N-BODY RESULTS ON DARK MATTER

Simon D. M. White
Steward Observatory
University of Arizona
Tucson, Arizona 85721
U.S.A.

ABSTRACT. The structure of the dominant "dark" component of the Universe may evolve primarily under the influence of gravity. A number of models for the evolution of the Universe make specific predictions for the statistical properties of density fluctuations at early times. N-body simulations can follow the nonlinear development of such fluctuations to the present day. A major difficulty arises because we cannot observe the present mass distribution directly. Recent N-body work has concentrated on models dominated by weakly interacting free elementary particles. Neutrino-dominated but otherwise conventional cosmologies pass rapidly from a smooth distribution to one dominated by lumps with masses greater than those of any known object. Cosmologies dominated by "cold dark matter" produce mass distributions which fit the observed galaxy distribution (i) if $\Omega = 0.1 - 0.2$ and galaxies follow the mass distribution, or (ii) if $\Omega = 1$, $H_0 < 50$ km/s/Mpc and galaxies form preferentially in high density regions. In the latter case, clumps form with flat rotation curves with about the amplitude and abundance expected for galaxy halos.

1. INTRODUCTION

Two major problems confront any attempt to make detailed evolutionary models for the distribution of dark matter in the Universe. We have no idea what the stuff is, and so cannot specify the physical interactions it is subject to. In addition, since we cannot see it, we have only indirect indications of where it is, and we do not know how much of it lies between galaxy groupings. The first difficulty may not be serious when considering recent evolution, because we have good evidence that the dark matter is presently in a non-gaseous, effectively collisionless form. These evolutionary phases can therefore be studied by N-body methods. However, although gravity may be the sole driver of evolution today, other interactions which depend on the nature of the dark matter condition the distribution of density fluctuations with which it emerged from the early universe. These fluctuations set the initial conditions from which later structure

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must grow by gravitational processes. In addition, other processes might have influenced the dark matter before it condensed into its present form. The situation is clearly simpler in the absence of any other significant processes. This is a major motivation for concentrating considerable effort on models which the dark matter is composed of freely moving elementary particles -- neutrinos, photinos or axions, for example -- even though they may seem inherently less plausible than "jupiters" or stellar remnants. For such models the astrophysical uncertainties associated with star formation do not affect the large-scale distribution of dark matter.

The uncertainties associated with galaxy formation cannot, however, be avoided when comparing the mass distribution predicted by any model with what we see in the sky. Some assumption about how galaxies "light up" the mass is required before such a comparison can be attempted. Most early N-body studies of structure formation assumed that "galaxies trace the mass", meaning that the statistical properties of the mass distribution in the models were identified directly with the corresponding properties of the observed galaxy distribution. This is an assumption of convenience which has little theoretical justification. We shall see below that during certain evolutionary phases of a neutrino-dominated universe, much of the mass lies in very large low density regions which contain no nonlinear structures, and thus no galaxies. In general, while it seems likely that regions where the mass density is high will form more galaxies than low density regions, there are many situations in which the overdensity in galaxies will be a nonlinear function of the overdensity in mass. In this case the morphology of the galaxy distribution will reflect that of the underlying mass distribution, but quantitative statistical properties such as correlation functions will differ. When trying to test N-body models it is important to remember the uncertainties introduced by our ignorance of how galaxies form, and to focus on those aspects of the "observed" mass distribution which are least subject to such uncertainties.

Early N-body simulations of the growth of structure in the universe were more concerned with elucidating the nonlinear clustering process than with the nature of the material that was clustering (Press and Schechter 1974, Peebles and Groth 1976, Aarseth, Gott and Turner 1979, Efstathiou 1979, Efstathiou and Eastwood 1981). Distributions of particles were evolved from initial conditions selected for simplicity, rather than on the basis of any theory for prior evolution, and particles were rather loosely identified as galaxies when making comparisons with observation. The first simulations of evolution from initial conditions with a coherence length were motivated by to theories for the origin of structure from adiabatic initial fluctuations. However, they still represented the theoretical predictions in an extremely schematic way (Klypin and Shandarin 1983, Centrella and Melott 1983, Frenk, White and Davis 1983). In addition these studies still identified the distribution of mass with that of the galaxies. Since 1983 it has become clear that modern N-body techniques can follow the nonlinear development of structure from a precise representation of the fluctuations predicted by detailed theories

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for the linear phases of evolution, albeit over a limited range of mass scales (Efstathiou et al. 1985). Studies to date have considered evolution from the linear fluctuation spectra predicted for universes dominated by massive neutrinos (White, Frenk and Davis 1983, Fry and Melott 1985) and by cold dark matter (Davis et al. 1985). When comparing with observation, this more recent work has made some attempt to account for uncertainties in how galaxies form. The rest of this review is based mainly on results from my collaborative research program with Marc Davis, George Efstathiou and Carlos Frenk.

