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Letter to the Editor

Energy deposition of relativistic electrons in a hot super-compressed plasma

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Abstract. The relativistic modified formula for the energy loss of the relativistic electron beam due to binary electron–electron collisions is obtained. Another important energy loss mechanism, the excitation of Langmuir collective plasma oscillation, is also treated within the relativistic framework. Then the relevant physics parameters in the fast-ignitor scenario, including the continuous winded range, the maximum penetration depth and the stopping time, have been calculated. The results obtained are much better than those from non-relativistic cases and even partially relativistic modified theories. Thus, we re-examine theoretically the possibility of igniting hot spots in a super-compressed deuterium–tritium plasma.

The advent of the chirped-pulse amplification (CPA) technique, coupled with the development of solid-state lasers capable of delivering ultra-short pulses, has opened up the new field of ultra-high-intensity laser physics [1, 2]. Ultra-high-intensity lasers can potentially be used in conjunction with conventional fusion lasers to ignite inertial confinement fusion (ICF) [3] capsules with a total energy of a few tens of kilojoules of laser light, and can possibly lead to high gain with as little as 100 kJ. The so-called fast-ignitor scenario (FIS) [4] proposed to ease the indirect drive approach to ICF of a hollow pellet target containing deuterium–tritium (DT) thermonuclear fuel has received considerable attention. In the FIS approach energy deposition by a relativistic electron beam (REB) in hot plasma plays a key part. The intense laser–target interaction may produce hot or super-hot plasma. So the dynamic parameters of the plasma should be taken as being relativistic. Recently, Deutsch et al. [5] have calculated analytically the REB interaction for a super-compressed thermonuclear fuel and examined the possibility of igniting a hot spot in the target. Unfortunately, there exist some mistakes in their calculations even if some errors have been corrected in the corrigendum [6]. In their calculations, (3) is an important expression in [5], which describes the basic process for the energy loss due to binary electron–electron collisions. There are some mistakes and defects in this expression, even if the authors of [5] had made corrections in [6], but it still exists as a mistake and a defect. First, the second term in the bracket of the equation should be $0.125(\tau/(\tau+1))^2$, while the revised one is just $0.125(\tau/(\tau+1))$. Second, the definition of the factor $\tau_{\min}$ according to [5] is $\lambda\!\!\!^{-}_{\mathrm{e}}/\lambda_{\mathrm{D}}$, which is obtained in the non-relativistic limit. In addition, in [5, (5)] two factors are missing, the Avogadro number, N_{A}, and the density of the DT target, e. Except for these revisions, we take the plasma as a relativistic one and then study the energy deposition of REB in the hot plasma. Though the intense laser–target interaction may produce a hot plasma, some studies still just took the dynamic parameters of the plasma as the non-relativistic ones, even treating the electrons as being non-relativistic particles [7–9].

In this letter, based on the relativistic dynamics we re-study the interaction between the REB and the super-compressed hot DT core. There are two basic processes for the energy loss due to the interactions of REB with a plasma [5, 10]: binary electron–electron collisions and the excitation of the Langmuir collective plasma oscillation.

The energy loss of the incident electron as a result of interaction with the free electron in the plasma may be calculated using Møller's cross section [11],

$$\left(\frac{d\sigma}{d\varepsilon}\right)^{-} = \frac{\chi}{E_0}\left[\frac{1}{\varepsilon^2} + \frac{1}{(1-\varepsilon)^2} + \left(\frac{\gamma-1}{\gamma}\right)^2 - \frac{(2\gamma-1)}{\gamma^2}\frac{1}{\varepsilon(1-\varepsilon)}\right] \tag{1}$$

where

$$\chi = \frac{2\pi r_0^2 m_{\mathrm{e}} c^2}{\beta^2}, \quad r_0 = \frac{e^2}{m_{\mathrm{e}} c^2}.$$

