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MONETARY RULES AND ENDOGENOUS GROWTH IN AN OPEN ECONOMY

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This paper develops a monetary endogenous growth model for an open economy. The salient feature of the model is that it is able to deal with various monetary policy rules, including money growth rate targeting, inflation rate targeting, and nominal income growth rate targeting. It is found that a rise in the pegged rate may either increase or decrease the balanced-growth rate under regimes of both money growth rate targeting and nominal income growth targeting. However, a rise in the pegged rate is sure to depress the balanced-growth rate under the regime of inflation rate targeting. It is also found that money growth rate targeting is fundamentally equivalent to nominal income growth rate targeting if a specific restriction is imposed, and inflation rate targeting is not qualitatively equivalent to either money growth rate targeting or nominal income growth rate targeting.

Keywords: Endogenous Growth, Monetary Policy Rules, Equivalence

1. INTRODUCTION

The effect of inflation on capital accumulation has long been one of the central topics in macroeconomics. Earlier works on this topic are restricted to the analysis of the effect of inflation *level* on capital accumulation. However, as pointed out in the Gylfason and Herbertsson (2001, p. 408) survey paper, empirical studies within the existing literature find a negative relationship between the *rates* of inflation and economic growth in the long run. The recent development of endogenous growth theory, initiated by Romer (1986) and Lucas (1988), provides an analytical

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framework for studying the effect of inflation rate on capital accumulation and hence is capable of explaining the existing empirical findings mentioned previously. Since the 1990s, the literature on monetary endogenous growth theories has expanded and developed at a rapid pace.

Most authors studying monetary endogenous growth confine the model setting to *closed economies*. For example, Marquis and Reffett (1991), Gomme (1993), and Mino (1997) introduce money into a two-sector Lucas (1988) model via the cash-in-advance constraint and emphasize how the money growth rate is related to the consumption–leisure decision. Van der Ploeg and Alogoskoufis (1994) and Mino and Shibata (1995) set out an overlapping generations model with money in the utility function and examine how a change in the money growth rate will affect the reallocation of resources between generations. Zhang (1996) highlights the role of money in facilitating transactions when a change in the rate of monetary growth alters the consumption–leisure decision and hence induces a change in transactions costs. Chang and Lai (2000) focus on the transitional responses of the growth rate of relevant macro variables to an anticipated permanent rise in the money growth rate.

As a consequence of globalization, frequent interaction among economies raises the importance of the role of international factors in relation to domestic macroeconomic performance. It is commonly believed that money is an indispensable factor in open-economy analyses. However, most of the authors studying endogenous growth in open economies, such as Razin and Yuen (1996), Turnovsky (1996, 1997, 2002a), Van der Ploeg (1996), and Eicher and Turnovsky (1999), develop their models from the *real perspective*. To be more specific, these studies unanimously downplay the role of money, and only deal with the economy of exchanges between export goods and import goods, and only with the movement of physical capital rather than of financial capital. With these specific simplifications, the existing literature on endogenously growing open economies cannot deal with the relationship between inflation and economic growth.

During the development of the literature on money and endogenous growth, it may appear strange that little attention was paid to an open-economy setting. The existing literature on monetary policy in open-economy models includes Palokangas (1997) and Shaw et al. (2005). Palokangas (1997) analyzes the effect of an unanticipated increase in the nominal interest rate on the long-run economic growth rate. He finds that the result is ambiguous and depends on the relative magnitudes of the elasticity of money holdings with respect to the interest rate and the elasticity of output with regard to the tax rate. Nonetheless, the transitional dynamics of macroeconomic variables is not examined in Palokangas (1997). Shaw et al. (2005) investigate the long-term as well as transitional effects of an anticipated domestic credit expansion. They point out that money is economic growth–retarding in both the intermediate term and the long run. A common feature in Palokangas (1997) and Shaw et al. (2005) is that they both focus on the effect of a single monetary rule and are silent on other important monetary rules that are currently implemented in many industrial and developing countries.

The debate over the choice of an appropriate monetary policy rule has a long history in macroeconomic analysis. In his pioneering paper, Poole (1970) focuses on the debate over whether the monetary authority should choose a pegged monetary stock or a pegged interest rate as its policy rule. Two Nobel laureates, Meade (1978) and Tobin (1980), propose nominal income as an alternative target for monetary policy. Recently, McCallum and Nelson (1999) presented a simulation analysis regarding the performance of nominal income targeting. Their results in calibrating U.S. quarterly data suggest that, in comparison with other targeting strategies, nominal income targeting exhibits better performance. Moreover, inflation targeting, which is one kind of monetary policy strategy, has been successfully implemented by a number of industrialized countries (including Australia, Canada, Finland, Israel, New Zealand, Spain, Sweden, and the United Kingdom) and by a growing number of emerging-market countries (including Chile, Brazil, the Czech Republic, Poland, and South Africa).

Until now there have been few studies devoted to discussing the equivalent relation among various monetary policy rules. Végh (2001) shows that, under certain conditions, a *k%*–money growth rule, a nominal interest rate rule combined with an inflation target, and a real interest rate rule combined with an inflation target are equivalent in the sense of dynamics. Schabert (2003) finds that, when prices are flexible, an active interest rate policy mimics a policy accommodating money growth rates, whereas a constant money growth rule is equivalent to a passive interest rate rule.[1] More recently, Yip and Li (2006) found that, under logarithmic preferences, a constant–money growth rule is identical to an interest rate–pegging rule. Although these studies provide their respective contributions on policy equivalence, they all confine their analysis to closed economies.

Based on the observation concerning academic progress on open-economy endogenous growth, this paper develops a monetary endogenous growth model for an open economy, and then uses it to address whether the monetary authorities will govern economic growth when they conduct various monetary policy rules, including money growth rate targeting, nominal income growth rate targeting, and inflation rate targeting.[2] In addition, this paper examines the equivalent relation among various monetary policy rules.

In the standard neoclassical (exogenous) growth models, the levels of both real money balances (the nominal money supply divided by the price level) and output converge to a specific value in the steady state. This implies that the money growth rate, the inflation rate, and the nominal income growth rate are identical in the steady state. As a consequence, at the stationary equilibrium, all money growth rate targeting, inflation rate targeting, and nominal income growth rate targeting are equivalent in the standard neoclassical (exogenous) growth model. However, in an endogenous growth model, the growth rates of real money balances and output converge to a specific value in the steady state. This implies that the money growth rate, the inflation rate, and the nominal income growth rate are not all equal to each other in the steady state. Accordingly, except via imposing specific restrictions, in general money growth rate targeting, inflation rate targeting, and nominal income growth rate targeting are not all equivalent in the endogenous growth model. This constitutes the motivation for our paper to set up an endogenous growth model and use it to highlight the possible difference among money growth rate targeting, inflation rate targeting, and nominal income growth rate targeting.

The remainder of this paper proceeds as follows. Section 2 develops a monetary endogenous growth model for an open economy. The prominent feature of the model is that it is able to deal with various monetary policy rules including money growth rate targeting, nominal income growth rate targeting, and inflation rate targeting. Section 3 discusses the economy's balanced-growth equilibrium and examines the relationship between the balanced-growth rate and the targeting rate under various monetary policy rules. Section 4 deals with the equivalent relation among money growth rate targeting, nominal income growth rate targeting, and inflation rate targeting. Finally, concluding remarks are provided in Section 5.

2. MODEL

The economy we consider is composed of three sectors: a household sector, a government, and a central bank. In what follows, we in turn describe the behavior of each of these sectors.

2.1. Households

The economy is populated by a large number of identical and infinitely lived households. For simplicity, population is normalized to unity. The household derives utility from consumption C; its lifetime utility can be specified as

$$\int_0^{\infty} \frac{C^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt, \tag{1}$$

where σ represents the inverse of the intertemporal elasticity of substitution in consumption and ρ is the constant rate of time preference.

In line with Rebelo (1991), output Y is produced using a stock of broad-concept productive capital K; that is, $Y = \Phi K$, where Φ (>0) stands for the total factor productivity. The household holds nominal money balance M to facilitate transactions of output. Let us denote $m(= M/P)$ as the real money balance with P representing the price level. Following Zhang (1996) and Suen and Yip (2005), to allow for a balanced-growth path, the transactions cost technology is summarized by a rate of loss in real output as follows:

$$t\left(\frac{m}{Y}\right) = \left[1 - \theta\left(\frac{m}{Y}\right)^{\alpha}\right]; \quad 0 < \theta < 1 \quad \text{and} \quad 0 < \alpha < 1. \tag{2}$$

As a result, the real resource costs required to facilitate transactions services in the economy are denoted by $T = tY$. Thus, total transactions costs and net output (the difference between output and total transaction costs) are given by[3]

$$T = t\left(\frac{m}{Y}\right)Y = \left[1 - \theta\left(\frac{m}{Y}\right)^{\alpha}\right]Y, \tag{3}$$

$$Y - T = \theta\left(\frac{m}{\Phi K}\right)^{\alpha}\Phi K = \Gamma K^{1-\alpha}m^{\alpha}; \quad \Gamma = \theta\Phi^{1-\alpha}. \tag{4}$$

The representative household accumulates physical capital involving adjustment costs (installation costs) with a quadratic convex function. In line with Hayashi (1982), Abel and Blanchard (1983), and Turnovsky (1996), the adjustment cost function is specified as

$$Z(I, K) = I\left(1 + \frac{h}{2}\frac{I}{K}\right); \quad h > 0, \tag{5}$$

where I denotes the level of investment, and h is a constant parameter of adjustment costs, expressing the sensitivity of the adjustment costs. Adjustment costs that depend on investment relative to the capital stock can be justified by learning by doing in the installation process. As addressed by Feichtinger, et al. (2001, p. 255), "if capital stock is large, a lot of machines have been installed in the past so that this firm has a lot of experience, implying that it is more efficient in installing new machines."

For simplicity, we assume that the capital stock does not depreciate, so that the representative household faces the following physical capital accumulation constraint:

$$\dot{K} = I. \tag{6}$$

At each instant in time, the representative household is bound by a flow constraint linking wealth accumulation to any difference between its gross income and its expenditure. Let b^* denote real holdings of foreign bonds measured in terms of domestic output. More specifically, $b^* = EB^*/P$, where B^* represents holdings of foreign bonds measured in terms of foreign currency and E is the nominal exchange rate. The household's flow budget constraint can then be expressed as

$$\dot{m} + \dot{b}^* = \Gamma K^{1-\alpha}m^{\alpha} + \mathrm{Tr} - C + (R^* + \varepsilon - \pi)b^* - \pi m - I\left(1 + \frac{h}{2}\frac{I}{K}\right), \tag{7}$$

where ε, Tr, R^*, and π denote the depreciation rate of the domestic currency, lump-sum transfers from the government to the private sector, the world nominal interest rate on foreign bonds, and the domestic inflation rate, respectively.