2. INITIAL CONDITIONS AND NUMBERICAL TECHNIQUES

The evolution of strucure in the early universe is usually discussed in terms of the linear gravitational instability of a Friedmann universe (see e.g. Peebles 1980). This is in accord with the intuitive expectations that gravitational effects will cause clumping to increase with time, and that protogalactic perturbations must therefore have had small amplitude when the mean density of the Universe exceeded present galactic densities. Further support comes from upper limits on fluctuations in the microwave background which show that the amplitude of large-scale density perturbations was $< 10^{-4}$ when the primeval plasma recombined at $z \approx 10^3$. Once the nature of the particle and radiation fields present in the early universe has been specified, linear theory can be used to calculate the amplitude of a plane wave perturbation of comoving spatial frequency $k = 2\pi/\lambda$ as a function of time --

$$\delta_k(t) = T(k,t)\ \delta_{k,p} \quad . \tag{1}$$

In this expression δ_k is the amplitude of the relative density fluctuation in some particle or radiation field, $\delta_{k,p}$ is the primordial amplitude with which it was generated, and $T(k,t)$ is a transfer function. If fluctuations are a result of quantum effects during an early inflationary epoch, then under very general conditions they are expected to have the scale-free Harrison-Zel'dovich constant curvature spectrum,

$$|\delta_{k,p}|^2 \propto k \quad , \tag{2}$$

with random phases. The inflationary model also predicts $\Omega = 1$, of course, unless some fine-tuning is invoked. (See Guth 1985 for a review of these questions.) Apart from an overall normalization, equations (1) and (2) determine the "initial" conditions for galaxy formation completely once the contents of the Universe are specified.

Figure 1 shows power spectra at late times in a universe now dominated by weakly interacting massive particles (from Bond and Szalay 1983, Bond and Efstathiou 1984). The quantity $k^3|\delta_k|^2$ is the total power in waves of scale $\lambda \sim 2\pi/k$. Objects of this size will condense out of the general expansion when $k^3|\delta_k|^2 \sim 1$. Such nonlinear behavior can in general be followed only by numerical simulation, but

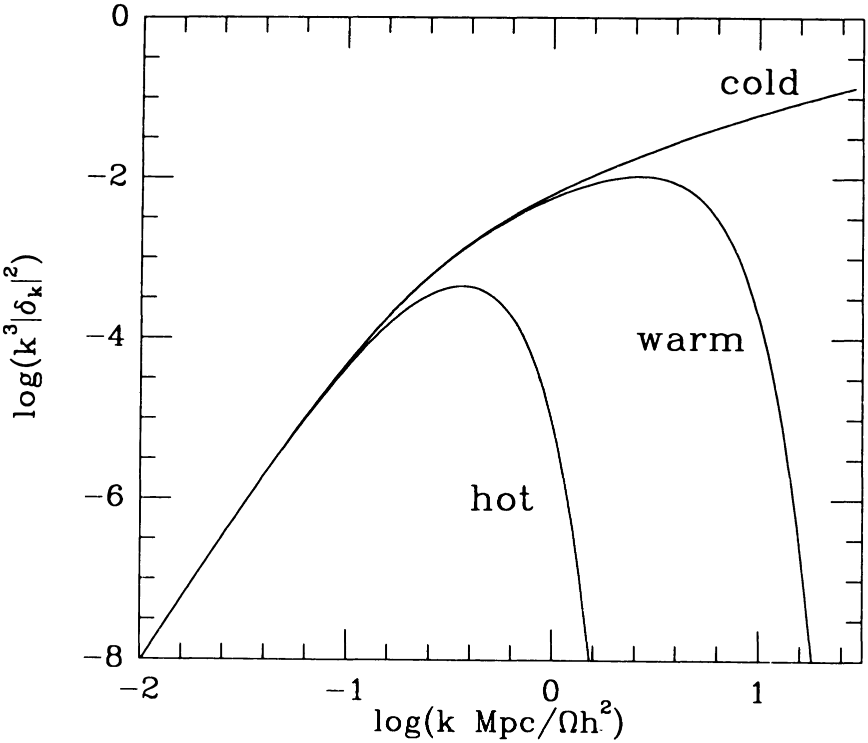


Figure 1. Power per decade as a function of spatial frequency in a universe dominated by collisionless elementary particles. These are the linear power spectra at late times in a universe which initially had the adiabatic constant curvature fluctuations predicted by inflationary models. The three cases are differentiated by the random velocities of the particles involved.

until nonlinear effects are important the spectra of Figure 1 evolve by increasing their amplitude while maintaining their shape. The three curves in the figure correspond to different types of weakly interacting particle. Neutrino-like particles can dominate the universe either if their mass is of the order of 30 eV, or if it is of order 3 GeV. In the former case the particles remain relativistic until $z \sim 10^5$ and all small-scale fluctuations in their density are wiped out as they move around. This effect is reflected in the high frequency cut-off of the curve labelled "hot"; note that the peak here corresponds to the rather large wavelength

