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# Coupling of the Okuda–Dawson model with a shear current-driven wave and the associated instability

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**Abstract.** It is pointed out that the Okuda–Dawson mode can couple with the newly proposed current-driven wave. It is also shown that the Shukla–Varma mode can couple with these waves if the density inhomogeneity is taken into account in a plasma containing stationary dust particles. A comparison of several low-frequency electrostatic waves and instabilities driven by shear current and shear plasma flow in an electron–ion plasma with and without stationary dust is also presented.

Long ago (D' Angelo 1965), a purely growing instability was proposed that is driven by the shear flow of plasma along the constant external magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$. A later investigation showed that the usual electrostatic drift wave becomes unstable due to shear flow of plasma (Saleem et al. 2007). In these investigations, plasma does not have a zero-order current because both the species i.e. electrons and ions flow parallel to $\mathbf{B}_0$ with the same shear flow velocity. Several authors have investigated the shear flow-driven waves and instabilities (Ishiguro et al. 1997; Ganguli et al. 2002; Eliasson et al. 2006). The effects of shear velocity on an ion cyclotron wave has been observed experimentally (Koepke et al. 1995; Amatucci et al. 1996). It has been pointed out that the shear flow of electrons along the ambient magnetic field, say $B_{0z}\hat{\mathbf{z}}$, introduces a shear current and hence the total zero-order magnetic field becomes space-dependent. In this case, a relatively high-frequency electrostatic wave may exist in the plasma which can become unstable under certain conditions in a heavier ion (like barium) plasma (Saleem and Eliasson 2011).

It has also been shown that the shear current introduces a low-frequency electrostatic drift-type wave which can become unstable in a homogeneous density plasma (Saleem 2011). If the wave vector parallel to $\mathbf{B}_0$ is zero ($k_\parallel = 0$), then the shear current gives rise to a flute-like mode (Saleem 2011) similar to the density gradient-driven Shukla–Varma mode (Shukla et al. 1993) in the plasma having stationary dust. In the absence of collisions, the convective cell mode (Okuda and Dawson 1973) is a stable low-frequency electrostatic mode which plays important role in the plasma transport. It has been shown to be modified in the presence of stationary dust (Shukla and Mamun 2002).

In the present work, our aim is to show that in the presence of stationary dust, the Okuda–Dawson mode (Okuda and Dawson 1973) couples with the current-driven drift-like mode (Saleem 2011) for $|v_{te}\partial_\parallel| << |\partial_t|$ (where $v_{te} = (T_e/m_e)^{1/2}$ is the electron thermal speed) and an instability appears which can play an important role in the cross-field plasma transport. This instability can also appear in an electron–ion plasma if the parallel shear flow velocities of ions and electrons are different ($v_{i0}(x) < v_{e0}(x)$) since they would cause a zero-order shear current.

Let us assume that both electrons and ions have the shear velocity $\mathbf{v}_{0i} = \mathbf{v}_{oe} = v_0(x)\hat{\mathbf{z}}$ along the initial background constant external magnetic field $B_{0z}\hat{\mathbf{z}}$ in the presence of stationary dust with the local equilibrium $n_{90} = n_{e0} + Z_d n_{d0}$, where $Z_d$ denotes the number of negative charges on the dust particles. The zero-order current $\mathbf{J}_0 = -eZ_d n_{d0} v_0 \hat{\mathbf{z}} \neq 0$ is non-zero which twists the initial field and, therefore, the total zero-order magnetic field becomes (Saleem and Eliasson 2011)

$$\mathbf{B}_0 = B_{0z}\hat{\mathbf{z}} + B_{0y}(x)\hat{\mathbf{y}} = B_0\hat{\mathbf{e}}_\parallel, \tag{1}$$

where $\hat{\mathbf{e}}_\parallel = \left(\hat{\mathbf{z}} + (x/L_B)\hat{\mathbf{y}}\right) / \left(1 + x^2/L_B^2\right)$ is the unit vector parallel to $\mathbf{B}_0$, $L_B = \kappa_B^{-1}$ and $\kappa_B = 1/B_0(dB_0/dx)$. Figure 1 shows the geometry of the electrostatic current-driven wave.

