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ELASTICITY OF SUBSTITUTION AND TECHNICAL PROGRESS: IS THERE A MISSPECIFICATION PROBLEM?

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In previous work, we estimated a dynamic model of the Italian economy, showing that its weakness in the past two decades is mainly due to the slowdown in total factor productivity growth. In those models, two parameters play a key role: technological progress and the elasticity of substitution. Recent estimates of those parameters are affected, in our opinion, by a specification problem: technological parameters are inherently long-run but their estimates are based on short-run data. Looking deeply into the estimation procedure, we show that the misspecification issue present in the estimates gives rise to a spurious regression bias (high R^2, low DW), because the standard approach does not incorporate frictions and rigidities. Our modeling strategy takes account of them. Although we cannot in general say that our framework gets rid of the serial correlation problem, the statistics for our model do show that residuals are not serially correlated.

Keywords: CES Production Function, Elasticity of Substitution, Income Distribution, Factor-Augmenting Technical Progress, ICT Technical Change

1. INTRODUCTION

In Saltari et al. (2012, 2013), we estimated a dynamic disequilibrium model of the Italian economy. The main result of those papers is that the weakness of the Italian economy in the past two decades has been the total factor productivity (TFP) slowdown. To investigate the roots of this productivity decline, we drew attention to the decreasing pace of capital accumulation. The modeling strategy in both papers is based on the distinction between traditional and innovative capital. In a nutshell, our main finding shows that there exists a structural and persistent gap between "optimal" and observed output, which, moreover, increased in the latter part of the sample period.

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In those models, two parameters play a crucial role. The first is the information and communication technologies (ICT)[1] capital stock in boosting the efficiency of both traditional capital and labor, and hence of the whole economy. Formally, the ICT efficiency contribution is captured in a multiplicative way through a weighting factor. The other key parameter to explain the Italian productivity decline is the elasticity of substitution. Since the introduction in the economic analysis by Hicks (1932) and its reformulation by Robinson (1933), the elasticity of substitution has attracted interest by both theoretical and empirical researchers for its central role in many fields, such as economic growth, fiscal policy, and development accounting. Recent analysis provides estimates consistently below 1—thus rejecting the traditional Cobb–Douglas production function—though there is no particular value on which the consensus converged.

The estimation of these two parameters is, however, tricky. This is because they are long-run in nature but their estimation is based on short-run data. In our opinion, the real issue is to bridge this gap. To get a satisfactory solution, it is crucial to focus on theoretical aspects underlying the modeling strategy.

Economic literature has addressed this problem in two ways. The first is based on statistical tools (such as cointegration, filtering, or simply assuming away the existence of the divergence) to recover long-run technological parameters from the short-run data. The second is to recognize the existence of short-run adjustment problems and to model them either explicitly, e.g., as in the Tobin's q framework, or implicitly, using ad hoc distributed lag processes not motivated by any form of optimization behavior [see Chirinko (2008) for a comprehensive survey of both lines of research]. However, both methods are in some sense inappropriate in that they do not explicitly incorporate the dynamic effect of these costs on the factor inputs in estimating the elasticity of substitution.

Our aim is to look deeply into the estimation procedure of these two technological parameters, the elasticity of substitution and the weight of ICT for the Italian economy. We proceed in two steps.

In the first, we stay within the standard framework and run a number of estimates, using both single- and system-equation approaches. The estimation procedure employs normalization as an instrument that allows us to properly identify the deep technological parameters through a suitable choice of a baseline point. Results show that single-equation approaches are largely unsuitable for jointly uncovering the elasticity of substitution and the weight of ICT.

We then build a system of two equations, the production function and the income share ratio derived from the two first-order conditions of factor inputs.

This is the most popular estimation method. It is an approach based on two assumptions: there is instantaneous adjustment of the marginal products to their user costs; it does not consider interactions with other markets. Within this framework, we get estimates for the elasticity of substitution and the weight of ICT. However, estimation results present a common fundamental feature in that the error term is serially correlated so the standard errors will be underestimated (i.e., biased downward). At the root of this result there is a specification problem:

the estimated models are static in nature and do not incorporate frictions and rigidities. Thus, for instance, the production function is estimated without any correction for the costs of rigidities [see, for instance, Mallick (2012)]. The same holds for the estimate of income share ratio, because it implicitly hypothesizes instantaneous adjustment between marginal products and input prices [as in Antras (2004)].

Our framework overcomes these difficulties by explicitly incorporating these costs into the theoretical model. The second step compares the results of our framework with those obtained using the standard estimation procedure.

Comparison suggests that the more popular approach of using a system with instantaneous adjustment is biased: for example, the weight of ICT appears to be underestimated. Our model is based on the idea that firms optimize their intertemporal profits subject to the production function; however, in the optimization process, firms are hindered by the presence of rigidities, adjustment costs, and other frictions. This gives rise to a dynamic model that enables the true parameters of the production function to be separated from the costs of adjustment, thus eliminating the autocorrelation in the residuals. As a consequence, the parameters are not biased by those costs. When we take account of these costs, we find an estimated elasticity well below unity, of about two-thirds.

A recent strand of literature [beginning from Solow (1987), Blanchard (1997), and McAdam and Willman (2013)] emphasizes a medium-run representation capable of explaining and reconciling protracted departures from the balanced growth path. Our disequilibrium approach is not too far from this representation. Indeed, both capture deviations from balanced growth and nest balanced growth as a special case. Moreover, both account for deviations introducing dynamics through factor adjustment costs. The difference resides in how these costs are modeled. In the medium-run approach a variety of adjustment mechanism is introduced that reflects structural frictions. In our disequilibrium framework the adjustment costs are embedded in a system of stochastic differential equations representing partial adjustment to long-run equilibrium, given a discrepancy between the long-run and actual value of a variable [Wymer (1996, 1997)].

The organization of the paper is as follows. The next section provides a brief literature review. Section 3 contains a short description of two issues related to the estimation of technological parameters. Section 4 gives the main empirical findings of our model. Section 5 reports the results of the traditional approach to the estimation of the technological parameters. Sections 6 and 7 compare our estimation procedure with the standard one, offering some insights for the solution of the misspecification issue. Section 8 concludes. Finally, the Online Appendix contains a short description of our model, data, and normalization procedure.