The representative household maximizes (1) subject to (6) and (7) by choosing $\{C, I, K, m, b^*\}_{t=0}^{\infty}$. Let λ_1 and λ_2 be the shadow values of the wealth $a\ (= b^* + m)$ and the physical capital stock K, respectively. The current-value Hamiltonian for

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the household's optimization is then given by

$$H = \frac{C^{1-\sigma} - 1}{1-\sigma} + \lambda_1 \left[\Gamma K^{1-\alpha} m^{\alpha} + \mathrm{Tr} - C + (R^* + \varepsilon - \pi) b^* - \pi m - I\left(1 + \frac{h}{2}\frac{I}{K}\right)\right] + \lambda_2 I. \tag{8}$$

The optimum conditions for the representative household with respect to the indicated variables are

$$C: \quad C^{-\sigma} = \lambda_1, \tag{9a}$$

$$I: \quad \lambda_1 \left(1 + \frac{hI}{K}\right) = \lambda_2, \tag{9b}$$

$$K: \quad \lambda_1 \left[(1-\alpha)\Gamma K^{-\alpha} m^{\alpha} + \frac{hI^2}{2K^2}\right] = -\dot{\lambda}_2 + \lambda_2 \rho, \tag{9c}$$

$$m: \quad \lambda_1 (\alpha \Gamma K^{1-\alpha} m^{\alpha-1} - \pi) = -\dot{\lambda}_1 + \lambda_1 \rho, \tag{9d}$$

$$b^*: \quad \lambda_1 (R^* + \varepsilon - \pi) = -\dot{\lambda}_1 + \lambda_1 \rho, \tag{9e}$$

together with equations (6) and (7) and the transversality conditions of m, b^*, and K: $\lim_{t\to\infty} \lambda_1 m e^{-\rho t} = 0$, $\lim_{t\to\infty} \lambda_1 b^* e^{-\rho t} = 0$, and $\lim_{t\to\infty} \lambda_2 K e^{-\rho t} = 0$.

Let $q = \lambda_2/\lambda_1$ be the market value of capital in terms of the price of wealth. Using equations (6) and (9b), we have

$$\frac{\dot{K}}{K} = \frac{I}{K} = \frac{q-1}{h}. \tag{10}$$

From equations (9c) and (9e), we can obtain

$$\frac{\dot{q}}{q} = R^* + \varepsilon - \pi - \frac{1}{q}\left[(1-\alpha)\Gamma K^{-\alpha} m^{\alpha} + \frac{hI^2}{2K^2}\right]. \tag{11}$$

2.2. Government and Central Bank

The government distributes seigniorage to the representative agent as a transfer payment in a lump-sum manner (i.e., $\dot{M}/P = \mathrm{Tr}$). Given $m = M/P$, the flow budget constraint of the government can be written as

$$\dot{m} = \mathrm{Tr} - \pi m. \tag{12}$$

Let μ denote the growth rate of the nominal money stock at a targeted level (i.e., $\mu = \dot{M}/M$). Then, by definition, the law of motion governing real cash balances is

$$\frac{\dot{m}}{m} = \mu - \pi. \tag{13}$$

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2.3. Decentralized Competitive Equilibrium

The domestic economy produces and consumes a single traded good, the foreign price of which is given in the world market. Let π^* denote the foreign inflation rate. In the absence of any impediments to trade, purchasing power parity is assumed to hold and can be described by

$$\pi = \pi^* + \varepsilon. \tag{14}$$

Given the definition $\dot{K} = I$, combining the government's budget constraint (12) and the household's budget constraint (7) yields the following aggregate resource constraint of the economy:

$$\dot{b}^* = \Gamma K^{1-\alpha} m^{\alpha} - C - I\left(1 + \frac{hI}{2K}\right) + (R^* + \varepsilon - \pi)b^*. \tag{15}$$

Equation (15) states that the economy's net accumulation of foreign bonds is the current account balance, which in turn equals the balance of trade plus the net interest income earned on the foreign bonds.

The full macroeconomic equilibrium for the economy is composed of the optimal conditions for the representative household, the government budget constraint, and the equilibrium condition for the goods market. The macroeconomic equilibrium of the economy can then be summarized by the following set of equations:

$$C^{-\sigma} = \lambda_1, \tag{16a}$$

$$\frac{I}{K} = \frac{q-1}{h}, \tag{16b}$$

$$\frac{\dot{q}}{q} = R^* + \varepsilon - \pi - \frac{1}{q}\left[(1-\alpha)\Gamma K^{-\alpha} m^{\alpha} + \frac{hI^2}{2K^2}\right], \tag{16c}$$

$$\lambda_1(\alpha \Gamma K^{1-\alpha} m^{\alpha-1} - \pi) = -\dot{\lambda}_1 + \lambda_1 \rho, \tag{16d}$$

$$\lambda_1(R^* + \varepsilon - \pi) = -\dot{\lambda}_1 + \lambda_1 \rho, \tag{16e}$$

$$\dot{K} = I, \tag{16f}$$

$$\dot{m} = \mathrm{Tr} - \pi m, \tag{16g}$$

$$\frac{\dot{m}}{m} = \mu - \pi, \tag{16h}$$

$$\pi = \pi^* + \varepsilon, \tag{16i}$$

$$\dot{b}^* = \Gamma K^{1-\alpha} m^{\alpha} - C - I\left(1 + \frac{hI}{2K}\right) + (R^* + \varepsilon - \pi)b^*. \tag{16j}$$

Solving equation (16e) with (16i) and equation (10), we have $\lambda_1 = \lambda_1(0)\exp[\rho t - (R^* - \pi^*)t]$ and $K = K(0)\exp[(q-1)t/h]$, where $\lambda_1(0)$ is the endogenously determined initial marginal utility and $K(0)$ is the given initial stock of domestic physical capital. Given $q = \lambda_2/\lambda_1$, the transversality condition can then be rewritten as

$$\lim_{t\to\infty} \lambda_2 K e^{-\rho t} = \lim_{t\to\infty} \lambda_1 q K e^{-\rho t} = \tilde{q}\lambda_1(0)K(0)$$
$$\times \exp\left\{\left[\frac{\tilde{q}-1}{h} - (R^* - \pi^*)\right]t\right\} = 0, \tag{17}$$

where $\tilde{q}$ is the stationary value of q determined later.

Based on equation (17), the restriction that the transversality condition is satisfied can be expressed as follows:

Condition TVC [the Transversality Condition].

$$(\tilde{q} - 1)/h < R^* - \pi^*. \tag{18a}$$

Equation (18a) states that the transversality condition requires that the real rate of interest on foreign bonds be greater than the rate of growth of domestic capital.

Based on equations (16b) and (16f), the restriction that an endogenously growing economy has a positive equilibrium growth rate of capital can be expressed as follows:

Condition PGRC [the Positive Growth Rate Condition].

$$\tilde{q} > 1. \tag{18b}$$

Both Condition TVC in equation (18a) and Condition PGRC in equation (18b) impose the restriction $1 < \tilde{q} < h(R^* - \pi^*) + 1$.

3. MONETARY POLICY RULES

3.1. Money Growth Rate Targeting

Under a regime of money growth targeting, the nominal money growth rate (μ) is kept constant. The macroeconomic model expressed in equations (16a)–(16j) determines the endogenous variables C, λ_1, q, K, m, I, π, Tr, ε, and b^*. By means of some simple manipulations to delete C, λ_1, I, π, Tr, ε, and b^*, the dynamic system in terms of q, K, and m can be described by

$$\frac{\dot{q}}{q} = R^* - \pi^* - \frac{1}{q}\left[(1-\alpha)\Gamma K^{-\alpha} m^{\alpha} + \frac{(q-1)^2}{2h}\right], \tag{19a}$$

$$\frac{\dot{K}}{K} = \frac{q-1}{h}, \tag{19b}$$

$$\frac{\dot{m}}{m} = \mu - \alpha\Gamma K^{1-\alpha} m^{\alpha-1} + R^* - \pi^*. \tag{19c}$$

In order to derive the dynamic equations that summarize the entire model, we define the transformed variable $z = m/K$. Using equations (19a), (19b), and (19c),

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q and z evolve as

$$\dot{q} = (R^* - \pi^*)q - (1-\alpha)\Gamma z^{\alpha} - \frac{(q-1)^2}{2h}, \tag{20a}$$

$$\dot{z} = z\left[\mu - \alpha\Gamma z^{\alpha-1} + R^* - \pi^* - \frac{(q-1)}{h}\right]. \tag{20b}$$

Let $\tilde{q}$ and $\tilde{z}$ be the stationary values of q and z. Then linearizing equations (20a) and (20b) around the steady-state equilibrium yields

$$\begin{bmatrix} \dot{q} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} J_q & J_z \\ F_q & F_z \end{bmatrix} \begin{bmatrix} q-\tilde{q} \\ z-\tilde{z} \end{bmatrix}, \tag{21}$$

where $J_q = R^* - \pi^* - (\tilde{q}-1)/h > 0$, $J_z = -(1-\alpha)\alpha\Gamma\tilde{z}^{\alpha-1} < 0$, $F_q = -\tilde{z}/h < 0$, and $F_z = \alpha(1-\alpha)\Gamma\tilde{z}^{\alpha-1} > 0$. It should be noted that, given that Condition TVC requires $R^* - \pi^* - (\tilde{q}-1)/h > 0$, $J_q > 0$ is ensured.

Let δ_1 and δ_2 be the two characteristic roots of the dynamic system. We then have

$$\delta_1 + \delta_2 = J_q + F_z > 0, \tag{22a}$$

$$\delta_1\delta_2 = J_q F_z - F_q J_z = \left(R^* - \pi^* - \tfrac{\tilde{q}-1}{h} - \tfrac{\tilde{z}}{h}\right)\alpha(1-\alpha)\Gamma\tilde{z}^{\alpha-1} \gtrless 0; \quad \text{if} \quad R^* - \pi^* - \tfrac{\tilde{q}-1}{h} \gtrless \tfrac{\tilde{z}}{h}. \tag{22b}$$

As claimed in the literature on dynamic rational expectations models (for example, Burmeister 1980; Buiter 1984; Turnovsky 2000), if the number of unstable (positive) roots equals the number of jump variables, then there exists a unique perfect-foresight equilibrium solution. Furthermore, in contrast, if the number of unstable (positive) roots is smaller than the number of jump variables, then the steady state is locally indeterminate. As indicated in equation (21), the dynamic system under the regime of money growth rate targeting has two jump variables, z and q. Based on the previously stated rule, if two characteristic roots have positive real parts, the steady-state equilibrium is locally determinate and there exists a unique growth path converging to it. However, if one of two characteristic roots has negative real part, the monetary equilibrium exhibits local indeterminacy.