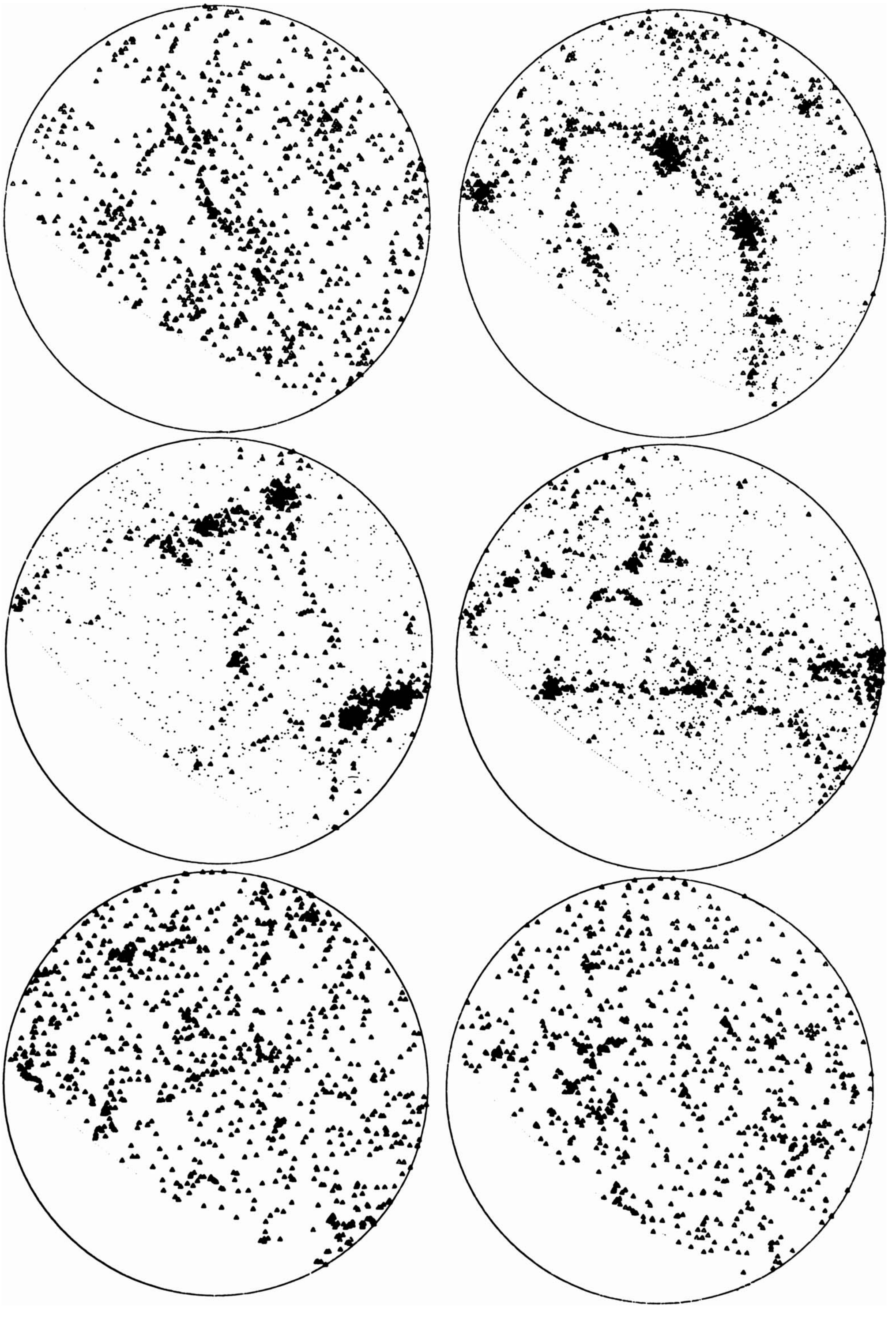
$$\lambda \approx 17/\Omega h^2 \text{ Mpc} \approx 17 \ (100 \text{ eV}/m_X) \text{ Mpc}, \qquad (3)$$

where Ω and h are the usual cosmological parameters and m_X is the mass of the particle. If the particles have a mass near 3 GeV, free streaming effects do not influence fluctuations on scales of interest; the particles are then a particular example of cold dark matter. Other cold dark matter candidates are axions and supersymmetric partners of certain known particles. These matters are reviewed by Turner in this volume. Turner also discusses the possible intermediate case of warm dark matter. Notice that in a hot dark matter universe the first objects to collapse will have a characteristic scale given by equation (3), whereas in a cold dark matter universe small objects will collapse first, and will then aggregate into larger systems. Thus structure in a neutrino-dominated universe will grow according to a variant of Zel'dovich's "pancake" scenario (Zel'dovich 1970) whereas an axion-dominated model will cluster hierarchically in the manner discussed by Peebles (1965, 1980).

When N-body methods are used to follow nonlinear evolution from the initial conditions summarized in Figure 1, a number of technical limitations arise. The mass of the particles used to represent the mass distribution in a "typical" region of the universe can be quite large. For example White et al. (1983) used 32768 particles to simulate the neutrino distribution in a comoving cubic region of present side $L = 65\ h^{-2}$ Mpc. In these $\Omega = 1$ models each particle thus represented $2 \times 10^{12}\ h^{-4}\ M_\odot$ or $10^{76}\ h^{-6}$ neutrinos. The smoothest possible way to put down N particles in a cubic region is to site them on a regular cubic lattice. Periodic boundary conditions can be used to embed this region in an infinite universe. When such a distribution is perturbed in order to obtain the desired linear power spectrum, the largest wavelength that can be represented is L, while the shortest is $2.0\ N^{-1/3}\ L$. Thus the initial conditions can be modelled over a dynamic range of 16 in length or 4000 in mass for N = 32768. As a simulation evolves, bound clumps cease to follow the general expansion and so shrink relative to the size of the computational volume. At late times the mass distribution can therefore be studied over a wider range of length scales, provided that the numerical scheme gives an accurate representation of the Newtonian force law at small separations. An N-body model is a useful representation of the evolution of clustering from the time when bound clumps of a few particles first condense out of the expansion until times when waves of scale L begin to go nonlinear. When the fluctuation spectrum, as plotted in Figure 1, is relatively flat, these two times can be uncomfortably close. This is the case for the cold dark matter spectrum on small scales. Efstathiou et al. (1985) give a detailed discussion both of the technical limitations of N-body codes, and of our technique for obtaining a random phase, growing mode realization of any given power spectrum. I now discuss the results that our group has obtained for neutrino-dominated and cold dark matter universes.

