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A BAYESIAN CLASSIFICATION APPROACH TO MONETARY AGGREGATION

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In this article we use Bayesian classification and finite mixture models to extract information from the MSI database (maintained by the Federal Reserve Bank of St. Louis) and construct a new set of non-nested monetary aggregates (under the Divisia aggregation procedure) based on statistical similarities and multidimensional structures. We also use recent advances in the fields of applied econometrics, dynamical systems theory, and statistical physics to investigate the relationship between the new money measures and economic activity. The empirical results offer practical evidence in favor of this approach to monetary aggregation.

Keywords: Monetary Aggregation, Monetary Policy

1. INTRODUCTION

In this paper we take a statistical approach to the problem of monetary aggregation. This is different from the economic approach to statistical index number theory pioneered by Diewert (1976) and used by Barnett (1980). We use an automatic classification program (AutoClass) for cluster analysis, developed by Stutz and Cheeseman (1996), to extract useful information from the MSI database maintained by the Federal Reserve Bank of St. Louis. AutoClass is an unsupervised classification system based on Bayesian theory. Instead of partitioning cases, as most clustering techniques do, the Bayesian approach searches in a model space for the "best" class descriptions based on statistical similarities and multidimensional structures. Bayesian classification theory is the recent focus of a study of the Bayes group at the Ames Research Center—see http://ic.arc.nasa.gov/ic/projects/bayes-group/autoclass for more details.

The Bayesian unsupervised classification approach avoids many of the limitations of traditional cluster analysis. Bayesian inference utilizes the data to generate "natural" classes by assigning a probability to class membership (called fuzzy classification). Moreover, this approach allows one to rank alternative class memberships. However, the Bayesian approach, like the traditional classification

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approaches, is also sensitive to the choice of attributes used to describe the object. Despite this proviso, however, Bayesian classification improves substantially on the subjectivity found in traditional classification approaches; conditional on the choice of descriptive attributes, class quantity and membership are generated by the data, instead of being arbitrary decisions made by the researcher.

The paper is divided into nine sections. Section 2 briefly discusses the Bayesian classification approach to monetary aggregation, based on finite mixture models. Section 3 discusses the data and presents the results of the Bayesian classification analysis. In Section 4, we discuss the problem of the definition (aggregation) of money and present three new monetary aggregates under the Divisia aggregation procedure. In Section 5, we summarize some key facts regarding the dynamic comovements between each of the three new money measures and U.S. industrial production, using the methodology suggested by Kydland and Prescott (1990) and Baxter and King (1999). In Section 6, we investigate the univariate and multivariate time series properties of the variables (since these properties are relevant for some problems of potential importance in the practical conduct of monetary policy as well as for estimation and hypothesis testing) and test for Granger causality. In Section 7, we examine the chaotic properties of the new monetary aggregates, using the Nychka et al. (1992) Lyapunov exponent estimator and its limit distribution. In Section 8, we apply the method of detrended fluctuation analysis (DFA)—introduced by Peng et al. (1994) and adapted to the analysis of long-range correlations in economic data by Serletis and Uritskaya (2007)—to investigate the fractal structure of the new aggregates. The final section concludes the paper and sketches the implications of our findings.

2. BAYESIAN CLASSIFICATION THEORY

Let $\mathbf{x} = \{\mathbf{x}_1, \ldots, \mathbf{x}_I\}$ be the data set, where $i = 1, \ldots, I$ indexes the number of cases (or instances). Each $\mathbf{x}_i$ contains a number of attributes (representing measurements on some instance properties common to all instances) indexed by k, $k = 1, \ldots, K$, so that each $\mathbf{x}_i$ is a $(1 \times K)$ vector of attribute values, $\{x_{i1}, \ldots, x_{iK}\}$, and $\mathbf{x}$ is an $(I \times K)$ matrix of data. It is assumed that $\mathbf{x}$ is sampled from a heterogeneous population, the classes (C) of which are indexed by j, $j = 1, \ldots, J$. The objective is to find the most probable classification—that is, to cluster the I instances into an optimal number of classes.

In using Bayesian inference we combine prior information with sample information, unlike classical statistical inference in which the observational (sample) data constitute the only relevant information. As already noted, it is assumed that the data are sampled from a population with J subgroups (or classes), with J unknown—and that for each class to be identified there is a probability distribution or density function for the attributes. We let $\mathcal{F} = \mathcal{F}_1, \ldots, \mathcal{F}_J$ denote the mathematical form of each of the J probability density functions and $\phi = \phi_1, \ldots, \phi_J$ the parameter set of the corresponding density function.

We assume that $\mathcal{F}_j$ is weighted by a mixture model T—that is, the probability distribution that any $\mathbf{x}_i$ is a member of class j, C_j, regardless of its attribute values. Letting $\mathbf{p} = p_1, \ldots, p_J$, with $\sum_{j=1}^{J} p_j = 1$, be the parameters of T indicating the proportion of the population that is from C_j, that is, $p_j = P(x_i \in C_j \mid \mathbf{p}, T)$; then the likelihood of (observation) $\mathbf{x}_i$ can be written as

$$\begin{aligned} P(\mathbf{x}_i \mid \phi, \mathbf{p}) &= p_1 P(\mathbf{x}_i \mid \phi_1, \mathcal{F}_1) + \cdots p_J P(\mathbf{x}_i \mid \phi_J, \mathcal{F}_J) \\ &= \sum_{j=1}^{J} p_j P(\mathbf{x}_i \mid \mathbf{x}_i \in C_j, \phi_j, \mathcal{F}_j). \end{aligned}$$

Let $\boldsymbol{\kappa} = (\phi, \mathbf{p})$ be the set of parameters of the entire model and $\mathcal{M} = (\mathcal{F}, T)$, with $\mathcal{M} \in S$ where S is the space of possible mixture models. Then the likelihood function of the whole sample $\mathbf{x}$ can be written as

$$P(\mathbf{x} \mid \boldsymbol{\kappa}, \mathcal{M}) = \prod_i \sum_{j=1} p_j P(\mathbf{x}_i \mid \mathbf{x}_i \in C_j, \phi_j, \mathcal{F}_j).$$

The joint density function of the sample and the parameters can be written as

$$\begin{aligned} P(\mathbf{x}, \boldsymbol{\kappa} \mid \mathcal{M}) &= P(\boldsymbol{\kappa} \mid \mathcal{M})\, P(\mathbf{x} \mid \boldsymbol{\kappa}, \mathcal{M}) \\ &= P(\boldsymbol{\kappa} \mid \mathcal{M}) \prod_i \sum_{j=1} p_j P(\mathbf{x}_i \mid \mathbf{x}_i \in C_j, \phi_j, \mathcal{F}_j), \end{aligned} \tag{1}$$

with the prior probability density function, incorporating all prior information, expressed as $P(\boldsymbol{\kappa} \mid \mathcal{M}) = P(\mathbf{p} \mid T) P(\phi \mid \mathcal{F})$, because $\mathbf{p}$ and ϕ are independent.

