONOMY

As a benchmark, it is convenient to begin with the case in which the government acts as a central planner and chooses all quantities directly to maximize intertemporal utility, (1a), subject to the production function (1b′), the aggregate resource constraint (2a), and the accumulation equations (2b) and (2c). With the normalization $N = 1$, the present-value Hamiltonian for this optimization is

$$H \equiv (1/\gamma)C^{\gamma}e^{-\rho t} + \nu e^{-\rho t}\left[(1-g)\alpha K_g^{\beta}K^{1-\beta} - C - \dot{K}\right] + \mu e^{-\rho t}\left[g\alpha K_g^{\beta}K^{1-\beta} - \dot{K}_g\right], \tag{3}$$

where ν is the shadow value (marginal utility) of private capital in the form of new output and μ is the shadow value of public capital. The analysis is simplified by using the shadow value of private capital as numeraire. Consequently, $q \equiv \mu/\nu$ is the value of public capital measured in terms of units of private capital. Likewise, the subsequent dynamics can be expressed conveniently in terms of quantities relative to the stock of private capital, namely, $c \equiv C/K$; $z \equiv K_g/K$.

The optimality conditions with respect to C, K, and K_g, for given g, thus can be expressed as

$$C^{\gamma-1} = \nu, \tag{4a}$$

$$(1-\beta)\alpha z^{\beta}[(1-g) + qg] = \rho - (\dot{\nu}/\nu), \tag{4b}$$

$$[(\beta\alpha z^{\beta-1})/q][(1-g) + qg] + (\dot{q}/q) = \rho - (\dot{\nu}/\nu). \tag{4c}$$

Equation (4a) equates the marginal utility of consumption to the shadow value of private capital. The left-hand side of equation (4b) describes the marginal return to investing in a unit of private capital. This comprises three components. The first is the gross marginal physical product $\partial Y/\partial K \equiv (1-\beta)\alpha z^{\beta}$. With government

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investment tied to output in accordance with (2c), private investment also induces an increase in public capital $g(\partial Y/\partial K)$, which is valued at its imputed real price q. Offsetting this are the resource costs embodied in the public capital, $g(\partial Y/\partial K)$, the price of which in terms of the numeraire is unity. In equilibrium, the sum of these components must equal the return to consumption, measured in terms of private capital as numeraire, and given by the right-hand side of (4b). Equation (4c) describes the analogous relationship for public capital. The only difference is that the return to investing in public capital, when expressed in terms of private capital as numeraire, includes the rate of capital gains, $\dot{q}/q$.

Dividing the public capital accumulation equation (2c) by K_g, while noting the production function, the growth rate of public capital, ϕ_g, may be expressed as

$$\phi_g \equiv \frac{\dot{K}_g}{K_g} = g\alpha z^{\beta-1}. \qquad \textbf{(5a)}$$

Likewise, dividing the goods market condition (2a) by K, and noting (5a), the growth rate of private capital, ϕ_k, becomes

$$\phi_k \equiv (\dot{K}/K) = (1-g)\alpha z^{\beta} - c. \qquad \textbf{(5b)}$$

Using these relationships, the dynamics of the centrally planned economy can be represented by the system of equations

$$(\dot{z}/z) = g\alpha z^{\beta-1} - (1-g)\alpha z^{\beta} + c, \qquad \textbf{(6a)}$$

$$\frac{\dot{c}}{c} = \frac{[(1-g)+qg]\alpha(1-\beta)z^{\beta} - \rho}{1-\gamma} - (1-g)\alpha z^{\beta} + c, \qquad \textbf{(6b)}$$

$$\dot{q} = [(1-\beta)zq - \beta]\alpha z^{\beta-1}[(1-g)+qg]. \qquad \textbf{(6c)}$$

The first of these equations describes the differential growth rate between public and private capital and is obtained by substituting (5a) and (5b) into the relationship $\dot{z}/z = \dot{K}_g/K_g - \dot{K}/K$. The second equation determines the differential growth rate between consumption and private capital. It is obtained from the relationship $\dot{c}/c = \dot{C}/C - \dot{K}/K$, where $\dot{C}/C$ is obtained by combining the time derivative of (4a) with (4b). The third equation is obtained by equating (4b) and (4c). To ensure that the intertemporal resource constraint is met, the following transversality conditions must hold:

$$\lim_{t\to\infty} \nu K e^{-\rho t} = 0; \qquad \lim_{t\to\infty} \mu K_g e^{-\rho t} = 0. \qquad \textbf{(7)}$$

3.1. Steady State

The steady state of this system is characterized by $\dot{z} = \dot{c} = \dot{q} = 0$, so that all real quantities grow at a common rate, with the shadow value of public capital being

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constant. This implies the following relationships, where tildes denote steady-states:

$$\tilde{c} = (1-g)\alpha\tilde{z}^{\beta} - g\alpha\tilde{z}^{\beta-1}, \tag{8a}$$

$$\frac{[(1-g)+\tilde{q}g]\alpha(1-\beta)\tilde{z}^{\beta}-\rho}{1-\gamma} - (1-g)\alpha\tilde{z}^{\beta} + \tilde{c} = 0, \tag{8b}$$

$$\tilde{z}\tilde{q} = \beta/(1-\beta). \tag{8c}$$

Thus, in steady-state equilibrium, the relative value of public to total capital, $\tilde{z}\tilde{q}/(1+\tilde{z}\tilde{q})$, equals β, the elasticity of public capital in the production function. Given $\tilde{q}$, the other two equations determine the ratios of consumption to private capital and public to private capital, consistent with a balanced-growth equilibrium.

However, equations (8a–c) may, or may not, be consistent with a well-defined steady state in which $\tilde{c} > 0$, $\tilde{z} > 0$. To see this, first substitute (8c) into (8b). The resulting equation, together with (8a) then defines a pair of nonlinear equations in $\tilde{c}$ and $\tilde{z}$. From this pair of equations, one can establish that a necessary and sufficient condition for these two loci to intersect in the positive quadrant and therefore for a well-defined steady state $\tilde{c} > 0$, $\tilde{z} > 0$ (and consequently $\tilde{q} > 0$) to obtain, is that[13]

$$\rho - \frac{\alpha\gamma g^{\beta}}{(1-g)^{\beta-1}} > 0. \tag{9}$$

This condition is clearly met if the utility function is logarithmic, but it may be violated if the share of output claimed by government investment is large and $\gamma > 0$. In this case, the large intertemporal elasticity of substitution indicates that the agent has a preference for consumption, leaving insufficient output available to enable private capital to maintain the same (high) growth rate as public capital, required to maintain a balanced-growth equilibrium. Henceforth, we assume that (9) is met, in which case, using (8), the corresponding (common) equilibrium growth rate of consumption and the two capital stocks may be expressed in the following equivalent forms:

$$\begin{aligned}\tilde{\phi} \equiv \frac{\tilde{\dot{C}}}{C} = \frac{\tilde{\dot{K}}}{K} = \frac{\tilde{\dot{K}}_g}{K_g} &= g\alpha\tilde{z}^{\beta-1} = (1-g)\alpha\tilde{z}^{\beta} - \tilde{c} \\ &= \frac{[(1-g)(1-\beta)\tilde{z} + \beta g]\tilde{z}^{\beta-1} - \rho}{1-\gamma}.\end{aligned} \tag{10}$$