3. NEUTRINO-DOMINATED UNIVERSES

White et al. (1983) used the hot dark matter spectrum of Figure 1 as an initial condition for simulations of a neutrino-dominated universe. This work superceded earlier studies which had used schematic and rather poor representations of the predictions of linear theory; furthermore it demonstrated that the major features of the



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subsequent evolution could be reproduced using different numerical techniques. Structure forms in these models very much as expected on the basis of earlier theoretical work (e.g., Doroshkevich et al. 1980). The first things to collapse are sheet-like structures which rapidly link up across the computational volume. Filaments form at the intersections of sheets, and clusters at the intersections of filaments. Although there is a clear tendency for uncondensed matter to flow primarily into sheets, for sheets to flow into filaments, and filaments into clusters, all three kinds of structure are present simultaneously. Evolution is quite rapid; by the time the universe has expanded by a factor $a/a_f = 3$ since the first formation of collapsed structure, almost half the total mass is associated with dense, massive, roughly spherical clusters. Thereafter the distribution evolves mainly by merging of clusters in a hierarchical fashion. When $a/a_f \approx 2.5$ the autocorrelation function of the mass distribution in the models has the same slope as the observed autocorrelation function of galaxies. The amplitudes agree provided the cosmological parameters satisfy $\Omega h \approx 1.0$. However, in the real universe we see manifestly nonlinear objects at $z > 3$; for $a/a_f > 4$ the model correlation function is steeper than that of galaxies and has comparable amplitude only for $\Omega h > 1.5$.

The major disagreement between observation and these models surfaces when we realize that galaxies can only form in regions where the local matter distribution has collapsed. Thus the many simulation particles which lie in low density regions between pancakes can represent neutrinos and uncondensed gas, but they cannot represent galaxies. White et al. (1983) found that if they calculated correlation functions using only particles in regions which had undergone local collapse, then for any value of a/a_f the result agreed in amplitude with the observed galaxy correlations only for $\Omega h > 2$. This unacceptable constraint reflects the fact that for acceptable cosmological parameters the amplitude of the correlation function of "galaxies" in the model exceeds that observed by a factor of at least 15. This discrepancy is illustrated in Figure 2 which compares the distribution of galaxies in the Center for Astrophysics survey with that predicted in three realizations of a flat neutrino-dominated universe with $a/a_f = 3.5$ and an age of 12 Gyr. The disagreement here is manifest. Although it depends on the identification of "galaxies" in this diagram, White et al. (1984) point out that for $a/a_f > 3.5$ the neutrino clumps contain most of the mass of the universe, are too

Figure 2. Equal area projections of the galaxy distribution in the northern sky and in five artificial catalogs made from simulations. The picture at top left is the CfA survey volume limited to 4000 km/s. The next three pictures show neutrino-dominated flat universes with $h = 0.54$ in which galaxy formation began at $z = 2.5$. Triangles show particles which could represent galaxies while dots show particles in uncollapsed regions which cannot represent galaxies. The bottom two pictures show catalogs made from CDM models with $\Omega = 0.2$ and $h = 1.1$ in which galaxies trace the mass.

massive to be identified with any known object and would be very hard to hide. Thus, regardless of the details of galaxy formation, the conventional neutrino-dominated model appears to be in severe difficulty.

4. COLD DARK MATTER UNIVERSES

From a numerical point of view cold dark matter (CDM) universes are considerably more difficult to simulate than neutrino-dominated universes. This is because the spectrum shown in Figure 1 has no pronounced characteristic length and is rather flat at high frequencies. As a result, during the early evolution of a CDM universe nonlinear structure is expected to form simultaneously on a wide range of scales. Davis et al. (1985) carried out a large series of simulations of CDM universes to study the evolution of the mass distribution on scales of a few Mpc. They modelled both open and Einstein-de Sitter universes and used a numerical scheme which reproduced the interparticle force correctly at separations greater than $\varepsilon = L/600$. Their simulations followed 32768 particles within comoving cubic regions of present size L = 32 and 64 $(\Omega h^2)^{-1}$ Mpc, giving individual particle masses of 3 and 23 x 10^{11} $(\Omega h^2)^{-2}$ $M_\odot$ respectively. They were thus able to resolve structures with scales intermediate between those of galactic halos and of superclusters of galaxies. Large filamentary structures, superclusters of clumps and large low-density regions appear at certain time in all of these models. The autocorrelation of their mass distribution steepens gradually with time, reflecting the lack of self-similarity expected as a result of the curvature of the linear CDM spectrum. Their three-point mass correlations are stronger relative to the two-point correlations than is the case for the observed galaxy distribution. Finally, dense clumps form in these models with a wide range of masses.

In order to compare these results with observation it is necessary to make some assumption about the location of galaxies. The simplest and most convenient choice is to assume that bright galaxies "trace the mass" and can therefore be identified with a random subset of the simulation particles. With this choice one is forced to consider open models in order to get an approximate match to the dynamics of observed groups and clusters. The open models of Davis et al. (1985) give a reasonable fit to the observed galaxy distribution with the parameter choice $\Omega = 0.2$, h = 1.1. Although this value of h is rather high, they would have been able to get a equally good fit to the data for smaller h if they had considered open models with a lower initial fluctuation amplitude. The bottom two plots in Figure 2 show sky maps of the distribution of "galaxies" in two of these open simulations for comparison with the plot of the CfA data. The qualitative agreement between observation and the models is quite good. A similar level of agreement is seen in wedge diagrams which plot galaxy redshift against one of the angular coordinates on the sky. This is shown in Figure 3 which compares part of the northern CfA survey with similarly shaped regions in catalogs constructed from two further models with $\Omega = 0.2$.