2. RELATED LITERATURE

Our paper is related to the modern growth literature [e.g., Acemoglu (2008), La Grandville (2009), Aghion and Howitt (2009)] that emphasizes the power of the CES production function. In recent years, the CES production technology has returned to the center of growth theory and increasingly empirical evidence shows that the nonunit elasticity of substitution allows recognizing the existence of biased technical change [see Chirinko et al. (1999), Klump et al. (2008), León-Ledesma et al. (2010)]. The wider use of CES technologies opens the door to a deeper understanding of the effects of variation in the elasticity of substitution on economic growth [Turnovsky (2002)].

As pointed out by Nelson (1965), the elasticity of substitution can be interpreted as an index of the rate at which diminishing marginal returns set in as one factor is increased with respect to the other. If the elasticity of substitution is high, then it is easy to substitute one factor for the other. Therefore, the higher the elasticity of substitution, the lower the drag caused by diminishing returns. From this interpretation, it is straightforward to notice that the elasticity of substitution will affect the growth rate of output when factors of production are increasing at different rates, so that their ratio is changing. The use of a Cobb–Douglas production function, as in most cases in the literature, is a misleading approximation for the behavior of the aggregate economy and hides the role of the elasticity of substitution not only as a source of increase in output but also as a source of technical change. If the elasticity of substitution in production is a measure of how easy it is to shift between factor inputs, typically labor and capital, it provides a powerful tool to answer questions about the distribution of national income between capital and labor.

The relevance of the elasticity of substitution and its relationship to economic growth and technical change has been established since Hicks (1932) and Solow (1957). However, it was after Arrow et al. (1961) that there was a boost to the theoretical and empirical issues involving the elasticity of substitution. More recently, La Grandville (1989) gave proof of the positive relationship between the elasticity of substitution and the output level.

Although the CES production technology seems relatively straightforward, its mathematical simplicity can be misleading. La Grandville (1989), Klump and La Grandville (2000), Klump and Preissler (2000), and Klump et al. (2008) have emphasized that the economic interpretation of the CES production technology requires attention, and they advocate the use of a normalized production function when analyzing the consequences of variation in the elasticity of substitution. To properly identify the elasticity of substitution, we employ normalization, which defines a family of CES functions whose members are distinguished only by different elasticities of substitution. Because the elasticity of substitution is originally defined as a point elasticity, one needs to fix benchmark values for the level of production, factor inputs, and the marginal rate of substitution, or equivalently for per capita production, capital deepening, and factor income shares. Normalization increases the usefulness of CES production functions for growth theorists, and this has led to its use in subsequent work such as Miyagiwa and Papageorgiou (2007) and Papageorgiou and Saam (2008).

3. TWO RELEVANT ISSUES

Before addressing the technical aspects, we deem it necessary to bring to the attention of the reader two far-reaching features of the economic environment's recent evolution in the main industrialized countries, which are relevant not only by themselves but also because they affect the estimation's robustness. This section discusses the role of ICT and the decline of labor's share.

Several recent studies have stressed the importance of ICT as a key factor behind the upsurge in U.S. productivity after 1995 [see, among others, Colecchia and Schreyer (2001); Stiroh (2002); Jorgenson et al. (2004)]. With regard to Europe, EU countries fall well below the United States in terms of ICT penetration [Timmer and van Ark (2005)]. Whereas there exists a huge literature for the U.S. economy, the literature is relatively scarce for Italy [see European Commission (2013)]. By now, it is an accepted fact that the setback of Italian labor productivity in the past twenty years is explained by two factors: a marked slowdown of capital deepening accompanied by a striking negative contribution of TFP.

To go a step further, notice that these two phenomena go hand in hand and are both relevant to explaining the standstill of labor productivity. Capital accumulation is important, at least since Solow (1957). In fact, what the data about capital deepening show is that in the Italian economy during the past twenty years there occurred a shift toward less capital-intensive techniques, thus reducing the efficiency of employment. This shift and the lack of adoption of new technologies, especially of the ICT variety, have been favored by the particular structure of the Italian specialization, skewed toward traditional sectors with low technological content and less skilled workers. That is, not only did the pace of investment decrease in the past 15 years, but also it was redirected toward traditional sectors rather than innovative ones. This change in capital accumulation mix explains why TFP and capital intensity rates decreased at the same time.

The last point is confirmed by the trend of capital accumulation in Italy, which is characterized by two main aspects. The first is that the dynamics of total capital accumulation mostly follows that of capital accumulation in the traditional (non-ICT) sector. The second is that the investment rate in the ICT sector accelerates up to the end of the 1980s, and then slows down, albeit with a recovery in the mid-1990s. It becomes negative in the most recent years.

The contribution of the ICT sector to productivity dynamics has not been modeled. The bulk of the literature assumes that technical progress occurs at a constant rate without giving a specific structure within which the ICT plays any role [a partial exception is Klump et al. (2008)]. In our model we take a stance about how ICT impacts technical progress: in particular, we assume that the productivity of the traditional capital stock is augmented by the ICT capital stock. This makes a difference with respect to the traditional approach in that the effect of ICT is not constant but reflects the pace of investment in innovative technologies.

Regarding the decline of labor share, evidence shows that since the 1980s the labor share has dramatically changed its behavior. Differently from the "stylized fact" of aggregate factor share constancy, the past three decades have seen a continuous decline of the labor share, thus casting doubt on share invariance. The decline of labor share is not limited to Italy but occurred in the large majority of industrialized countries [see Elsby et al. (2013); Karabarbounis and Neiman (2014)]. Empirically, this is a problem, for it implies nonstationarity in income shares, an issue difficult to deal with. The stability of the income share of labor is a key foundation in most macroeconomic models, taken for granted until very recently.