Given the results reported in equations (22a) and (22b) that $\delta_1 + \delta_2 > 0$ and $\delta_1\delta_2 \gtrless 0$, we can thus conclude that the monetary equilibrium is locally determinate when the gap between the foreign real interest rate and the growth rate of physical capital is sufficiently large (i.e., $R^* - \pi^* - (\tilde{q}-1)/h > \tilde{z}/h$); otherwise, the monetary equilibrium exhibits local indeterminacy. The result leads us to establish the following proposition:

PROPOSITION 1. *Under a regime of money growth rate targeting, the monetary equilibrium is locally determinate only when the gap between the foreign real*

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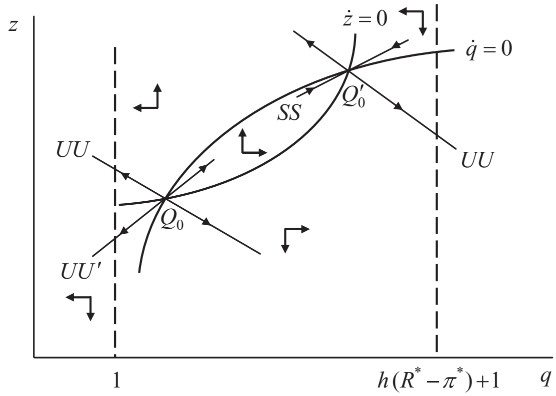

FIGURE 1. Phase diagram under a regime of money growth rate targeting.

interest rate and the growth rate of physical capital is sufficiently large. Otherwise, the monetary equilibrium is locally indeterminate.

A graphical presentation will be helpful to our understanding of the feature of a dynamic system. The evolution of both q and z can be illustrated by a consideration of the phase diagram. It is quite obvious from equations (20a) and (20b) that the slopes of loci $\dot{q}=0$ and $\dot{z}=0$ displayed in the q and z plane are

$$\left.\frac{\partial z}{\partial q}\right|_{\dot{q}=0} = \frac{-J_q}{J_z} = \frac{(R^*-\pi^*)-[(\tilde{q}-1)/h]}{(1-\alpha)\alpha\Gamma\tilde{z}^{\alpha-1}} > 0, \tag{23a}$$

$$\left.\frac{\partial z}{\partial q}\right|_{\dot{z}=0} = \frac{-F_q}{F_z} = \frac{(\tilde{z}/h)}{(1-\alpha)\alpha\Gamma\tilde{z}^{\alpha-1}} > 0. \tag{23b}$$

Equations (23a) and (23b) indicate that both the $\dot{q}=0$ locus and the $\dot{z}=0$ locus are upward-sloping.[4] In Figure 1, the $\dot{q}=0$ schedule and the $\dot{z}=0$ schedule intersect twice, at Q_0 and Q_0'. Moreover, in Figure 1, we draw a dotted vertical line, which defines the threshold value of $\tilde{q}$ to conform to $(\tilde{q}-1)/h = R^*-\pi^*$. As is obvious, to satisfy Condition TVC, the economy should be located to the left of the dotted vertical line. There are thus two potential balanced-growth equilibria. As indicated by the directions of the arrows in Figure 1, we can sketch all possible trajectories. It is clear from Figure 1 that the low-growth equilibrium (point Q_0) is locally determinate and the high-growth equilibrium (point Q_0') is locally indeterminate. The result can be described by the following proposition:

PROPOSITION 2. *Under a regime of money growth rate targeting, dual balanced-growth equilibria may emerge. At the low-growth equilibrium the economy is characterized by local determinacy and at the high-growth equilibrium the economy is characterized by local indeterminacy.*

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The rationale for local indeterminacy at the high-growth equilibrium can be understood intuitively. When the representative household generates optimistic expectations regarding having a higher future (shadow) price of physical capital, two conflicting effects would work. First, in response to a rise in the market value of physical capital, the household will be inclined to shift its holdings of foreign bonds to physical capital. This induces a reduction in the net marginal product of physical capital (henceforth referred to as NMPK),[5] and hence in turn leads to a fall in the market value of physical capital. As a result, the portfolio substitution effect has a negative impact on the market value of physical capital. Second, a higher market value of physical capital raises the total wealth of the household. The representative household will increase its money holdings and then NMPK will increase in response.[6] The higher NMPK is, the higher will be the market value of physical capital. As a consequence, the income effect has a positive impact on the market value of physical capital. If the income effect outweighs the portfolio substitution effect (i.e., the rate of return on foreign bonds is relatively small, $R^* - \pi^* < (\tilde{q} - 1)/h + \tilde{z}/h$), an actual rise in the market value of physical capital is present. This result reveals that the representative household's initial optimistic expectations become self-fulfilling.[7]

At the balanced-growth equilibrium, the economy is characterized by $\dot{q} = \dot{z} = 0$. Recall that $\tilde{q}$ and $\tilde{z}$ are the stationary values of q and z, respectively. It follows from (20a) and (20b) that the steady-state values $\tilde{q}$ and $\tilde{z}$ satisfy the following stationary relationships:

$$(R^* - \pi^*)\tilde{q} - (1 - \alpha)\Gamma\tilde{z}^{\alpha} - \frac{(\tilde{q} - 1)^2}{2h} = 0, \tag{24a}$$

$$\mu - \alpha\Gamma\tilde{z}^{\alpha-1} + R^* - \pi^* - \frac{(\tilde{q} - 1)}{h} = 0. \tag{24b}$$

Given $z = m/K$, $\dot{z} = 0$ implies that both real balances and the capital stock grow at a common rate $\tilde{\gamma}$ along the balanced-growth equilibrium. Moreover, based on $Y = \Phi K$, we can also infer that output also grows at a common rate $\tilde{\gamma}$. Then, it follows from equation (10) that, in the balanced-growth equilibrium, the growth rate of the economy is given by

$$\tilde{\gamma} = \frac{\tilde{q} - 1}{h}. \tag{25}$$

It is easy to see from equations (24a), (24b), and (25) that

$$\frac{d\tilde{\gamma}}{d\mu} = \frac{-\tilde{z}/h}{(R^* - \pi^*) - (\tilde{q} - 1)/h - \tilde{z}/h} \gtrless 0 \quad \text{if} \quad R^* - \pi^* - \frac{\tilde{q} - 1}{h} \lessgtr \frac{\tilde{z}}{h}. \tag{26}$$

Equation (26) indicates that a rise in the nominal money growth rate may either increase or decrease the economic growth rate in the long run, depending on whether the balanced-growth equilibrium is locally indeterminate or determinate.

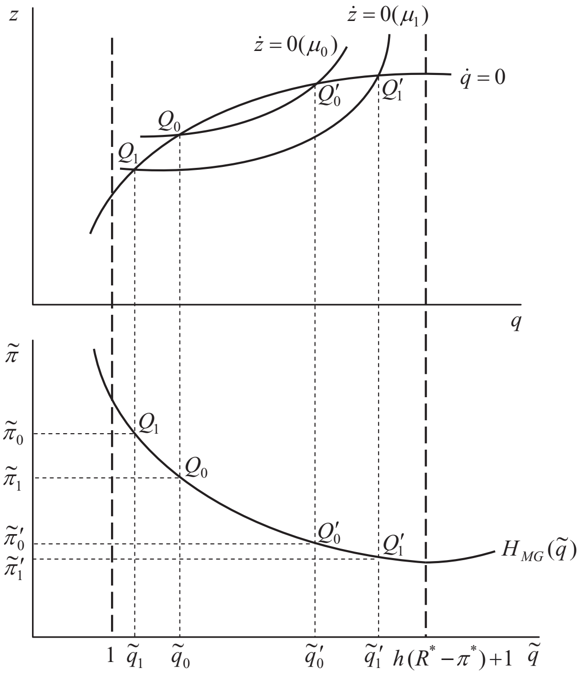

FIGURE 2. The effect of a rise in the money growth rate.

The upper panel of Figure 2 presents a diagram to illustrate the relationship between the anchor of the money growth rate and the balanced-growth rate.[8] In response to a rise in the money growth rate from μ_0 to μ_1, the $\dot{z} = 0(\mu_0)$ schedule shifts rightward to $\dot{z} = 0(\mu_1)$. Under the situation $R^* - \pi^* - (\tilde{q} - 1)/h > \tilde{z}/h$ in which the economy is at the low-growth equilibrium initially, the balanced-growth equilibrium changes from point Q_0 to Q_1. At the new stationary equilibrium, $\tilde{q}$ falls from $\tilde{q}_0$ to $\tilde{q}_1$, and, based on equation (25), the balanced-growth rate is lowered from its initial level to a new level. In contrast, under the situation $R^* - \pi^* - (\tilde{q} - 1)/h < \tilde{z}/h$, in which the economy is at the high-growth equilibrium initially, the balanced-growth equilibrium changes from point Q_0' to Q_1'. At the new stationary equilibrium, $\tilde{q}$ rises from $\tilde{q}_0'$ to $\tilde{q}_1'$, and, based on equation (25), the balanced-growth rate is increased from its initial level to a new level.[9]

We are now ready to discuss how the inflation rate will react following a change in the money growth rate. From equations (16d) and (16e) we obtain the following expression:

$$z = \frac{m}{K} = \left(\frac{\alpha\Gamma}{R^* - \pi^* + \pi}\right)^{\frac{1}{1-\alpha}}. \tag{27a}$$

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Substituting equation (27a) into (24a) yields

$$\tilde{\pi} = \frac{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{\left[(R^*-\pi^*)\tilde{q} - \frac{(\tilde{q}-1)^2}{2h}\right]^{\frac{1-\alpha}{\alpha}}} - (R^*-\pi^*) \equiv H_{\mathrm{MG}}(\tilde{q}), \qquad \textbf{(27b)}$$

where the subscript "MG" denotes the regime of money growth rate targeting. From (27b) we have

$$H'_{\mathrm{MG}}(\tilde{q}) = \frac{-(1-\alpha)^{\frac{1}{\alpha}}\Gamma^{\frac{1}{\alpha}}\left(R^*-\pi^*-\frac{\tilde{q}-1}{h}\right)}{\left[(R^*-\pi^*)\tilde{q}-\frac{(\tilde{q}-1)^2}{2h}\right]^{\frac{1}{\alpha}}} \gtreqless 0 \quad \text{if} \quad \tilde{q} \gtreqless h(R^*-\pi^*)+1. \qquad \textbf{(27c)}$$

Based on equations (27b) and (27c), as exhibited in the lower panel of Figure 2, we can sketch the $H_{\mathrm{MG}}(\tilde{q})$ curve under the regime of money growth rate targeting, which represents all pairs of $\tilde{\pi}$ and $\tilde{q}$ that satisfy equation (27b). It should be noted that at the lowest point on the $H_{\mathrm{MG}}(\tilde{q})$ curve the level of $\tilde{q}$ is associated with $h(R^*-\pi^*)+1$.

From the upper panel of Figure 2, we learn that, if the economy is located at the low-growth equilibrium initially, the balanced-growth equilibrium changes from point Q_0 to Q_1 and $\tilde{q}$ is reduced from $\tilde{q}_0$ to $\tilde{q}_1$. As exhibited in the lower panel of Figure 2, we can infer from the $H_{\mathrm{MG}}(\tilde{q})$ curve that the domestic inflation rate should rise from $\tilde{\pi}_0$ to $\tilde{\pi}_1$ when the money growth rate is increased from μ_0 to μ_1. However, in the upper panel of Figure 2, if the economy is located at the high-growth equilibrium initially, the balanced-growth equilibrium changes from point Q'_0 to Q'_1 and $\tilde{q}$ rises from $\tilde{q}'_0$ to $\tilde{q}'_1$. Then, based on the $H_{\mathrm{MG}}(\tilde{q})$ curve in the lower panel of Figure 2, we learn that the domestic inflation rate should decline from $\tilde{\pi}'_0$ to $\tilde{\pi}'_1$ when the money growth rate is increased from μ_0 to μ_1.