The objective is to find the posterior probability density function of the parameters and the maximum posterior (MAP) parameter values. The posterior probability density function of the parameters is

$$P(\boldsymbol{\kappa} \mid \mathbf{x}, \mathcal{M}) = \frac{P(\mathbf{x}, \boldsymbol{\kappa} \mid \mathcal{M})}{P(\mathbf{x} \mid \mathcal{M})} = \frac{P(\mathbf{x}, \boldsymbol{\kappa} \mid \mathcal{M})}{\int P(\mathbf{x}, \boldsymbol{\kappa} \mid \mathcal{M})\, d\boldsymbol{\kappa}},$$

and the posterior probability density function of the model given the sample is

$$\begin{aligned} P(\mathcal{M} \mid \mathbf{x}) &= \frac{P(\mathcal{M}, \mathbf{x})}{P(\mathbf{x})} = \frac{\int P(\mathbf{x}, \boldsymbol{\kappa} \mid \mathcal{M}) P(\mathcal{M}) d\boldsymbol{\kappa}}{P(\mathbf{x})} \\ &\propto \int P(\mathbf{x}, \boldsymbol{\kappa} \mid \mathcal{M})\, d\boldsymbol{\kappa} = P(\mathbf{x} \mid \mathcal{M}). \end{aligned} \tag{2}$$

The proportionality in (2) holds when we assume the prior probability $P(\mathcal{M})$ to be uniform; this is a reasonable assumption, because we have no reason to favor one model over another. For more details regarding the Bayesian classification method, see Stutz and Cheeseman (1996).

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3. BAYESIAN CLASSIFICATION ANALYSIS

The natural place to begin is with the list of 26 assets that the Federal Reserve currently uses in the construction of monetary aggregates—see Table 1. We use monthly data from 1959:1 to 2002:12 obtained from the St. Louis MSI database, maintained by the Federal Reserve Bank of St. Louis as a part of the Bank's Federal Reserve Economic Database (FRED). The data contain many missing values, because some of the assets did not exist over the entire 44-year period (see Table 1). AutoClass, however, is able to handle the missing values by treating them as valid rather than an error in collection, thereby clustering the entire data set, not just the data that are observed.

The four attributes that we used in AutoClass to classify each of the 26 monetary asset cases include the monetary asset value (x_t), its percent change at an annual rate (μ_t), the asset's user cost (π_t), and its velocity (V_t). The percent change at an annual rate between month $t - 1$ and the current month t is calculated as

TABLE 1. The 26 monetary assets from the Federal Reserve

	Monetary asset	Period
1	Currency	1959:01–2002:12
2	Travelers' checks	1959:01–2002:12
3	Demand deposits	1959:01–2002:12
4	Other checkable deposits at commercial banks	1974:01–1985:12
5	Other checkable deposits at thrift institutions	1959:01–1985:12
6	Super now accounts at commercial banks	1983:01–1985:12
7	Super now accounts at thrift institutions	1983:01–1985:12
8	Other checkable deposits and super now accounts at banks	1986:01–2002:12
9	Other checkable deposits and super now accounts at thrifts	1986:01–2002:12
10	Money market deposit accounts at commercial banks	1982:01–1991:08
11	Money market deposit accounts at thrift institutions	1982:01–1991:08
12	Savings deposits at commercial banks	1959:01–1991:08
13	Savings deposits at thrift institutions	1959:01–1991:08
14	Savings deposits and money market deposit accounts at banks	1991:09–2002:12
15	Savings deposits and money market deposit accounts at thrifts	1991:09–2002:12
16	Retail money funds	1973:02–2002:12
17	Small denomination time deposits at commercial banks	1959:01–2002:12
18	Small denomination time deposits at thrift institutions	1959:01–2002:12
19	Repurchase agreements	1959:10–2002:12
20	Eurodollars	1959:01–2002:12
21	Large denomination time deposits	1959:01–2002:12
22	Institutional money funds	1974:01–2002:12
23	Saving bonds	1959:01–2002:12
24	Short-term treasury securities	1959:01–2002:12
25	Bankers' acceptances	1959:01–2002:12
26	Commercial paper	1959:01–2002:12

$1{,}200 \times [(x_t/x_{t-1}) - 1]$, the user cost series were obtained from the MSI database, and in calculating velocity we constructed a synthetic monthly GNP series by multiplying the industrial production index (IPI) by the consumer price index (CPI), and then calculated velocity as $V_i = (\text{IPI} \times \text{CPI})/x_i$. Thus, we used 176 attributes—the 44 annual averages of each of the quantity, growth rate, user cost, and velocity series.

The results of the Bayesian classification analysis are as follows:

Number of classes	2	3	4	5
Log probability	−33,877.264	−33,660.219	−33,771.350	−33,827.509

where numbers in the second row are log posterior probabilities of possible classification models. Clearly, the classification with the largest probability groups the data into three clusters, as shown in Table 2, with classes 1 and 2 having 10 assets each and class 3 having 6 assets. In Table 3 we compare the results of the Bayesian classification analysis with the Federal Reserve's groupings. The main difference between the two classification systems is that the Federal Reserve concentrates on six nested levels of aggregation—M1A, M1, MZM, M2, M3, and L—whereas the Bayesian classification is not nested, but rather three distinct groups of liquid

TABLE 2. AutoClass results

Class 1	Class 2	Class 3
Travelers' checks	Currency	Other checkable deposits at commercial banks
Money market deposit accounts at commercial banks	Demand deposits	Other checkable deposits at thrift institutions
Money market deposit accounts at thrift institutions	Savings deposits at commercial banks	Super now accounts at commercial banks
Savings deposits and money market deposits at banks	Savings deposits at thrift institutions	Super now accounts at thrift institutions
Savings deposits and money market deposits at thrifts	Small denomination time deposits at banks	Checkable deposits and super now accounts at banks
Retail money funds	Small denomination time deposits at thrifts	Checkable deposits and super now accounts at thrifts
Repurchase agreements	Large denomination time deposits	
Eurodollars	Savings bonds	
Institutional money funds	Short-term treasury securities	
Bankers' acceptances	Commercial paper	