Finally, it is straightforward to establish that this equilibrium is consistent with the transversality condition (7), so that it is intertemporally viable.[14]

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3.2. Equilibrium Dynamics

Because the dynamic system (6) is nonlinear, we proceed by considering the linearized dynamics about steady-state:

$$\begin{pmatrix} \dot{z} \\ \dot{c} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} -\alpha[g(1-\beta)+(1-g)\beta\tilde{z}]\tilde{z}^{\beta-1} & \tilde{z} & 0 \\ \dfrac{[(\gamma-\beta)(1-g)+\tilde{q}g(1-\beta)]\alpha\beta\tilde{z}^{\beta-1}\tilde{c}}{1-\gamma} & \tilde{c} & \dfrac{g\alpha(1-\beta)\tilde{z}^{\beta}\tilde{c}}{1-\gamma} \\ (1-\beta)\tilde{q}[(1-g)+\tilde{q}g]\alpha\tilde{z}^{\beta-1} & 0 & (1-\beta)[(1-g)+\tilde{q}g]\alpha\tilde{z}^{\beta} \end{pmatrix} \times \begin{pmatrix} z-\tilde{z} \\ c-\tilde{c} \\ q-\tilde{q} \end{pmatrix}. \qquad \textbf{(11)}$$

One can show that the linearized system (11) has two unstable eigenvalues and one stable eigenvalue, $\lambda < 0$. It is therefore a saddlepoint. As usual, the shadow value, q, can respond instantaneously to new information, and the consumption-to-capital ratio, c, is also free to jump. With both types of capital being constrained to continuous accumulation, we assume that, in general, their ratio, z, also is constrained to continuous adjustments. However, in the case in which all resources are under the control of the central planner, and there are no adjustment costs, we permit the central planner to choose the initial ratio, z_0, by transforming one type of capital to another; see Section 3.3.

Starting from a given initial ratio of the two types of capital, z_0, the stable solution in the centralized economy is

$$z(t) - \tilde{z} = (z_0 - \tilde{z})e^{\lambda t}, \qquad \textbf{(12a)}$$

$$c(t) - \tilde{c} = (1/\tilde{z})[g(1-\beta)\alpha\tilde{z}^{\beta-1} + (1-g)\beta\alpha\tilde{z}^{\beta} + \lambda](z - \tilde{z}), \qquad \textbf{(12b)}$$

$$q(t) - \tilde{q} = -\frac{(1-\beta)\tilde{q}[(1-g)+g\tilde{q}]\alpha\tilde{z}^{\beta-1}}{[(1-\beta)[(1-g)+g\tilde{q}]\alpha\tilde{z}^{\beta} - \lambda]}(z - \tilde{z}). \qquad \textbf{(12c)}$$

Equation (12c) implies that as the ratio of public capital to private capital increases, the shadow value of public capital in terms of that of private capital declines.

3.3. Optimal Government Expenditure

The nature of the equilibrium changes dramatically when instead of setting the public investment share of output, g, arbitrarily, the government does so optimally. Setting $\partial H/\partial g = 0$ in (3) leads to the condition $q = 1$, implying that $\dot{q} \equiv 0$. It then follows from (6c) that $z = \beta/(1-\beta)$, so that $\dot{z} \equiv 0$. With z constant, (6b), together with the transversality condition, implies that c must also be constant.

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Thus the first-best equilibrium, where the government chooses its expenditure optimally, requires that the system always be at its steady state [in this case denoted by ($\hat{\ }$)]:

$$\hat{q} = 1, \qquad \hat{z} = \frac{\beta}{(1-\beta)}, \qquad \hat{c} = \frac{\rho - \alpha\gamma(1-\beta)\hat{z}^{\beta}}{(1-\beta)(1-\gamma)}, \tag{13}$$

in which the shadow values of the two capital stocks are equal, and the share of public capital to the total capital stock equals β, its elasticity in production. The planner transforms one form of existing capital to the other so as to attain the optimal ratio as determined in (13) and there are no dynamics. The corresponding share of output devoted to government expenditure and the steady-state growth rate are given by

$$\hat{g} = \beta - \frac{\beta\hat{c}}{\alpha\hat{z}^{\beta}} < \beta, \qquad \hat{\phi} = \frac{(1-\beta)\alpha\hat{z}^{\beta} - \rho}{1-\gamma}. \tag{14}$$

3.4. Long-Run Effects of Fiscal Expansion

The long-run effects of a fiscal expansion in the centrally planned economy are described by the following expressions:

$$\frac{d\tilde{z}}{dg} = \frac{\tilde{z}\left[1 + \dfrac{1}{1-\gamma}[(1-\beta)\tilde{z} - \beta]\right]}{(1-\beta)\left[g + \dfrac{\beta}{1-\gamma}[(1-g)\tilde{z} - g]\right]}, \tag{15a}$$

$$\frac{d\tilde{c}}{dg} = \frac{\gamma\tilde{z}^{\beta}(g-\beta)}{(1-\beta)(1-\gamma)\left[g + \dfrac{\beta}{1-\gamma}[(1-g)\tilde{z} - g]\right]}, \tag{15b}$$

$$\frac{d\tilde{\phi}}{dg} = \frac{\tilde{z}^{\beta}(\beta - g)}{(1-\gamma)\left[g + \dfrac{\beta}{1-\gamma}[(1-g)\tilde{z} - g]\right]}, \tag{15c}$$

where recalling (8a) (and $\tilde{c} > 0$) we have $(1-g)\tilde{z} > g$. The most significant result is that the long-run growth-maximizing rate of government expenditure, $\bar{g}$ say, is determined where $\bar{g} = \beta$, as in the flow model of Barro (1990). Comparing $\bar{g} = \beta$ with (14), we find that as in Futagami et al. (1993), this value exceeds the welfare-maximizing value of government expenditure. This contrasts with the flow model when the two quantities coincide. The difference is accounted for by the fact that, when government expenditure influences production as a flow, maximizing the growth rate of capital is equivalent to maximizing the growth rate of consumption and therefore to maximizing its level at each instant of time, thereby maximizing overall intertemporal welfare. By contrast, when government expenditure affects output as a stock, consumption is foregone in the process of accumulating public capital. Maximizing the common growth rate of the two types

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of capital involves a consumption loss. The central planner is better off reducing the growth rate, thereby enjoying more consumption.[15]

Equation (15a) implies that if $\tilde{z} \geq \hat{z}$, so that the economy does not have a shortage of public capital, then if the central planner increases the share of output devoted to public capital, the long-run ratio of public to private capital is increased. However, if $\tilde{z} < \hat{z}$, one cannot rule out the possibility that the short-run growth in private capital generated by the increase in g will be sufficiently great as to reduce the steady-state ratio of public to private capital.

4. DECENTRALIZED ECONOMY

We now consider the representative agent in a decentralized economy. The objective of the agent is to maximize his constant elasticity utility function, (1a), subject to his accumulation of private capital, (2a), and flow budget constraint, represented by (recalling the normalization $N = 1$)

$$\dot{K} + \dot{B} = r(1-\tau)B + (1-\tau)Y - (1+\omega)C - T, \tag{16}$$

where B is the stock of government bonds paying an interest rate r, τ is the rate of income tax, ω is the rate of consumption tax, and T denotes time-varying lump-sum taxes. In performing this optimization, the agent is assumed to treat the stock of public capital as given and independent of his own decisions. Thus in (1b′) K_g and the aggregate capital stock K are taken as given, though with the normalization, the condition $k = K$ is assumed to hold in equilibrium.