The dynamics of aggregate labor share in Italy, France, Germany, and the United States until the 1970s was approximately constant in all these countries, thus confirming one of the stylized facts highlighted by Kaldor (1961). Starting from the following decade, the decline of labor share becomes evident. In the period 1980–2011, the reduction was 11 percent for Italy and France, 8 percent in Germany, and 6 percent in the United States. Obviously, this downward trend will not last forever. It seems that in the past thirty years the income shares dynamics has been (at least locally) nonstationary; in other words, it is likely that this process will come to a halt. The local nonstationarity will create problems because it is an independent source of serial correlation. This is a critical issue often neglected in the estimation of the technological parameters of the production function, and especially in that of the elasticity of substitution. Though this is a relevant question, it is not clear which way out can be adopted.

4. THE MODEL

We assume that the Italian economy can be described by behavioral functions derived by the intertemporal optimization of the objective (profit) function. Institutional and market structures are incorporated into the model as constraints. These constraints represent the adjustment costs that hamper the instantaneous equality between factor marginal products and their prices. For instance, differently from the traditional approach, the capital stocks adjust more slowly to their marginal products. These rates of adjustment reflect the costs and risks of firms changing their capital stocks. Analogously, it is not assumed that the labor market instantaneously clears but rather that there are imperfections and frictions.[2]

To understand our methodology, let us have a look at the second-order (time) derivative of the log of traditional capital, which implicitly defines the investment equation in our model:

$$\dot{k} = \alpha_1 \left[\alpha_2 \left(\frac{\partial f}{\partial K} - (r - \beta_7 D \ln p + \beta_8) \right) - (k - \mu_K) \right], \tag{1}$$

where $k = D \ln (K)$, $\dot{k} = D^2 \ln (K)$, D denotes the operator d/dt, and μ_K is the growth rate of labor efficiency. Inside the parentheses, we model the short-run

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adjustment of the capital marginal product to its user cost, defined by the real interest rate $(r - \beta_7 D \ln p)$ plus a risk premium (β_8). The speed at which firms make this adjustment is given by α_2 or, in terms of its reciprocal, how long it takes to adjust the existing capital stock to its desired level. As time goes by, however, the desired level of capital stock changes at the velocity μ_K. Inside the square brackets we find the long-run adjustment process, which runs at α_1, the speed of the accumulation process. Of course, as the estimates in Saltari et al. (2012) confirm, the first adjustment takes a much shorter time than the second.

All the other equations in our model are specified in a similar manner, i.e., as dynamic equations (the basic model equations are summarized in the Online Appendix). This implies that the model is recursive, in the sense that it is expressed as a system of differential equations in which the derivative of each endogenous variable depends on the levels of all the other variables.[3]

Formally, these assumptions give rise to a system of stochastic differential equations that is estimated by the full-information maximum likelihood method (FIML). It is important to note that the parameters of the production function occur throughout the model under the various marginal product conditions that arise from cost minimization. The way in which they occur varies with the specific marginal functions.[4]

The aggregate production function is given by

$$\begin{aligned} Y_t &= f\,(C_t, K_t, Ls_t, Lu_t) \\ &= \beta_3 \left[(C_t{}^{\gamma_1} K_t)^{-\beta_1} + \left(\beta_2\, e^{\mu_K t}\, Ls_t{}^{\gamma_s}\, Lu_t{}^{\gamma_u}\right)^{-\beta_1} \right]^{-1/\beta_1}. \end{aligned} \tag{2}$$

In equation (2), $\mu_K = \lambda_K + \gamma_1 \lambda_C$ is the labor efficiency growth rate, whereas λ_K and λ_C are the rates of technical progress in the use of traditional capital stock K and innovative capital C. These parameters may be interpreted as an indication of the expected long-run term rates of growth, provided the system is stable. The coefficient β_2 is the labor-augmenting technical progress, whereas β_3 is a measure of the total factor productivity. The efficiency of traditional capital stock is augmented by ICT capital, C, with a weighting factor equal to γ_1. The elasticity of substitution is given by $\sigma_1 = \frac{1}{1+\beta_1}$. Finally, L_s is skilled labor and L_u is unskilled labor.

Defining total employment as a Cobb–Douglas function of the skilled and unskilled labor components, $L = L_s{}^{\gamma_s} L_u^{\gamma_u}$, the production function can be written as[5]

$$Y_t = \beta_3 \left[(C_t{}^{\gamma_1} K_t)^{-\beta_1} + \left(\beta_2\, e^{\mu_K t}\, L_t\right)^{-\beta_1} \right]^{-1/\beta_1}. \tag{3}$$

Two features of the production function (3) are worth noting. First, as emphasized earlier, the specification of factor-augmenting technical progress is based on the key role played by ICT on the productivity dynamics in industrialized countries since the 1990s. ICT is particularly relevant for Italy, although in a negative sense.

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TABLE 1. Parameter estimates (asymptotic standard errors in parentheses)

β_1	σ_1	β_2	β_3	γ_1	γ_S	γ_U	λ_K	λ_C
0.519 (0.0045)	0.658 (0.020)	27.075 (3.598)	0.869 (0.031)	0.048 (0.013)	0.027 (0.005)	0.971 (0.010)	0.001 (0.001)	0.036 (0.005)

However, as demonstrated in Diamond et al. (1978), it is impossible to identify this role separately from that of the elasticity of substitution unless one imposes a specific structure of technical change. In defining this structure, we abandon the traditional specification of technical progress as growing at a constant rate. This is the second feature of the production function. Our model assumes that the efficiency of traditional capital stock is augmented by ICT capital according to γ_1. Because the labor-augmenting index is defined as $\mu_K = \lambda_K + \gamma_1 \lambda_C$, the same factor also increases the labor efficiency. In this way, we are assuming that ICT investment improves labor productivity as well. Hence, we explicitly introduce the ICT capital stock as a capital-augmenting efficiency factor which also affects the efficiency of labor. To our knowledge, this specification of technical progress was first introduced in the growth model of Kaldor (1957).[6]

In our model, parameters of the production function are not the result of single-equation estimation, as of equation (3). Rather, they are obtained by the estimation of a structural dynamic model of general and ICT investment functions, skilled and unskilled labor sectors, and price determination under imperfect competition.[7] The parameters' estimates of the production functions are reported in Table 1.

