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Propagation characteristics of Hermite-cosh-Gaussian laser beam in a rippled density plasmas

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Abstract

In the present research work, the authors have investigated the self-focusing and defocusing of Hermite-cosh-Gaussian laser (HChG) beam in an inhomogeneous rippled density plasmas. By taking the relativistic non-linearity into account, an equation for envelope is set up and solved using Wentzel–Kramers–Brillouin and the paraxial ray approximation. An ordinary non-linear differential equation governing the beam width parameter as a function of propagation distance is set up for different mode structures of the beam. Further, a numerical study of this differential equation is carried for suitable set of plasma and laser parameters. The beam undergoes periodic self-focusing/defocusing due to relativistic non-linearity. We also report the comparison between self-focusing/defocusing of HChG beam in the absence and presence of density ripple. Presence of ripple does not only leads to substantial increase in self-focusing length, but also results in oscillatory character with decreasing *f*. In a relativistic case, strong oscillatory self-focusing and defocusing is observed. Further, self-focusing is enhanced with increased value of decentered parameter.

Keywords: Beam width parameter; Density ripple; Hermite-cosh-Gaussian beam; Relativistic self-focusing

1. INTRODUCTION

Self focusing of laser beam at high laser intensities is an important nonlinear phenomenon from last three decades due to its technological application such as high harmonic generation (Ganeev *et al.*, 2015), attosecond pulse generation (Ma *et al.*, 2015), generation of x- rays (Arora *et al.*, 2014), inertial confinement fusion (Michel *et al.*, 2015) and charged particle acceleration (Lotov *et al.*, 2014). For practical realization of these applications, it is desirable that high-power laser beam should propagate over long distance with high precession to directionality. Among the fundamental processes, self-focusing of laser beam is a genuinely non-linear phenomenon, which has become important and active area of research during the last five decades. Self-focusing is counterbalanced by the tendency of the beam to spread because of diffraction. In the absence of non-linearity, the beam will spread substantially in a Rayleigh length $R_d(\approx k_0 a_0^2)$, where k_0 is the wave number, and a_0 is the spot size of the laser beam.

If the peak power is high enough, then a balance between self-focusing and diffraction can provide a condition for laser beam to propagate over several Rayleigh lengths. The basic physical mechanism responsible for self-focusing is non-linearity of the media such as plasmas, liquids, dielectric, semiconductors (Luther *et al.*, 1994; Boyd *et al.*, 2009; Rubenchik *et al.*, 2009), which originates in its interaction with the laser field. For example, when a Gaussian laser beam propagates in a plasma, then the refractive index becomes larger on the axis compared with the other parts of the wavefront due to maximum intensity on the axis. As a result, the wavefront of the beam creates refractive index profile across its own intensity profile, develops curvature, and focuses on its own. An intense laser or electromagnetic beam is capable of introducing thermal (Litvak, 1966), ponderomotive and relativistic non-linearity (Hora, 1975) during its propagation in the plasma. If the frequency of the laser beam exceeds the natural frequency of electron oscillations in a plasma, then the ponderomotive force comes into play. The ponderomotive force pushes the electron from axial region, that is, from the high-intensity region to the low-intensity region, thus reducing the local electron density, which further focuses the laser beam.

Extensive research work has been carried out on relativistic self-focusing of the laser beam (Singh & Walia, 2010; Irani *et al.*, 2012; Sharma, 2015). The phenomenon appears at the intensities when electron mass experience relativistic changes. With rapid evolution of ultra-short and ultra-intense

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compact lasers using the chirped pulse amplification techniques (Ren *et al*., 2007), the physics of the laser–plasma interaction in relativistic regimes has been identified as an emerging area in the recent years (Umstadter, 2003). For ultra-fast laser pulses, the drift velocity of the electrons in the plasma can be comparable with the velocity of light, causing significant increase in the mass of the electrons and hence the dielectric constant of the plasma. The non-linearity in the dielectric constant arises on account of relativistic variation of mass for arbitrary magnitude of intensity. The relativistic self-focusing has been investigated in the relativistic regime theoretically and experimentally (Asthana *et al*., 1994; Mainfray, 1995).

Beyond the focusing point, the nonlinear refraction starts weakening with the result that laser spot size increases. This is followed by oscillating self-focusing/de-focusing. Suk *et al*. (2001) proposed a scheme for electron trapping by making use of density transition. Gupta *et al*. (2007) applied this concept by introducing a slowly increasing density ramp leading to the enhancement of self-focusing. Later on Bonabi *et al*. (2009) used proper plasma density ramp profile to show significant improvement of the relativistic self-focusing.

However, there is another way to increase the self-focusing effect by introducing density ripple in the plasma. Different lasers are used to create density ripples in the plasma. Lin *et al*. (2006) used longitudinal spatial structure using laser matching with light modulator to achieve arbitrary plasma structures. Kuo *et al*. (2007) reported enhancement of relativistic harmonic generation by using a preformed periodic plasma waveguide. Resonant dependence of harmonic intensity on the plasma density and density modulation parameters was observed. Kaur and Sharma (2009) studied numerically the non-linear propagation of intense short Gaussian laser beam in a plasma containing large amplitude density ripple. Self-focusing due to relativistic and ponderomotive effect is undergoes significant enhancement in the region of high plasma density. Liu & Tripathi (2008) investigated the third-harmonic generation of a short pulse laser in a plasma density ripple. It was reported that the energy conversion efficiency scaled as the square of the plasma density and square of the depth of density ripple.