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TABLE 3. A comparison of the Federal Reserve and AutoClass groupings

MIA
Currency
Demand deposits
Travelers' checks

M1 = M1A + the following
Other checkable deposits at commercial banks
Other checkable deposits at thrift institutions
Super now accounts at commercial banks
Super now accounts at thrift institutions
Checkable deposits and super now accounts at banks
Checkable deposits and super now accounts at thrifts

MZM = M1 + the following
Money market deposit accounts at commercial banks
Money market deposit accounts at thrift institutions
Savings deposits at commercial banks
Savings deposits at thrift institutions
Savings deposits and money market deposits at banks
Savings deposits and money market deposits at thrifts
Retail money funds

M2 = MZM + the following
Small denomination time deposits at commercial banks
Small denomination time deposits at thrift institutions

M3 = M2 + the following
Repurchase agreements
Eurodollars
Large denomination time deposits
Institutional money funds

L = M3 + the following
Saving bonds
Short-term treasury securities
Bankers' acceptances
Commercial paper

Class I
Travelers' checks
Money market deposit accounts at commercial banks
Money market deposit accounts at thrift institutions
Savings deposits and money market deposits at banks
Savings deposits and money market deposits at thrifts
Retail money funds
Repurchase agreements
Eurodollars
Institutional money funds
Bankers' acceptances

Class 2
Currency
Demand deposits
Savings deposits at commercial banks
Savings deposits at thrift institutions
Small denomination time deposits at commercial banks
Small denomination time deposits at thrift institutions
Large denomination time deposits
Saving bonds
Short-term treasury securities
Commercial paper

Class 3
Other checkable deposits at commercial banks
Other checkable deposits at thrift institutions
Super now accounts at commercial banks
Super now accounts at thrift institutions
Checkable deposits and super now accounts at banks
Checkable deposits and super now accounts at thrifts

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assets based on statistical similarities and multidimensional structures. Note that our Class 3 is identical to the Fed's M1 net of M1A money measure.

4. THE NEW MONETARY AGGREGATES

The next problem in the construction of monetary aggregates is the selection of a monetary aggregation procedure. The monetary aggregates currently in use by the Federal Reserve are simple-sum indices in which all monetary components are assigned a constant and equal (unitary) weight. This summation index, however, implies that all monetary components contribute equally to the money total and it views all components as dollar for dollar perfect substitutes. Barnett (1980), however, derived the theoretical linkage between monetary theory and aggregation and index number theory and constructed monetary aggregates based upon Diewert's (1976) class of superlative quantity index numbers. The new aggregates are Divisia quantity indices that are elements of the superlative class—see Barnett et al. (1992), Barnett and Serletis (2000b), or Serletis (2007) for more details regarding the source and underlying microeconomic theory of the Divisia index.

Here, we use the theoretically consistent Divisia index to construct our Class 1, Class 2, and Class 3 monetary aggregates. Figure 1 provides graphical representations of these three money measures under the Divisia aggregation procedure. We also calculated percent changes at an annual rate, $1200 \times [(M_t/M_{t-1}) - 1]$, and percent changes from one year ago, $100 \times [(M_t/M_{t-12}) - 1]$, and noted that money growth (irrespective of which money measure is used) was less volatile shortly after the change in the Fed's operating procedures and the deemphasis of monetary aggregates in October 1982. In fact, during the October 1982–late 1992 period when the Fed used borrowed reserves (discount loan borrowings) as an operating target and during the federal funds targeting regime since late 1992, the Fed produced surprisingly smooth money growth, according to the Class 1, Class 2, and Class 3 money measures.

5. THE STYLIZED MONEY FACTS

In this section we investigate the basic stylized facts of the (Divisia) Class 1, Class 2, and Class 3 money measures, using stationary cyclical deviations based on the Hodrick and Prescott (1980) and the Baxter and King (1999) filters; see Hodrick and Prescott (1980) and Baxter and King (1999) for more details. In doing so, we define the Class 1, Class 2, and Class 3 cycle regularity as the dynamic comovement of the cyclical component of each of these money measures and the cycle. In particular, the business cycle regularities that we consider are autocorrelations and dynamic cross correlations between the cyclical component of each money measure, on the one hand, and the cyclical component of U.S. industrial production, on the other.

We measure the degree of comovement of each money measure with the cycle by the magnitude of the correlation coefficient $\rho(j)$, $j \in \{0, \pm 1, \pm 2, \ldots\}$. The