The estimated Italian net domestic product (NDP) for the period 1980:Q2–2005:Q1, a total of 100 quarters,[8] is reproduced, together with the actual NDP, in Figure 1.

The whole system of nonlinear stochastic differential equations allows us to estimate the production function (or production frontier) subject to all the constraints mentioned in the preceding. The estimated parameters of the production function give the output at any given level of inputs. This is the amount of output that would be achieved at equilibrium, if inputs were used efficiently on the production frontier.

A visual inspection of the figure reveals that the model replicates fairly well what happened in Italy in the period under observation (the correlation coefficient is 0.99). However, a persistent gap exists between the estimated and observed dynamics of the Italian NDP, which tends to widen toward the end of the sample period after the burst of the dot-com bubble; in contrast, at the beginning of the 80s the estimated output falls, and thus the gap narrows, because of the crisis of the industrial system. On the average over the sample period, the gap between the estimated and observed NDP is about 3 percent.

Our estimation of the elasticity of substitution ($\sigma_1 = 0.658$) is confirmed by latest econometric studies. These contributions find values of σ_1 that are

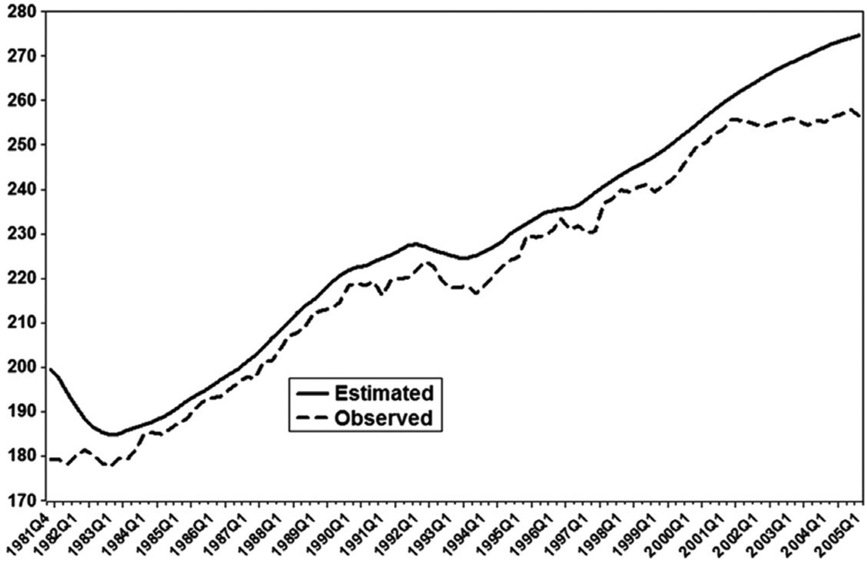

FIGURE 1. The dynamics of estimated and observed NDP in the Italian economy.

consistently below unity, but a great deal of variation in the results persists. Pereira (2003) surveyed major papers in the field from the past 40 years and found that, in general, elasticity values were below unity. Chirinko (2008) surveyed recent studies on the elasticity parameter and found considerable variation in cross-study results. However, the weight of the evidence suggests a range of σ_1 between 0.4 and 0.6, with the assumption of Cobb–Douglas being strongly rejected. Klump et al. (2008) estimated a long-run supply model for the euro area over the period 1970–2005 and found an aggregate elasticity of substitution below unity (around 0.7). Mallick (2012) obtained the elasticity parameters for 90 countries by estimating the CES production function for each country separately using time series spanning the period 1950–2000. The mean value for all 90 countries is 0.338. For the OECD countries the mean is 0.340. One problem with interpreting these cross-study results is that the various analyses are not all measuring the same thing: the results found are generally sensitive to sample size and estimation techniques. La Grandville (1989) and Klump and La Grandville (2000) emphasize the role of normalization of the CES production function because it makes more consistent cross-study estimates of the elasticity parameter.

In the following sections we compare the estimates of technological parameters σ_1 and γ_1 reported in Table 1 with those obtained employing the most frequently approaches: single-equation and two- and three-equation systems. Single-equation estimates concentrate either on the production function or on the first-order conditions, whereas the system approach combines them, exploiting cross-equation restrictions.

5. ESTIMATION RESULTS

5.1. Single-Equation Approach

The single-equation approach has been used for parameter estimation following two alternative routes: the production function and the optimizing behavior implicit in the equations of the income shares. These estimates are discussed in the following subsections.[9]

Technology. Let us begin with the estimation of the production function. After normalization, the production function in log form is

$$\ln (y_t) = -\frac{1}{\hat{\beta}_1} \ln \left[\pi_0 kit_t^{-\hat{\beta}_1} + (1-\pi_0)\, e^{\mu_K\,(t-t_0)} l_t^{-\hat{\beta}_1} \right], \tag{4}$$

where $\hat{\beta}_1$ is the estimated parameter and small capital letters indicate capital expressed in efficiency units and labor in index number form $kit_t = \frac{KIT_t}{KIT_0} = \frac{K_t C_t^{\gamma_1}}{K_0 C_0^{\gamma_1}}$ and $l_t = \frac{L_t}{L_0}$ (see the Online Appendix for details).

We first estimate only β_1, setting the other parameters $(\gamma_1, \lambda_K, \lambda_C)$ at their values in Table 1, with nonlinear least squares. This produces an estimate of $\hat{\beta}_1$ equal to 8.9. The implied value of the elasticity of substitution is $\hat{\sigma}_1 = \frac{1}{1+\hat{\beta}_1} = 0.1$, with an R^2 equal to 0.98 and a standard error of 0.026 (see Table 2). Notwithstanding the high significance level and the good fit, the estimate presents at least two problems. First, the implied level of σ_1 is quite low and "unrealistic." Second, and more importantly, the Durbin–Watson statistic is very low (DW = 0.19), indicating the existence of positive serial correlation in the residuals.[10] The strong residual autocorrelation affects the $\hat{\beta}_1$ estimate in that the estimate is unbiased but its standard error is biased downward. In other words, there is a loss of efficiency.