The earlier investigation on self-focusing has been confined to cylindrical symmetric Gaussian laser beam. The propagation of Gaussian beam (Wang & Zhou, 2011), Hermite–Gaussian beam (Takale *et al.,* 2009; Kant *et al*., 2012), cosh-Gaussian beam (Patil *et al.,* 2009, 2010, 2012; Patil & Takale, 2013; Nanda & Kant, 2014), Hermite-cosh-Gaussian (HChG) beam (Patil *et al*., 2007), etc. are recently studied by researchers due to its wide ranging applications. Saini & Gill (2006) studied the non-linear self-focusing and self-phase modulation of elliptic Gaussian laser beam in collisionless magnetoplasma with the help of variational approach. They discussed the effect of two beam width parameters along *x*- and *y*-directions on the cross-focusing of the beam. The focusing of one beam results in defocusing of another. When we increase the magnetic field, enhanced oscillating self-focusing with increased intensity is observed.

Purohit *et al.* (2016) numerically investigate the effect of relativistic–ponderomotive non-linearity and decentered parameter on the propagation of two cosh-Gaussian beams in collision less plasma. The decentered parameter plays a crucial role to enhance the focusing of cosh and HChG beams in a non-linear medium. Habibi and Ghamar (2015) studied strong relativistic self-focusing of cosh-Gaussian beam through dense plasmas of density ramp profile by using higher-order paraxial approximation. The periodic self-focusing/defocusing of cosh-Gaussian beam due to relativistic–ponderomotive non-linearity in a ripple density plasma has been observed by Aggarwal *et al.* (2014). Singh *et al.* (2013) explored that by the beating of two cosh-Gaussian laser beam in a rippled density, terahertz radiations are generated. They have discussed that by choosing appropriate decentered parameter, terahertz radiations can be focused at a specific position. The self focusing of cosh-Gaussian beams in a plasma with weakly relativistic and ponderomotive nonlinearity is studied by Gill *et al.* (2011) where they have discussed self phase modulation, self trapping of the cosh-Gaussian beams and impact of decentered parameter b on focusing of the laser beam has been studied. They found that oscillatory self-focusing takes place for a higher value of decentered parameter $b = 1$ and sharp self-focusing exhibits for $b = 2$.

Recently, Nanda and Nitikant (2014) studied that plasma density ramp effects the self focusing of HChG beams. They have discussed that decentered parameter is responsible for enhanced relativistic self focusing of HChG beams. Nanda *et al.* (2013a, b) studied the strong self-focusing of HChG beam in magnetoplasma of ramp density profile. The propagation properties of HChG beams in semiconductors for first three mode indices have been investigated analytically by Patil *et al.* (2008a, b, c). The focusing of HChG laser beams in a magnetoplasma with ponderomotive nonlinearity is studied by Patil *et al.* (2010). They demonstrated that additional self focusing is obtained for higher decentered parameter. Similarly mode index also affects the focusing of the laser beams.

In this paper, we present an investigation of self-focusing of HChG laser beam in rippled density plasmas taking into account of relativistic nonlinearity. HChG beam is one of the solutions of paraxial wave equation and it can be obtained in the laboratory by the superposition of two decentered Hermite–Gaussian beams. Further, HChG can possess high power in comparison with that of a Gaussian laser beam. Moreover, the self-focusing phenomenon of such beams is very sensitive to the decentered parameter b and different mode indices. Decentered parameter plays a crucial role in propagation characteristics of these beams. The capability of fabricating gas/plasma density structure is important for the development of plasma devices for high field applications. As inferred from the recent experiments, density ripples have been used to achieve phase-matched harmonic generation from plasmas. Non-linear parabolic partial differential equation governing the evolution of complex envelope in slowly varying approximation is solved using paraxial approximation in a periodically modulated density profile. The present work is valid for paraxial approximation, because the irradiance of the HChG beams depends upon mode indices and decentered parameter. For small values of decentered parameter, $(m + 1)$ lobes are formed in the intensity profile. For $m = 0$, intensity of the beam is maximum on the axis and for $m = 2$, intensity profile contains two decentered lobes and one centred lob on the axis. But in case of $m = 1$, the intensity is minimum on axis and maximum on the decentered positions. For $m = 1$ and 2, paraxial ray theory is only approximate one. Still, the paraxial ray theory is used in a number of investigation because of its mathematical simplicity Nanda and Nitikant (2014) & Patil *et al.* (2008a, b, c). The paper is as follows. in Section 2 self-focusing and defocusing of HChG beams are studied analytically. In this case, the wave equation for the beam width parameter is solved numerically for the relevant parameter and in Section 3, we discuss the results followed by conclusion.

2. SELF-FOCUSING AND DEFOCUSING OF THE BEAM

Let the equilibrium electron density n_0 be sinusoidal, $n = n_0(1 + \alpha_2 \cos qz)$, where n_0 is the maximum electron density and $\alpha_2 = n_2/n_0$ is the depth of density modulation and q is the ripple wave number. A laser beam launched into the plasma, which propagates through the plasma along $\hat{z}$,

$$E = A(r, z)e^{i(\omega t - kz)}, \tag{1}$$

where $k = (\varepsilon_{\mathrm{rel}})^{1/2}\ (\omega/c)$, is in the absence of density transition and $k = (\varepsilon(z))^{1/2}\ (\omega/c)$, is in the presence of density transition.