Two further points concern the specification of the tax rates. First, for the present, we assume that τ and ω are fixed through time, being subject to, at most, once-and-for-all policy changes at discrete times. As we show in Section 5, to replicate the first-best optimum, τ will need to be time-varying.

The agent's decisions are to choose his consumption level, C, private capital stock, K, and holdings of government bonds, B, leading to the optimality conditions

$$(C^*)^{\gamma-1} = \nu^*(1+\omega), \tag{17a}$$

$$r = \alpha(1-\beta\sigma)(z^*)^{\beta}, \tag{17b}$$

$$(1-\tau)\alpha(1-\beta\sigma)(z^*)^{\beta} = \rho - (\dot{\nu}^*/\nu^*), \tag{17c}$$

where the asterisk identifies the decentralized equilibrium. Equation (17a) equates the marginal utility of consumption to the consumption tax-adjusted marginal utility of wealth. Equation (17b) asserts that the equilibrium interest rate equals the marginal physical product of private capital. The latter reflects the fact that more (relative) congestion, i.e., a smaller σ, raises the productivity of private capital in the sense that it increases the quantity of the productive services of public capital derived by the individual agent, as he increases k. The final equation equates the after-tax marginal physical product of capital (and the corresponding real return on bonds) to the rate of return on consumption.

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The dynamics of the equilibrium in the decentralized economy are represented by

$$(\dot{z}^*/z^*) = g\alpha(z^*)^{\beta-1} - (1-g)\alpha(z^*)^{\beta} + c^*, \tag{6a$'$}$$

$$\frac{\dot{c}^*}{c^*} = \frac{(1-\tau)\alpha(1-\beta\sigma)(z^*)^{\beta} - \rho}{1-\gamma} - (1-g)\alpha(z^*)^{\beta} + c^*, \tag{6b$'$}$$

and the relevant transversality conditions are now

$$\lim_{t\to\infty} \nu^* K^* e^{-\rho t} = 0; \qquad \lim_{t\to\infty} \nu^* B e^{-\rho t} = 0. \tag{7$'$}$$

The difference between the evolution of the decentralized economy is that the dynamics now are represented by a pair of equations in z^* and c^* and proceed independently of the shadow value of public capital, q. This is because the private agent in the decentralized economy responds to the given tax rate, τ, in contrast to the central planner who takes account of the endogenously evolving shadow value of public capital, q.

4.1. Steady State

Steady-state equilibrium is determined by

$$\tilde{c}^* = (1-g)\alpha(\tilde{z}^*)^{\beta} - g\alpha(\tilde{z}^*)^{\beta-1}, \tag{8a$'$}$$

$$\frac{(1-\tau)\alpha(1-\beta\sigma)(\tilde{z}^*)^{\beta} - \rho}{1-\gamma} - (1-g)\alpha(\tilde{z}^*)^{\beta} + \tilde{c}^* = 0, \tag{8b$'$}$$

which jointly determine the equilibrium values of $\tilde{z}^*$ and $\tilde{c}^*$. Analogous to the centrally planned economy, there may or may not be a viable solution to these equations, and a condition similar to (9) can be derived.[16] The steady-state equilibrium growth rate now is given by the first pair of equations in (10). The transversality condition now requires $(1-\tau)(1-\beta\sigma)\tilde{z}^* > g$, or equivalently, that $\rho > \gamma g\alpha(\tilde{z}^*)^{\beta-1}$. As long as $\gamma > 0$, this imposes a restriction on the government policy parameters g and τ, though the condition certainly will be met when these policy parameters are chosen optimally so as to replicate the equilibrium of the centrally planned economy; see equation (20).

4.2. Equilibrium Dynamics

The linearized equilibrium dynamics for the decentralized economy are described by

$$\begin{pmatrix} \dot{z}^* \\ \dot{c}^* \end{pmatrix} = \begin{pmatrix} -\alpha[g(1-\beta) + (1-g)\beta\tilde{z}^*](\tilde{z}^*)^{\beta-1} & \tilde{z}^* \\ \left[\frac{(1-\tau)\alpha(1-\beta\sigma)\beta(\tilde{z}^*)^{\beta-1}}{1-\gamma} - (1-g)\alpha\beta(\tilde{z}^*)^{\beta-1}\right]\tilde{c}^* & \tilde{c}^* \end{pmatrix} \times \begin{pmatrix} z^* - \tilde{z}^* \\ c^* - \tilde{c}^* \end{pmatrix}, \tag{11$'$}$$

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which again can be shown to be a saddlepoint, with the stable eigenvalue being $\lambda^* < 0$. The dynamic paths for z^* and c^* are described by relationships analogous to (12a) and (12b), namely,

$$z^*(t) - \tilde{z}^* = \left(z_0^* - \tilde{z}^*\right)e^{\lambda^* t}, \quad \textbf{(12a}'\textbf{)}$$

$$c^*(t) - \tilde{c}^* = (1/\tilde{z}^*)[g(1-\beta)\alpha(\tilde{z}^*)^{\beta-1} + (1-g)\beta\alpha(\tilde{z}^*)^{\beta} + \lambda^*](z^* - \tilde{z}^*). \quad \textbf{(12b}'\textbf{)}$$

It is convenient to focus on the transitional dynamics in terms of the growth rates of the two types of capital. Recalling (5a) and (5b), the respective growth rates in the decentralized economy are

$$\phi_g^* \equiv \left(\dot{K}_g^*/K_g^*\right) = g\alpha(z^*)^{\beta-1}, \quad \textbf{(5a}'\textbf{)}$$

$$\phi_k^* \equiv (\dot{K}^*/K^*) = (1-g)\alpha(z^*)^{\beta} - c^*. \quad \textbf{(5b}'\textbf{)}$$

In steady-state equilibrium, the ratio of public to private capital remains constant, so that both types of capital grow asymptotically at the same rate $\tilde{\phi}^*$. Thus, the linearized transitional paths followed by the growth rates of the capital stocks are

$$\phi_g^* - \tilde{\phi}^* \equiv \left(\dot{K}_g^*/K_g^*\right) - \tilde{\phi}^* = -g\alpha(1-\beta)(\tilde{z}^*)^{\beta-2}(z^* - \tilde{z}^*), \quad \textbf{(18a)}$$

$$\phi_k^* - \tilde{\phi}^* \equiv (\dot{K}^*/K^*) - \tilde{\phi}^* = -(1/\tilde{z}^*)[g\alpha(1-\beta)(\tilde{z}^*)^{\beta-1} + \lambda^*](z^* - \tilde{z}^*). \quad \textbf{(18b)}$$

These are illustrated in Figure 1. The locus *XX* corresponds to the stable transitional adjustment path in the growth of private capital and can be shown to be positively sloped[17]; the corresponding path for public capital is illustrated by the negatively sloped locus, *YY*. The key feature of the adjustment is that, during any transition, the growth rates of the two forms of capital are moving in opposite directions. This is because the transitional dynamics are driven by the ratio of public to private capital, z^*, and the fact that, as this ratio increases, the productivity of private capital rises, whereas that of public capital declines.