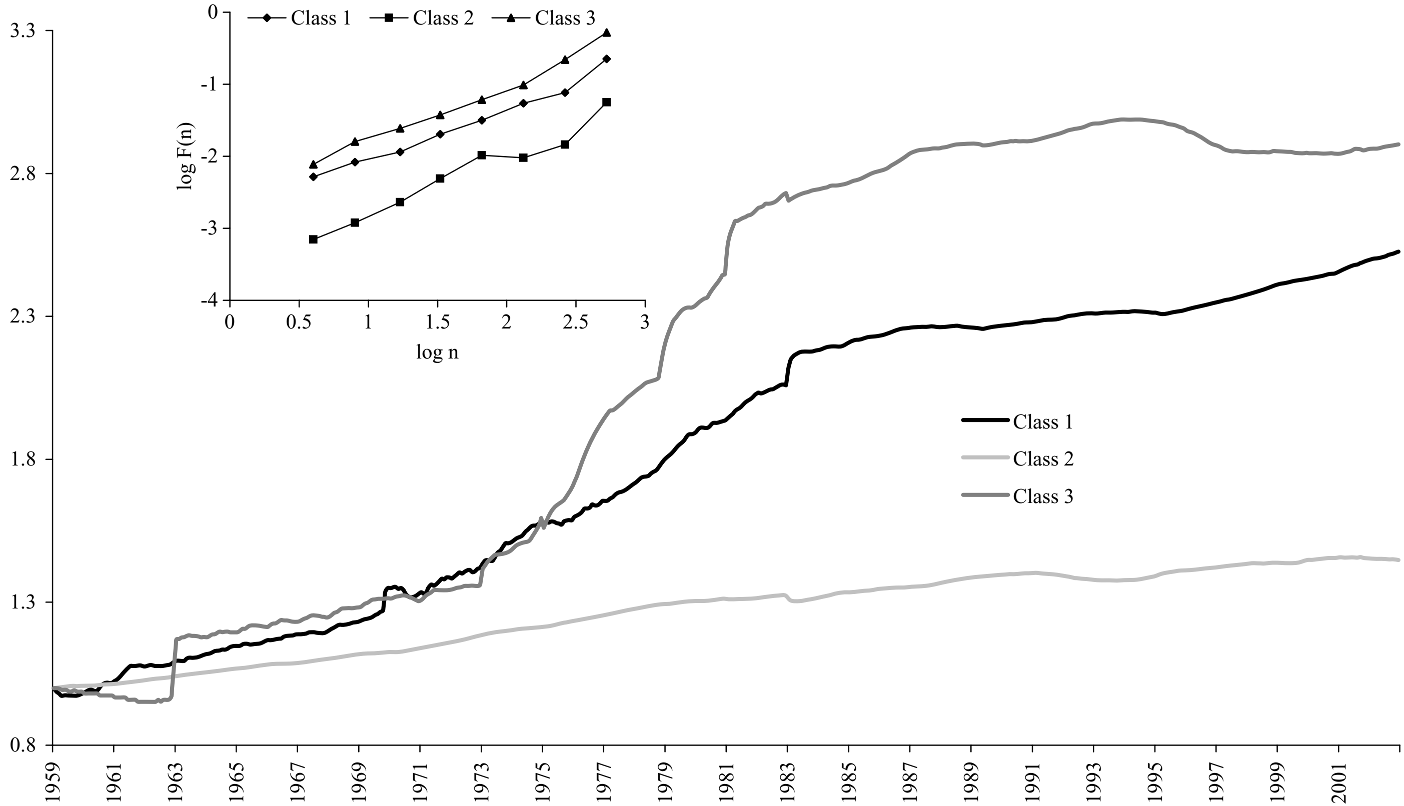

FIGURE 1. (Divisia) Class 1, Class 2, and Class 3 money measures.

contemporaneous correlation coefficient—$\rho(0)$—gives information on the degree of contemporaneous comovement. In particular, if $\rho(0)$ is positive, zero, or negative, we say that the series is procyclical, acyclical, or countercyclical, respectively. The cross-correlation coefficient, $\rho(j)$, $j \in \{\pm 1, \pm 2, \ldots\}$, gives information on the phase shift of money relative to the cycle. If $|\rho(j)|$ is maximum for a positive, zero, or negative j, we say that the cycle of money is leading the cycle by j periods, is synchronous, or is lagging the cycle by j periods, respectively.

Table 4 reports the contemporaneous correlations as well as the cross correlations based on the Hodrick–Prescott (Panel A) and Baxter–King (Panel B) filters, at lags and leads of 1, 2, 3, 6, 9, and 12 months, between the cyclical component of each of the (Divisia) Class 1, Class 2, and Class 3 money measures and the cyclical component of U.S. industrial production. Clearly, irrespective of the filter used, these money measures appear to be acyclical. This is consistent with the evidence reported by Serletis and Uritskaya (2007) using (monthly data and) the monetary aggregates most commonly used by the Federal Reserve.

With these results in mind, in the next section we investigate whether monetary impulses Granger cause the level of economic activity—in doing so, we interpret causality in terms of predictability and not as suggesting the existence of underlying structural relationships between the variables.

6. GRANGER CAUSALITY TESTS

The first step in testing for Granger causality is to test for the presence of a stochastic trend in the autoregressive representation of each (logged) individual time series. In Table 5 we report p-values [based on the response surface estimates given by MacKinnon (1994)] for the augmented weighted symmetric (WS) unit root test [see Pantula et al. (1994)], the augmented Dickey–Fuller (ADF) test [see Dickey and Fuller (1981)], and the nonparametric $Z(t_{\hat{\alpha}})$ test of Phillips (1987) and Phillips and Perron (1988)—all the unit root regression equations include deterministic components. For the WS and ADF tests, the optimal lag length is taken to be the order selected by the Akaike information criterion (AIC) plus 2—see Pantula et al. (1994) for details regarding the advantages of this rule for choosing the number of augmenting lags. The $Z(t_{\hat{\alpha}})$ test is done with the same Dickey–Fuller regression variables, using no augmenting lags.

Based on the p-values for the WS, ADF, and $Z(t_{\hat{\alpha}})$ unit root tests reported in Panel A of Table 5, the null hypothesis of a unit root in levels cannot be rejected and we conclude that all series have at least one unit root. We also test the null hypothesis of a second unit root (in Panel B of Table 5) by testing the null hypothesis of a unit root in the (logarithmic) first differences of the series. We conclude that the differenced series are stationary, except for the CPI series, which appears to have two unit roots [suggesting that the inflation rate is integrated of order one, or $I(1)$ in the terminology of Engle and Granger (1987)]. The decision on the order of integration of the series is documented in the last column of Table 5.

TABLE 4. Cyclical correlations of Divisia Class 1, Class 2, and Class 3 money measures with industrial production

	$\rho(x_t, y_{t+j})$, $j = -12, -9, -6, -3, -2, -1, 0, 1, 2, 3, 6, 9, 12$												
	$j = -12$	$j = -9$	$j = -6$	$j = -3$	$j = -2$	$j = -1$	$j = 0$	$j = 1$	$j = 2$	$j = 3$	$j = 6$	$j = 9$	$j = 12$
Panel A. Hodrick and Prescott filter													
Class 1	−.017	−.072	−.058	−.002	−.000	.005	.004	.006	.005	.012	.012	.002	−.044
Class 2	−.143	−.100	−.032	.042	.077	.119	.160	.203	.232	.250	.271	.253	.174
Class 3	−.174	−.081	.013	.094	.109	.123	.128	.145	.157	.155	.085	.001	−.012
Panel B. Baxter and King bandpass filter													
Class 1	.005	−.059	−.010	.030	−.022	.006	.008	−.013	−.003	.030	.011	−.030	−.063
Class 2	−.025	.062	.020	−.002	−.072	.160	−.157	.120	.008	−.057	−.039	−.010	.202
Class 3	.098	−.036	.047	−.036	−.095	.218	−.228	.152	−.012	−.067	−.080	−.089	.061

Note: Sample period, monthly data 1959:1–2002:12. x_t = money, y_t = industrial production.

TABLE 5. Unit root test results

	A. Log levels			B. Logged differences			
Variable	WS	ADF	$Z(t_{\hat{\alpha}})$	WS	ADF	$Z(t_{\hat{\alpha}})$	Decision
Class 1	0.990	0.987	0.995	0.000	0.000	0.000	I(1)
Class 2	0.999	0.988	0.999	0.000	0.000	0.000	I(1)
Class 3	0.998	0.995	0.999	0.000	0.000	0.000	I(1)
CPI	0.999	0.813	0.985	0.483	0.407	0.000	I(2)
CPI × IPI	0.999	0.992	0.994	0.000	0.000	0.000	I(1)
IPI	0.938	0.111	0.529	0.000	0.000	0.000	I(1)

Notes: Sample period, monthly data 1959:1–2002:12. Numbers in the WS, ADF, and $Z(t_{\hat{\alpha}})$ columns are tail areas of unit root tests.