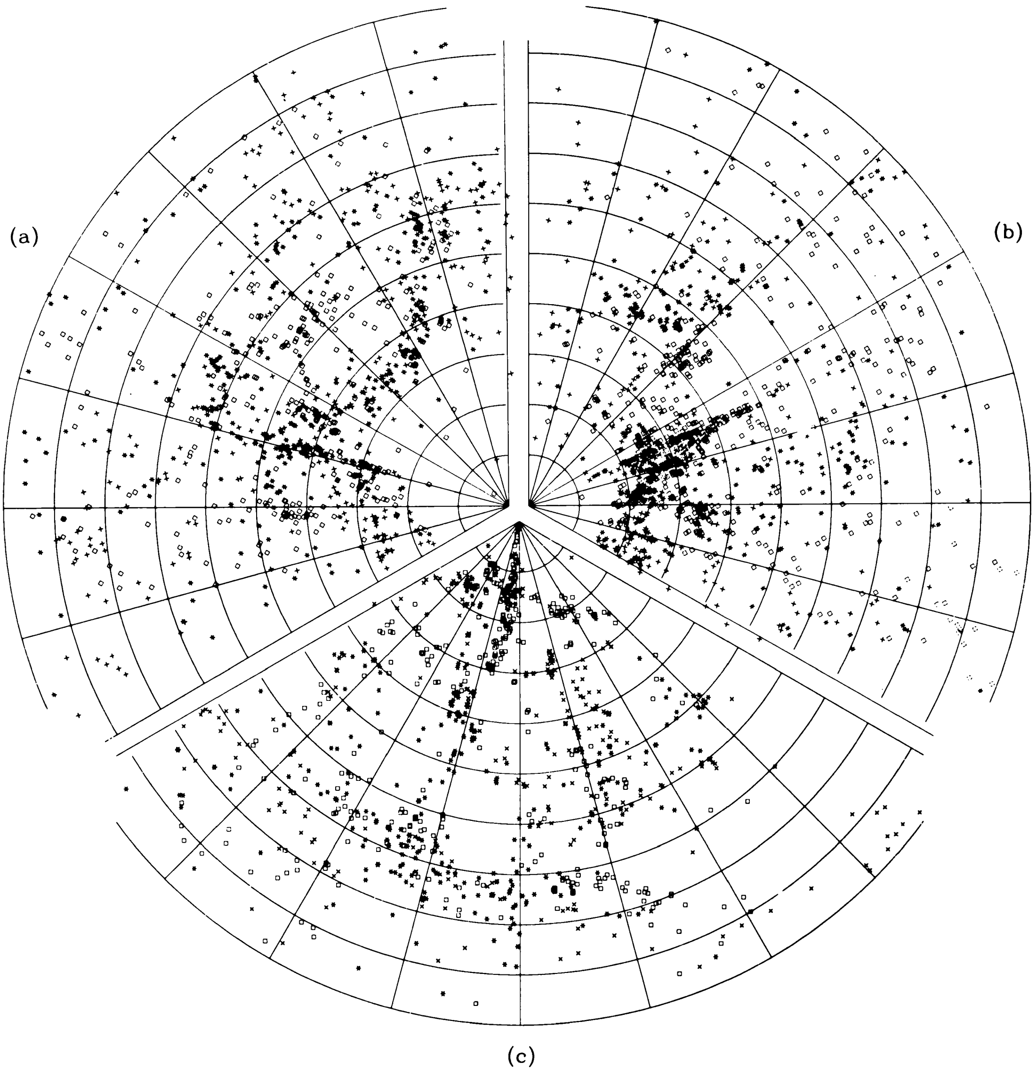

Figure 3. Wedge diagrams for the CfA survey (c), and for two catalogs from CDM models with $\Omega = 0.2$ and $h = 1.1$ in which galaxies trace the mass. "Galaxies" with $0° < \delta < 45°$ are plotted. Circles are placed at intervals of 1000 km/s and radial lines are at intervals of one hour in R.A. The three symbols refer to three 15° ranges in δ.

Notice the fingers-of-God effect both in the models and in the real data, as well as the large filament in model (a) which is transverse to the redshift coordinate and is similar to the Coma supercluster in the CfA catalog. These model catalogs differ from the real data mainly in that their clusters are somewhat tighter and the associated velocities are somewhat higher. This shows up quantitatively in a two-point correlation function which is too steep on small scales, in overly strong three-point correlations, in an rms relative velocity of pairs which is too large on small scales, and in apparent M/L ratios for groups which are about a factor of 2 too large (see Nolthenius and White, this volume). It is not clear how serious these discrepancies are, because physical effects related to the neglected internal structure of galaxies and their halos may become important on small scales (see, for example Barnes 1985). In addition some of the problems would be alleviated in open models with a lower initial fluctuation amplitude.

If Ω is indeed unity, as suggested by inflation, galaxies cannot trace the mass. Rather they must be over represented by a factor of about 5 in the dense regions from which dynamical mass estimates are obtained. Such a bias arises if galaxies form only near high peaks of the linear density field. This effect is discussed from a theoretical point of view by Kaiser (1985) and Bardeen (1985), and was demonstrated explicitly by Davis et al. (1985) in their simulations. It is intuitively attractive to assume that galaxies form preferentially at peaks of the dark matter density field, but it is far from obvious that a sufficiently sharp threshold will be imposed for the bias to be large (see for example, Rees 1985). The bias is illustrated in Figure 4 which compares the mass distribution in an Einstein-de Sitter CDM model with the distribution of "galaxies" which were initially placed at 2.5σ peaks of the smoothed linear density field. The same structure is evident in both pictures but it has much higher contrast in the "galaxies". Davis et al. found that for $h \approx 0.45$ the "galaxy" distribution in these $\Omega = 1$ models provided a somewhat better fit to observation than the unbiased $\Omega = 0.2$ models discussed above. In particular, the three-point correlations and the properties of groups fitted better in the biased models. Thus it would clearly be premature to conclude that the observed kinematics of galaxies on 0.5 - 5 h^{-1} Mpc scales requires an open universe.