The results displayed in Figure 2 can be summarized by the following proposition:

PROPOSITION 3. *Under a regime of money growth targeting, at the high-growth equilibrium a rise in the money growth target increases the balanced-growth rate, whereas at the low-growth equilibrium a rise in the money growth target decreases the balanced-growth rate.*

The economic intuition behind Proposition 3 can also be explained by means of the relative extent of the portfolio effect and the income effect. A rise in the nominal money growth rate expands the lump-sum transfers from the government to the private sector. On one hand, a rise in the nominal money growth rate stimulates the inflation rate and hence raises the opportunity cost of holding money. The representative household will shift its money holdings to foreign bonds and then NMPK will be reduced.[10] The lower NMPK, the lower will be the market value of physical capital. In response to a fall in the market value of physical capital,

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the household will be inclined to lower its accumulation of physical capital. As a result, the portfolio substitution effect has a negative effect on the growth rate. On the other hand, a rise in the lump-sum transfer raises the total wealth of the household. The representative household will increase its money holdings and then NMPK will increase. The higher NMPK, the higher will be the market value of physical capital. In response to a rise in the market value of physical capital, the household will be inclined to increase its accumulation of physical capital. As a result, the income effect has a positive effect on the growth rate. As mentioned earlier, at the high-growth equilibrium the income effect dominates the portfolio substitution effect. Consequently, a rise in the nominal money growth rate leads to a rise in the balanced economic growth rate at the high-growth equilibrium. In contrast, at the low-growth equilibrium the income effect falls short of the portfolio substitution. Accordingly, a rise in the nominal money growth rate is associated with a fall in the balanced economic growth rate at the low-growth equilibrium.

Two points concerning the difference between the open economy and the corresponding closed economy should be mentioned here. First, differentiating equation (16a) with respect to time and using equations (16e) and (16i), we can derive the growth rate of consumption:

$$\frac{\dot{C}}{C} = \frac{R^* - \pi^* - \rho}{\sigma}. \tag{28}$$

Obviously, when the finance channel from the world capital market is opened, the growth rate of consumption exhibits a different pace than the growth rate of output. As pointed out by Turnovsky (2002b, p. 319), "[this result] contrasts to the closed economy in which, constrained by the growth of its own resources, all real variables, including consumption and output would ultimately have to grow at the same rate."

Second, in the context of a *closed-economy* model, Itaya and Mino (2003, 2007), Lai et al. (2005), Suen and Yip (2005), and Yip and Li (2006) show that the rule of money growth rate targeting may result in local indeterminacy.[11] Itaya and Mino (2003, 2007) find that local indeterminacy may emerge when labor externalities are sufficiently large. Lai et al. (2005) specify that the representative household holds both domestic bonds and money balances as assets, and find that the elasticity of the domestic interest rate with respect to the real balances–output ratio is crucial for the emergence of indeterminacy. Jha et al. (2002), Suen and Yip (2005), Itaya and Mino (2007), and Chen and Guo (2008) set up a monetary endogenous growth model where money is introduced via either a transactions cost technology or a cash-in-advance constraint. Their analyses find that the intertemporal elasticity of substitution in consumption is an important factor determining the presence of indeterminacy. In the context of an *open-economy* model without labor externalities, by the channel of the opening up of the domestic economy to the international capital market, our analysis shows that the presence of local indeterminacy is not related to the intertemporal elasticity of substitution.

3.2. Nominal Income Growth Rate Targeting

Under a regime of nominal income growth rate targeting, the following relationship must hold:

$$\pi + \frac{\dot{Y}}{Y} = v, \tag{29a}$$

where v is the government's target for the nominal income growth rate.[12] Given $Y = \Phi K$ (hence $\dot{Y}/Y = \dot{K}/K$), from equation (29a) we have

$$\frac{\dot{K}}{K} = v - \pi. \tag{29b}$$

When the monetary authorities target a specific nominal income growth rate, the nominal income growth rate (v) is kept constant. Under such a scenario, the macroeconomic model can be expressed by equations (16a)–(16j) and (29b), and the endogenous variables are $C, \lambda_1, q, K, m, I, \pi, \mu$, Tr, ε, and b^*. By substituting the expressions for C, λ_1, K, m, I, π, μ, Tr, ε, and b^*, the following dynamic system in q is obtained:

$$\dot{q} = (R^* - \pi^*)q - (1-\alpha)\Gamma \left\{ \frac{\alpha\Gamma}{v + R^* - \pi^* - [(q-1)/h]} \right\}^{\frac{\alpha}{1-\alpha}} - \frac{(q-1)^2}{2h}. \tag{30a}$$

Linearizing equation (30a) around the balanced-growth equilibrium yields

$$\dot{q} = J_q(q - \tilde{q}), \tag{30b}$$

where

$$J_q = R^* - \pi^* - \frac{\tilde{q}-1}{h} - \frac{1}{h} \left\{ \frac{\alpha\Gamma}{v + R^* - \pi^* - [(\tilde{q}-1)/h]} \right\}^{\frac{1}{1-\alpha}} \gtrless 0.$$

Let δ be the characteristic root of the dynamic system. We then have

$$\delta = J_q \gtrless 0 \text{ if } R^* - \pi^* - \frac{\tilde{q}-1}{h} \gtrless \Theta, \tag{31}$$

where

$$\Theta = \frac{1}{h} \left\{ \frac{\alpha\Gamma}{v + R^* - \pi^* - [(\tilde{q}-1)/h]} \right\}^{\frac{1}{1-\alpha}}.$$

Given that $\delta \gtrless 0$ and q is a jump variable, we can thus conclude that the balanced-growth equilibrium is locally determinate when the gap between the foreign real

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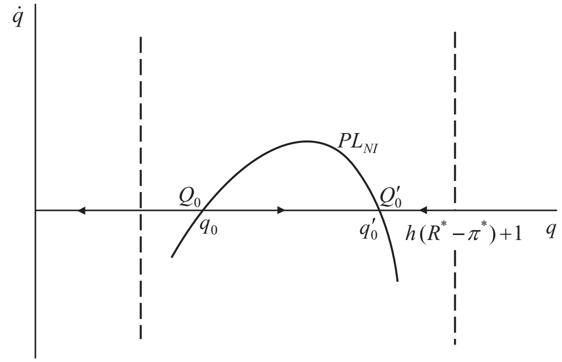

FIGURE 3. Phase diagram under a regime of nominal income growth rate targeting.

interest rate and the growth rate of physical capital is sufficiently large (i.e., $R^* - \pi^* - (\tilde{q} - 1)/h > \Theta$); otherwise, the balanced-growth equilibrium is locally indeterminate. Summing up the preceding discussions, we have the following proposition:

PROPOSITION 4. *Under a regime of nominal income targeting, the monetary equilibrium is characterized by local determinacy only when the gap between the foreign real interest rate and the growth rate of physical capital is sufficiently large. Otherwise, the monetary equilibrium is characterized by local indeterminacy.*

The phase diagram depicted in Figure 3 can be taken to illustrate the evolution of q. In that figure, the phase locus for nominal income targeting, $\mathrm{PL_{NI}}$, depicts all combinations of $\dot{q}$ and q that satisfy equation (30a). It is straightforward from equation (30a) to infer that:

$$\frac{\partial \dot{q}}{\partial q} = (R^* - \pi^*) - \frac{(\tilde{q} - 1)}{h} - \Theta \gtrless 0 \text{ if } R^* - \pi^* - \frac{\tilde{q} - 1}{h} \gtrless \Theta. \tag{32a}$$

$$\frac{\partial^2 \dot{q}}{\partial q^2} = -\frac{\Theta}{(1-\alpha)\{v + R^* - \pi^* - [(\tilde{q} - 1)/h]\}h} - \frac{1}{h} < 0. \tag{32b}$$

Equations (32a) and (32b) indicate that the $\mathrm{PL_{NI}}$ locus is concave downward.

In Figure 3, the $\mathrm{PL_{NI}}$ schedule and the horizontal axis intersect twice at Q_0 and Q_0'. This indicates that two potential balanced-growth equilibria may emerge. As indicated by the directions of the arrows in Figure 3, it is clear that the low-growth equilibrium (point Q_0) is locally unstable and the high-growth equilibrium (point Q_0') is locally stable. Given that q is a jump variable, we can thus conclude that the low-growth equilibrium is characterized by local determinacy and the high-growth equilibrium is characterized by local indeterminacy.[13] The result can be described by the following proposition:

PROPOSITION 5. *Under a regime of nominal income targeting, dual balanced-growth equilibria may emerge. At the low-growth equilibrium the* *economy is characterized by local determinacy and at the high-growth equilibrium the economy is characterized by local indeterminacy.*

At the steady-growth equilibrium, the economy is characterized by $\dot{q} = 0$. It follows from (30a) that the steady-state value $\tilde{q}$ satisfies the following stationary relationship:

$$(R^* - \pi^*)\tilde{q} - (1-\alpha)\Gamma\left\{\frac{\alpha\Gamma}{v + R^* - \pi^* - [(\tilde{q}-1)/h]}\right\}^{\frac{\alpha}{1-\alpha}} - \frac{(\tilde{q}-1)^2}{2h} = 0. \tag{33}$$

Thus, from equations (33) and (25), we can infer the following result:

$$\frac{d\tilde{\gamma}}{dv} = \frac{-\Theta}{(R^* - \pi^*) - (\tilde{q}-1)/h - \Theta} \gtrless 0;\ \text{if } R^* - \pi^* - \frac{\tilde{q}-1}{h} \lessgtr \Theta. \tag{34}$$

Equation (34) reveals that a rise in the nominal income growth rate may either encourage or discourage the balanced-growth rate, depending upon whether the gap between the growth rate of physical capital and the foreign real interest rate is sufficiently large or not.

The upper panel of Figure 4 illustrates the relationship between the anchor of the nominal income growth rate and the balanced-growth rate.[14] In response to a rise in the nominal income growth rate from v_0 to v_1, the $PL_{NI}(v_0)$ schedule shifts upward to $PL_{NI}(v_1)$.[15] Under the situation $R^* - \pi^* - (\tilde{q}-1)/h > \Theta$, in which the economy is at the low-growth equilibrium initially, the balanced-growth equilibrium changes from point Q_0 to Q_1. At the new stationary equilibrium, $\tilde{q}$ falls from $\tilde{q}_0$ to $\tilde{q}_1$, and, based on equation (25), the balanced-growth rate is lowered in response. In contrast, in the situation $R^* - \pi^* - (\tilde{q}-1)/h < \Theta$, in which the economy is at the high-growth equilibrium initially, the balanced-growth equilibrium moves from point Q'_0 to Q'_1. At the new stationary equilibrium, $\tilde{q}$ rises from $\tilde{q}'_0$ to $\tilde{q}'_1$, and, based on equation (25), the balanced-growth rate is increased in response.