The propagation of field amplitude of HChG laser beam in a plasma is given by

$$E(r, z) = \frac{E_0}{f(z)}\left[H_m\left(\frac{\sqrt{2}r}{r_0 f(z)}\right)\right] e^{b^2/4}\ \left\{e^{-([r/r_0 f]+[b/2])^2} + e^{-([r/r_0 f]-[b/2])^2}\right\}, \tag{2}$$

where mode index of the Hermite polynomial H_m is given by m, maximum amplitude of HChG beam for central position at $r = z = 0$ is E_0, the initial spot size of the laser beam is r_0, decentred parameter is b and dimensionless beam width parameter of the laser beam is $f(z)$.

The general wave equation of laser beam propagating is written as,

$$\nabla^2 E - \frac{\omega^2}{c^2}\varepsilon E + \nabla\left(\frac{E \cdot \nabla(\varepsilon)}{\varepsilon}\right) = 0. \tag{3}$$

For $\left(1/k^2\right)\nabla^2(\ln \varepsilon)\ \ll 1$, we get

$$\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \varepsilon\frac{\omega^2}{c^2}E = 0. \tag{4}$$

The solution of this equation is given by (1). Propagation of laser beam through plasmas causes longitudinal oscillatory velocity to electrons given by $v = eE/m_0\ \omega c$, where m_0, ω, and e are the rest mass of electrons, angular frequency of incident beam, and charge on electron; and c is the speed of light in vacuum, $\gamma = \sqrt{1 + +\alpha EE^*}$ depends upon laser intensity with α relativistic factor given by $\alpha = e^2/m_0^2\ \omega^2 c^2$. The effective permittivity of the plasma is given by (Sodha *et al.* 1976),

$$\varepsilon = \varepsilon_0 + \varphi(EE^*),$$

where $\varepsilon_0 = 1 - (\omega_{\mathrm{p}}^2/\omega^2)$ is the linear part, φ is non-linear part of dielectric constant and $\omega_{\mathrm{p}} = \left(4\pi n_0 e^2/m_0\right)^{1/2}$ is the equilibrium plasma frequency. Here, we are considering only the relativistic mass of the electrons, $m_{\mathrm{e}} = m_0\gamma$. In the absence of density transition, the intensity-dependent non-linear part of dielectric constant is given by,

$$\varphi(EE^*) = \left(\frac{\omega_{\mathrm{p0}}^2}{\omega^2}\right)\left[1 - \frac{1}{(1 + \alpha EE^*)^{1/2}}\right]. \tag{5}$$

Now, in the presence of density ripple $n = n_0(1 + \alpha_2 \cos qz)$ the dielectric constant of the plasma is represented as

$$\varepsilon(z) = 1 - \left(\frac{\omega_{\mathrm{p0}}^2}{\gamma\omega^2} + \frac{\omega_{\mathrm{p0}}^2}{\gamma\omega^2}\alpha_2 \cos qz\right).$$

Non-linear part is given by

$$\varphi(EE^*) = \left[\frac{\omega_{\mathrm{p0}}^2}{\gamma\omega^2} + \frac{\omega_{\mathrm{p0}}^2}{\gamma\omega^2}\alpha_2 \cos qz\right]\left[1 - \frac{1}{(1 + \alpha EE^*)^{1/2}}\right]. \tag{6}$$

For nearly spherical wave front, the complex amplitude $A(r, z)$ may be expressed as

$$A(r, z) = A_0(r, z)e^{-ik(z)S(r,z)}, \tag{7}$$

where A_0 and S are the real functions of r and z and eikonal S is given by

$$S = \frac{r^2}{2}\beta(z) + \varphi(z), \tag{8}$$

where $\beta(z)$ can be expressed as $(1/f)(df/dz)$ and it represents the curvature of the wave front. Now substituting the values of Eq. (1) and (7) in Eq. (4) and neglecting $\partial^2 A/\partial z^2$, we get a complex differential equation with real and imaginary parts.

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In the presence of density ripple, the real part is given by

$$-2\frac{\partial S}{\partial z}-\frac{S\omega^2\alpha_2 q\sin qz}{c^2k^2}\frac{\omega_{p0}^2}{\gamma\omega^2}-\frac{zS\omega^4\alpha_2^2q^2\sin^2 qz}{2c^4k^4}\left(\frac{\omega_{p0}^2}{\gamma\omega^2}\right)^2$$
$$-\frac{z\omega^2\alpha_2 q\sin qz}{c^2k^2}\left(\frac{\omega_{p0}^2}{\gamma\omega^2}\right)\left(\frac{\partial S}{\partial z}\right)\frac{z\omega^2\alpha_2 q\sin qz}{c^2k^2}\left(\frac{\omega_{p0}^2}{\gamma\omega^2}\right)$$
$$-\frac{z^2\omega^4\alpha_2^2q^2\ \sin^2 qz}{4c^4k^4}\left(\frac{\omega_{p0}^2}{\gamma\omega^2}\right)^2+\frac{1}{2A_0^2k^2}\frac{\partial^2 A_0^2}{\partial r^2}-\left(\frac{\partial S}{\partial r}\right)^2 \tag{9}$$
$$+\frac{1}{2rA_0^2k^2}\frac{\partial A_0^2}{\partial r}-\frac{1}{4A_0^2k^2}\left(\frac{\partial A_0^2}{\partial r}\right)^2=0$$