The Okuda–Dawson mode (Okuda and Dawson 1973) appears in the limit $|v_{te}\partial_\parallel| < |\partial_t|$ and hence the electrons do not follow the Boltzmann density distribution. The equation of motion of electrons is, therefore, given by

$$m_e n_e(\partial_t + \mathbf{v}_e.\nabla)\mathbf{v}_e = -en_e\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_e \times \mathbf{B}_0\right). \tag{2}$$

The perpendicular component of the electron fluid velocity from (2) for $|\partial_t| < \Omega_e$ yields

$$\mathbf{v}_{e\perp} \approx (c/B_0)\hat{\mathbf{e}}_\parallel \times \nabla_{\perp\phi}. \tag{3}$$

The parallel equation of motion becomes

$$(\partial_t + \mathbf{v}_e.\nabla)v_{ez} = \frac{e}{m_e}(\partial_z\phi + S), \tag{4}$$

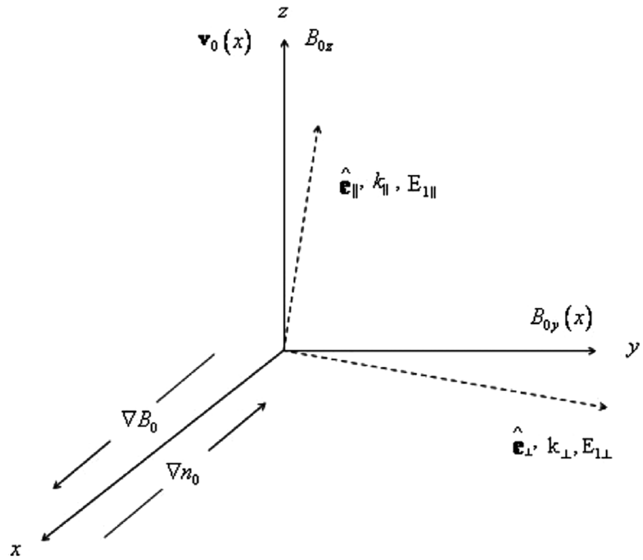


**Figure 1.** The wave geometry is shown.

where $S = (1/\Omega_e) dv_{e0}/dx$ is the dimensionless shear flow parameter and $\Omega_e = eB_0/m_e c$. The equation of motion for cold ions becomes

$$m_i n_i (\partial_t + \mathbf{v}_i.\nabla)\mathbf{v}_i = e n_i \left(\mathbf{E} + \frac{1}{c}\mathbf{v}_i \times \mathbf{B}_0\right). \quad (5)$$

The perpendicular component of the ion fluid velocity from (5) yields

$$\mathbf{v}_{i\perp} \approx \mathbf{v}_E - \frac{c}{B_0 \Omega_i}(\partial_t + \mathbf{v}_E.\nabla)\nabla_\perp \phi, \quad (6)$$

where the first term on the right-hand side $\mathbf{v}_E = (c/B_0)\hat{\mathbf{e}}_\parallel \times \nabla_\perp \phi$ is the $\mathbf{E} \times \mathbf{B}$ drift and the usual limit $\partial_t << \Omega_i = eB_0/m_i c$ has been used for low-frequency waves. Using (3) and (6), the continuity equations for electrons and ions can be written respectively as

$$\partial_t n_e - \frac{c n_{e0}}{B_0}(\hat{\mathbf{e}}_\parallel \times \nabla_\perp \phi.\kappa_B) + n_{e0}\partial_\parallel v_{ez} + v_0 \partial_\parallel n_e = 0 \quad (7)$$