As we saw in the preceding, ICT played a key role in explaining the Italian economic dynamics. Hence, we try to fix the specification problem by extending the estimation to the weight of ICT, γ_1. Consequently, we jointly estimate the elasticity of substitution and the weight of ICT, obtaining $\hat{\beta}_1 = 11.6$ and $\hat{\gamma}_1 = 0.037$, with an R^2 equal to 0.98 for each. This slightly increases the estimate of $\hat{\beta}_1$—the elasticity of substitution becomes $\hat{\sigma}_1 = 0.08$—but the serial correlation remains high.

Up to now we have estimated the parameters using observed variables, without taking into account that the estimation refers to long-run relations. One of the methods frequently used in the literature to recover "desired" or long-run values is to filter the time series (filtering results in a reduction of the sample period to 1984Q4–2002Q3). The adopted procedure transforms variables in the frequency domain, excluding medium and high frequencies, keeping only low frequencies. In the time domain this allows us to get back long-run values. So what do we get from filtering? We tried several filters—the Baxter–King and Christiano–Fitzgerald versions, with different hypotheses about the trend—but results seem insensitive to these transformations. These are $\hat{\beta}_1 = 49.3$ ($\hat{\sigma}_1 = 0.02$) and $\hat{\gamma}_1 = -0.027$.

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TABLE 2. Single equation estimation: Technology

	Nonfiltered		Filtered
$\hat{\sigma}_1$	0.1 (0.026)	0.08 (0.014)	0.02 (0.002)
$\hat{\gamma}_1$	Calibrated	0.037^{**} (0.004)	-0.027^{**} (0.008)
R^2	0.98	0.98	0.98
DW	0.19	0.24	0.39

Note: Robust standard errors in parentheses; standard errors of $\hat{\sigma}$ computed by the delta method.
** Significant at 1%.

However, the ICT weight has an implausible negative sign,[11] and, above all, the residuals are still serially correlated.[12]

It is worth noticing that all these regressions have a common feature: they present a combination of a low DW and a high R^2. This combination, which will recur in all the subsequent regressions, seems to suggest a spurious relation between variables.

The problem with estimating a production function as a single equation is that it assumes that output is on the production frontier. It may also have a simultaneous equation bias because it assumes that, throughout the sample, output is determined by the supply side only. However, it is likely that the past few years have shown that output is demand-driven. If so, however, it is y_t that is causing kit_t and l_t, not vice versa. Besides, although the representation and estimation of a production function are important, on its own it is a purely technical relation.

In addition, the approach of trying to adjust the explanatory variables, kit_t and l_t, with filtering techniques loses information and may leave one not knowing what is really being lost. Also, the standard errors of the parameters of the estimated production function are usually incorrect, as they are based on the adjusted or filtered values of kit_t and l_t, not the actual ones.

Income shares. Let us now turn to the estimation of the first-order conditions related to firm's optimizing behavior. We use the income share equations that embody first-order conditions. In writing the production function in index form, we employed the mid-sample as a reference period. Income shares at the baseline value were determined as follows:

$$1 - \pi_0 = (\beta_3)^{-\beta_1} \left(\frac{Y_0}{\beta_2 L_0} \right)^{\beta_1}.$$

More generally, the labor share in period t can be written as

$$1 - \pi_t = (\beta_3)^{-\beta_1} \left[\frac{Y_t}{\beta_2 e^{\mu_K (t-t_0)} L_t} \right]^{\beta_1},$$

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where $\mu_K = \lambda_K + \gamma_1 \lambda_C$. Dividing the last two equations side by side, we obtain

$$1 - \pi_t = (1 - \pi_0) \left[\frac{y_t}{e^{\mu_K (t - t_0)} l_t} \right]^{\beta_1}.$$

This equation has a straightforward economic interpretation: the labor income share is directly related, via β_1 and thus the elasticity of substitution, to the productivity of labor expressed in efficiency units.

Taking logs, we get

$$\ln (1 - \pi_t) = \ln (1 - \pi_0) + \beta_1 \ln \left[\frac{y_t}{e^{\mu_K (t - t_0)} l_t} \right]. \tag{5}$$

As in the case of production function estimation, we set the lambdas at the values specified in Table 1 and estimate the two deep parameters γ_1 and σ_1. Using both observed and filtered data for the variables involved in the previous equation, we get almost the same values for β_1, equal to 0.2 ($\hat{\sigma}_1 = 0.83$), whereas the weight of ICT is 0.39 for unfiltered data and 0.42 for the filtered ones. What remains unchanged is the high serial correlation in the residuals.

An analogous estimation can be done for the capital income share. The equation estimated in log form is

$$\ln (\pi_t) = \ln (\pi_0) + \beta_1 \ln \left(\frac{y_t}{k_t c_t^{\gamma_1}} \right). \tag{6}$$

As in the case of labor, the capital income share is directly linked to the productivity of capital, expressed in efficiency units through the ICT capital contribution. Estimating this equation as before with observed and "long-run" data, we obtain $\hat{\beta}_1 = 3.81$ ($\hat{\sigma}_1 = 0.21$) and $\hat{\beta}_1 = 1.14$ ($\hat{\sigma}_1 = 0.47$), respectively. As before, what appears counterintuitive in both cases is the negative sign of the estimated γ_1. However, such a finding is not uncommon in the literature [see, for instance, Antras (2004)]. The estimate of the elasticity of substitution based on the labor income share is higher than that based on the capital income share, an empirical regularity confirmed also in other studies.

Finally, we estimate the ratio of income shares, which in log form is

$$\ln \left(\frac{\pi_t}{1 - \pi_t} \right) = \ln \left(\frac{\pi_0}{1 - \pi_0} \right) + \beta_1 \ln \left[\frac{e^{\mu_K (t - t_0)} l_t}{k_t c_t^{\gamma_1}} \right]. \tag{7}$$

In words, the ratio of capital to labor income share is inversely related to the labor–capital ratio, both expressed in efficiency units.