We are now in a position to trace how the inflation rate will respond following a rise in the nominal income growth rate. From equations (25) and (29b), we can infer the stationary relation $\tilde{\pi} = v - (\tilde{q}-1)/h$. Substituting this relation into equation (33) yields

$$\tilde{\pi} = \frac{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{\left[(R^* - \pi^*)\tilde{q} - \frac{(\tilde{q}-1)^2}{2h}\right]^{\frac{1-\alpha}{\alpha}}} - (R^* - \pi^*) \equiv H_{NI}(\tilde{q}), \tag{35}$$

where the subscript "NI" denotes the regime of nominal income growth rate targeting.

As is evident, $H_{NI}(\tilde{q})$ in equation (35) under the regime of nominal income growth rate targeting is identical to $H_{MG}(\tilde{q})$ in equation (27b) under the regime

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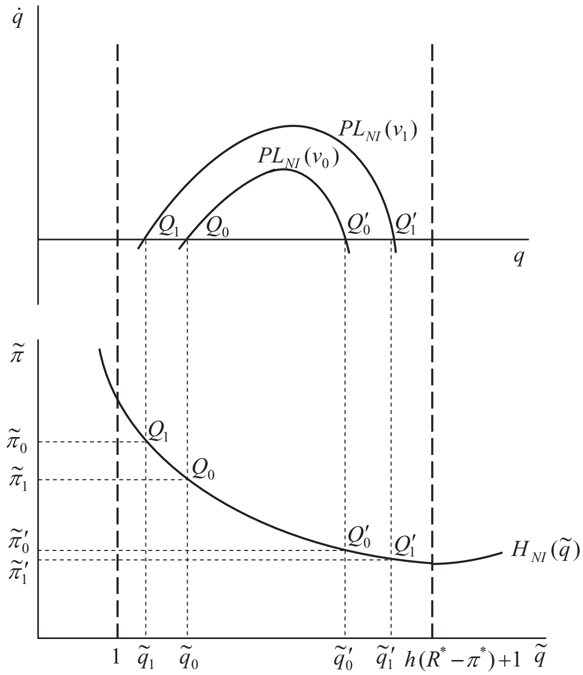

FIGURE 4. The effect of a rise in the nominal income growth rate.

of money growth rate targeting. We can thus depict the $H_{\mathrm{NI}}(\tilde{q})$ curve in the lower panel of Figure 4, which is the same as the $H_{\mathrm{MG}}(\tilde{q})$ curve in the lower panel of Figure 2.

From the upper panel of Figure 4, we learn that, if the economy is established at the low-growth equilibrium initially, the balanced-growth equilibrium changes from point Q_0 to Q_1 and $\tilde{q}$ is lowered from $\tilde{q}_0$ to $\tilde{q}_1$. As shown in the lower panel of Figure 4, based on the $H_{\mathrm{NI}}(\tilde{q})$ curve we find that the domestic inflation rate should rise from $\tilde{\pi}_0$ to $\tilde{\pi}_1$ when the nominal income growth rate increases from v_0 to v_1. However, in the upper panel of Figure 4, if the economy is established at the high-growth equilibrium initially, the balanced-growth equilibrium moves from point Q'_0 to Q'_1 and $\tilde{q}$ goes up from $\tilde{q}'_0$ to $\tilde{q}'_1$. Then, from the $H_{\mathrm{NI}}(\tilde{q})$ curve in the lower panel of Figure 4, we can infer that the domestic inflation rate should fall from $\tilde{\pi}'_0$ to $\tilde{\pi}'_1$ when the nominal income growth rate rises from v_0 to v_1.

Summing up this description, we establish the following proposition:

PROPOSITION 6. *Under a regime of nominal income targeting, at the high-growth equilibrium a rise in the nominal income growth target raises the balanced-growth rate, whereas at the low-growth equilibrium a rise in the nominal income growth target lowers the balanced-growth rate.*

The economic intuition behind Proposition 6 is similar to that in the regime of money growth rate targeting, which can be briefly explained as follows. When the authorities raise the nominal income growth rate, they should also expand the nominal money growth rate and the lump-sum transfers from the government to the private sector in response. On one hand, the portfolio substitution effect has a negative effect on the growth rate. On the other hand, the income effect has a positive effect on the growth rate. At the low-growth equilibrium, the foreign real interest rate is sufficiently large (i.e., $R^* - \pi^* > (\tilde{q} - 1)/h + \Theta$), implying that the portfolio substitution effect surmounts the income effect. Accordingly, a rise in the nominal income growth rate is associated with a fall in the balanced economic growth rate at the low-growth equilibrium. In contrast, at the high-growth equilibrium, the foreign real interest rate is relatively small (i.e., $(\tilde{q} - 1)/h > R^* - \pi^* - \Theta$), implying that the portfolio substitution effect is smaller than the income effect. As a consequence, a rise in the nominal income growth rate leads to an increase in the balanced economic growth rate at the high-growth equilibrium.

3.3. Inflation Rate Targeting

When the monetary authorities target a specific inflation rate, the inflation rate (π) is kept constant. Under a regime of inflation rate targeting, the macroeconomic model can be expressed by equations (16a)–(16j), and the endogenous variables are C, λ_1, q, K, m, I, μ, Tr, ε, and b^*. By substituting the expressions for C, λ_1, K, m, I, μ, Tr, ε, and b^*, the following dynamic system in q is obtained:

$$\dot{q} = (R^* - \pi^*)q - (1-\alpha)\Gamma\left(\frac{\alpha\Gamma}{\pi + R^* - \pi^*}\right)^{\frac{\alpha}{1-\alpha}} - \frac{(q-1)^2}{2h}. \qquad \textbf{(36a)}$$

Linearizing equation (36a) around the balanced-growth equilibrium gives

$$\dot{q} = J_q(q - \tilde{q}),\; J_q = R^* - \pi^* - \frac{\tilde{q} - 1}{h} > 0. \qquad \textbf{(36b)}$$

It should be noted that, given that Condition TVC requires $R^* - \pi^* - (\tilde{q} - 1)/h > 0$, $J_q > 0$ is ascertained.

Let δ be the characteristic root of the dynamic system. Then, from equation (36b), we have

$$\delta = J_q > 0. \qquad \textbf{(37)}$$

Given that $\delta > 0$ and q is a jump variable, we can thus conclude that the balanced-growth equilibrium is locally determinate. This result leads to the following proposition:

PROPOSITION 7. *Under a regime of inflation rate targeting, the balanced-growth equilibrium is characterized by local determinacy.*

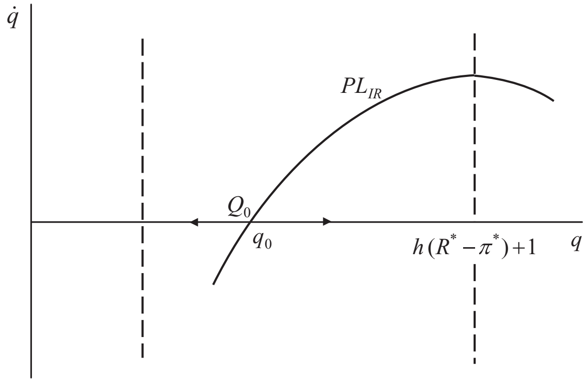

FIGURE 5. Phase diagram under a regime of inflation rate targeting.

The rationale for local determinacy under the regime of inflation rate targeting can be explained intuitively. Similarly to money growth targeting and nominal income growth rate targeting, two conflicting effects emerge when the representative household expects a higher future price of physical capital. The portfolio substitution effect has a negative impact on the market value of physical capital, whereas the income effect has a positive impact on the market value of physical capital. Running in sharp contrast to money growth targeting and nominal income growth rate targeting, in the regime of inflation rate targeting the restriction of TVC requires that the portfolio substitution effect definitely outweigh the income effect, and hence lead to an actual decline in the market value of physical capital. As a consequence, the households' initial optimistic expectations fail to be self-fulfilling, and hence the economy displays equilibrium determinacy.[16]

We are now in a position to trace the evolution of q. In Figure 5, the phase locus for inflation rate targeting, $\mathrm{PL_{IR}}$, depicts all combinations of $\dot{q}$ and q that satisfy equation (36a). It is quite easy to infer from equation (36a) that

$$\frac{\partial \dot{q}}{\partial q} = R^* - \pi^* - \frac{(\tilde{q}-1)}{h} > 0, \tag{38a}$$

$$\frac{\partial^2 \dot{q}}{\partial q^2} = -\frac{1}{h} < 0. \tag{38b}$$

Equations (38a) and (38b) indicate that the $\mathrm{PL_{IR}}$ locus is upward-sloping and concave downward.

The $\mathrm{PL_{IR}}$ schedule is depicted in Figure 5. As shown in Appendix C, the $\mathrm{PL_{IR}}$ schedule reaches its highest point at which the level of $\tilde{q}$ is $h(R^* - \pi^*) + 1$. Based on this feature and the restriction $1 < \tilde{q} < h(R^* - \pi^*) + 1$, the $\mathrm{PL_{IR}}$ schedule and the horizontal axis intersect once at point Q_0 inside the interval $(1, h(R^* - \pi^*) + 1)$. This indicates that only one balanced-growth equilibrium may emerge.[17] As exhibited by the directions of the arrows in Figure 5, it is clear that the balanced-growth equilibrium (point Q_0) is locally unstable. Given that q is a jump variable, we can thus conclude that the balanced-growth equilibrium is characterized by local determinacy. The result can be stated in the following proposition:

PROPOSITION 8. *Under a regime of inflation rate targeting, a unique balanced-growth equilibrium is presented.*

At the balanced-growth equilibrium, the economy is characterized by $\dot{q} = 0$. It follows from (36a) that the steady-state value $\tilde{q}$ satisfies the following stationary relationship:

$$(R^* - \pi^*)\tilde{q} - (1-\alpha)\Gamma\left(\frac{\alpha\Gamma}{\pi + R^* - \pi^*}\right)^{\frac{\alpha}{1-\alpha}} - \frac{(\tilde{q}-1)^2}{2h} = 0. \tag{39}$$

Then, based on equations (39) and (25), we have the following result:

$$\frac{d\tilde{\gamma}}{d\pi} = \frac{-\left[(\alpha\Gamma)/(\pi + R^* - \pi^*)\right]^{\frac{1}{1-\alpha}}}{h(R^* - \pi^*) - (\tilde{q}-1)} < 0. \tag{40}$$

Equation (40) indicates that a rise in the inflation rate lowers the economic growth rate in the long run. Gylfason and Herbertsson (2001) summarize the empirical findings in the existing literature and conclude that there exists a negative relationship between inflation and economic growth in the long run. The result in equation (40) can be viewed as a plausible basis for explaining the empirical findings in the existing literature.