and

$$\partial_t n_i - \frac{c n_{i0}}{B_0}(\hat{\mathbf{e}}_\parallel \times \nabla_\perp \phi.\kappa_B) + v_0 \partial_\parallel n_i = 0, \quad (8)$$

where $(\mathbf{v}_0.\nabla n_j) \simeq v_0 \partial_\parallel n_j$ has been used. Since the current-driven instability does not require the density gradient; therefore, we first study the uniform density plasma with stationary negatively charged dust. The equilibrium demands $n_{i0} = n_{e0} + Z_d n_{d0}$. Let the linear perturbations be proportional to $exp\ \{\imath(\mathbf{k}_\perp.\gamma_\perp + \mathbf{k}_\parallel.\gamma_\parallel - \omega t)\}$. Then subtracting (7) and (8), using Poisson's equation ($\nabla^2\phi = 4\pi e(n_e - n_i)$) and ignoring the ion parallel velocity, we obtain the following linear dispersion relation for the coupled current-driven (Saleem 2011) and modified Okuda–Dawson mode (Shukla and Mamun 2002)

$$\left(1 + \frac{\Omega_i^2}{\omega_{pi}^2}\right)\Omega^2 + \left\{\frac{4\pi c e z_d n_{d0}}{B_0}\frac{\Omega^2}{\omega_{pi}^2}(\hat{\mathbf{e}}_\parallel \times \nabla ln B_0.\mathbf{k}_\perp)\right\}$$
$$\times \Omega - \frac{n_{e0}}{n_{i0}}(\Omega_i \Omega_e)\frac{k_z^2}{k_\perp^2}\left(1 - \frac{k_\perp}{k_z}S\right) = 0. \quad (9)$$

Here, $\Omega = (\omega - \omega_0)$ and $\omega_0 = v_0 k_\parallel$. The modified Okuda–Dawson mode has frequency (Shukla and Mamun 2002)

$$\omega_{cc} = \left(\frac{n_{e0}}{n_{i0}}(\Omega_i \Omega_e)\right)^{\frac{1}{2}} \frac{k_\parallel}{k_\perp}, \quad (10)$$

for $k_\parallel/k_\perp < m_e/m_i$, and the current-driven mode propagating perpendicular to $B_0$ has frequency

$$\omega_s^* = \frac{4\pi c e Z_d n_{d0}}{B_0(1 + \omega_{pi}^2/\Omega_i^2)}\frac{\kappa_B}{\kappa_\perp}, \quad (11)$$

which is (10) of Saleem (2011). Using the above notation, (9) can be expressed as

$$\Omega^2 - \omega_s^* \Omega - \omega_{cc}^2\left(1 - \frac{k_\perp}{k_\parallel}S\right) = 0. \quad (12)$$

If $k_\parallel = 0$, then (12) gives $\omega = \omega_s^*$ and if $\mathbf{J}_0 = 0$, then (12) yields $\omega = \omega_{cc}$. The two roots of (12) are

$$\Omega_\pm = \frac{1}{2}\left\{\omega_s^* \pm \sqrt{4\omega_{cc}^2\left(1 - \frac{k_\perp}{k_\parallel}S\right)}\right\}, \quad (13)$$

and one of the roots (with positive sign) becomes unstable for

$$\frac{k_\parallel}{k_\perp} < S. \quad (14)$$

It is important to note that the present electrostatic instability takes place for $(v_{Te}\partial_\parallel) < |\partial_t|$ when the cold electrons do not follow the Boltzmann density distribution and ions parallel motion is neglected because $v_{Ti} << v_{Te}$. On the other hand, when electrons follow the Boltzmann density distribution and ions parallel dynamics is taken into account for $|\partial_t| < |v_{Te}\partial_\parallel|$, then instead of (12) one obtains

$$\left(\frac{n_{e0}}{n_{i0}} + \rho_s^2 k_\perp^2\right)\Omega^2 - \omega_s^*\Omega - c_s^2 k_\parallel^2\left(1 - \frac{k_\perp}{k_\parallel}A\right) = 0, \quad (15)$$