In the period considered, income shares are not stationary, in contrast with an accepted stylized fact in the economics literature. This stylized fact is negated, as there has been a continuous decrease in the labor share—not only in Italy but also for most industrialized countries, as already discussed. Even if the labor share seems to be stabilized at a new lower level in recent years, it creates an estimation

TABLE 3. Single equation estimation: Income shares

	Labor income share		Capital income share		Income share ratio	
	Nonfiltered	Filtered	Nonfiltered	Filtered	Nonfiltered	Filtered
$\hat{\sigma}_1$	0.83 (0.1)	0.83^{*} (0.015)	0.21^{**} (0.056)	0.47^{**} (0.131)	1.33 (0.65)	0.58^{**} (0.02)
$\hat{\gamma}_1$	0.39 (0.24)	0.42^{**} (0.05)	-0.16^{**} (0.028)	-0.53^{**} (0.14)	0.46^{**} (0.12)	0.61^{**} (0.04)
R^2	0.74	0.94	0.66	0.80	0.88	0.95
DW	0.01	0.03	0.07	0.02	0.03	0.03

Note: Robust standard errors in parentheses; standard errors of $\hat{\sigma}$ computed by the delta method.
** Significant at 1%.
* Significant at 5%.

problem not easy to solve, because the nonstationarity is a potential cause of serial correlation (as shown by the results in Table 3). Indeed, we made attempts, both with raw and filtered data, without obtaining satisfactory results: for instance, the estimated weight of ICT is implausibly high (above 40%), whereas the actual weight of the ICT capital stock in the total capital stock is in the range of 3–6%. Moreover, residuals remain serially correlated.

To sum up the results discussed so far, the estimates computed in the single-equation approaches are all potentially subject to a spurious regression bias, as they present a low DW and a high R^2.

As is well known, in this case the estimates are consistent only if there exists a cointegrating relationship. In all the previous regressions we tested for this possibility—using different hypotheses on trend and constant specifications.[13] We were not able to obtain unambiguous results, because the presence of a cointegrating relation very much depends on the number of lags included. In our opinion, these findings suggest a misspecification issue.

The problem behind the one-equation approach—such as $Y = f(K, L)$ alone—is that it assumes the observations are taken from a static economy at equilibrium. We can see no way that can hold. These three variables are perhaps the most heavily interrelated in theory: both K and L will be functions of demand, and demand (for given prices and wages) must be met from domestic output, imports, or variations in stocks. K will depend on some investment function, which alone will lead to lags. L will depend almost certainly on demand and the current (installed) production frontier, so even leaving aside simultaneous equation bias, there will be some form of serial correlation (probably moving average disturbances) within the model. Similar problems arise in the context of income share estimation.

5.2. System Approach

Because the single-equation approach seems unsuitable for jointly estimating the two technical parameters of interest, we turn our attention to the system approach, which is also the most frequently used in the literature.

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TABLE 4. System estimation

	System 1		System 2	
$\hat{\sigma}_1$	—		0.384^{**} (0.056)	
$\hat{\gamma}_1$	0.05^{**} (0.004)		0.075^{**} (0.017)	
	R^2	DW	R^2	DW
Production function	0.990	1.04	0.994	1.24
Income share ratio	0.998	0.17	0.998	1.30
Q-statistic	$132.87 > \chi^2_{(0.95,98)}$		$89.38 < \chi^2_{(0.95,98)}$	

Note: Robust standard errors in parentheses; standard errors of $\hat{\sigma}$ computed by the delta method.
** Significant at 1%.

The system estimated is composed of two equations: (4) and (7). It is estimated using nonlinear SUR. The estimation gives wrong signs for both parameters and strongly indicates the presence of serial correlation (the DW is close to 0). The residual autocorrelation is confirmed by the multivariate Box–Pierce/Ljung–Box Q-statistics. To correct for this problem, we decide to add autoregressive components. We run unit root tests both for the NDP and for the income share ratio, which indicate that the former is $I(1)$, whereas the latter is $I(2)$. This leads us to include in the system (indicated in Table 4 as System 1) one autoregressive term in the first equation and two in the second equation. This procedure reduces, but does not solve, the serial correlation problem; furthermore, it gives us a wrong sign in the $\hat{\beta}_1$ estimate.[14] A visual inspection of the residual correlogram leads us to increase the number of autoregressive terms in the equation for the income share ratio. Increasing the order of the autoregressive process of the system (indicated in Table 4 as System 2) seems to solve the serial correlation problem, because the Q-statistic is lower than the value of the critical χ^2 distribution with 98 degrees of freedom at the 5 percent level, which is equal to 122.1. However, in our opinion the economic content of this econometric manipulation has very limited value.[15]

Moreover, econometric theory tells us that if the original system was well specified, then correcting for serial correlation should change the standard errors, but the estimated coefficients should not change very much (because both the original and corrected results are unbiased). However, they went from negative to positive. This could be interpreted as an informal signal that the system was not well specified in the original estimate.

6. THE THEORETICAL ROOTS OF THE ESTIMATION PROBLEM

As seen in the preceding, the traditional approach has some weaknesses. Indeed, a key issue arises in estimating the technological parameters. To see the problem at hand in the simplest way, suppose that the production function underlying

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the economy may be represented as $y = f(K, L, \theta)$, where θ is a vector of parameters. These are technological parameters indicating the way in which factors of production are brought together to produce output.

If the economy has frictions, rigidities, etc., that reduce the efficiency of production, these rigidities must be taken into account in the estimation of the parameters of the production function, for otherwise, the standard errors of the estimates will be biased. Some rigidities will be unavoidable; it takes time to install capital, build a new plant, etc., but it can be assumed that firms will take whatever steps they can to minimize costs associated with those rigidities. The same will apply to regulations; although firms are assumed to minimize costs by choosing the optimal point on the production frontier, depending on factor costs, rigidities will encourage or force the firm to operate at some other, suboptimal point. This suboptimal point may be at a different point on the same "isotechnology" frontier, or the whole frontier may be suboptimal.