Next we explore for shared stochastic trends between each of the Class 1, Class 2, and Class 3 money measures and each of the other $I(1)$ variables, using methods recommended by Engle and Granger (1987). That is, we test the null hypothesis of no cointegration (against the alternative of cointegration) between each money measure and the inflation rate (INFL), nominal income, and real income, using the Engle and Granger (1987) two-step procedure. In particular, we regress one variable against the other (including a constant and a trend variable in the regression) to obtain the OLS regression residuals $\widehat{\zeta}_t$. A test of the null hypothesis of no cointegration is then based on testing for a unit root in $\widehat{\zeta}_t$, using the ADF test (with the number of augmenting lags being chosen based on the AIC+2 rule mentioned earlier), and for asymptotic p-values, using the coefficients in MacKinnon (1994). The cointegration tests are first done with one variable as the dependent variable in the cointegrating regression and then repeated with the other variable as the dependent variable. The results in Table 6 indicate that the null hypothesis of no cointegration between each of the (Divisia) Class 1, Class 2, and Class 3 money measures and each of the INFL, CPI×IPI, and IPI series cannot be rejected (at conventional significance levels).

Since we are not able to find evidence of cointegration, to avoid the spurious regression problem we test for Granger causality in the context of the

TABLE 6. Marginal significance levels of cointegration tests between money and INFL, CPI × IPI, and IPI

	Money and INFL		Money and CPI × IPI		Money and IPI	
	Money	INFL	Money	CPI × IPI	Money	IPI
Class 1	0.376	0.352	0.599	0.683	0.994	0.245
Class 2	0.775	0.518	0.218	0.265	0.996	0.269
Class 3	0.347	0.241	0.711	0.716	0.997	0.254

Notes: Sample period, monthly data 1959:1–2002:12. Low p-values indicate cointegration.

TABLE 7. Marginal significance levels of Granger causality tests between money and INFL, CPI × IPI, and IPI

	Money to INFL		Money to CPI × IPI		Money to IPI	
	AIC lags	*p*-value	AIC lags	*p*-value	AIC lags	*p*-value
Class 1	(12,3)	0.001	(3,1)	0.000	(7,1)	0.000
Class 2	(12,1)	0.005	(3,1)	0.000	(5,1)	0.000
Class 3	(12,8)	0.008	(3,1)	0.000	(7,1)	0.000

Notes: Sample period, monthly data 1959:1–2002:12. Numbers in parentheses indicate the optimal (based on the AIC) lag specification. Low p-values imply strong marginal predicitve power.

system

$$\Delta z_t = \alpha_1 + \sum_{j=1}^{r} \alpha_{11}(j)\Delta z_{t-j} + \sum_{j=1}^{s} \alpha_{12}(j)\Delta m_{t-j} + \varepsilon_{zt}, \tag{3}$$

where α_1, $\alpha_{11}(j)$, and $\alpha_{12}(j)$ are all parameters and ε_{zt} is a white noise disturbance. We use m_t to denote logged money and z_t to denote the inflation rate (INFL) and the logarithm of each of CPI × IPI and IPI.

In the context of (3) the causal relationship between z_t and m_t can be determined by first fitting equation (3) by ordinary least squares and obtaining the unrestricted sum of squared residuals, SSR_u. Then, by running another regression equation under the null hypothesis that all the coefficients of the lagged values of Δm_t are zero, the restricted sum of squared residuals, SSR_r, is obtained. The statistic

$$\frac{(\text{SSR}_\text{r} - \text{SSR}_\text{u})/s}{\text{SSR}_\text{u}/(T - 1 - r - s)}$$

has an asymptotic F-distribution with numerator degrees of freedom s and denominator degrees of freedom $(T - 1 - r - s)$. T is the number of observations, r represents the number of lags of Δz_t in equation (3), s represents the number of lags for Δm_t, and 1 is subtracted out to account for the constant term in equation (3). If the null hypothesis cannot be rejected, then the conclusion is that the data do not show causality. If the null hypothesis is rejected, then the conclusion is that the data do show causality.

We used the AIC with a maximum value of 12 for each of r and s in (3) and by running 144 regressions for each bivariate relationship we chose the one that produced the smallest value for the AIC. We present these optimal lag length specifications in Table 7 together with p-values for Granger causality F-tests based on the optimal specifications. Clearly, the Class 1, Class 2, and Class 3 money measures cause the inflation rate, as well as CPI × IPI and IPI (at the 1% level). This is also consistent with most of the empirical evidence reported in the literature regarding commonly used monetary aggregates.

7. CHAOTIC DYNAMICS

Barnett and Chen (1988) claimed successful detection of chaos in the U.S. Divisia monetary aggregates. This published claim of successful detection of chaos has generated considerable controversy and also motivated a number of other investigations—see, for example, Barnett et al. (1995, 1997), Serletis (1995), and Serletis and Andreadis (2000), among others—often with contradictory results. Barnett and Serletis (2000a) provide an extensive discussion of the controversies that have arisen about the available tests and results.

Sensitive dependence on initial conditions is the most relevant property of chaos and its characterization in terms of Lyapunov exponents is the most satisfactory from a computable perspective. Lyapunov exponents measure average exponential divergence or convergence between trajectories that differ only in having an "infinitesimally small" difference in their initial conditions and remain well-defined for noisy systems. A bounded system with a positive Lyapunov exponent is one operational definition of chaotic behavior. Over the years, a number of methods have been introduced for calculating Lyapunov exponents. Until recently, however, it was not possible to investigate the statistical significance of the sign of the Lyapunov exponent point estimates. Thus, it was difficult to tell whether the positive Lyapunov exponents were evidence of chaotic behavior.

More recently, Serletis and Shintani (2006) have followed the contributions by Whang and Linton (1999) and Shintani and Linton (2003, 2004) to construct the standard error for the Nychka et al. (1992) dominant Lyapunov exponent and tested for chaos in Canadian and U.S. simple-sum, Divisia, and currency equivalent money and velocity measures—see Rotemberg et al. (1995) regarding the currency equivalent money measures. They have reported statistically significant evidence against low-dimensional chaos. As Barnett (2006) puts it, the Serletis and Shintani (2006) paper "is important, since it resolves some of the problems associated with a long standing controversy. In fact the paper is close to being the 'last word' on the subject."