4.3. Steady-State Fiscal Effects

The long-run effects of fiscal policies on the relative capital stock, z^*, consumption ratio, c^*, and the growth rate, ϕ^*, are obtained by considering (8a′), (8b′), and (10). We discuss the effects of changes in the income tax rate; and the share of government expenditure, assuming that the government budget constraint is met through an appropriate adjustment in debt, or equivalently in lump-sum taxes. The equilibrium is independent of the consumption tax, ω, which therefore also operates as a lump-sum tax, and also may serve as the balancing item in the government budget.

Omitting details, the following results can be established:

$$\frac{\partial \tilde{z}^*}{\partial \tau} = \frac{(\tilde{z}^*)^2}{(1-\gamma)J} > 0; \qquad \frac{\partial \tilde{z}^*}{\partial g} = \frac{\tilde{z}^*}{(1-\beta\sigma)J} > 0, \quad \textbf{(19a)}$$

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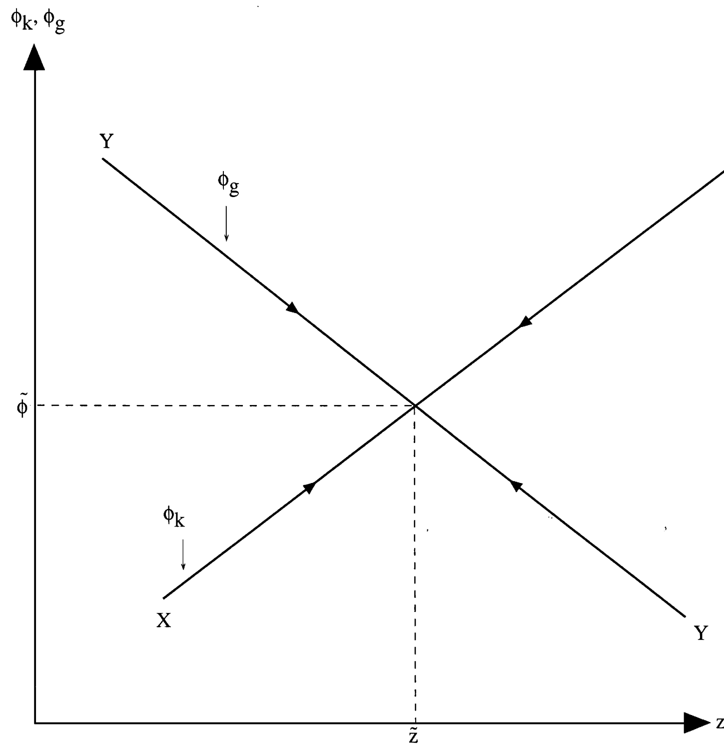

Figure 1. Stable adjustment paths for growth rates of public and private capital.

$$\frac{\partial \tilde{c}^*}{\partial \tau} = \frac{\alpha (\tilde{z}^*)^\beta [(1-g)\beta \tilde{z}^* + g(1-\beta)]}{(1-\gamma)J} > 0;$$
$$\frac{\partial \tilde{c}^*}{\partial g} = \frac{\alpha (\tilde{z}^*)^\beta \left[\left(\frac{\beta - g}{1-\beta\sigma}\right) - \frac{(1-\tau)\beta(1+\tilde{z}^*)}{1-\gamma}\right]}{J}, \tag{19b}$$

$$\frac{\partial \tilde{\phi}^*}{\partial \tau} = -\frac{g\alpha(1-\beta)(\tilde{z}^*)^\beta}{(1-\gamma)J} < 0; \qquad \frac{\partial \tilde{\phi}^*}{\partial g} = \frac{\alpha\beta(1-\tau)(\tilde{z}^*)^\beta}{(1-\gamma)J} > 0, \tag{19c}$$

where

$$J \equiv \left(\frac{1-\beta}{1-\beta\sigma}\right) g + \frac{\beta(1-\tau)\tilde{z}^*}{(1-\gamma)} > 0.$$

A reduction, say, in the income tax rate, τ, raises the net rate of return to private capital, thereby inducing investors to switch from consumption to saving, thus lowering the consumption/capital ratio and increasing the growth rate of private capital. This positive effect on the return to private capital favors its accumulation and leads to a long-run decline in the ratio of public to private capital.[18]

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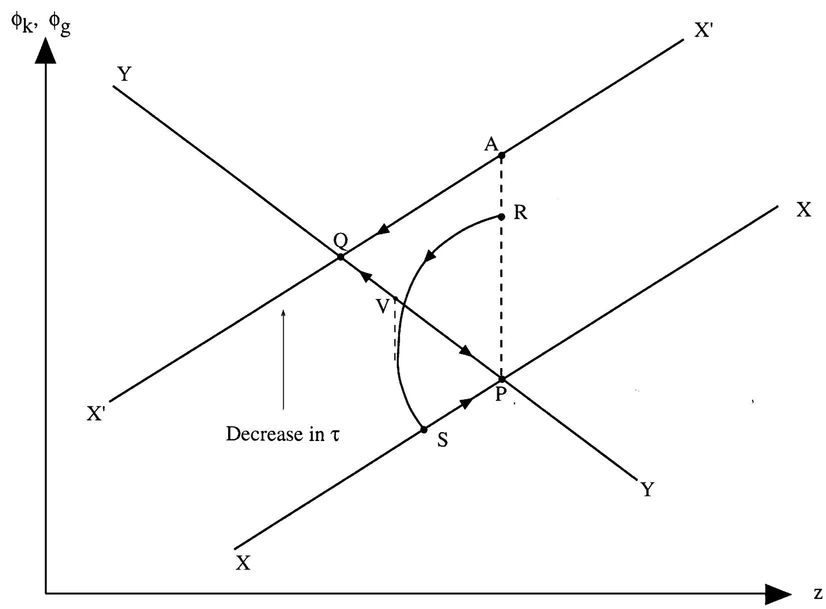

FIGURE 2. Transitional dynamics of capital: tax cut.

In contrast to the centrally planned economy, an increase in the share of output claimed by the government, financed by a lump-sum tax (or bonds), raises the equilibrium growth rate of capital unambiguously. This is because lump-sum taxation avoids the excess burden associated with distortionary taxes, which the central planner is effectively imposing.[19]

The case considered by Futagami et al. (1993) in which government expenditure is determined by tax revenues, corresponds to $g = \tau$ and hence $dg = d\tau$. It is straightforward to verify that such a balanced-budget increase in g leads to responses that are qualitative similar to those in the centrally planned economy. In particular, whereas the increase in g raises the growth rate, the corresponding increase in τ has the opposite effect, rendering a net effect that depends upon $(\beta - g)$, precisely as in the centralized economy; see (15c).

4.4. Transitional Dynamics

Figure 2 illustrates the transitional dynamics in private and public capital in response to a fiscal expansion taking the form of a cut in the income tax rate, financed by a lump-sum tax. Both permanent and temporary tax cuts are discussed.

Suppose that the economy is initially in steady-state equilibrium at the point P and that a *permanent* tax cut is introduced. The immediate effect of the lower tax is to raise the net return to private capital, inducing agents to reduce their level of consumption and to increase their rate of accumulation of private capital.