and imaginary part is given by,

$$-\frac{1}{A_0^2}\frac{\partial A_0^2}{\partial z}-\frac{2\pi^2 z\alpha_2 q\ \sin qz}{\lambda^2A_0^2k^2}\frac{\partial A_0^2}{\partial z}-\frac{4\pi^2\alpha_2 q\ \sin qz}{\lambda^2k^2}\frac{\lambda^2}{\gamma\lambda_p^2}$$
$$+\frac{4\pi^4 z\alpha_2^2q^2\ \sin^2 qz}{\lambda^4k^4}\left(\frac{\lambda^2}{\gamma\lambda_p^2}\right)^2-\frac{2z\pi^2\alpha_2 q^2\cos qz}{\lambda^2k^2}\frac{\lambda^2}{\gamma\lambda_p^2} \tag{10}$$
$$-\frac{1}{A_0^2}\frac{\partial A_0^2}{\partial r}\frac{\partial S}{\partial r}-\frac{\partial^2 S}{\partial r^2}-\frac{1}{r}\frac{\partial S}{\partial r}=0.$$

Substituting A_0^2 as the solution of Eqs. (9) and (10) is given by

$$A_0^2=\frac{E_0^2}{f^2(z)}\left[H_m\left(\frac{\sqrt{2}r}{r_0 f}\right)^2\right]e^{b^2/2}$$
$$\left\{e^{-2([r/r_0f(z)]+[b/2])^2}-e^{-2([r/r_0f(z)]-[b/2])^2}+2e^{-([2r^2/r_0^2f^2(z)]+[b^2/2])}\right\}. \tag{11}$$

Using the values of Eqs. (8) and (11) in Eq. (9), we get the equation governing the evolution of beam width parameter. In the absence of ripple density, the beam width reduces to that of Eqs (11b), (12b), (13b) (Nanda and Nitikant, 2014) for the case of $m=0$ and $m=1$ & 2. In the presence of ripple density, for mode indices $m=0$, the equation of beam width parameter is given by, where $q'=qR_d$, $\xi=z/R_d$ is the normalized propagation distance, R_d is the diffraction length.

$$\left[1+\frac{\xi\alpha_2\,q'\sin(q'\xi)(\omega_{p0}^2/\gamma\omega^2)}{2\left(1-\left[\omega_{p0}^2/\gamma\omega^2\right]-\left[\omega_{p0}^2/\gamma\omega^2\right]\alpha_2\cos(q'\xi)\right)}\right]\frac{d^2f}{d\xi^2}$$
$$+\left[1+\frac{\xi\alpha_2\,q'\sin(q'\xi)[\omega_{p0}^2/\gamma\omega^2]}{2\left(1-\left[\omega_{p0}^2/\gamma\omega^2\right]-\left[\omega_{p0}^2/\gamma\omega^2\right]\alpha_2\cos(q'\xi)\right)}\right]$$
$$\times\left(\frac{\alpha_2\,q'\sin(q'\xi)\ [\omega_{p0}^2/\gamma\omega^2](\partial f/\partial\xi)}{2\left(1-\left[\omega_{p0}^2/\gamma\omega^2\right]-\left[\omega_{p0}^2/\gamma\omega^2\right]\alpha_2\cos(q'\xi)\right)}\right) \tag{12a}$$
$$-\frac{\xi\alpha_2\,q'\sin(q'\xi)[\omega_{p0}^2/\gamma\omega^2]}{2\left(1-\left[\omega_{p0}^2/\gamma\omega^2\right]-\left[\omega_{p0}^2/\gamma\omega^2\right]\alpha_2\cos(q'\xi)\right)}\frac{1}{f}\left(\frac{df}{d\xi}\right)^2$$
$$-\frac{(4-4b^2)}{f^3}+\frac{4\alpha E_0^2}{f^3}\left(\frac{\omega_{p0}^2}{\omega^2}+\frac{\omega_{p0}^2}{\omega^2}\alpha_2\cos(q'\xi)\right)\left(\frac{\omega r_0}{c}\right)^2$$
$$\times\left(1+\frac{4\alpha E_0^2}{f^2}\right)^{-3/2}e^{b^2/2}=0.$$

In our earlier investigation, we have studied the self-focusing for mode indices $m=0$. Moreover, in the present investigation the differential equation 12(a) governing the evolution of beam width parameter is different from equation (12) of Kaur and Kaur (2016). This paper was partial extension of Nanda and Nitikant (2014) where we have introduced density ripple. Further, Eq. (12) agrees with those of Nanda and Nitikant (2014) in the absence of density ripple. Equation (12) is approximately true, as the two terms written below were missing in Nanda and Nitikant (2014). In the present investigation, we have taken all the terms contained in the eikonal s, which are appearing in Equation 12(a) and we have taken care of both the terms in beam width parameter equation, that is,

$$\left[\begin{matrix}\left[1+\left(\xi\alpha_2\,q'\sin(q'\xi)\dfrac{\omega_{p0}^2}{\gamma\omega^2}/2\left(1-\dfrac{\omega_{p0}^2}{\gamma\omega^2}-\dfrac{\omega_{p0}^2}{\gamma\omega^2}\alpha_2\cos(q'\xi)\right)\right)\right]\\ \left(\alpha_2\,q'\sin(q'\xi)\ \dfrac{\omega_{p0}^2}{\gamma\omega^2}\left(\dfrac{\partial f}{\partial\xi}\right)/2\left(1-\dfrac{\omega_{p0}^2}{\gamma\omega^2}-\dfrac{\omega_{p0}^2}{\gamma\omega^2}\alpha_2\cos(q'\xi)\right)\right)\end{matrix}\right].$$