In this section, we test for chaos in the Class 1, Class 2, and Class 3 monetary aggregates. In doing so, we follow Serletis and Shintani (2006) and construct the standard error for the Nychka et al. (1992) dominant Lyapunov exponent for the logged first differenced money measures, thereby providing a statistical test for chaos. As in Serletis and Shintani (2006), we report both global and local Lyapunov exponents—it has been argued that local Lyapunov exponents provide a more detailed description of the system's dynamics, in the sense that they can identify differences in short-term predictability among regions in the state space. See Serletis and Shintani (2006) regarding the technical details of the Whang and Linton (1999) and Shintani and Linton (2003, 2004) approach to testing for chaos.

Lyapunov exponent point estimates, along with their t-statistics (in parentheses), are displayed in Tables 8–10 for the logarithmic first differences of the data, as in Serletis and Shintani (2006). The results are presented for dimensions 1 through 6, with the optimal value of the number of hidden units (k) in the neural net

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TABLE 8. Lyapunov exponent estimates for Class 1

NLAR lag (m)	Number of hidden units								
	$k = 1$			$k = 2$			$k = 3$		
	BIC	Full	Block	BIC	Full	Block	BIC	Full	Block
1	−7.822	−3.461	−3.328	−7.989	−2.296	82.151	−7.988	−1.531	−1.529
		(−19.214)	(−6.153)		(−17.346)	(−5.892)		(−14.628)	(−5.509)
		[<0.000]	[<0.000]		[<0.000]	[<0.000]		[<0.000]	[<0.000]
2	−7.986	−0.534	−0.434	−7.939	−0.499	−0.480	−7.983	−0.018	−0.018
		(−6.464)	(−2.032)		(−7.190)	(−4.031)		(−0.292)	(−0.164)
		[<0.000]	[0.026]		[<0.000]	[0.005]		[0.385]	[0.436]
3	−7.809	−0.887	−0.741	−8.068	0.084	0.144	−8.079	−0.084	0.045
		(−20.146)	(−3.704)		(0.990)	(0.651)		(−1.182)	(0.244)
		[<0.000]	[<0.000]		[0.839]	[0.742]		[0.119]	[0.596]
4	−8.121	−0.589	−0493	−8.311	0.215	0.344	−8.438	0.163	0.285
		(−14.155)	(−2.453)		(5.205)	(2.532)		(5.673)	(3.116)
		[<0.000]	[0.007]		[1.000]	[0.994]		[1.000]	[0.999]
5	−7.901	−0.501	−0.369	−8.115	0.255	0.383	−8.167	0.482	0.637
		(−8.577)	(−1.903)		(7.113)	(2.892)		(9.784)	(3.834)
		[<0.000]	[0.029]		[1.000]	[0.998]		[1.000]	[1.000]
6	−8.153	−0.399	−0.216	−8.400	0.263	0.356	−8.349	0.510	0.643
		(−8.952)	(−1.772)		(7.462)	(3.237)		(12.004)	(3.652)
		[<0.000]	[0.038]		[1.000]	[0.999]		[1.000]	[1.000]

Note: Sample size $T = 525$. For the full sample estimation (Full), the largest Lyapunov exponent estimates are presented with t statistics in parentheses and p-value for $H_0 : \lambda \geq 0$ in brackets. For the block estimation (Block), median values are presented; the number of blocks was set equal to 10. QS kernel with optimal bandwidth (Andrews, 1991) is used for the heteroskedasticity and autocorrelation consistent covariance estimation.

TABLE 9. Lyapunov exponent estimates for Class 2

NLAR lag (m)	Number of hidden units: $k = 1$ BIC	$k = 1$ Full	$k = 1$ Block	$k = 2$ BIC	$k = 2$ Full	$k = 2$ Block	$k = 3$ BIC	$k = 3$ Full	$k = 3$ Block
1	−7.822	−1.021	−0.957	−7.989	−0.475	−0.478	−7.988	−0.389	−0.433
		(−20.839)	(−7.132)		(−8.263)	(−5.124)		(−5.074)	(−2.640)
		[<0.000]	[<0.000]		[<0.000]	[<0.000]		[<0.000]	[0.006]
2	−7.986	−1.772	−1.806	−7.939	−0.477	−0.602	−7.983	−0.588	−0.768
		(−10.916)	(−3.675)		(−3.268)	(−1.554)		(−3.883)	(−1.948)
		[<0.000]	[<0.000]		[0.001]	[0.061]		[<0.000]	[0.026]
3	−7.809	0.091	0.126	−8.090	0.092	0.127	−8.143	0.063	0.095
		(4.892)	(2.015)		(5.024)	(2.043)		(2.075)	(1.419)
		[1.000]	[0.978]		[1.000]	[0.979]		[0.981]	[0.922]
4	−8.278	−0.115	−0.047	−8.288	0.166	0.343	−8.307	−0.070	−0.039
		(−5.479)	(−0.675)		(4.245)	(3.047)		(−2.528)	(−0.606)
		[<0.000]	[0.258]		[1.000]	[0.999]		[0.006]	[0.286]
5	−7.908	−0.321	−0.129	−8.240	0.032	0.127	−8.166	0.030	0.129
		(−5.615)	(−1.409)		(1.358)	(2.130)		(1.266)	(2.139)
		[<0.000]	[0.080]		[0.913]	[0.983]		[0.897]	[0.984]
6	−8.457	−0.237	−0.079	−8.337	0.059	0.177	−8.344	0.060	0.180
		(−5.707)	(−0.789)		(3.624)	(3.073)		(3.650)	(3.147)
		[<0.000]	[0.216]		[1.000]	[0.999]		[1.000]	[0.999]

Note: Sample size $T = 525$. For the full sample estimation (Full), the largest Lyapunov exponent estimates are presented with t statistics in parentheses and p-value for $H_0 : \lambda \geq 0$ in brackets. For the block estimation (Block), median values are presented; the number of blocks was set equal to 10. QS kernel with optimal bandwidth [Andrews (1991)] is used for the heteroskedasticity and autocorrelation consistent covariance estimation.