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This increase in the growth of private capital causes the ratio of public to private capital, z^*, to begin to decline. As z^* declines, the average productivity of private capital, $\alpha(z^*)^\beta$ falls, causing its growth rate to decrease; see (5b′). The transitional adjustment in the growth rate of private capital is illustrated by the initial jump from P to A, on the new stable arm $X'X'$, followed by the continuous decline AQ, to the new steady state at Q. With the growth of public capital being tied through aggregate output to the capital stocks in accordance with (2b), the growth rate of public capital does not respond instantaneously to the lower tax rate, τ. The stable arm YY remains fixed. Instead, as z^* declines, the average productivity of public capital $\alpha(z^*)^{\beta-1}$ rises, causing the growth rate of public capital to rise gradually over time along the path PQ.

Now, suppose that the tax decrease announced and implemented at time 0 is only *temporary*, lasting until time T. Being only temporary, it is discounted partially by the private agent, so that the initial decline in consumption is reduced, thereby inducing a smaller initial increase in the growth rate of private capital, ϕ_k^*. This takes it to the point R lying below A. At that point the economy begins to follow the unstable locus RS, reaching the point of intersection S with the original stable saddlepath XX at time T, when the tax rate is restored to its original level. Thereafter, the growth rate of the private capital stock follows the locus XX back to the original steady-state equilibrium P.

With the initial increase in the growth rate, ϕ_k^*, being scaled down, the initial rate of decline in the ratio of public to private capital is reduced, implying that the rate of decline of private capital growth is larger than if the tax cut were permanent. Indeed, by the time the growth rate of private capital reaches the point S during the transition, ϕ_k^* has been driven below its initial equilibrium. The elimination of the tax cut at that time causes consumption to begin rising, so that z^* begins to increase as well; i.e., $\dot{z}^* > 0$ see (6a′). This gradually increases the productivity of private capital so that the growth rate of private capital begins to increase; see (18b).

The response of the growth rate of public capital to the temporary tax cut is more gradual, moving continuously along PQ on the YY locus. As the ratio of public to private capital begins to decline, the growth rate of public capital gradually increases, reaching a maximum at the point V, which corresponds to the minimum value of the ratio z^* along the transitional path, RS. Thereafter, as z^* increases, ϕ_g^* retraces its steps along the path VP, back toward the original equilibrium at P.

The paths for the two capital stocks over time are illustrated in Figure 3. It is clear that the tax being imposed on private capital income generates more volatile behavior in private capital than it does in public capital, the response of which is rather gradual. This is particularly true insofar as a temporary tax cut is concerned.[20]

It is straightforward to conduct the same analysis for government expenditure. In that case we again find that the two types of capital approach their respective equilibrium growth rates from opposite directions, though the pattern of adjustment is reversed. Now, the growth rate of public capital initially overshoots its long-run increase, whereas the growth rate of private capital always undershoots

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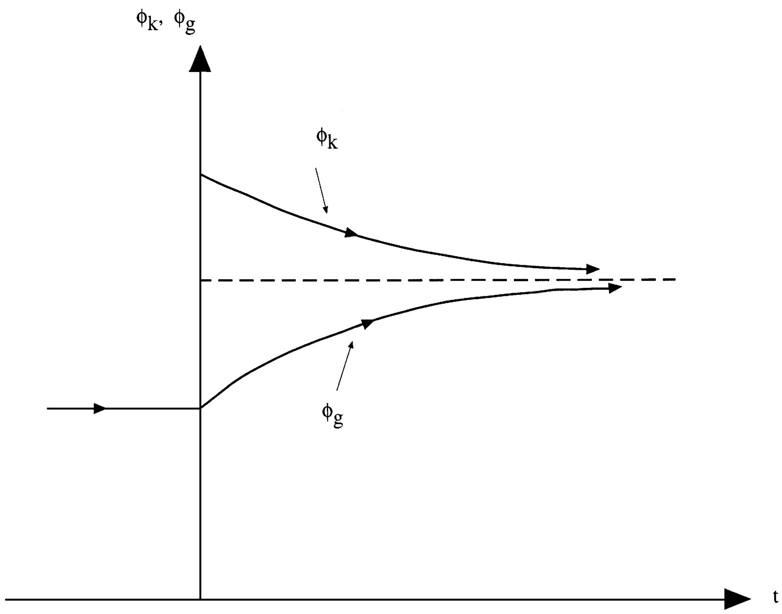
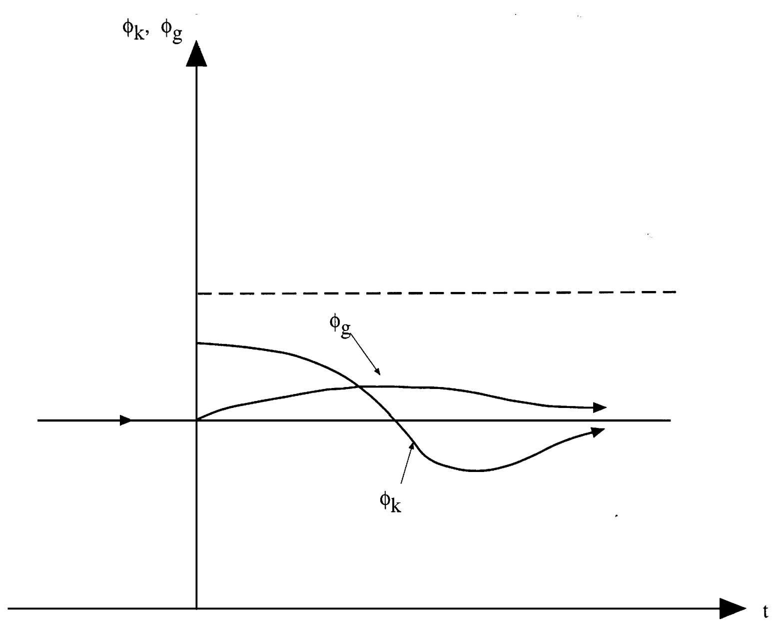


A) Permanent tax cut



B) Temporary tax cut

FIGURE 3. Time paths for private and public capital.

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on impact—and indeed may respond perversely—before gradually increasing to its new equilibrium. Public capital now exhibits greater volatility, especially in response to a temporary expenditure shock.[21]

5. OPTIMAL TAX

We turn now to the determination of the tax structure that will enable the decentralized economy to replicate the first-best outcome of the centrally planned economy. There are two aspects to consider. The first is that the decentralized economy must attain the steady state of the centralized economy. The second is that, having replicated the steady state, the transitional adjustment path to that equilibrium also must be followed. Because the dynamics generating z and c are of the same form in the two economies [cf. (12a,b) and (12a′,12b′)], this will be attained—at least to the linear approximation representation—by replicating the stable eigenvalue. That is, the solution for λ to (11) must equal the solution for λ^* to (11′). In general, this requires the income tax, τ, to be time-varying.

To see this, consider first the case in which the income tax rate remains constant through time at the rate $\tau = \bar{\tau}$. Comparing the corresponding relationships (8a–c) with (8a′) and (8b′), we see that the steady-state equilibrium values $(\tilde{z}^*, \tilde{c}^*)$ will replicate the first-best optimum in the centrally planned economy, $(\tilde{z}, \tilde{c})$, if and only if $\bar{\tau}$ satisfies

$$(1 - \bar{\tau})(1 - \beta\sigma) = (1 - g + \tilde{q}g)(1 - \beta),$$

where $\tilde{q} = [\beta/(1 - \beta)\tilde{z}]$ and $\tilde{z}$ is determined by (8a–c). Simplifying this relationship, the optimal steady-state income tax can be expressed as

$$\bar{\tau} = g(1 - \tilde{q}) + \frac{\beta(1 - \sigma)}{1 - \beta\sigma}. \qquad \textbf{(20)}$$

Setting the income tax in accordance with (20) ensures that the steady-state equilibrium of the centrally planned economy will be replicated. We discuss the significance of this tax policy presently, but before doing so we show how, if τ is maintained at $\bar{\tau}$ during the transition, the adjustment path followed by the decentralized equilibrium will fail to mimic that of the first-best optimum. To see this, we consider the respective eigenvalues and show how in this circumstance $\lambda^* \neq \lambda$.