The upper panel of Figure 6 illustrates the relationship between the inflation rate anchor and the balanced-growth rate. Following a rise in the inflation rate from π_0 to π_1, the $\mathrm{PL_{IR}}(\pi_0)$ schedule shifts upward to $\mathrm{PL_{IR}}(\pi_1)$, and the balanced-growth equilibrium moves from point Q_0 to Q_1. At the new stationary equilibrium, $\tilde{q}$ falls from $\tilde{q}_0$ to $\tilde{q}_1$, and, based on equation (25), the balanced-growth rate is lowered in response.

The relationship between the inflation rate and the balanced-growth rate can be exhibited by another graphical apparatus. From equation (39), it is quite easy to derive the result

$$\pi = \frac{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{\left[(R^* - \pi^*)\tilde{q} - \frac{(\tilde{q}-1)^2}{2h}\right]^{\frac{1-\alpha}{\alpha}}} - (R^* - \pi^*) \equiv H_{\mathrm{IR}}(\tilde{q}), \tag{41}$$

where the subscript "IR" denotes the regime of inflation targeting.

From equations (27b), (35), and (41) we can find that the right-hand sides of these three equations (i.e., $H_{\mathrm{MG}}(\tilde{q})$, $H_{\mathrm{NI}}(\tilde{q})$, and $H_{\mathrm{IR}}(\tilde{q})$) are identical. The only difference among these three equations is that the left-hand side in equation (41) is the inflation targeting parameter π, whereas the left-hand side in equations (27b) and (35) is the stationary value of the inflation rate $\tilde{\pi}$. We can plot the graph of

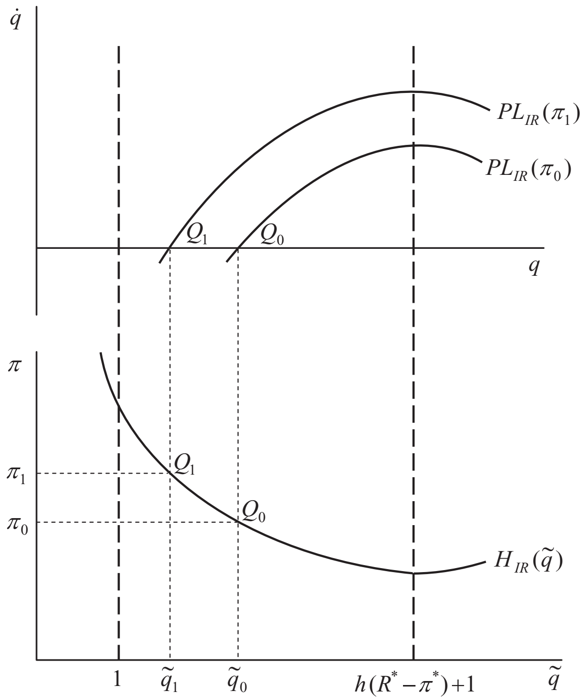

FIGURE 6. The effect of a rise in the inflation rate.

the $H_{\rm IR}(\tilde{q})$ curve in the lower panel of Figure 6, which is the same as both the $H_{\rm MG}(\tilde{q})$ curve in the lower panel of Figure 2 and the $H_{\rm NI}(\tilde{q})$ curve in the lower panel of Figure 4.

In Figure 6, the economy is initially established at point Q_0 in the upper panel, which is the point of intersection of the $\mathrm{PL_{IR}}(\pi_0)$ curve and the horizontal axis. By tracing over the $H_{\rm NI}(\tilde{q})$ schedule, the initial equilibrium is represented by point Q_0 in the lower panel. In response to a rise in the inflation rate from π_0 to π_1, the balanced-growth equilibrium moves from point Q_0 to Q_1 and $\tilde{q}$ goes down from $\tilde{q}_0$ to $\tilde{q}_1$ in the upper panel. By tracing over the $H_{\rm NI}(\tilde{q})$ schedule, the new equilibrium is represented by point Q_1 in the lower panel, where the inflation rate is π_1 and the market value of physical capital is $\tilde{q}_1$.

Summing up this discussion, we can establish the following proposition:

PROPOSITION 9. *Under a regime of inflation rate targeting, a rise in the inflation rate target deters the balanced-growth rate.*

The economic intuition for Proposition 9 can be briefly described as follows. When the monetary authorities raise the inflation rate anchor, they should also expand the nominal money growth rate and the lump-sum transfers from the government to the private sector as a response. Similarly to money growth targeting and nominal income growth rate targeting, the portfolio effect and the income effect emerge from monetary manipulation and government activity. The portfolio substitution effect has a negative effect on the growth rate, whereas the income effect has a positive effect on the growth rate. In contrast to money growth targeting and nominal income growth rate targeting, in the regime of inflation rate targeting the restriction of TVC requires that the portfolio substitution effect definitely dominate the income effect.[18] As a result, a rise in the inflation rate anchor reduces the long-run economic growth rate.

Two points related to domestic inflation rate targeting should be noted here. First, in our paper, the purchasing power parity reported in equation (14) requires that $\pi = \pi^* + \varepsilon$. In a small open economy, all foreign variables including π^* are treated as given. In consequence, an inflation rate targeting rule is equivalent to an exchange rate targeting policy.

Second, our framework can easily be extended to analyze nominal interest rate targeting. Consider that the representative household can hold domestic bonds, in addition to real money balances, physical capital, and foreign bonds, as its assets, and assume that domestic bonds and foreign bonds are perfectly substitutable assets. Then, under flexible exchange rates with perfect capital mobility, the nonarbitrage condition between domestic bonds and foreign bonds requires that the domestic nominal interest rate R equal the sum of the foreign nominal interest rate R^* and the depreciation rate of domestic currency ε (i.e., uncovered interest rate parity holds). This implies that $R = R^* + \varepsilon$. Integrating purchasing power parity $\pi = \pi^* + \varepsilon$ with uncovered interest rate parity $R = R^* + \varepsilon$ yields the following expression:

$$R = R^* + \pi - \pi^*. \tag{42}$$

In a small open economy, all foreign variables, including both R^* and π^*, are treated as given. Accordingly, the target for the domestic nominal interest rate is equivalent to the target for the domestic inflation rate. So all the results of domestic inflation rate targeting in our paper can be applied to domestic interest rate targeting, and hence our paper does not deal with the nominal interest rate rule.

4. EQUIVALENCE

We are now ready to examine equivalence relations among the three monetary policy rules. Three types of equivalence are discussed in what follows in turn.

4.1. Qualitative Equivalence

Qualitative equivalence, proposed by Wang and Yip (1992), requires that monetary policy rules lead to the same comparative static results on the balanced-growth rate *in terms of signs*. Equations (26), (34), and (40) indicate that, because of the possibility of local indeterminacy, a rise in the pegged rate has an ambiguous effect on the balanced-growth rate under both money growth rate targeting and nominal income growth rate targeting, whereas a rise in the pegged rate has a negative effect on the balanced-growth rate under inflation rate targeting. As a result, inflation rate targeting is not qualitatively equivalent to either money growth rate targeting or nominal income growth rate targeting. However, if we rule out the possibility of local indeterminacy, a rise in the pegged rate generates a negative effect on the balanced-growth rate under all three policy rules. Under such a situation, all three policy rules are qualitatively equivalent.

4.2. Fundamental Equivalence

Fundamental equivalence is a stronger form than qualitative equivalence. It requires that monetary policies yield the same balanced-growth rate. Based on equations (26) and (34), after some manipulation, we can show that both money growth rate targeting and nominal income growth rate targeting yield the same long-run economic growth rate if we impose a constraint $\mu = v$ in equilibrium.[19] Accordingly, there exists a fundamental equivalence between money growth rate targeting and nominal income growth rate targeting. The intuition for fundamental equivalence between money growth rate targeting and nominal income growth rate targeting is quite apparent. Recall that money growth rate targeting requires that $\pi + \dot{m}/m = \mu$ and that nominal income growth rate targeting requires that $\pi + \dot{Y}/Y = v$. Along the balanced-growth path, both real money balances and output grow at the same rate (i.e., $\dot{m}/m = \dot{Y}/Y$ at the balanced-growth equilibrium), implying that both targeting rules are subject to the same requirement if we impose a restriction $\mu = v$.

Moreover, equations (26), (34), and (40) indicate that $d\tilde{\gamma}/d\mu = d\tilde{\gamma}/dv \neq d\tilde{\gamma}/d\pi$ even if we impose a restriction $\mu = v$ in equilibrium,[20] and, accordingly, inflation rate targeting is not fundamentally equivalent to either money growth rate targeting or nominal income growth rate targeting.

4.3. Dynamic Equivalence

Dynamic equivalence requires the same macroeconomic equilibrium dynamics. Equations (22a), (22b), (31), and (37) reveal that there are distinct characteristic roots that govern the dynamic adjustment of relevant macro variables among the regimes of money growth rate targeting, nominal income growth rate targeting, and inflation rate targeting. This implies that the three monetary policy rules do not exhibit the same equilibrium dynamics. As a result, dynamic equivalence is not established among the three monetary policy rules.

The preceding discussions on equivalence relations lead to the following two propositions:

PROPOSITION 10. *Because local indeterminacy may occur under both money growth rate targeting and nominal income growth rate targeting, inflation rate* *targeting is not qualitatively equivalent to either money growth rate targeting or nominal income growth rate targeting.*

PROPOSITION 11. *Money growth rate targeting is fundamentally equivalent to nominal income growth rate targeting if a specific restriction is imposed.*

5. CONCLUDING REMARKS

This paper sets out a monetary endogenous growth model for an open economy. The salient feature of the model is that it is able to deal with various monetary policy rules, including money growth rate targeting, inflation rate targeting, and nominal income growth rate targeting. We then use the model to examine the consequence of adjusting the target rate of various monetary policy rules in relation to the balanced-growth rate.

Several main findings emerge from the analysis. First, because of the possibility of local indeterminacy, a rise in the pegged rate may either increase or decrease the balanced economic growth rate under both money growth rate targeting and nominal income growth rate targeting. Second, under a regime of inflation rate targeting, the balanced-growth equilibrium is characterized by local determinacy. Third, a higher inflation rate is associated with a lower balanced-growth rate. Fourth, because local indeterminacy may occur under both money growth rate targeting and nominal income growth rate targeting, inflation rate targeting is not qualitatively equivalent to either money growth rate targeting or nominal income growth rate targeting.

NOTES

1. Schabert (2003) specifies that the monetary authorities raise the interest rate in response to an increase in the inflation rate. The interest rate policy is active (passive) when the percentage change in the interest rate is greater (less) than that in the inflation rate.

2. In Section 3.3 we will highlight that inflation rate targeting, domestic nominal interest rate targeting, and exchange rate targeting are all equivalent, and this is why our paper does not deal with the domestic nominal interest rate rule and the exchange rate rule.