For mode indices $m=1$, the equation of beam width parameter can be written as

$$\left[1+\frac{\xi\alpha_2\,q'\sin(q'\xi)\left[\omega_{p0}^2/\gamma\omega^2\right]}{2\left(1-\left(\omega_{p0}^2/\gamma\omega^2\right)-\left(\omega_{p0}^2/\gamma\omega^2\right)\alpha_2\cos(q'\xi)\right)}\right]$$
$$\left(\frac{\alpha_2\,q'\sin(q'\xi)\ \left[\omega_{p0}^2/\gamma\omega^2\right](\partial f/\partial\xi)}{2\left(1-\left(\omega_{p0}^2/\gamma\omega^2\right)-\left(\omega_{p0}^2/\gamma\omega^2\right)\alpha_2\cos(q'\xi)\right)}\right)$$
$$-\frac{\xi\alpha_2\,q'\sin(q'\xi)\left[\omega_{p0}^2/\gamma\omega^2\right]}{2\left(1-\left[\omega_{p0}^2/\gamma\omega^2\right]-\left[\omega_{p0}^2/\gamma\omega^2\right]\alpha_2\cos(q'\xi)\right)}\frac{1}{f}\left(\frac{df}{d\xi}\right)^2 \tag{12b}$$
$$-\frac{(4-4b^2)}{f^3}-\frac{8\alpha E_0^2}{f^3}\left(\frac{\omega_{p0}^2}{\omega^2}+\frac{\omega_{p0}^2}{\omega^2}\alpha_2\cos(q'\xi)\right)\left(\frac{\omega r_0}{c}\right)^2$$
$$\times(2-b^2)e^{b^2/2}=0.$$

For mode indices $m=2$

$$\left[1+\frac{\xi\alpha_2\,q'\sin(q'\xi)\ \left[\omega_{p0}^2/\gamma\omega^2\right]}{2\left(1-\left[\omega_{p0}^2/\gamma\omega^2\right]-\left[\omega_{p0}^2/\gamma\omega^2\right]\alpha_2\cos(q'\xi)\right)}\right]\frac{d^2f}{d\xi^2}$$
$$+\left[1+\frac{\xi\alpha_2\,q'\sin(q'\xi)\left[\omega_{p0}^2/\gamma\omega^2\right]}{2\left(1-\left[\omega_{p0}^2/\gamma\omega^2\right]-\left[\omega_{p0}^2/\gamma\omega^2\right]\alpha_2\cos(q'\xi)\right)}\right]$$
$$\left(\frac{\alpha_2\,q'\sin(q'\xi)\ \left[\omega_{p0}^2/\gamma\omega^2\right](\partial f/\partial\xi)}{2\left(1-\left[\omega_{p0}^2/\gamma\omega^2\right]-\left[\omega_{p0}^2/\gamma\omega^2\right]\alpha_2\cos(q'\xi)\right)}\right)$$
$$-\frac{\xi\alpha_2\,q'\sin(q'\xi)\left[\omega_{p0}^2/\gamma\omega^2\right]}{2\left(1-\left[\omega_{p0}^2/\gamma\omega^2\right]-\left[\omega_{p0}^2/\gamma\omega^2\right]\alpha_2\cos(q'\xi)\right)}$$
$$\frac{1}{f}\left(\frac{df}{d\xi}\right)^2-\frac{(4-4b^2)}{f^3}+\frac{16\alpha E_0^2}{f^3}\left(\frac{\omega_{p0}^2}{\omega^2}+\frac{\omega_{p0}^2}{\omega^2}\alpha_2\cos(q'\xi)\right)$$
$$\left(\frac{\omega r_0}{c}\right)^2\left(1+\frac{16\alpha E_0^2}{f^2}\right)^{-3/2}e^{b^2/2}\ (5-2b^2)=0, \tag{12c}$$

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3. RESULTS AND DISCUSSION

In this section, we will study the self-focusing and defocusing of HChG beams in plasma with density transition for mode indices $m = 0$, 1, and 2. Equations (12a)–(12c) are the fundamental second-order differential equations for first three mode indices. These equations governing the self-focusing/defocusing of HChG beam in a plasma with density transitions. To analyse these equations numerically, we apply necessary and sufficient initial boundary conditions $f = 1$ and $df/d\xi = 0$ at $\xi = 0$. There are several terms in Eqs. (12a)–(12c). Analytical solutions of these equations are not possible. We therefore, seek numerical computational techniques to study the beam dynamics. Equations (12b) and (12c) represent evolution of beam width parameter as a function of dimensionless distance of propagation respectively for mode index $m = 1$ and 2. It is clear that Eq. (12a) is a non-linear ordinary differential equation, in which left-hand side contains several parameters such as plasma density, γ relativistic factor, q' relative wave number, α_2 as depth of density modulation, b as decentered parameter as well as intensity parameter of incident laser beam. Each of these physical entities contributes to the dynamics of beam width parameter for a given mode structure. With this in mind, we have carried out numerical simulation for various mode structure and beam width parameter with following choice of laser and plasma parameters (Lin *et al.*, 2006), $r_0\omega/c = 100$, $\omega_p^2/\omega^2 = 0.02$, $\alpha E_0^2 = 0.1$, $\alpha_2 = 0.2$, $q' = 30$, 65, and $b = 0$, 0.5, 0.9. Figure 1 exhibits the graphs of beam width parameter as a function of dimensionless distance of propagation both, in the presence of ripple as well as in absence of ripple when mode parameter is $m = 0$. In the absence of ripple, oscillatory self-focusing is observed. Our equation reduces to those of Nanda and Nitikant (2014) Eqs. 11(b), 12(b), 13(b) and our results agrees with those obtained in the upper mentioned reference. However, with the presence of density ripple as shown by dotted curve ($q' = 65$) leads to enhanced self-focusing, but oscillatory character is destroyed due to the appearance of $n_0\alpha_2 \cos qz$. When ripple is introduced the solution of Eqs. 12(a)–(c) as exhibited in figure, focusing is substantially enhanced leading to increase in the intensity in the focused zone. Further, there is increase in distance of propagation to the extent of two Rayleigh length. As observed, the initial changes are small; however, the introduction of density ripple leads rapid oscillatory and strong self-focusing. However, no further significant distance of propagation is observed as strong defocusing results set in beyond $\xi = 0.9$.