For notational convenience, we denote the elements of the matrix of coefficients in the linearized centralized economy by (a_{ij}). These elements can be immediately identified by referring to (11). The equilibrium eigenvalue in the centralized economy is the negative solution to the cubic equation

$$F(\lambda) \equiv (a_{33} - \lambda)[(a_{11} - \lambda)(\tilde{c} - \lambda) - a_{21}\tilde{z}] + \tilde{z}a_{23}a_{31} = 0. \qquad \textbf{(21)}$$

Using this notation and if the tax rate $\bar{\tau}$ in the decentralized economy is set in accordance with (20), thereby replicating the first-best steady state, then the

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corresponding eigenvalue, λ^*, in the decentralized economy is determined where

$$G(\lambda^*) \equiv \left(a_{11} - \lambda^*\right)(\tilde{c} - \lambda^*) - a_{21}\tilde{z} = 0. \qquad \textbf{(21')}$$

Combining (21) and (21′), we find that $F(\lambda^*) = \tilde{z}a_{23}a_{31} > 0$. It then follows from the fact that the function $F(\cdot)$ is cubic in λ and that λ, λ^* are stable eigenvalues that the relationship $\lambda^* < \lambda < 0$ must hold. In other words, if the tax rate is fixed over time at $\tau = \overline{\tau}$ as in (20), then the ratio of public to private capital in the decentralized economy, z^*, determined by (12a′) will converge too rapidly, relative to the optimal rate of adjustment as described by (12a).

The intuition for this result is straightforward and is a consequence of the fact that the private agent treats τ as fixed and does not respond to changes in the shadow value of public capital, q, as does the central planner. Suppose some change occurs causing z to increase from z_0 to $\tilde{z}$. During the transition as z is increased, the shadow value of public capital declines. This, however, is not reflected by a fixed τ, so that during the transition $\overline{\tau}$ overstates the proper social value of public capital. Accordingly, private capital is taxed too much and there is an overinvestment in public capital relative to private capital along the transitional path in the decentralized economy, but, although the relationship $z^* > z$ holds along the transitional path, asymptotically $z^* \to \tilde{z}$.

We now propose modifying the income tax rate to

$$\tau(t) = \overline{\tau} + \theta[z^*(t) - \tilde{z}^*], \qquad \textbf{(22)}$$

where $\overline{\tau}$ is given by (20) and θ is a constant to be determined. The income tax rate as specified by (22) is time-varying, tracking the evolution of the economy as the relative stocks of capital change over time.[22] Intuitively, the time-varying tax rate $\tau(t)$ in effect permits the representative agent to track the endogenous shadow value of public capital. Because θ is relevant only along the transitional path (when $z^* \neq \tilde{z}^*$), it has no impact on the steady-state equilibrium. Consequently, setting $\overline{\tau}$ in accordance with (20) still will replicate the steady-state capital and consumption ratios, $\tilde{z}$, $\tilde{c}$, of the first-best optimum.

However, θ will affect the eigenvalue λ^* in the decentralized economy and therefore the speed of adjustment along the transitional path. The critical modification to be made is to the linearization of (6b′) [appearing as the second row in (11′)], which now becomes

$$\frac{\dot{c}^*}{c^*} = \left[\frac{(1-\overline{\tau})\alpha(1-\beta\sigma)\beta(\tilde{z}^*)^{\beta-1} - \alpha(1-\beta\sigma)\theta(\tilde{z}^*)^{\beta}}{1-\gamma}\right](z^* - \tilde{z}^*) + (c^* - \tilde{c}^*).$$

If $\overline{\tau}$ is set in accordance with (20), the eigenvalue, λ^*, in the decentralized economy is now determined by

$$G(\lambda^*) \equiv \left(a_{11} - \lambda^*\right)(\tilde{c} - \lambda^*) - \tilde{z}\left[a_{21} - \frac{\alpha(1-\beta\sigma)\theta\tilde{z}^{\beta}\tilde{c}}{1-\gamma}\right] = 0. \qquad \textbf{(21'')}$$

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It then follows that the speed of adjustment in the decentralized economy will exactly replicate that in the centralized economy (i.e., $\lambda^* = \lambda$) if and only if

$$\theta = -g\tilde{q}\left(\frac{1-\beta}{1-\beta\sigma}\right)\frac{a_{33}}{(a_{33}-\lambda)\tilde{z}} < 0, \tag{23}$$

where $a_{33} = (1-\beta)[(1-g)+\tilde{q}g]\alpha\tilde{z}^{\beta}$.

Thus the time-varying income tax rate (22) where $\bar{\tau}$ is determined by (20) and θ is determined by (23) will replicate the first-best optimum in the sense that both the steady state and the transitional path will be attained. Having set the distortionary income taxes optimally, any combination of lump-sum taxes, consumption tax, and bonds satisfying the flow government budget constraint and consistent with the consumer transversality condition, (7′), and therefore the intertemporal government budget constraint, will replicate the first-best optimal path. Note further that, with the availability of a full set of tax instruments, the problem of time inconsistency of optimal policy does not arise. With the target value for the income tax rate at each instant of time being determined by the time path followed by the first-best optimum, the government always will want to choose the income tax rate to attain that given and unchanging target path.

We return to the optimal steady-state tax rate, $\bar{\tau}$, given in (20). The intuition behind this optimum can be understood by comparing the *social* and *private* returns to private capital accumulation in the presence of public capital. Recalling (4b), the social return to accumulating a marginal unit of private capital is

$$r_s \equiv [(1-g)+qg](1-\beta)\alpha z^{\beta}.$$

This takes account of the fact that, because the government maintains a fixed expenditure ratio, gY, the accumulation of private capital indirectly causes the government to increase its rate of investment.

By contrast, the individual in the decentralized economy computes the marginal physical product of private capital on the assumption that the value of the public capital, K_g, remains unaffected by his individual decision. Thus, the after-tax private rate of return on private capital is

$$r_p \equiv (1-\tau)(1-\beta\sigma)\alpha z^{\beta},$$

which takes account of the degree of congestion associated with the public capital. The optimal tax rate $\bar{\tau}$ is set so as to equate r_p to r_s. The income tax rate thus corrects for two potential sources of externality: the size of the government relative to its social optimum, and the degree of congestion.

Suppose that there is no congestion, so that $\sigma = 1$ and that $\tilde{q} > 1$, i.e., $\tilde{z} < \hat{z}$ so that the relative stock of government capital is less than optimal. In this case, the optimal tax on private capital income is $\bar{\tau} < 0$; see (20). Because private investment increases output and therefore has the desirable effect of increasing the size of public capital, it generates a positive externality and therefore should be

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encouraged through a subsidy. On the other hand, if $\tilde{q} < 1$ and the government is too large relative to the optimum, the accumulation of capital generates a negative externality and should be discouraged through a positive tax on capital income. Finally, if $\tilde{q} = 1$, so that the size of the government sector is optimal, the induced change in government expenditure is just worth its cost. There is no externality and so private capital income should be untaxed. The first-best optimum can be reached either through lump-sum taxation alone, or equivalently through a consumption tax.