Any consistent model for the growth of structure must explain not only the properties of groups and clusters of galaxies, but also the unrelaxed filaments and voids seen on very large scales, as well as the structure of individual galaxy halos on small scales. Frenk et al. (1985) have begun a program to study the second of these questions by simulating the evolution of small regions of a CDM universe. Figure 5 shows results from a simulation which followed a comoving cubic region of side $L = 3.0\ h^{-2}$ Mpc from $z = 6$ to the present day in an Einstein-de Sitter universe. The fluctuation amplitude was chosen to agree with that needed to fit galaxy clustering (for $h = 0.45$) in the "biased" models; with this scaling each particle in the model weighs $6 \times 10^9\ M_\odot$. The three frames on the left show projections of the whole simulation at $z = 2.5$, $z = 1$, and $z = 0$, while those on the right show the

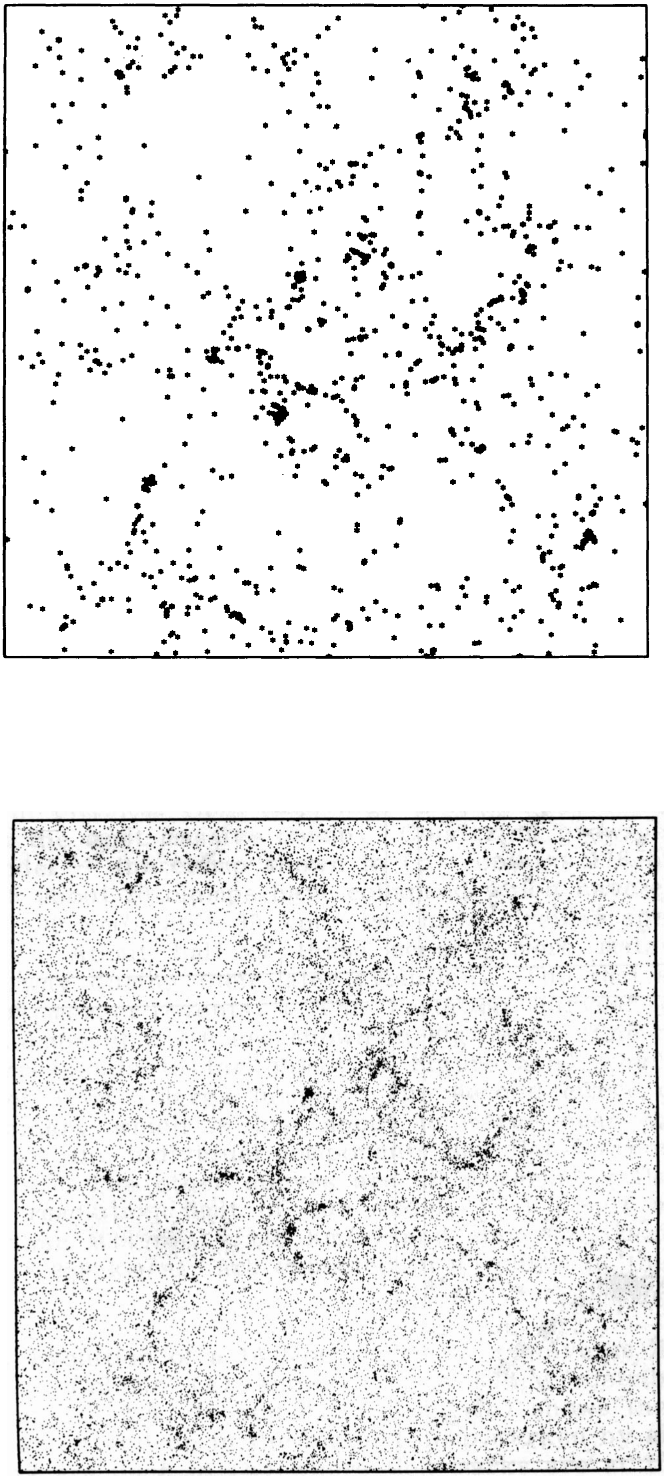

Figure 4. The projected distribution of all particles (left) and of "galaxies" (right) in a simulation of a flat CDM universe. The side of the box is 160 Mpc for h = 0.45. The "galaxies" correspond to 2.5σ peaks of the smoothed linear density field.