In Figure 2, Eq. (12b) is solved numerically for relevant set of parameters mentioned above and it is pertinent to mention that for $m = 1$ strong defocusing is observed in all cases of intensity, ripple, and other parameter. For small values of decentered parameter, the beam intensity is minimum on the axis as compare to off-axis. Thus, the diffraction effect dominates over focusing term due to which beam shows steady defocusing in the plasma. As the decentered parameter increases, the self-focusing term dominates that leads to focusing of the beam in the plasma. However, a slight variation is observed when ripple is considered (dotted line). The defocusing observed is quite similar to the one observed in case of Nanda and Nitikant (2014). Figure 3 displays the self-focusing of beam width parameter f versus ξ for mode indices $m = 2$. It is noticed that the behavior is quite similar to the case of $m = 0$, Eq. 12(a). Further, it may be noted that the distance of propagation is substantially enhanced with strong oscillatory self-focusing. When $m = 0$ case is considered, $\xi = 0.9$ was obtained. However, for mode $m = 2$, self-focusing distance increases to nearly two times. This contrast the case of Nanda and Nitikant (2014) where $\xi = 0.003$ is achieved with density transition. In the present investigation, density ripple yields superior propagation characteristics.

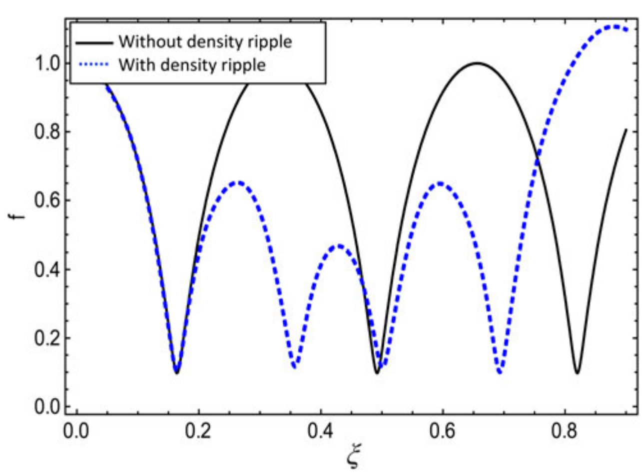

Fig. 1. Variation of beam width parameter (f) with normalized distance of propagation (ξ) for $m = 0$ with density ripple (dotted line) and without density ripple (solid line). The other parameters are $\omega_p^2/\omega^2 = 0.02$, $\alpha_2 = 0.2$, $\alpha E_0^2 = 0.1$, normalized ripple wave number $q' = 65$, $b = 0.5$, $\omega r_0/c = 100$.

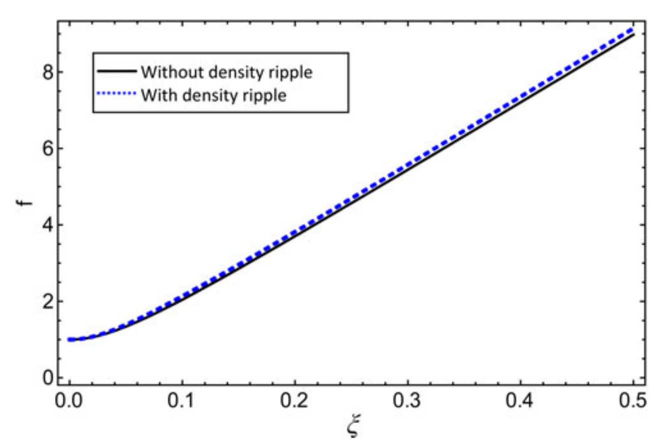

Fig. 2. Variation of beam width parameter (f) with normalized distance of propagation (ξ) for $m = 1$ with ripple (dotted line) and without ripple (solid line). The other parameters are $\omega_p^2/\omega^2 = 0.02$, $\alpha_2 = 0.2$, $\alpha E_0^2 = 0.1$, normalized ripple wave number $q' = 30$, $b = 0.5$ and $\omega r_0/c = 100$.