Suppose now that $\sigma = 0$, so that congestion is proportional. If the stock of public capital is at its social optimum, $\tilde{q} = 1$, the income from private capital now should be taxed at the rate $\overline{\tau} = \beta$, the proportion of public capital in the overall social optimum; see (13). Because at the optimum $\hat{g} < \beta$ [see (14)], this implies $\overline{\tau} > \hat{g}$. Thus, to attain the first-best optimum in the presence of proportional congestion, the government should impose an income tax in excess of its current investment costs, rebating the excess either as a lump-sum tax, or equivalently as a consumption tax.

The idea that the presence of congestion favors an income tax over lump-sum taxation or a consumption tax has been shown previously by Barro and Sala-i-Martin (1992) and Turnovsky (1996a). In models in which government expenditure appears as a flow, the optimal tax rate turns out to be $\overline{\tau} = \hat{g}$, so that the expenditure is exactly self-financing. In the present case, because congestion in public capital enhances the return to private capital, a larger tax is required to offset this incentive to overaccumulate private capital.

Because $\theta < 0$, the transitional component of the tax rate, $\tau(t)$ is a subsidy as long as $\tilde{z}^* > z^*(t)$, favoring the accumulation of private capital. In the absence of such a subsidy, the ratio of public capital in the decentralized economy will accumulate too fast relative to the social optimum and the effect of $\theta < 0$ is to slow down the speed of adjustment. Notice that as the ratio z approaches its steady state along the path, the magnitude of the subsidy declines. If $\tilde{z}^* < z^*(t)$, it is a tax slowing down the contraction of z (i.e., speeding up the relative contraction of private capital). In the case of the first-best optimum government expenditure, in which the centrally planned economy is always in steady state [see (13), (14)], this can be replicated in the decentralized economy by letting $\theta \to \infty$.[23]

6. CONCLUSIONS

The role of public expenditure in determining the productive performance of the economy has become an important issue in both academic research and policy debates. Virtually all of the analytical work addressing this issue has introduced government expenditure as a flow in the production function, and therefore has been subject to the criticism that insofar as it is intended to represent the infrastructure of the economy, what is really relevant is the accumulated stock of publicly provided capital. This paper has introduced both public and private capital in a model of endogenous growth. Some aspects of the model, such as the presence of transitional dynamics and the noncoincidence of the growth and welfare-maximizing tax rates,

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familiar from previous models having multiple capital goods, apply here as well. However, the extension to congestion and the more complete set of fiscal instruments introduces new insights.

First, during the transition, the two capital stocks always approach their common equilibrium growth rate from opposite directions. In response to a permanent income tax cut the growth rate of private capital initially overshoots its new (higher) long-run level before declining during the subsequent transition. By contrast, the growth rate of public capital increases, although only gradually, in response to a tax cut. This pattern of adjustment is reversed in response to a permanent increase in government expenditure. Second, government policy induces the more volatile response in the capital stock upon which it impinges most directly; private capital in the case of a tax, public capital in the case of expenditure. This relative volatility is more pronounced in the case of temporary policy shocks. Third, whereas an expansion in government expenditure financed by a lump-sum tax always increases the equilibrium growth rate in a decentralized economy, its effect in a centralized economy depends upon the size of government, g, relative to its growth-maximizing level, β.

Finally, we have characterized a time-varying income tax that enables the decentralized economy to replicate both the first-best transitional dynamics and steady-state equilibrium of a centrally planned economy. The steady-state component has a simple structure aimed at correcting for potential externalities due to the deviation in government expenditure from its social optimum, and the effects of congestion associated with public capital. One interesting aspect of this is that, in the case of proportional congestion ($\sigma = 0$), the government should impose an income tax in excess of its current investment costs, rebating the excess. The transitional component is aimed at inducing the representative agent to take proper account of the fact that the shadow value of public capital varies inversely with the changing ratio of public to private capital along the adjustment path. With the consumption tax essentially operating as a lump-sum tax, the issue of time inconsistency does not arise. Given an unchanging time path characterizing the first-best optimum, the policy maker will have no incentive to deviate from it.

NOTES

1. A comprehensive review of recent empirical literature is provided by Gramlich (1994).

2. See, e.g., Barro (1990), Jones and Manuelli (1990), Rebelo (1991), Saint-Paul (1992), Jones et al. (1993), Turnovsky (1996a,b). Recently, this implication of new growth models has become a source of criticism because of the suggestion that it runs counter to the empirical evidence. See Jones (1995), who proposes a new class of nonscale growth models as an alternative.

3. There is a substantial literature of two-sector endogenous-growth models in which the two capital goods are human and nonhuman capital; see, e.g., Lucas (1988), Mulligan and Sala-i-Martin (1993), and Davereux and Love (1995). The present analysis shares some of the characteristics of these models, in particular transitional dynamics.

4. Glomm and Ravikumar (1994) also emphasize congestion. Their model is one in which private capital fully depreciates each period, rather than being subject to gradual depreciation. This enables the dynamics of the system to be represented by a single state variable alone, so that it behaves like

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the Barro model in which government expenditure is introduced as a flow. In particular, under constant returns to scale in the reproducible factors there are no transitional dynamics and the economy is always on a balanced growth path.

5. The function (1c) is the standard specification in the median voter model of congestion; see, e.g., Edwards (1990). It implies decreasing marginal congestion provided $\sigma < 1$.

6. A natural alternative specification of congestion is to assume that it is of the absolute form $K_g^s = K_g K^{\sigma-1}$. However, this formulation is, in general, inconsistent with an equilibrium of ongoing endogenous growth.

7. Previous studies to analyze the effects of congestion on optimal tax policy include Barro and Sala-i-Martin (1992) and Turnovsky (1996a).

8. In the case $\sigma = 0$, the good is like a private good in that the median voter receives his proportionate share.

9. The case $\sigma < 0$ can be interpreted as describing an extreme situation in which the congestion of the public good is faster than the growth of the economy. There is substantial empirical evidence supporting this case; see Edwards (1990). Although we do not discuss it, one can easily interpret our results where $\sigma < 0$.

10. The dependence of the growth rate on population size is emphasized by Glomm and Ravikumar (1994). In note 18, we determine the effect of the degree of congestion on the equilibrium growth rate. One motivation of the nonscale models proposed by Jones (1995) and others is to eliminate such scale effects from long-run growth rates.

11. This specification is a standard one in the growth literature. For example, it is adopted by Devereux and Love (1995) in their analysis of government consumption expenditure. As long as g remains constant, the government is claiming a fixed share of the growing output for investment, so that an increase in the share, g, parameterizes an expansionary fiscal policy in a growing economy.

12. Barro (1990), Rebelo (1991), and Futagami et al. (1993) in effect parameterize government expenditure in this fashion by assuming that all income tax revenues are spent; i.e., $G = \tau Y$.