3. The specification of the net output function reported in equation (4) is similar to that of Suen and Yip (2005).

4. From equations (23a) and (23b), we can further infer the following results:

$$\left.\frac{\partial^2 z}{\partial q^2}\right|_{\dot{q}=0} = \frac{-\mu}{h(1-\alpha)[\mu + R^* - \pi^* - (\tilde{q}-1)/h]^2} < 0; \qquad \left.\frac{\partial^2 z}{\partial q^2}\right|_{\dot{z}=0} = \frac{(2-\alpha)\tilde{z}^{3-2\alpha}}{[h(1-\alpha)\alpha\Gamma]^2} > 0.$$

These results indicate that, as exhibited in Figure 1, the $\dot{q} = 0$ locus is concave downward and the $\dot{z} = 0$ locus is concave upward.

5. From equation (4) we obtain NMPK $= \partial(Y-T)/\partial K = \partial(\Gamma K^{1-\alpha} m^{\alpha})/\partial K = (1-\alpha)\Gamma K^{-\alpha} m^{\alpha}$.

6. Given that NMPK $= \partial(\Gamma K^{1-\alpha} m^{\alpha})/\partial K = (1-\alpha)\Gamma K^{-\alpha} m^{\alpha}$ and $0 < \alpha < 1$, we can infer the result $\partial\text{NMPK}/\partial m = (1-\alpha)\alpha\Gamma K^{-\alpha} m^{\alpha-1} > 0$.

7. Our intuitive explanation is similar to those of Guo and Harrison (2004) and Chen and Guo (2008): the household's optimistic expectations of a higher future return on capital lead to an actual rise in the return on capital, implying that the household's initial optimistic expectations become self-fulfilling.

8. Appendix A provides a detailed derivation for the restrictions in the presence of dual balanced-growth equilibria under the regime of money growth rate targeting. The question was raised by an anonymous referee, to whom we are grateful.

9. It should be noted that in the upper panel of Figure 2 the stationary values of z and q in association with both the low-growth and the high-growth equilibrium should simultaneously satisfy equations (24a) and (24b) under the regime of money growth rate targeting.

10. Recall that NMPK $= \partial(\Gamma K^{1-\alpha} m^{\alpha})/\partial K = (1-\alpha)\Gamma K^{-\alpha} m^{\alpha}$ and $0 < \alpha < 1$. We can then infer that NMPK will decline in response as m decreases.

11. Using a monetary endogenous growth model with money in the utility function, Chang and Lai (2000) assert that the rule of money growth rate targeting does not lead to local indeterminacy.

12. Most of the existing studies on nominal income targeting focus on closed economies. Guender and Tam (2004) instead set up an open-economy model to deal with the merits of nominal income targeting. However, in a departure from our optimizing model, the Guender and Tam (2004) model is an ad hoc framework by nature.

13. The intuition for local indeterminacy in the regime of nominal income growth rate targeting is similar to that in the regime of money growth rate targeting, so there is no need to repeat this here.

14. Appendix B provides a detailed discussion for the restrictions in association with dual balanced-growth equilibria under the regime of nominal income targeting. The question was raised by an anonymous referee, to whom we are grateful.

15. Let $\hat{q}$ be the value such that $\partial\dot{q}/\partial q = 0$ holds. Equation (32a) indicates that the following relation is satisfied:

$$(R^* - \pi^*) - \frac{(\hat{q}-1)}{h} - \frac{1}{h}\left\{\frac{\alpha\Gamma}{v + R^* - \pi^* - [(\hat{q}-1)/h]}\right\}^{\frac{1}{1-\alpha}} = 0.$$

It is easy to infer from this equation that

$$\frac{\partial\hat{q}}{\partial v} = \frac{h\Theta}{(1-\alpha)\{v + R^* - \pi^* - [(\hat{q}-1)/h]\} + \Theta} > 0.$$

This result indicates that, as shown in Figure 4, the value of q in association with the highest point along the $\mathrm{PL}_{\mathrm{NI}}(v_1)$ line is greater than that along the $\mathrm{PL}_{\mathrm{NI}}(v_0)$ line.

16. Under both money growth rate targeting rules and nominal income growth targeting rules, even with the restriction of TVC, the portfolio substitution effect may either dominate or fall short of the income effect.

17. Appendix C gives a detailed discussion of the restrictions in association with unique balanced-growth equilibrium under the regime of inflation rate targeting. The question was raised by an anonymous referee, to whom we are grateful.

18. Under both money growth rate targeting and nominal income growth targeting, even if TVC is restricted, the portfolio substitution effect may either dominate or fall short of the income effect. As a consequence, a rise in the anchor under both regimes may either decrease or increase the balanced-growth rate.

19. Appendix D provides a detailed proof.

20. Appendix E provides a detailed proof.

21. It is obvious our graphical analysis in A.1 assumes $\mu^{C1} < \mu^{C2}$. However, if the difference between 1 and $h(R^* - \pi^*) + 1$ becomes smaller (i.e., in Figure A.1 the vertical dotted line $\tilde{q} = h(R^* - \pi^*) + 1$ moves leftward and is more close to the vertical dotted line $\tilde{q} = 1$), it is possible that $\mu^{C1} > \mu^{C2}$ would happen. To save space, we do not depict the graph associated with this situation.

22. Similarly to the graphical analysis under the regime of money growth targeting exhibited in Figure A.1, to save space our graphical analysis in Figure B.1 only deals with the case $v^{C1} < v^{C2}$, and does not discuss the scenario associated with $v^{C1} > v^{C2}$.

23. Based on equations (C.1) and (C.2) with $R^* - \pi^* > 0$, we can infer the following result:

$$\pi^{C2} = \frac{\alpha[2(1-\alpha)]^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{[h(R^*-\pi^*)+2(R^*-\pi^*)]^{\frac{1-\alpha}{\alpha}}} - (R^*-\pi^*) < \frac{\alpha[2(1-\alpha)]^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{[2(R^*-\pi^*)]^{\frac{1-\alpha}{\alpha}}} - (R^*-\pi^*)$$

$$= \frac{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{(R^*-\pi^*)^{\frac{1-\alpha}{\alpha}}} - (R^*-\pi^*) = \pi^{C1}.$$

24. Let $\hat{q}$ be the value such that $\partial\dot{q}/\partial q = 0$ is true. From equation (36a) we then have

$$\frac{\partial\dot{q}}{\partial q} = R^* - \pi^* - \frac{(\hat{q}-1)}{h} = 0; \text{ if } \hat{q} = h(R^*-\pi^*)+1.$$

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APPENDIX A

Based on equations (24a) and (24b), we can define two threshold values of μ, namely μ^{C1} and μ^{C2}, that satisfy $\tilde{q} = 1$ and $\tilde{q} = h(R^* - \pi^*) + 1$ at the balanced-growth equilibrium, respectively. These two threshold values can be expressed as follows:

$$\mu^{C1} = \frac{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{(R^* - \pi^*)^{\frac{1-\alpha}{\alpha}}} - (R^* - \pi^*), \qquad \textbf{(A.1)}$$

$$\mu^{C2} = \frac{\alpha[2(1-\alpha)]^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{[h(R^* - \pi^*) + 2(R^* - \pi^*)]^{\frac{1-\alpha}{\alpha}}}. \qquad \textbf{(A.2)}$$

As exhibited in Figure A.1, in association with μ^{C1}, the $\dot{q} = 0$ schedule crosses the $\dot{z} = 0(\mu^{C1})$ schedule at point Q^{C1}, where the level of $\tilde{q}$ is equal to 1. Moreover, in association with μ^{C2}, both $\dot{q} = 0$ and $\dot{z} = 0(\mu^{C2})$ schedules intersect at point Q^{C2}, where $\tilde{q}$ is at the level $h(R^* - \pi^*) + 1$.

Condition TVC requires that $\tilde{q} < h(R^* - \pi^*) + 1$ and Condition PGRC requires that $\tilde{q} > 1$. These two conditions impose the restriction $1 < \tilde{q} < h(R^* - \pi^*) + 1$. As a consequence, the balanced-growth equilibrium should be ruled out if its level of $\tilde{q}$ lie outside the interval $(1, h(R^* - \pi^*) + 1)$. Three possible situations should be considered:

First, as indicated in Figure A.1, if the money growth rate is μ_a and $\mu_a < \min(\mu^{C1}, \mu^{C2})$, the $\dot{q} = 0$ schedule and the $\dot{z} = 0(\mu_a)$ schedule intersect twice, at Q_a (low-growth equilibrium) and Q'_a (high-growth equilibrium). As is clear, the levels of $\tilde{q}$ associated with Q_a and Q'_a are $\tilde{q}_a$ and $\tilde{q}'_a$, respectively, and both $\tilde{q}_a$ and $\tilde{q}'_a$ lie within the interval $(1, h(R^* - \pi^*) + 1)$. This situation reveals that the economy is characterized by dual balanced-growth equilibria.

Second, if the money growth rate is μ_b and $\min(\mu^{C1}, \mu^{C2}) < \mu_b < \max(\mu^{C1}, \mu^{C2})$, both $\dot{q} = 0$ and $\dot{z} = 0(\mu_b)$ cross twice, at Q_b (low-growth equilibrium) and Q'_b (high-growth equilibrium). The corresponding levels of $\tilde{q}$ at these two points are $\tilde{q}_b$ and $\tilde{q}'_b$, respectively. As Figure A.1 shows, $\tilde{q}_b$ lies outside the interval $(1, h(R^* - \pi^*) + 1)$

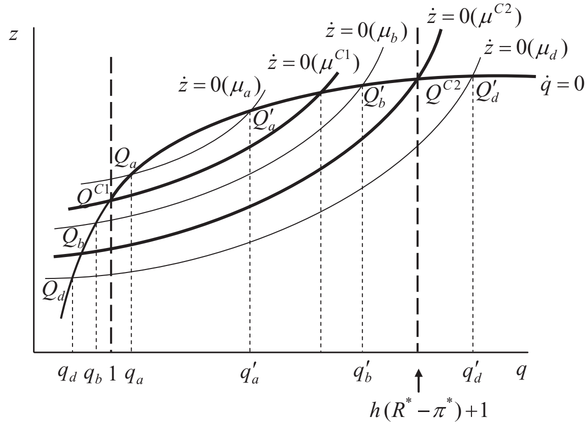

FIGURE A.1. The threshold values of the money growth rate.

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and point Q_b (low-growth equilibrium) should be excluded from the balanced-growth equilibrium. The economy thus is characterized by a unique growth equilibrium.