$\beta = v_0/c$ and $\gamma = (1-\beta^2)^{-1/2}$ are the Lorentz parameters of the incident electron, m_{e} is the rest mass of the electron and v_0 is the speed of the incident electron. The incident electron total energy is $E = \gamma m_{\mathrm{e}} c^2$, the kinetic energy is $E_0 = (\gamma - 1)m_{\mathrm{e}}c^2$ and ε is the energy transfer in units of E_0. Since the outgoing electron of higher energy is by definition the primary electron, the maximum energy transfer is $\varepsilon_{\max} = \frac{1}{2}$; while in the plasma the minimum energy transfer should be $\varepsilon_{\min} = \frac{1}{2}(\hbar^2\omega_{\mathrm{p}}'^2/E_0 kT)$, where $\omega_{\mathrm{p}}' = (4\pi n_{\mathrm{p}} e^2/m_{\mathrm{e}}\gamma'^3)^{1/2}$ denotes the relativistic modified frequency [12] for the hot plasma, kT is the plasma temperature, γ' is the Lorentz factor of the electron in the hot plasma and n_{p} is the plasma density. In the non-relativistic limit, we can see $\varepsilon_{\min}^{1/2}$ is just equal to $\lambda\!\!\!^{-}_{\mathrm{e}}/\lambda_{\mathrm{D}}$, where $\lambda\!\!\!^{-}_{\mathrm{e}}$ is the de Broglie wavelength of the incident electron and λ_{D} is the Debye length of the plasma, which is consistent with other previous works [5, 10].

Therefore, due to hard collisions the average energy loss per atom of Z electrons may be expressed as follows:

$$ZE_0\int_{\varepsilon_{\min}}^{1/2}\varepsilon\left(\frac{d\sigma}{d\varepsilon}\right)^{-} d\varepsilon = Z\chi\left[\ln\frac{1}{4\varepsilon_{\min}} + 1 - \frac{2\gamma-1}{\gamma^2}\ln 2 + \frac{1}{8}\left(\frac{\gamma-1}{\gamma}\right)^2\right]. \tag{2}$$

Thus, using (2) we can easily obtain the rate of average energy loss per unit path length x in a medium with n_{p} atoms per unit volume for the binary collisions:

$$-\frac{dE}{dx} = \frac{2\pi n_{\mathrm{p}} Z e^4}{m_{\mathrm{e}}\beta^2 c^2}\left[\ln\frac{m_{\mathrm{e}}c^2 kT(\gamma-1)}{\hbar^2{\omega_{\mathrm{p}}'}^2} - \frac{2\gamma-1}{\gamma^2}\ln 2 + \frac{1}{8}\left(\frac{\gamma-1}{\gamma}\right)^2 + 1 - \ln 2\right], \tag{3}$$

where $Z = 1$ for the hot super-compressed DT core and n_{p} is the plasma density.

In addition, to the energy loss due to the binary collisions one must add the contribution from the excitation of the Langmuir collective plasma oscillation [13],

$$-\frac{dE}{dx} = \frac{2\pi n_{\mathrm{p}} e^4}{m_{\mathrm{e}}\beta^2 c^2 \gamma'^3}\ln\left[\frac{v_0}{\omega_{\mathrm{p}}'\lambda_{\mathrm{D}}'}\left(\frac{2}{3}\right)^{1/2}\right]^2, \tag{4}$$

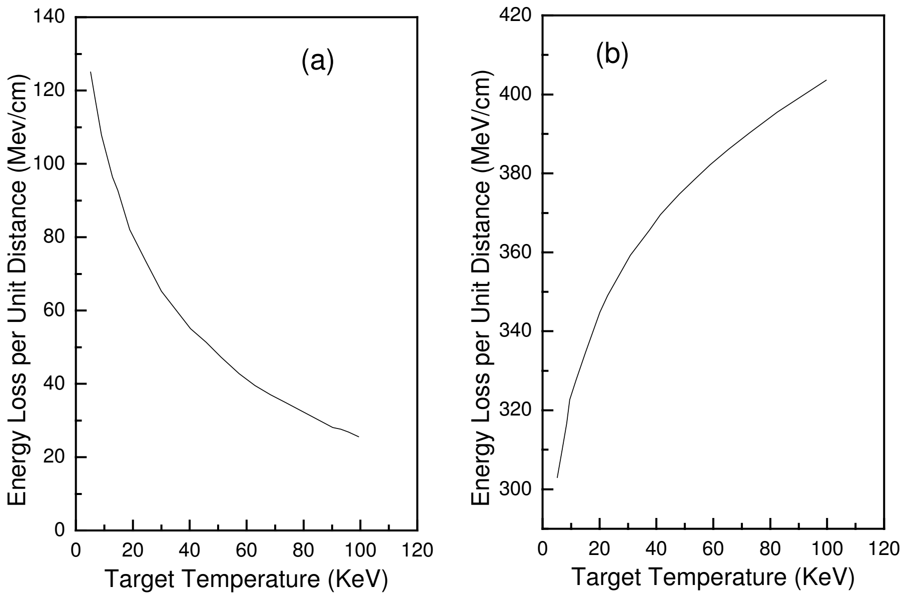

Figure 1. The rate of energy loss per unit distance of the REB with 1 MeV energy in the DT target at a density of 300 g cm^{-1} and a temperature in the range of 5–100 keV caused by (a) collective excitation and (b) two-body collisions.

where $\lambda'_{\mathrm{D}} = [\frac{1}{3}(\langle v_i^2\rangle_{\mathrm{av}}/\omega_{\mathrm{p}}'^2)]^{1/2}$ is the relativistic modified 'Debye length'. $\langle v_i\rangle_{\mathrm{av}}$ is the average speed of the electron in the plasma.