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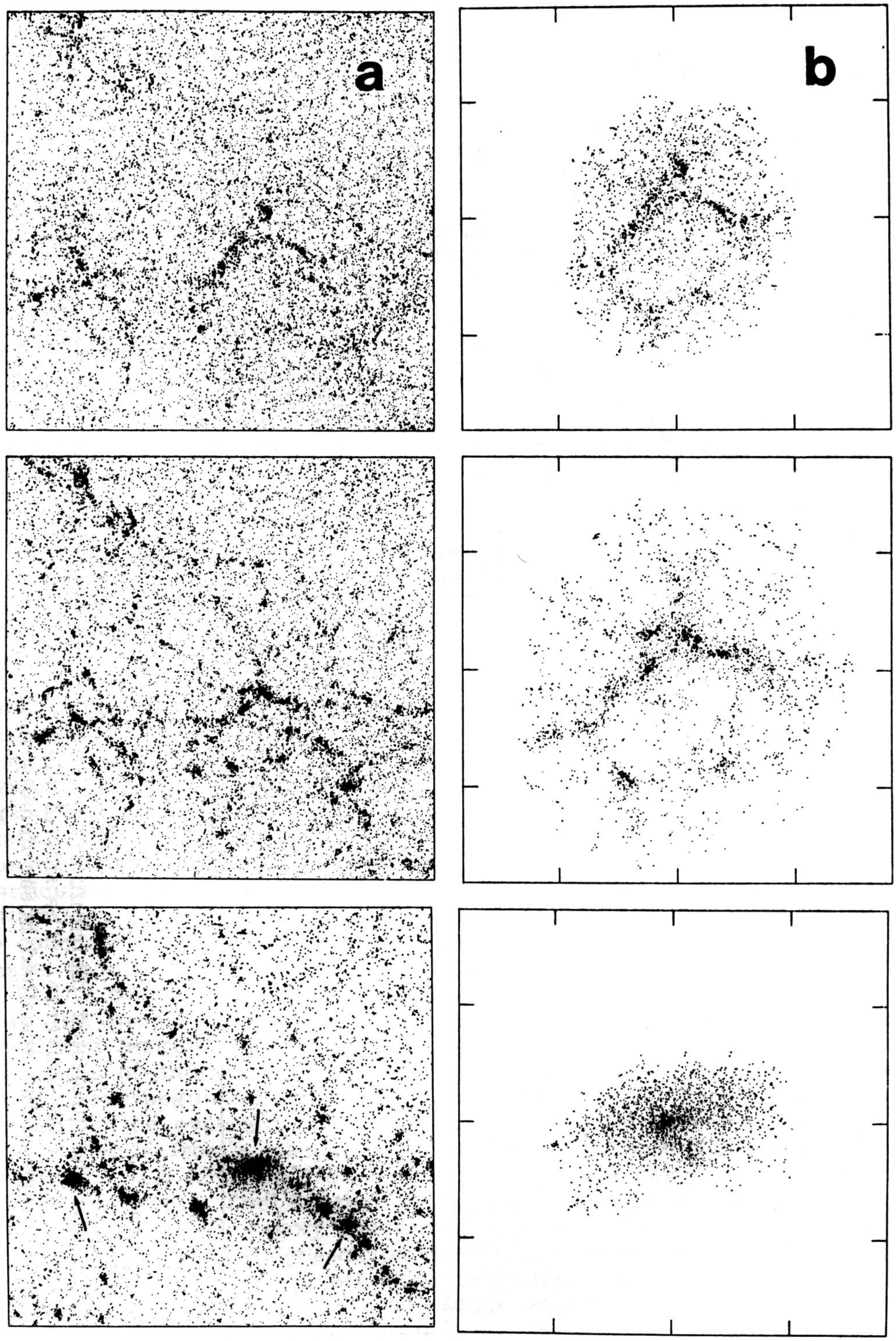

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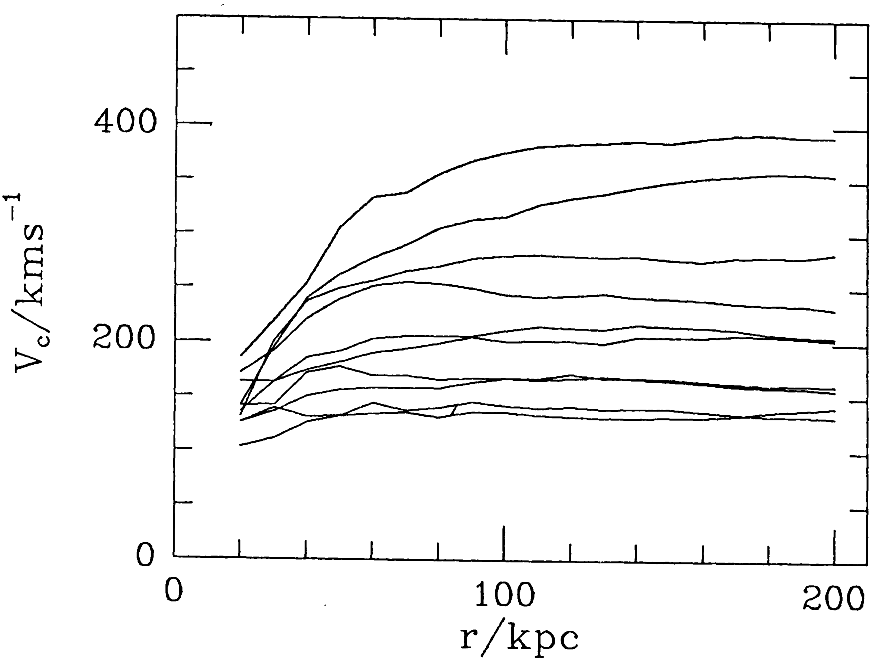

Figure 6. Rotation curves defined by $V_c^2 = GM(r)/r$ for the ten most massive lumps at z = 0 in the simulation shown in Figure 5.

positions in physical (non-comoving) coordinates of the particles which end up in the most massive clump. Clustering grows extremely rapidly in this model -- the clump shown in (b) contains 1/8 of the total mass. By z = 2.5 about 20 clumps have collapsed which have circular speeds in excess of 100 km/s at ~ 30 kpc; two have circular speeds in excess of 200 km/s. About half of these accrete more material in a rather quiescent way during their later evolution. The rest are involved in a variety of merger events. The "rotation curves" of the ten most massive clumps at z = 0 are shown in Figure 6. The highest amplitude curve corresponds to the object of Figure 5(b) which formed by the merging of 4 or 5 large lumps. The second curve corresponds to a massive binary which is just about to merge near the bottom left of Figure 5(a); the individual components had flat rotation curves with V_c = 220 km/s. The other curves are all remarkably flat outside r = 20 kpc, the resolution limit of our model. A priori about 10 galaxies brighter than M33 ($V_c \approx$ 110 km/s) and about 2 galaxies brighter than M31 ($V_c \approx$ 250 km/s) are expected in a volume of this size. Thus it

Figure 5. Evolution of a 14 Mpc region of a universe like that shown in Figure 4. Column (a) shows the projected distribution of all the particles at z = 2.5, 1. and 0. Column (b) shows the evolution of the particles which end up in the biggest lump marked by an arrow. Physical coordinates are used in (b) where tick marks are at 1 Mpc intervals.