TABLE 10. Lyapunov exponent estimates for Class 3

NLAR lag (m)	Number of hidden units $k = 1$ BIC	Full	Block	$k = 2$ BIC	Full	Block	$k = 3$ BIC	Full	Block
1	−7.822	−1.675 (−10.694) [<0.000]	−1.528 (−4.979) [<0.000]	−7.987	−0.063 (−0.385) [0.350]	0.189 (0.558) [0.712]	−7.987	0.108 (1.210) [0.887]	0.212 (1.985) [0.924]
2	−7.985	−1.252 (−40.193) [<0.000]	−1.171 (−10.116) [<0.000]	−7.939	−0.542 (−13.204) [<0.000]	−0.458 (−3.667) [<0.000]	−7.983	−0.400 (−7.434) [<0.000]	−0.241 (−2.191) [<0.014]
3	−7.806	−0.411 (−13.940) [<0.000]	−0.370 (−3.276) [0.001]	−8.076	0.421 (3.713) [1.000]	0.649 (1.930) [0.972]	−8.141	0.489 (4.148) [1.000]	0.739 (2.387) [0.991]
4	−8.299	−0.064 (−2.358) [0.009]	0.006 (0.049) [0.520]	−8.335	0.248 (5.335) [1.000]	0.297 (3.410) [0.999]	−8.407	0.286 (5.210) [1.000]	0.281 (2.564) [0.995]
5	−7.903	−0.453 (−4.954) [<0.000]	−0.271 (−0.940) [0.174]	−8.129	0.304 (6.482) [1.000]	0.380 (2.930) [0.998]	−8.163	0.417 (4.609) [1.000]	0.605 (2.627) [0.996]
6	−8.294	−0.201 (−2.411) [0.008]	0.080 (0.253) [0.600]	−8.559	0.343 (8.947) [1.000]	0.388 (3.498) [1.000]	−8.275	0.540 (4.982) [1.000]	0.775 (2.590) [0.994]

Note: Sample size $T = 525$. For the full sample estimation (Full), the largest Lyapunov exponent estimates are presented with t statistics in parentheses and p-value for $H_0 : \lambda \geq 0$ in brackets. For the block estimation (Block), median values are presented; the number of blocks was set equal to 10. QS kernel with optimal bandwidth [Andrews (1991)] is used for the heteroskedasticity and autocorrelation consistent covariance estimation.

being chosen by minimizing the BIC criterion. p-values for the null hypothesis $H_0 : \lambda \geq 0$ are also reported in brackets. The Full column under each value of k shows the estimated largest Lyapunov exponent using the full sample. The Block column shows median values for the block estimation, with the number of blocks (B) being set equal to 8.

In general, the reported Lyapunov exponent point estimates are negative and we reject the null hypothesis of chaotic behavior. Of course, the estimates depend on the choice of the dimension parameter m. As m increases, the Lyapunov exponent point estimates increase in value. The presence, however, of dynamic noise makes it difficult and perhaps impossible to distinguish between (noisy) high-dimensional chaos and pure randomness. For this reason, as in Serletis and Shintani (2006), we do not pursue the investigation of high-dimensional chaos in the present paper.

8. FRACTAL STRUCTURE

Recently, Serletis and Uritskaya (2007) have extended the work in Serletis and Shintani (2006) by using a statistical physics approach—namely detrended fluctuation analysis (DFA), introduced by Peng et al. (1994)—to investigate the temporal fractal structure of sum, Divisia, and CE money and velocity measures in the United States. DFA is known as a powerful statistical tool for detecting fractal correlations in various types of data, including financial, geophysical, and physiological signals.

In this section, we present evidence on the fractal structure of the (Divisia) Class 1, Class 2, and Class 3 money measures, using DFA analysis as in Serletis and Uritskaya (2007), and compare the results to those reported by Serletis and Uritskaya for the commonly used monetary aggregates. Let us denote the logarithmic first differences of the data by $z(t)$, $t = 1, \ldots, N$. The first step of the DFA technique consists of creating a running sum of the $z(t)$ fluctuations,

$$y(k) = \sum_{t=1}^{k} [z(t) - \langle z \rangle],$$

in which $\langle z \rangle$ is the average value of the series $z(t)$ and $k = 1, \ldots, N$. $y(k)$ is then divided into M nonoverlapping boxes of equal length n—the boxes are indexed by $m = 1, \ldots, M$ and their starting times are denoted as $k_{n,m}$, where $k_{n,m=1} = 1$ and $k_{n,m+1} = k_{n,m} + l + 1$. For each mth box of size n, the least-squares line $y_{n,m}(k)$ representing a local linear trend in that box is fitted to the data. Next, the integrated series $y(k)$ is detrended by subtracting $y_{n,m}(k)$, and its root-mean-square fluctuation is calculated as

$$F(n) = \frac{1}{M} \sum_{m=1}^{M} \sqrt{\frac{1}{N} \sum_{k=k_{n,m}}^{k_{n,m}+n} [y(k) - y_{n,m}(k)]^2}.$$

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This computation is repeated over all box sizes in order to characterize the relationship between the average detrended fluctuation $F(n)$ and the time scale n.

Typically, $F(n)$ will increase with the box size. A linear relationship between $F(n)$ and n on a log–log plot indicates the presence of power law (fractal) scaling $F \sim n^{\alpha}$. The value of α is related to the slope β of the $1/f^{\beta}$ power spectrum of the growth rate time series $z(t)$ by $\beta = 2\alpha - 1$ and to the fractal dimension of the original series $x(t)$ by $D = 2 - \alpha$. In particular, if $\alpha = 0.5$ (and $\beta = 0$), $z(t)$ is completely uncorrelated (white noise), and the original monetary aggregate $x(t)$ can be represented as a $1/f^2$ noise with $D = 1.5$—a random walk series.

The detrended fluctuation analysis functions, $F(n)$, for the (Divisia) Class 1, Class 2, and Class 3 money measures are shown in Figure 1—the analysis has been applied to logged first differenced series as explained above. In all cases, $F(n)$ has an approximate linear form in double logarithmic coordinates, indicating its power-law temporal scaling. Estimates of the fractal exponent α are 0.7269, 0.8225, and 0.8050 for the Class 1, Class 2, and Class 3 money measures, respectively.

As already noted, α characterizes long-range correlations in the studied time series. Clearly the estimated exponents are far from that of a random walk (0.5). Comparing our results with those reported by Serletis and Uritskaya (2007), we note that they present evidence showing that simple-sum and Divisia money measures have almost integer fractal dimensions, whereas the currency equivalent monetary aggregates have fractal dimensions close to that of a random walk. This suggests that the (Divisia) Class 1, Class 2, and Class 3 money measures have different correlation structures than the monetary aggregates commonly used by the Fed and that this structure could potentially be exploited for monetary policy purposes.