In the following, we calculate the rate of energy loss per unit distance of the REB with 1 MeV energy interacting with the super-compressed DT target at a density of 300 g cm^{-3} and a temperature in the range 5–100 keV. The results are shown in Figs 1(a) and (b). With the increase of the plasma temperature, the energy loss per unit distance caused by collective excitation gradually decreases while that caused by the two-body collisions opposes it, but the total energy loss almost stays unchanged at about 430 MeV cm^{-1}. The reason may be as follows. In a dense electron gas we know that for phenomena involving distances greater than the 'Debye length', the system behaves collectively; for distance shorter than this length, it may be treated as a collection of approximately free individual particles. Here, the increase of the plasma temperature, as well as that of the velocity of relativistic electrons, lead to a decrease of the plasma frequency ω'_{p} and an increase of the 'Debye length'. Thus, the increase of the 'Debye length' causes a decrease in the number of electrons participating in the collective oscillation and an increase of the number of electrons participating in the random thermal motion. But the homeostasis process in the plasma makes the total energy loss almost unchanged.

For a stopping target, by putting the stopping contributions (3) and (4) together, we may calculate the continuous winded range

$$R = \int \frac{dx}{dE}\,dE = \frac{(m_{\mathrm{e}}c^2)^2}{4\pi n_{\mathrm{p}}e^4}\int \frac{V\,dV}{(1-V)^{3/2}}D(V)^{-1} \tag{5}$$

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with $V = \beta^2$ and

$$D(V) = \ln \frac{m_e c^2 kT(1/\sqrt{1-V} - 1)}{\hbar^2 \omega_p'^2} - (2\sqrt{1-V} - 1 + V)\ln 2 + \frac{(1-\sqrt{1-V})^2}{8} + 1 - \ln 2 + \frac{1}{\gamma'^3} \ln \frac{2c^2 V}{3\omega_p'^2 \lambda_D'^2}.$$

Here, the value of R describes the actual path length of the incident electron during its passage through the DT target. The quasielastic and highly erratic motion for the incident relativistic electrons make them experience multiple scattering on the target. Such a process is essentially described by the square average deflection per unit path length ($Z = 1, A = 2$) [14],

$$\lambda^{-1}(\mu\mathrm{m}^{-1}) = 8\pi \left(\frac{e^2}{m_e c^2}\right)^2 \frac{Z(Z+1)N_A \rho}{A\beta^4}(1-\beta^2) \times \left[\ln\left(\frac{137\beta}{Z^{1/3}(1-\beta^2)^{1/2}}\right) + \ln 1.76 - \left(1 + \frac{\beta^2}{4}\right)\right], \quad (6)$$

where N_A is the Avogadro number equal to 6.022×10^{23} mol^{-1}; ρ, Z and A are the density, atomic number and the atomic weight of the DT target, respectively.

In fact, we really need an efficient packing mechanism to wind the projectile trajectories within a smaller domain in the compressed core. This winding process is easily described by the maximum penetration depth l_0. The simple relationship between the continuous winded range (the stopping range) R and the maximum penetration depth l_0 is given by [15]

$$R = l_0 + \frac{1}{2}\frac{l_0^2}{\lambda} + \frac{1}{2}\frac{l_0^3}{\lambda^2}. \quad (7)$$

In order to qualify the FIS as a coherent ignition scenario, a sufficiently short stopping time

$$t_{\mathrm{stop}} = \frac{1}{c}\int_{E_{\max}/10}^{E_{\max}} \frac{1 + E/m_e c^2}{[(E/m_e c^2)(E/m_e c^2 + 2)]^{1/2}} \frac{dE}{dE/dx} \quad (8)$$

is required.

In order to calculate the values of R, l_0 and t_{stop}, here for the super-compressed DT core we take a typical value of the density as 300 g cm^{-3} and a temperature of being 5 keV. Therefore, according to the expressions (5)–(8), for a 90% energy loss of 1 MeV REB we can obtain $R = 18.226$ μm, $l_0 = 13.95$ μm and $t_{\mathrm{stop}} = 8.45 \times 10^{-14}$ s. For comparison, we quote the relevant results given by [5, 6] as follows: $R = 42.66$ μm, $l_0 = 11$ μm, $t_{\mathrm{stop}} = 1.6 \times 10^{-13}$ s in [5] and $R = 34.66$ μm in [6]. Obviously, having made a more careful relativistic treatment, our results are much better than those given in [5, 6]†.