If data on costs were available, it might be possible to build them into the production function, but generally that is not the case, at the aggregate level anyway. Also, to the extent that firms take steps to reach the optimal position from their current suboptimal position, estimation of the production function is likely to result in autocorrelated errors.[16]

The divergence between suboptimal and optimal variables is often cast in terms of the difference between observable short-run data and their long-run values. Observable data do not include adjustment costs, whereas the long-run values—on which the estimates should be based—are already cost-adjusted but unobservable.

The divergence between optimal and suboptimal positions is a problem with time-series analysis; autocorrelated errors are often eliminated statistically, but if they have an economic cause originating from a misspecification problem, the model should be respecified accordingly.

Our model is based on the idea that firms optimize their intertemporal profits subject to the production function, but taking account of adjustment costs and other frictions. This produces the dynamic model, which allows us to separate the true parameters of the production function from the costs of adjustment. The parameters θ then are not biased by those costs.

Once the model specified to include these costs has been estimated, the parameters will be unbiased and the partial equilibrium of the economy may be calculated under the assumption that costs of rigidities are zero. It is worthwhile to notice that the calculated values from the unbiased estimates exclude these costs, so that estimated output will be higher than observed (as in Figure 1).

A more formal way of looking at this issue from a general point of view is to think of a theoretical function such as

$$\tilde{y}(t) = f[x(t), \theta] + u(t), \tag{8}$$

where the $u(t)$ are a set of errors that would arise if this relationship, and in particular $\tilde{y}(t)$, could be observed. It could then be estimated directly. If this relationship is subject to adjustment costs, rigidities, frictions, etc., the preceding function could be considered as embedded in a more general relationship, as, for instance,

$$y(t) = g\{f[x(t), \theta], z(t), \omega\} + \xi\,(t)\,. \tag{9}$$

Function (9) may depend on other variables $z(t)$ and parameters ω, thus giving a better representation of the economy. Hence, $y(t)$ is the variable that is observed. In that case, it is equation (9) that should be estimated for all of the parameters θ and ω. If this equation is the correct specification of the model that produces the observed $y(t)$, estimating the first equation on the assumption that $\tilde{y}(t) = y(t)$ would produce biased estimates of the parameters vector θ.

If (9) were the true model but (8) is estimated using the observed values of $y(t)$, it is likely that residuals in (8) will be serially correlated because of model misspecification. Take, for instance, the behavior of factor markets. These are very often characterized by frictions and rigidities arising from many sources that affect the adjustment process. In many countries, the employment protection legislation is evidence of the existence of institutional factors that delay or hinder the achievement of equilibrium in the labor market; at the same time, they make the wage unresponsive to the excess of demand or supply. Similarly, the optimal or "desired" capital stock cannot be instantaneously obtained given a variety of adjustment costs. Some scholars [see, for example, Antras (2004), Leon-Ledesma et al. (2010)] do not consider the presence of those frictions and rigidities, assuming that the economic system is in equilibrium at any point in time. Although this assumption may be convenient for theoretical work, it causes an error in the specification of the structure underlying the model, thus giving rise to serially correlated residuals.

As this has an economic cause, that is it is due to a misspecification of using $y(t)$ with (8) rather than (9), it should be eliminated by using the correct specification rather than by some statistical means. Because of the dynamics in the true model, if we wished to use values calculated from (9) to reestimate (8) directly, we would need to calculate "observations" of these variables from (9) first, but such estimates would almost certainly be inconsistent.[17] In the following we will see that the standard approach suffers from such a misspecification problem.

7. DEALING WITH THE MISSPECIFICATION PROBLEM

Our model is formulated as a dynamic disequilibrium system in continuous time. The model is based on the idea that firms optimize their intertemporal profits subject to the production function but are hindered by rigidities, adjustment costs, and other frictions. Hence, firms cannot optimize instantaneously, as in the traditional approach, but only at a finite speed—formally represented by the alphas, as in equation (1). This produces the dynamic model, which, at least to an approximation,[18] enables the true parameters of the production function to be separated from the costs of adjustment. The parameters then are not biased by those costs.

These particular features may help, at least in principle, in solving the residual correlation and misspecification problems. As the reader may recall, the misspecification derives from the "fundamental tension," as Chirinko (2008) dubs it, between the short-run observable data and the long-run nature of the elasticity of substitution. As the estimation results showed, the ways out of this problem proposed in the literature have not been useful. As said earlier, our modeling strategy takes into account, though implicitly through the alphas, a variety of adjustment costs without leaving out the optimization hypothesis. The standard procedure assumes instead that there are no lags or frictions hampering the equality between input prices and their marginal products in estimating the technical parameters.

One may ask what results would be obtained following the standard procedure. There are two main points characterizing the traditional methodology of estimating the elasticity of substitution and technical change. First, because of the impossibility of identifying the parameters separately, a specific structure is imposed on technical progress: it is assumed that the factor input efficiency grows at a constant rate. Second, it is assumed that the adjustment speed of the factor marginal productivities at their rental prices tends to infinity. Antras (2004) is perhaps one of the best recent papers that exemplify the standard approach for the U.S. economy. His main result is that the elasticity of substitution is well below 1, and hence the aggregate production function is not of Cobb–Douglas type. However, from the very beginning, he has to deal with the same problem emphasized in the preceding, i.e., a combination of high R^2 and low Durbin–Watson statistics pointing toward a spurious regression bias. To solve this problem, he employs a number of econometric techniques (besides OLS, FGLS, GIV, and cointegration) without substantial improvements.

Assuming finite adjustment speed and a different form of technical change, in our opinion, provides a better representation of the economic system. Although our framework cannot always guarantee the solution to the specification problem, in this case it turns out that residuals are not serially correlated. The multivariate Ljung–Box Q test is equal to 101.4 for the first two autocorrelations.[19] As a consequence, the null hypothesis that the residuals are not serially correlated cannot be rejected because the Q-statistic is below the critical value in the region of the upper tail.[20] This is not a surprising result, because a second-order differential equation model gives rise to a second-order moving-average error process that is taken into account explicitly in the estimation procedure. To the extent that observations generated by a second-order system inherently incorporate a first- or second-order moving average process, depending on whether the variables are stocks or flows, at least in a linear model and to an approximation in a nonlinear model, that too can be taken into account and variables transformed to remove the serial correlation—see Wymer (1972).