Third, if the money growth rate is μ_d and $\mu_d > \max(\mu^{C1}, \mu^{C2})$, both $\dot{q} = 0$ and $\dot{z} = 0(\mu_d)$ intersect twice, at Q_d (low-growth equilibrium) and Q'_d (high-growth equilibrium). The levels of $\tilde{q}$ associated with points Q_d and Q'_d are $\tilde{q}_d$ and $\tilde{q}'_d$, respectively. As is evident, both $\tilde{q}_d$ and $\tilde{q}'_d$ lie outside the interval $(1, h(R^* - \pi^*) + 1)$, and hence both points Q_d and Q'_d should be excluded from the balanced-growth equilibrium. The economy thus features zero balanced-growth equilibrium.[21]

APPENDIX B

From equation (33) we can define two threshold values of the nominal income growth rate, namely v^{C1} and v^{C2}, that satisfy $\tilde{q} = 1$ and $\tilde{q} = h(R^* - \pi^*) + 1$ at the balanced-growth equilibrium, respectively. These two threshold values are given by

$$v^{C1} = \frac{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{(R^* - \pi^*)^{\frac{1-\alpha}{\alpha}}} - (R^* - \pi^*), \tag{B.1}$$

$$v^{C2} = \frac{\alpha[2(1-\alpha)]^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{[h(R^* - \pi^*) + 2(R^* - \pi^*)]^{\frac{1-\alpha}{\alpha}}}. \tag{B.2}$$

In Figure B.1, in association with v^{C1}, the $\mathrm{PL_{NI}}(v^{C1})$ curve and the horizontal axis intersect at point Q^{C1}, with the level of $\tilde{q}$ being $\tilde{q} = 1$. Moreover, in association with v^{C2}, the $\mathrm{PL_{NI}}(v^{C2})$ curve and the horizontal axis cross at point Q^{C2}, with the level of $\tilde{q}$ being $\tilde{q} = h(R^* - \pi^*) + 1$.

With reasoning similar to the case of money growth rate targeting, the balanced-growth equilibrium under the regime of nominal income growth targeting should be ruled out if its level of $\tilde{q}$ lie outside the interval $(1, h(R^* - \pi^*) + 1)$. Three possible situations should be taken into account. First, as exhibited in Figure B.1, if the nominal income growth rate is

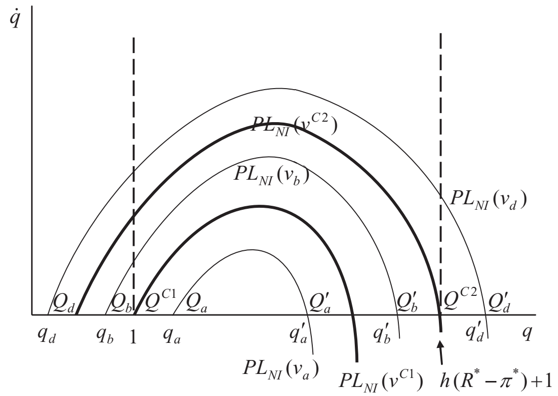

FIGURE B.1. The threshold values of the nominal income growth rate.

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v_a and $v_a < \min(v^{C1}, v^{C2})$, the $\mathrm{PL_{NI}}(v_a)$ line and the horizontal axis intersect twice, at Q_a (low-growth equilibrium) and Q'_a (high-growth equilibrium). At these two points, the levels of $\tilde{q}$ are $\tilde{q}_a$ and $\tilde{q}'_a$, respectively, and both $\tilde{q}_a$ and $\tilde{q}'_a$ lie within the interval $(1, h(R^*-\pi^*)+1)$. This result implies that the economy is characterized by dual balanced-growth equilibria. Second, if the nominal income growth rate is v_b and $\min(v^{C1}, v^{C2}) < v_b < \max(v^{C1}, v^{C2})$, the $\mathrm{PL_{NI}}(v_b)$ curve crosses the horizontal axis twice, at Q_b (low-growth equilibrium) and Q'_b (high-growth equilibrium). The levels of $\tilde{q}$ associated with Q_b and Q'_b are $\tilde{q}_b$ and $\tilde{q}'_b$, respectively. As exhibited in Figure B.1, $\tilde{q}_b$ lies outside the interval $(1, h(R^* - \pi^*) + 1)$, and hence point Q_b (low-growth equilibrium) should be excluded from the balanced-growth equilibrium. The economy thus features unique growth equilibrium. Third, if the money growth rate is v_d and $v_d > \max(v^{C1}, v^{C2})$, both $\mathrm{PL_{NI}}(v_d)$ and the horizontal axis intersect twice at Q_d (low-growth equilibrium) and Q'_d (high-growth equilibrium). The corresponding levels of $\tilde{q}$ at points Q_d and Q'_d are $\tilde{q}_d$ and $\tilde{q}'_d$, respectively. Given the fact that both $\tilde{q}_d$ and $\tilde{q}'_d$ lie outside the interval $(1, h(R^* - \pi^*) + 1)$, both points Q_d and Q'_d should be excluded from the balanced-growth equilibrium. The economy thus features zero balanced-growth equilibrium.[22]

APPENDIX C

From equation (39), we can define two threshold values of the domestic inflation rate, namely π^{C1} and π^{C2}, that satisfy $\tilde{q} = 1$ and $\tilde{q} = h(R^* - \pi^*) + 1$ at the balanced-growth equilibrium, respectively. These two threshold values can be expressed as follows:

$$\pi^{C1} = \frac{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{(R^*-\pi^*)^{\frac{1-\alpha}{\alpha}}} - (R^*-\pi^*), \tag{C.1}$$

$$\pi^{C2} = \frac{\alpha[2(1-\alpha)]^{\frac{1-\alpha}{\alpha}}\Gamma^{\frac{1}{\alpha}}}{[h(R^*-\pi^*)+2(R^*-\pi^*)]^{\frac{1-\alpha}{\alpha}}} - (R^*-\pi^*). \tag{C.2}$$

It is quite easy to show the result $\pi^{C1} > \pi^{C2}$.[23] In Figure C.1, in association with π^{C1}, the $\mathrm{PL_{IR}}(\pi^{C1})$ curve intersects the horizontal axis at point Q^{C1}, where the level of $\tilde{q}$ is equal to $\tilde{q} = 1$. Moreover, in association with π^{C2}, the $\mathrm{PL_{IR}}(\pi^{C2})$ curve is tangent to the horizontal axis at point Q^{C2}, where the level of $\tilde{q}$ is equal to $\tilde{q} = h(R^* - \pi^*) + 1$. It should be noted that, as exhibited in Figure C.1, the $\mathrm{PL_{IR}}$ curve in association with any value of π reaches its highest point, at which the level $\tilde{q}$ is $h(R^* - \pi^*) + 1$.[24]

Similarly to the case of money growth rate targeting and nominal income growth targeting, the balanced-growth equilibrium under the regime of inflation rate targeting should be ruled out if its corresponding level of $\tilde{q}$ lies outside the interval $(1, h(R^* - \pi^*) + 1)$. Three possible situations should be taken into consideration. First, as displayed in Figure C.1, if the inflation is π_a and $\pi_a < \pi^{C2}$, the $\mathrm{PL_{IR}}(\pi_a)$ curve does not intersect the horizontal axis, and the economy is characterized by zero balanced-growth equilibrium. Second, if the inflation rate is π_b and $\pi^{C2} < \pi_b < \pi^{C1}$, both $\mathrm{PL_{IR}}(\pi_b)$ and the horizontal axis intersect twice at Q_b (low-growth equilibrium) and Q'_b (high-growth equilibrium), with the levels of $\tilde{q}$ being $\tilde{q}_b$ and $\tilde{q}'_b$, respectively. As is apparent, $\tilde{q}'_b$ lies outside the interval

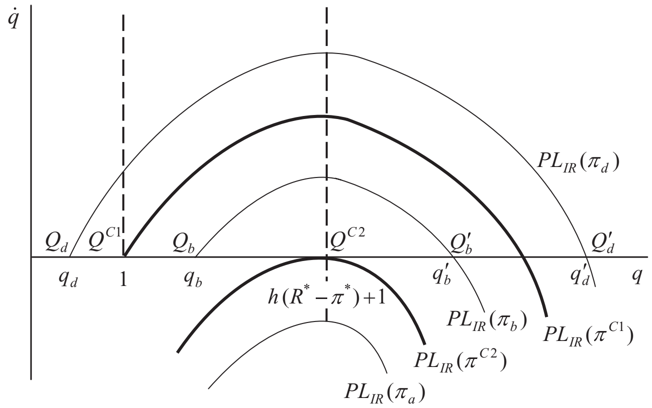

FIGURE C.1. The threshold values of the inflation rate.

$(1, h(R^* - \pi^*) + 1)$ and point Q'_b (high-growth equilibrium) should be excluded from the balanced-growth equilibrium. The economy thus is characterized by a unique low-growth equilibrium. Third, if the money growth rate is π_d and $\pi_d > \pi^{C1}$, both $\mathrm{PL_{NI}}(\pi_d)$ and the horizontal axis intersect twice at Q_d (low-growth equilibrium) and Q'_d (high-growth equilibrium). The levels of $\tilde{q}$ associated with Q_d and Q'_d are $\tilde{q}_d$ and $\tilde{q}'_d$, respectively. As is evident, both $\tilde{q}_d$ and $\tilde{q}'_d$ lie outside the interval $(1, h(R^* - \pi^*) + 1)$, and both points Q_d (low-growth equilibrium) and Q'_d (high-growth equilibrium) should be excluded from the balanced-growth equilibrium. The economy thus also features zero balanced-growth equilibrium. Based on this discussion, to make our analysis meaningful we impose the restriction $\pi \in (\pi^{C2}, \pi^{C1})$.

APPENDIX D

This Appendix shows that $d\tilde{\gamma}/d\mu = d\tilde{\gamma}/dv$ if we impose a restriction $\mu = v$ in the balanced-growth equilibrium.

From equation (24b), we have

$$\tilde{z} = \left\{ \frac{\alpha\Gamma}{\mu + R^* - \pi^* - [(\tilde{q} - 1)/h]} \right\}^{\frac{1}{1-\alpha}}. \tag{D.1}$$

Substituting equation (D.1) into (26) yields

$$\frac{d\tilde{\gamma}}{d\mu} = \frac{-\Xi}{(R^* - \pi^*) - (\tilde{q} - 1)/h - \Xi}, \tag{D.2}$$

where

$$\Xi = \frac{1}{h} \left\{ \frac{\alpha\Gamma}{\mu + R^* - \pi^* - [(\tilde{q} - 1)/h]} \right\}^{\frac{1}{1-\alpha}}.$$

Comparing equation (D.2) with (34), we have $d\tilde{\gamma}/d\mu = d\tilde{\gamma}/dv$ if $\mu = v$.

APPENDIX E

This Appendix shows that $d\tilde{\gamma}/d\mu = d\tilde{\gamma}/dv \neq d\tilde{\gamma}/d\pi$ even if we impose a restriction $\mu = v$ in the balanced-growth equilibrium.

From equation (40), we have

$$\frac{d\tilde{\gamma}}{d\pi} = \frac{-\Psi}{(R^* - \pi^*) - (\tilde{q} - 1)/h}, \tag{E.1}$$

where

$$\Psi = \frac{1}{h}\left(\frac{\alpha\Gamma}{\pi + R^* - \pi^*}\right)^{\frac{1}{1-\alpha}}.$$

From equations (D.2), (34), and (E.1), we have $d\tilde{\gamma}/d\mu = d\tilde{\gamma}/dv \neq d\tilde{\gamma}/d\pi$ even if the restriction $\mu = v$ is imposed.

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