We calculate the values of R and l_0 for the energy range $0.5 <$ (MeV) < 1.5 of the relativistic electrons with 90% energy loss, with a super-compressed DT target density of 300 g cm^{-3} and a temperature of 5 keV. The results are shown in Fig. 2(a).

† In [5, 6] the formula for λ^{-1} is not cited correctly, when it is corrected according to [14], the results should be: $R = 42.66$ μm, $l_0 = 25.24$ μm, $t_{\mathrm{stop}} = 1.6 \times 10^{-13}$ s in [5] and, $R = 34.66$ μm, $l_0 = 22.07$ μm in [6].

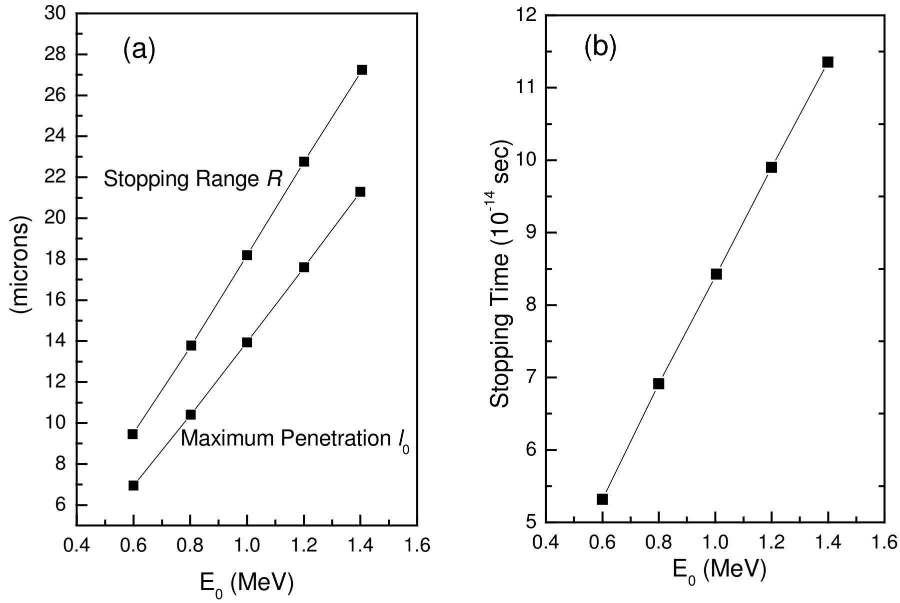

Figure 2. (a) REB stopping range R (μm) and maximum penetration depth l_0 (μm) in a 300 g cm^{-3} DT target at 5 keV temperature and $0.5 < E_0$ (MeV) < 1.5. (b) Corresponding stopping time t_{stop}.

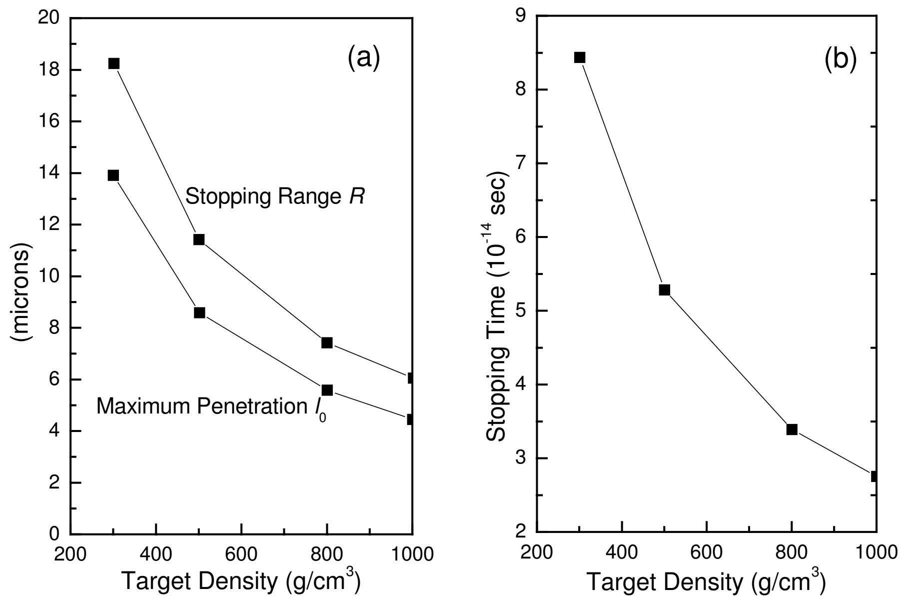

Figure 3. (a) R and l_0 for 1 MeV REB passing through a DT target with density ranging from 300 to 1000 g cm^{-3}, at 5 keV temperature. (b) Corresponding stopping time t_{stop}.