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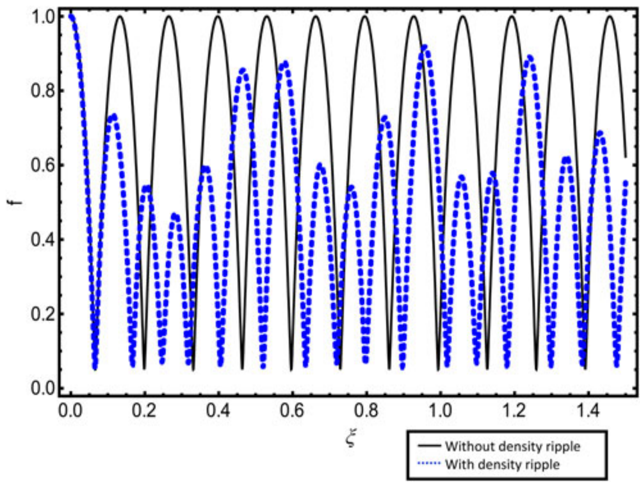

Fig. 3. Variation of beam width parameter (f) with normalized distance of propagation (ξ) for $m = 2$ with ripple (dotted line) and without ripple (solid line). The other parameters are $\omega_p^2/\omega^2 = 0.02$, $\alpha_2 = 0.2$, $\alpha E_0^2 = 0.1$, normalized ripple wave number $q' = 65$, $b = 0.5$, $\omega r_0/c = 100$.

Figure 4 represents the variation for the relativistic case. Earlier investigations reported above were limited to case when $\alpha E_0^2 = 0.1$ was considered, which is clearly a non-relativistic case of self-focusing. However, we consider relativistic non-linearity by taking $\alpha E_0^2 = 1.0$ as observed in Figure 4. Strong oscillatory self-focusing is observed upto $\xi = 0.8$ when density ripple is considered (dotted line). This is further accompanied by oscillatory defocusing. However, oscillatory self-focusing is observed in the absence of ripple and the propagation of several Rayleigh lengths is obtained. To highlight the role of decentered parameter in relativistic case, we have plotted f as a function of ξ for three different values of decentered parameter in Figure 5. Oscillatory self-focusing is observed in all the three cases ($b = 0$ dotted, $b = 0.5$ semi-dotted, $b = 0.9$ solid line). However,

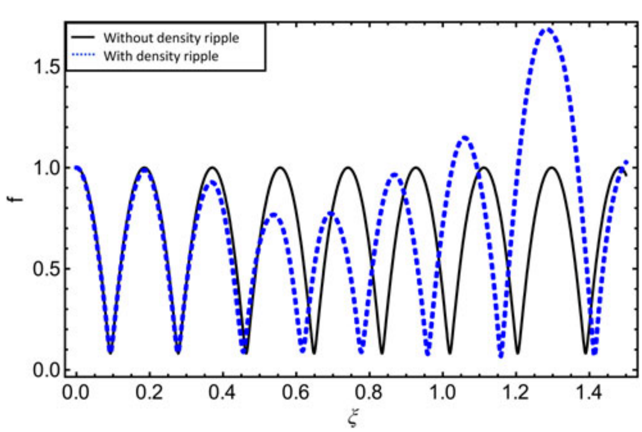

Fig. 4. Variation of beam width parameter (f) with normalized distance of propagation (ξ) for $m = 2$ with density ripple (dotted line) and without density ripple (solid line) for relativistic case. The other parameters are $\omega_p^2/\omega^2 = 0.02$, $\alpha_2 = 0.2$, $\alpha E_0^2 = 1$, normalized ripple wave number $q' = 65$, $b = 0.5$, $\omega r_0/c = 100$.

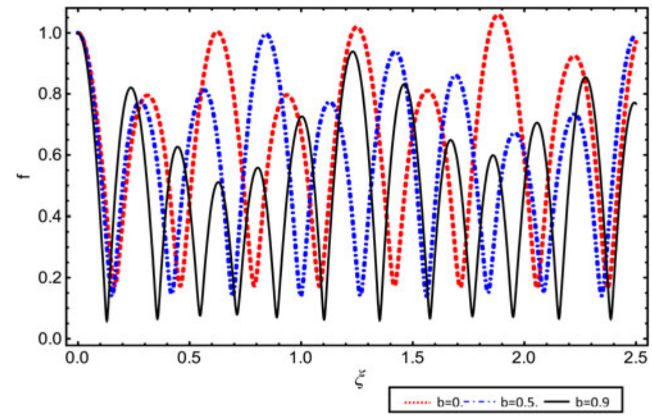

Fig. 5. Variation of beam width parameter (f) with normalized distance of propagation (ξ) for $m = 0$ with density ripple in relativistic case for different values of decentered parameter $b = 0, 0.5, 0.9$. The other parameters are $\omega_p^2/\omega^2 = 0.02$, $\alpha_2 = 0.2$, $\alpha E_0^2 = 1$, $q' = 30$, and $\omega r_0/c = 100$.

when decentered parameter $b = 0$, we get least focusing. As decentered parameter is increased, self-focusing is enhanced. Thus role of increment of decentered parameter in the presence of density ripple ($q' = 30$) not only increase the self-focusing, but also enhances the distance of propagation to several Rayleigh lengths. The effect of decentered parameter on relativistic self-focusing of high-intensity HChG laser beam has been investigated by Nanda *et al.* (2013a, b) and reported the occurrence of strong self-focusing at $\xi \approx 1.02$. In our case, we found that with the introduction of density ripple in the plasma, strong self-focusing of beam occurs earlier at lower values of distance of propagation ($\xi \approx 0.1$). In Figure 6, we consider the mode $m = 2$ in relativistic case $\left(\alpha E_0^2 = 1\right)$ for $q' = 65$. We seek the effect of decentered parameter ($b = 0$ dotted, $b = 0.5$ semi-dotted, $b = 0.9$ solid line) on f beam width parameter. Surprisingly, we obtain contrasting results to the case of $m = 0$. Here for $m = 2$, we observe oscillatory strong defocusing with ξ on increasing decentered parameter b. Nanda *et al.* (2013a, b)