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appears that galaxy halos form with flat rotation curves of about the right amplitude in a biased CDM model with Ω = 1. Frenk et al. sketch a theory for the origin of the Hubble sequence based on the evolutionary properties of this model. If it is correct, it requires galaxy formation to be a long drawn-out process occurring predominantly at quite recent redshifts (z = 1-3).

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DISCUSSION

YAHIL: I have a question about the normalization of the power spectrum. The nice thing about the cold dark matter scenario is that it predicts the shape of the initial perturbations, but the normalization of those perturbations is up to you. I am concerned because the calculations were carried out over only a small range of the expansion factor. If I understood correctly, you put the perturbation spectrum down at a relatively low initial redshift. I understand that computer time limitations make it difficult to do the calculations over longer periods of time, but do you have any feeling about what would happen if you took the same shape and started it earlier with a lower amplitude?

WHITE: It's not a question of computer time. It costs nothing to do the early stages. All that would happen if we took a lower amplitude and started earlier would be that the initial condition would look more like a purely regular grid. In fact, the real limitation is the number of particles we used, not the epoch at which we started.

ZUREK: I am very concerned about the range over which your calculations reproduce the evolution of the cold dark matter universe. The problem is not the smoothing of the potential, but rather the initial conditions. At $z = 6$, much of the structure on the galactic ($\sim 10^{12}\ M_\odot$) mass scale is already nonlinear ($\delta M/M \sim 1$, and in the peaks of the density distribution which are presumably future galaxies, $\delta M/M > 1$). Therefore, if one starts the simulation at $z \sim 5$ with an unevolved cold dark matter spectrum, one does not give galactic and smaller scales enough time to evolve. Consequently, I would expect that your calculation underestimates the richness of the structure on galactic and smaller scales.

WHITE: The fundamental resolution limit of our model is set by the mass of the individual particles ($7 \times 10^9\ M_\odot$). At the start, structures made up of ~ 10 particles are not expected to have collapsed, but they do so soon thereafter. It is clear that we cannot model the full richness of substructure expected on scales below $10^{10}\ M_\odot$. However, the binding energy invested in such substructure is very small, and I don't believe it will significantly influence the structure of the much more strongly bound objects which form later; these we identify with galaxy halos.

BARNES: I've been asked to comment on the following question: are massive halos a massive liability for galaxies in groups? N-body simulations of groups, with each galaxy modeled by many particles (Barnes 1985, M.N.R.A.S., **215**, 517), show that galaxies with extensive dark halos merge very quickly. Low-velocity encounters in groups allow the halos to stick together, trapping the luminous galaxies, which then merge in a few local dynamical times. Thus, groups of galaxies with massive halos are dynamically unstable.

We observe plenty of groups with short crossing times ($\sim 0.1\ H_0^{-1}$), so it is logical to ask if groups of halo galaxies last long enough. If all the dark mass is initially in individual halos, the galaxies merge too quickly, but models with 0.5 - 0.75 of the DM in a common group halo pass this test. Even so, the merging rates are rather high, and one might argue that groups of halo galaxies yield too many isolated merger remnants. But it is not clear to me (1) how many remnants are too many, (2) whether they would really be isolated, and (3) what to identify them with: ellipticals, cD galaxies, or things like V Zw 311. There is also some uncertainty in choosing the initial conditions. Finally, an old merger remnant may later accrete enough gas to form a new disk and disguise itself as a spiral galaxy.

While I have tried to argue that rapid merging in groups may not be unreasonable, other people have reached very different conclusions. Ishizawa (this conference) has assumed that multiple mergers within a group yield a cD, and concluded that individual galactic halos cannot extend beyond ~ 40 kpc. Mamon (Ph. D. thesis, Princeton) has argued that most compact groups are really chance projections within loose groups. Coming back to my original question, I feel that there is as yet no convincing answer one way or the other, and I hope the rest of you are now as confused as I am.

B. JONES: When looking at the "galaxies" in your simulations, should one worry about two-body effects modifying the density profiles and the rotation curves? This could be quite serious for the smaller galaxies which show the flat rotation curves in your simulations.

WHITE: The very central parts of the final "halos" do indeed contain only a few tens of particles, as do the first clumps that form near the start of the simulation. However, structure evolves so rapidly to large mass scales in this model that I don't believe there is sufficient time for two-body effects to produce any substantial rearrangement of binding energy. The only way to demonstrate this conclusively would be to repeat the model with a much larger number of particles.

DEKEL: Cold dark matter seems successful on galactic scales, and one can even get the galaxy correlation function right if $\Omega \sim 0.2$ or if the galaxies are biased. But could you reproduce the large-scale structure with cold dark matter alone? In particular, I am afraid that you can't reproduce the observed cluster correlation function, and that you don't have the pronounced filamentary structure observed on scales of 20 - 200 h^{-1} Mpc. This seems a severe difficulty for the $\Omega = 1$ biased model, which is somewhat reduced if $\Omega = 0.2$, because the length scale is stretched accordingly.

WHITE: This is an important question which we have begun to address but have not yet fully investigated. We seem to get roughly the right abundances of rich clusters in our models, but they seem less correlated than the results of Bahcall and Soneira suggest. Most of our models concentrate on too small a volume of space to give any useful information about the existence or otherwise of very large-scale structures such as the void in Bootes or the Perseus-Pisces Supercluster chain.