The corresponding stopping time t_{stop} is shown in Fig. 2(b). From Fig. 2 we can see the maximum penetration l_0 is on a microns scale. One knows that the fuel ignition requires the density radius of the hot DT spot to obey $\rho r > 0.3$ g cm^{-2} [4, 16].

For a DT target with a density of $300\,\mathrm{g\,cm^{-3}}$, if we take $\rho r \sim 0.45\,\mathrm{g\,cm^{-2}}$, the target radius r is about 15 µm. Our results show the maximum penetration to be $l_0 = 13.95$ µm, while the REB at 1 MeV suffers from an energy loss of 90%. Thus, the REB can efficiently transfer their energy to the plasma of the DT core. The corresponding stopping time $t_{\mathrm{stop}} \sim 10^{-14}$ s is short enough to ignite the target effectively. These results exhibit that FIS is possible and hopeful.

On the other hand, in Fig. 3 we also investigated the target density dependence of the above results for a 1 MeV REB and with the target density ranging from 300 to 1000 g $\mathrm{cm^{-3}}$, with a 5 keV temperature. From Fig. 3 we can see R, l_0 and t_{stop} reduce drastically with increasing target density.

In conclusion, we have studied the REB energy deposition during passage through a super-compressed DT core within the framework of relativistic theory. The results are much better than those works that did not take relativistic effects into consideration completely. We re-examine theoretically the possibility of igniting hot spots in a super-compressed DT target and the answer is that the FIS is able to yield thermonuclear ignition in the target.

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References

[1] Blanchot, N., Rouyer, C. and Sauteret, C. 1995 *Opt. Lett.* **20**, 395.

[2] Rouyer, C., Blanchot, N., Allais, I., Mazataud, E., Miquel, J. L., Nail, M., Pierre, A. and Sauteret, C. 1996 *J. Opt. Soc. Am.* B **13**, 55.

[3] Nuckolls, J. H., Wood, L., Thiessen, A. and Zimmermann, G. B. 1972 *Nature* **239**, 129.

[4] Tabak, M., Hammer, I., Glinsky, M. E., Kruer, W. L., Wilks, S. C., Woodworth, J., Campbell, E. M., Perry M. D. and Mason, R. J. 1994 *Phys. Plasma* **1**, 1626.

[5] Deutsch, C., Furukawa, H., Mima, K., Murakami, M. and Nishihara, K. 1996 *Phys. Rev. Lett.* **77**, 2483.

[6] Deutsch, C., Furukawa, H., Mima, K., Murakami, M. and Nishihara, K. 2000 *Phys. Rev. Lett.* **85**, 1140.

[7] Macchi, A., Cornolti, F., Pegoraro, F., Liseikina, T. V., Ruhl, H. and Vshivkov, V. A. 2001 *Phys. Rev. Lett.* **87**, 205004.

[8] Spitkovsky, A and Chen, P. 2000 *AIP Conf. Proc.*, Vol. 569. New York: AIP, p. 183.

[9] Barov, N., Rosenzweig, J. B. and Thompson, M. C. 2004 *Phys. Rev. ST Accel.* Beams **7**, 061301.

[10] Nardi, E. and Zinamon, Z. 1978 *Phys. Rev.* A **18**, 1246.

[11] Møller, C. 1932 *Ann. Phys.* **14**, 531.

[12] Ma, J. Y., Qiu, X. J. and Zhu, Z. Y. 2003 *Chin. Phys. Lett.* **20** 1306.

[13] Ma, J. Y., Qiu, X. J. and Zhu, Z. Y. 2004 *Chin. Phys.* **13**, 373.

[14] Hubbel, H. H. and Birkhoff, R. D. 1982 *Phys. Rev.* A **26**, 2460.

[15] Hemmer, P. C. and Farquahar, I. E. 1968 *Phys. Rev.* **168**, 294.

[16] Slutz, S. A., Vesey, R. A., Shoemaker, I. and Mehlhorn, T. A. 2004 *Phys. Plasmas* **11**, 3483.