which is the same as (16) of Saleem (2011) for a homogeneous density plasma. Here, $\rho_s = c_s/\Omega_i$ and $c_s = \sqrt{T_i/m_i}$. Note that $A = 1/\Omega_i \left|dv_{0/dx}\right|$ in (15). If the plasma has a non-uniform density, then (12) is modified as

$$\Omega^2 - (\omega_s^* + \omega_{sv}^*)\Omega - \omega_{cc}^2\left(1 - \frac{k_\perp}{k_\parallel}S\right) = 0, \quad (16)$$

where

$$\omega_{sv}^* = \frac{4\pi c e z_d n_{d0}/B_0}{\left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right)}\frac{\kappa_{nd}}{\kappa_\perp} \quad (17)$$

is the frequency of the Shukla–Varma mode (Shukla et al. 1993) and $\kappa_{nd} = |(1/n_{d0})dn_{d0}/dx|$. Again for $S < k_\parallel/k_\perp$, an instability appears. Now, we present another interesting comparison of (12) and (15). If the plasma does not have stationary dust and hence $\mathbf{J}_0 = 0$, then (15) yields a purely growing unstable D'Angelo's mode (D' Angelo 1965)

$$\Omega = \frac{c_s k_\parallel}{(1 + \rho_s^2 k_\perp^2)^{1/2}}\left(1 - \frac{k_\parallel}{k_\perp}A\right)^{1/2}. \quad (18)$$

If $A < k_\parallel/k_\perp$, then we do not obtain a stable ion acoustic wave. Similarly for $\mathbf{J}_0 \neq 0$, (16) yields a purely growing mode

$$\Omega^2 = \omega_{cc}^2\left(1 - \frac{k_\perp}{k_\parallel}S\right), \quad (19)$$

if (14) is satisfied along with $\omega_s^* << \omega_{cc}^*$ and consequently we do not obtain a stable Okuda–Dawson mode.

It is pertinent to mention here that several years ago Shukla et al. (2002) investigated the electron Landau contribution on the instability of dust ion acoustic drift waves in the presence of ion parallel velocity gradient. The electron pressure contribution, however, was crucial in their investigation. If the electron Landau contribution is neglected and ions are assumed to be cold, then (10) of Shukla and Mamun (2002) reduces to (18) for $dB_0/dx = 0$ and $n_{e0}/n_{i0} = 1$ (which implies a pure electron–ion plasma). Note that (15) is structurally similar to (11) of Shukla et al. (2002) for $\rho_s^2 k_\perp^2 << 1$ with the difference that here $\omega_s^*$ comes owing to the spatial inhomogeneity of the equilibrium magnetic field gradient. We further emphasize that the Okuda–Dawson mode appears when electron pressure is small and is neglected in the parallel equation of motion of electrons. In that case the electrons do not follow the Boltzmann density distribution in the fluid limit and the electron density fluctuations are estimated by substituting $v_{ez}$ from the parallel equation of motion into the continuity equation. Therefore, the findings of the work by Shukla et al. (2002) should be compared with (15) of the current investigation and not with (16) of the present work.

To summarize, we have shown in this paper that the shear flow-driven wave couples with the modified Okuda–Dawson mode in a cold plasma as given in (12). An electrostatic instability appears if condition (14) is satisfied. In the presence of density inhomogeneity, the Shukla-Varma mode has been shown to couple with these waves and the oscillation frequency is $\omega_r = \omega_s^* + \omega_{sv}^*$. The waves have been found to become unstable if condition (14) holds. A comparison of the dispersion relations of the shear flow-driven waves and instabilities in hot and cold electron plasmas has also been presented. The free energy available in the form of flow and current can give rise to several kinds of waves and instabilities, which can play significant roles in particle and energy transport in plasmas.

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