13. This is the counterpart in the centrally planned economy to Proposition 1 of Futagami et al. (1993).

14. Solving (4c) and (10), the transversality condition on K can be expressed as $\lim_{t\to\infty} \nu(t) K(t)e^{-\rho t} = \lim_{t\to\infty} \nu(0)K_0 e^{-(\tilde{\theta}-\tilde{\phi})t} = 0$, where $\tilde{\theta} \equiv (1-\beta)\alpha\tilde{z}^{\beta}[1-g+\tilde{q}g]$ and $\tilde{\phi}$ is defined in (10). Thus, the transversality condition will hold if and only if $\tilde{\theta} > \tilde{\phi}$. Recalling (8a) and (8c), this reduces to $\tilde{c} > 0$, a condition that is ensured by (9). An analogous condition holds within respect to the transversality condition on K_g.

15. In Turnovsky (1996b), where we introduce government expenditure as a flow, we assume that it not only improves the productivity of existing capital, but also that it reduces the cost of adjustment associated with investment. This latter aspect also leads to the result that the growth-maximizing rate of government expenditure exceeds the welfare-maximizing rate.

16. The condition is $\alpha[(1-g)(1-\gamma)-(1-\tau)(1-\beta\sigma)](g/(1-g)^{\beta}+\rho > 0$.

17. Using the fact that λ^* is an eigenvalue of (11′), one can establish

$$[g\alpha(1-\beta)(\tilde{z}^*)^{\beta-1}+\lambda^*] = \frac{\alpha\beta(\tilde{z}^*)^{\beta}[(1-g)\lambda^* - \tilde{c}^*(1-\tau)(1-\beta\sigma)]}{(1-\gamma)(\tilde{c}^*-\lambda^*)} < 0.$$

18. In the case in which N is not normalized to unity, one can show that

$$\frac{\partial\tilde{\phi}^*}{\partial\sigma} = \frac{g\tilde{z}^{\beta}(1-\tau)\alpha\beta}{(1-\beta\sigma)(1-\gamma)J}[(\beta-1)+(1-\beta\sigma)\ln N].$$

Thus, an increase in the degree of congestion (lower σ) associated with the public good has two offsetting effects on the equilibrium growth rate. On the one hand, to maintain g constant, the government must increase its rate of investment and this has a positive effect on the growth rate. However, this is offset by the reduction in productivity of capital because of the higher congestion and the adverse impact this has on the growth rate.

https://doi.org/10.1017/S1365100597004045 Published online by Cambridge University Press

19. This result does not mean that the government can increase the growth rate indefinitely by increasing g indefinitely. There are constraints because $g \leq 1$ and the transversality conditions must be met.

20. It is straightforward to introduce an investment tax credit (ITC), x say. All that happens is that the after-tax income factor $(1 - \tau)$ is replaced by $[(1 - \tau)/(1 - x)]$, so that the ITC is qualitatively identical to a reduction in the income tax rate. Hence, Figure 3 implies that a temporary ITC also induces instability in the growth of private capital. The ITC is discussed in detail by Turnovsky (1996b).

21. The dynamics of permanent and temporary government expenditure shocks are analyzed in an expanded version of this paper.

22. The time-varying tax rate (22) is assumed to be a function of the ratio of the aggregate stock of public to private capital and therefore is taken as given by the representative agent.

23. It is possible to replicate both the steady state and the time path of the centrally planned economy with a fixed income tax and a fixed tax/subsidy on *total* capital usage (e.g., xz^*). In this case, the tax on capital does affect the steady state, so that a corresponding adjustment in the income tax rate is required. The more familiar investment tax credit (i.e., $x\dot{z}^*$) will not work, because it is equivalent to an adjustment in τ and therefore does not provide the necessary extra independent policy instrument.

REFERENCES

Arrow, K.J. & M. Kurz (1970) *Public Investment, the Rate of Return, and Optimal Fiscal Policy.* Baltimore, MD: Johns Hopkins University Press.

Aschauer, D.A. (1988) The equilibrium approach to fiscal policy. *Journal of Money, Credit, and Banking* 20, 41–62.

Aschauer, D.A. (1989a) Is public expenditure productive? *Journal of Monetary Economics* 23, 177–200.

Aschauer, D.A. (1989b) Does public capital crowd out private capital? *Journal of Monetary Economics* 24, 171–188.

Barro, R.J. (1989) The neoclassical approach to fiscal policy. In R.J. Barro (ed.), *Modern Business Cycle Theory*, pp. 178–235. Cambridge, MA: Harvard University Press.

Barro, R.J. (1990) Government spending in a simple model of endogenous growth. *Journal of Political Economy* 98, S103–S125.

Barro, R.J & X. Sala-i-Martin (1992) Public finance in models of economic growth. *Review of Economic Studies* 59, 645–661.

Barro, R.J. & X. Sala-i-Martin (1995) *Economic Growth*. New York: McGraw–Hill.

Baxter, M. & R.G. King (1993) Fiscal policy in general equilibrium. *American Economic Review* 83, 315–334.

Devereux, M.B. & D.R.F. Love (1995) The dynamic effects of government spending policies in a two-sector endogenous growth model. *Journal of Money, Credit, and Banking* 27, 232–256.

Edwards, J.H.Y. (1990) Congestion function specification and the "publicness" of local public goods. *Journal of Urban Economics* 27, 80–96.

Futagami, K., Y. Morita & A. Shibata (1993) Dynamic analysis of an endogenous growth model with public capital. *Scandinavian Journal of Economics* 95, 607–625.

Glomm, G. & B. Ravikumar (1994) Public investment in infrastructure in a simple growth model. *Journal of Economic Dynamics and Control* 18, 1173–1187.

Gramlich, E.M. (1994) Infrastructure investment: A review essay. *Journal of Economic Literature* 32, 1176–1196.

Jones, C. (1995) Time series tests of endogenous growth models. *Quarterly Journal of Economics* 110, 495–527.

Jones, L.E. & R.E. Manuelli (1990) A convex model of equilibrium growth: Theory and policy implications. *Journal of Political Economy* 98, 1008–1038.

Jones, L.E., R.E. Manuelli & P.E. Rossi (1993) Optimal taxation in models of endogenous growth. *Journal of Political Economy* 101, 485–517.

https://doi.org/10.1017/S1365100597004045 Published online by Cambridge University Press

Lucas, R.E. (1988) On the mechanics of economic development. *Journal of Monetary Economics* 22, 3–42.
Mulligan, C.B. & X. Sala-i-Martin (1993) Transitional dynamics in two-sector models of endogenous growth. *Quarterly Journal of Economics* 108, 739–773.
Rebelo, S. (1991) Long-run policy analysis and long-run growth. *Journal of Political Economy* 99, 500–521.
Romer, P.M. (1986) Increasing returns and long-run growth. *Journal of Political Economy* 94, 1002–1037.
Saint-Paul, G. (1992) Fiscal policy in an endogenous growth model. *Quarterly Journal of Economics* 107, 1243–1259.
Turnovsky, S.J. (1996a) Optimal tax, debt, and expenditure policies in a growing economy. *Journal of Public Economics* 60, 21–44.
Turnovsky, S.J. (1996b) Fiscal policy, adjustment costs, and endogenous growth. *Oxford Economic Papers* 48, 361–381.
Turnovsky, S.J. & W.H. Fisher (1995) The composition of government expenditure and its consequences for macroeconomic performance. *Journal of Economic Dynamics and Control*, 19, 747–786.

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