8. CONCLUSIONS

A growing number of papers have shown that the elasticity of substitution is a key technological parameter for boosting economic efficiency. Perhaps the most innovative and interesting result of this literature is that the elasticity of substitution is well below 1, i.e., the Cobb–Douglas assumption is biased upward.

However, in our opinion, these new estimates are affected by a theoretical weakness. The elasticity of substitution is a long-run technological parameter whose estimation is constrained by the availability of short-run data. This problem has been solved by employing two different econometric strategies: on one hand, making use of a theoretical framework to account for the delayed adjustment to long-run optimizing relationships; on the other, filtering the data in such a way as to retain only long-run components.

This paper has emphasized that these estimates have a serial correlation problem deriving from unsolved theoretical issues: it is difficult to explicitly specify the appropriate adjustment costs, and data filtering is subject to the usual ad hoc criticism. The model we proposed is a tentative solution to these problems in that it incorporates frictions and, as a disequilibrium model, it is intrinsically dynamic. The test results seem to confirm that the strategy is effective.

A distinctive feature of our model is the capital-augmenting technical progress which gives a key role to the ICT capital stock, differently from the existing literature, where it is generally assumed to be constant. The next step of our research project is to extend the model by endogenizing the ICT sector. We also believe that a deeper understanding of the issues we have been examining can come from the introduction of a variable elasticity of substitution to the specification of technology (which will be more responsive to price changes).

NOTES

1. By ICT we refer to ICT-producing sectors: hardware, software, and office equipment.
2. The adjustment process may take two forms. The first-order process assumes that the variable under consideration adjusts to its partial equilibrium level in the following way:

$$Dy(t) = \alpha\,[\hat{y}(t) - y(t)],$$

where $\hat{y}(t)$ is the equilibrium or desired level, α is the speed of adjustment, and D is the operator d/dt. The second-order adjustment assumes instead that it is the rate of change of the variable that adjusts to its equilibrium level:

$$D^2 y(t) = \alpha_1 \{\alpha_2 [\hat{y}(t) - y(t)] - Dy(t)\},$$

where the first term in parentheses describes the adjustment of the variable to its desired level and $Dy(t)$ has a damping effect on the adjustment process.

3. More details on dynamic disequilibrium models can be found in Gandolfo (1981) and Wymer (1996).
4. The FIML estimator used ensures that all of the cross-equation constraints implicit in these functions are imposed in the estimation, and hence the parameter estimates are consistent across the

model. This increases the (statistical) efficiency of the estimates; i.e., they have a lower asymptotic standard error.

5. This production function can easily be transformed into the well-known form introduced into the literature by Arrow et al. (1961):

$$Y_t = q\left[w\left(C_t{}^{\gamma_1} K_t\right)^{-\beta_1} + (1-w)\left(\beta_2 e^{\mu_K t} L_t\right)^{-\beta_1}\right]^{-1/\beta_1},$$

where the "efficiency" parameter is defined as $q = \frac{\beta_3}{\left(1+\beta_2^{-\beta_1}\right)^{1/\beta_1}}$ and the "distribution" parameter as $w = \frac{1}{1+\beta_2^{-\beta_1}}$.

6. Kaldor (1957) is explicit in affirming that one specific characteristic of his growth model is that "it eschews any distinction between changes in techniques (and in productivity) which are induced by changes in the supply of capital relative to labor and those induced by technical invention or innovation—i.e., the introduction of new knowledge. The use of more capital per worker (whether measured in terms of the value of capital at constant prices, in terms of tons of weight of the equipment, mechanical power, etc.) inevitably entails the introduction of superior techniques" (p. 595).

7. The model assumes that the market environment is one of imperfect competition where firms have similar production functions but different endowments and their products are sufficiently differentiated so that they are monopolistic competitors in the short run, setting their own prices. Thus, they may set prices according to their marginal costs plus some mark-up. As a consequence, each firm is assumed to be a "quantity-taker" and aims to supply the amount demanded.

8. In our estimation period there are 100 quarters, but 4 have been discarded for estimation reasons.

9. The normalization used is described in the Online Appendix.

10. Here, and in what follows, we tested for residual correlation computing Q-statistics, ADF, and Breusch–Godfrey statistics. The tests always confirm the DW results. For brevity, these tests are not reported.

11. A negative sign for γ_1 is economically meaningless, as it would imply technological regression when ICT capital stock increased.

12. We also replicated the specification of Mallick (2012), who assumes Hicks-neutral technical progress, obtaining very similar results for Italy ($\sigma_1 = 0.15$). However, he does not address the serial correlation problem.

13. We used Engle–Granger and Phillips–Ouliaris single-equation residual-based cointegration tests. Results are available upon request from the authors.

14. The strategy of filtering data does not improve the results.

15. Specifically, we included autoregressive components at lags 1 up to 5, and at lags 9 and 10 for the equation of income share ratio; for the production function, we add one autoregressive element.

16. For a theoretical discussion of "dynamic misspecification" in the context of dynamic programming and how it can generate serial correlated errors, see Hansen and Sargent (2008, pp. 26–27).

17. As all the FIML or similar estimators are asymptotic, a vector of parameters is consistent if and only if the probability limit of the estimate of the vector of parameters θ equals the true value of θ as the sample size goes to infinity for the estimate to be consistent. This condition is roughly asymptotically equivalent to—but not the same as—the condition in OLS that a parameter is unbiased if and only if the expected value of its estimate is equal to the true value.

18. The approximation derives from the fact that, at a macro level, it is not possible to give an analytic representation of rigidities, frictions, and adjustment costs.

19. The augmented Dickey–Fuller statistic may also be run for the single equations of the model. Although these are not appropriate for a FIML estimator, the single-equation results, for what they are worth, show no relevance to nonlinear differential equation systems.

20. Approximate critical values of the critical χ^2 distribution with 98 degrees of freedom are at the 5 percent level equal to 122.1, whereas at the 1 percent level they are equal to 133.5.

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