9. CONCLUSION

This article exploits a new automated Bayesian classification system to construct monetary aggregates. We started with a list of 26 monetary assets that go into the Federal Reserve's monetary aggregates. Based on four attributes (value, percent change, user cost, and velocity), AutoClass sorts the monetary assets into three nonoverlapping groups, and then a Divisia aggregate is constructed from the components of each group. At the end we have three new (non-nested) monetary aggregates (under the Divisia aggregation procedure) whose subcomponents have similar properties. This is a novel and interesting way to construct monetary aggregates, especially given the ability of AutoClass to handle missing values and new instruments.

We found evidence that the new money measures are not cointegrated with the price level and nominal income (suggesting that velocity and real money balances are nonstationary quantities and that monetary targeting will be problematic). The estimation, however, of autoregressive causality models showed that the new aggregates are useful in anticipating future movements in macroeconomic activity. We have also investigated the dynamic structure of the new money measures to

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address disputes about the presence of chaos in monetary aggregates. We have found statistically significant evidence against low-dimensional chaos, consistent with the evidence reported by Serletis and Shintani (2006) for the aggregates most commonly used by the Fed. Finally, we have used a statistical physics technique to investigate the correlation structure of the new aggregates and compared our results to those presented by Serletis and Uritskaya (2007) for the MSI sum, Divisia, and currency equivalent money measures. Overall, the empirical results offer practical evidence in favor of this new approach to monetary aggregation.

REFERENCES

Andrews, D.W.K. (1991) Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica* 59, 817–858.

Barnett, W.A. (1980) Economic monetary aggregates: An application of aggregation and index number theory. *Journal of Econometrics* 14, 11–48.

Barnett, W.A. (2006) Comments on "Chaotic monetary dynamics with confidence." *Journal of Macroeconomics* 28, 253–255.

Barnett, W.A. and P. Chen (1988) The aggregation-theoretic monetary aggregates are chaotic and have strange attractors: An econometric application of mathematical chaos. In W.A. Barnett, E. Berndt, and H. White (eds.), *Dynamic Econometric Modeling*, Proceedings of the Third International Symposium in Economic Theory and Econometrics, pp. 199–246. Cambridge: Cambridge University Press.

Barnett, W.A., D. Fisher, and A. Serletis (1992) Consumer theory and the demand for money. *Journal of Economic Literature* 30, 2086–2119.

Barnett, W.A., A.R. Gallant, M.J. Hinich, J.A. Jungeilges, D.T. Kaplan, and M.J. Jensen (1995) Robustness of nonlinearity and chaos tests to measurement error, inference method, and sample size. *Journal of Economic Behavior and Organization* 27, 301–320.

Barnett, W.A., A.R. Gallant, M.J. Hinich, J.A. Jungeilges, D.T. Kaplan, and M.J. Jensen (1997) A single-blind controlled competition among tests for nonlinearity and chaos. *Journal of Econometrics* 82, 157–192.

Barnett, W.A. and A. Serletis (2000a) Martingales, nonlinearity, and chaos. *Journal of Economic Dynamics and Control* 24, 703–724.

Barnett, W.A. and A. Serletis (2000b) *The Theory of Monetary Aggregation*. Amsterdam: North-Holland.

Baxter, M. and R.G. King (1999) Measuring business cycles: Approximate band-pass filters for economic time series. *The Review of Economics and Statistics* 81, 575–593.

Dickey, D.A. and W.A. Fuller (1981) Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica* 49, 1057–1072.

Diewert, W.E. (1976) Exact and superlative index numbers. *Journal of Econometrics* 4, 115–146.

Engle, R.F. and C.W. Granger (1987) Cointegration and error correction: Representation, estimation and testing. *Econometrica* 55, 251–276.

Hodrick, R.J. and E.C. Prescott (1980) Post-War U.S. Business Cycles: An Empirical Investigation. Working paper, Carnegie Mellon University.

Kydland, F.E. and E.C. Prescott (1990) Business cycles: Real facts and a monetary myth. *Quarterly Review, Federal Reserve Bank of Minneapolis* 3–18.

MacKinnon, J.G. (1994) Approximate asymptotic distribution functions for unit-root and cointegration tests. *Journal of Business and Economic Statistics* 12, 167–176.

Nychka, D.W., S. Ellner, A.R. Gallant, and D. McCaffrey (1992) Finding chaos in noisy systems. *Journal of the Royal Statistical Society* B 54, 399–426.

Pantula, S.G., G. Gonzalez-Farias, and W. Fuller (1994) A comparison of unit-root test criteria. *Journal of Business and Economic Statistics* 12, 449–459.

Peng, C.-K., S.V. Buldyrev, S. Havlin, M. Simmons, H.E. Stanley, and A.L. Goldberger (1994) Mosaic organization of DNA nucleotides. *Physical Review E* 49, 1685–1689.

Phillips, P.C.B. (1987) Time series regresion with a unit root. *Econometrica* 55, 277–301.

Phillips, P.C.B. and P. Perron (1987) Testing for a unit root in time series regression. *Biometrica* 75, 335–346.

Rotemberg, J., J.C. Driscoll, and J.M. Poterba (1995) Money, output and prices: Evidence from a new monetary aggregate. *Journal of Business and Economic Statistics* 13, 67–83.

Serletis, A. (1995) Random walks, breaking trend functions, and the chaotic structure of the velocity of money. *Journal of Business and Economic Statistics* 13, 453–458.

Serletis, A. (2007) *The Demand for Money: Theoretical and Empirical Approaches*, 2nd ed. New York: Springer-Verlag.

Serletis, A. and I. Andreadis (2000) Chaotic analysis of U.S. money and velocity measures. *International Journal of Systems Science* 31, 161–169.

Serletis, A. and M. Shintani (2006) Chaotic monetary dynamics with confidence. *Journal of Macroeconomics* 28, 228–252.

Serletis, A. and O.Y. Uritskaya (2007) Detecting signatures of stochastic self-organization in US money and velocity measures. *Physica A* 385, 281–291.

Shintani, M. and O. Linton (2003) Is there chaos in the world economy? A nonparametric test using consistent standard errors. *International Economic Review* 44, 331–358.

Shintani, M. and O. Linton (2004) Nonparametric neural network estimation of Lyapunov exponents and direct tests for chaos. *Journal of Econometrics* 120, 1–33.

Stutz, J. and P. Cheeseman (1996) AutoClass: A Bayesian approach to classification. In Skilling, J. and S. Sibisi (eds.), *Maximum Entropy and Bayesian Methods*. Kluwer Academic Publishers.

Whang, Y.-J. and O. Linton (1999) The asymptotic distribution of nonparametric estimates of the Lyapunov exponent for stochastic time series. *Journal of Econometrics* 91, 1–42.