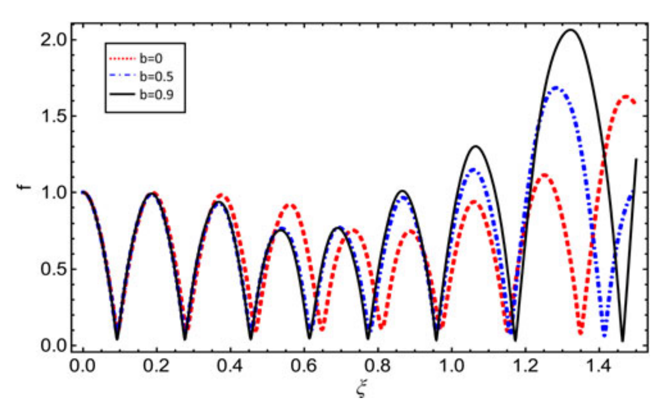

Fig. 6. Variation of beam width parameter (f) with normalized distance of propagation (ξ) for $m = 2$ with density ripple in relativistic case for different values of decentered par b = 0, 0.5, 0.9. The other parameters are $\omega_p^2/\omega^2 = 0.02$, $\alpha_2 = 0.2$, $\alpha E_0^2 = 1$, $q' = 65$, and $\omega r_0/c = 100$.

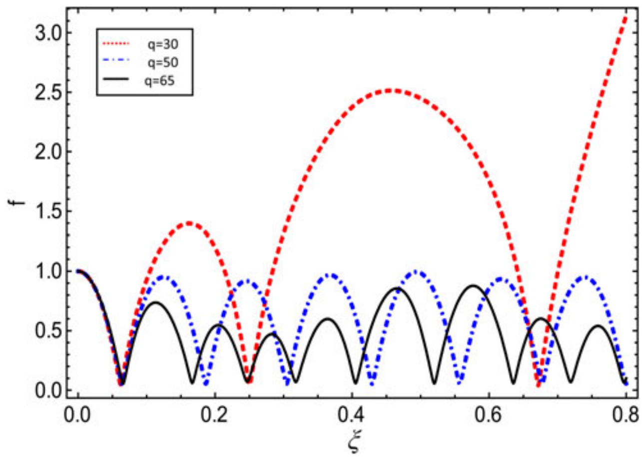

Fig. 7. Variation of beam width parameter (f) with normalized distance of propagation (ξ) for $m = 2$ with different values of normalized ripple wave numbers $q' = 30, 50, 65$. The other parameters are $\omega_p^2/\omega^2 = 0.02$, $\alpha_2 = 0.2$, $\alpha E_0^2 = 0.1$, $b = 0.5$, and $\omega r_0/c = 100$.

reported that with the increase in decentered parameter, divergence terms dominates due to which weak focusing is observed at $\xi = 1.02$ for $b = 1.642$. In the present work, we observed that for small values of decentered parameter, the focusing term dominates over diffraction term and strong self-focusing is observed at $\xi = 0.08$ for $b = 0.9$. In last, we have observed the effect of ripple wave number on the propagation characteristics of the laser beam for mode index ($m = 2$) (Figure 7). For this purpose, we have plotted $f(\xi)$ as a function of three values of ripple wave number q' viz. dotted (red) for $q' = 30$, dotted for $q' = 50$ semi-dotted for $q' = 65$ solid line. Significant enhancement of self-focusing and propagation is achieved on increasing the ripple wave number. Similar results are obtained when relativistic case is considered for $m = 2$ mode structure. Thus self-focusing/defocusing of HChG laser beam in rippled density plasma can be controlled by choosing the appropriate mode indices, decentered parameter and ripple wave number. Out of these, the decentered parameter and ripple number are responsible for the focusing of the beam.

4. CONCLUSION

In present research work, we have studied self-focusing of HChG laser beam in a rippled density plasmas for mode index $m = 0$, 1 and $m = 2$. The other parameters are $r_0\omega/c = 100$, $\omega_p^2/\omega^2 = 0.02$, $\alpha E_0^2 = 0.1$, $\alpha_2 = 0.2$, $q' = 30$, 50, 65 and $b = 0$, 0.5, 0.9. In the absence of ripple, oscillatory self-focusing is observed. However, the presence of density ripple ($q' = 65$), leads to enhanced self-focusing, but oscillatory character is destroyed due to appearance of $n_0\alpha_2\cos qz$ for $m = 0$ and $m = 2$ mode indices. For $m = 1$ mode with small value of decentered parameter, diffraction effect dominates over focusing of the laser beam. Strong self-focusing is observed upto $\xi = 0.8$ by considering relativistic, which is further accompanied by oscillatory defocusing. As decentered parameter increases, self-focusing is enhanced upto several Rayleigh lengths, but for higher mode $m = 2$ with increased decentered parameter strong oscillatory defocusing is observed. Significant enhancement of self-focusing and propagation is achieved on increasing the ripple wave number. The presence of density ripples in a plasma significantly modulates the phenomenon of self-focusing with increase in intensity in the focused zone. Further, there is increase in distance of propagation to the extent of two Rayleigh length. We have taken the paraxial ray approximation, which is appropriate for small values of decentered parameter: $b = 0$, 0.5, and 0.9. When we increase the decentered parameter, non-paraxial study will be more appropriate as compare with paraxial study. The present work may add additional information in the field of laser-driven fusion to improve the focusing quality of the beam with an adequate intensity and decentered parameter.

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