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A new family of spherical parallel manipulators

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SUMMARY

In the literature, 3-RRPRR architectures were proposed to obtain pure translation manipulators. Moreover, the geometric conditions, which 3-RRPRR architectures must match, in order to make the end-effector (platform) perform infinitesimal (elementary) spherical motion were enunciated. The ability to perform elementary spherical motion is a necessary but not sufficient condition to conclude that the platform is bound to accomplish finite spherical motion, i.e. that the mechanism is a spherical parallel manipulator (parallel wrist). This paper demonstrates that the 3-RRPRR architectures matching the geometric conditions for elementary spherical motion make the platform accomplish finite spherical motion, i.e. they are parallel wrists (3-RRPRR wrist), provided that some singular configurations, named translation singularities, are not reached. Moreover, it shows that 3-RRPRR wrists belong to a family of parallel wrists which share the same analytic expression of the constraints which the legs impose on the platform. Finally, the condition that identifies all the translation singularities of the mechanisms of this family is found and geometrically interpreted. The result of this analysis is that the translation singularity locus can be represented by a surface (singularity surface) in the configuration space of the mechanism. Singularity surfaces drawn by exploiting the given condition are useful tools in designing these wrists.

KEYWORDS: Kinematics; Parallel mechanisms; Spherical manipulators, Mobility analysis; Translation singularity.

1. INTRODUCTION

Spatial mechanisms with three degrees of freedom (dof) are used in many industrial applications, because many manipulation tasks need only three dof. For instance, either translation or orientation of a rigid body requires three dof. Three-dof spatial mechanisms can be obtained by means of either serial or parallel architectures. Nevertheless, if a reduced workspace and high stiffness are required, parallel architectures will be favored.

Parallel architectures consist of two rigid bodies, one movable (platform) and the other fixed (base), connected by means of a number of kinematic chains (legs). The leg number usually is equal to the dof number and only one kinematic pair per leg is actuated. Moreover, if the legs are constituted of equal kinematic chains, the manufacturing process will need a reduced set of different components, thus an easier and cheaper process will result.

In the literature, three-dof parallel architectures were proposed either for pure translation[1–5] or for spherical motion[6–9] or for mixed three-dof motion[10,11] of the platform with respect to the base. The presented three-dof parallel mechanisms can be grouped into two sets: Mechanisms with repeated constraints[2,6,8,10] (overconstrained mechanisms) and mechanisms with independent constraints.[1,3–5,7,9,11] These two sets of mechanisms behave in a different way when geometric errors occur: Overconstrained mechanisms become structures (often hyperstatic structures), whereas independent constraint mechanisms still are mechanisms, but their positioning precision worsens. As a consequence, independent constraint mechanisms are to be preferred to the overconstrained ones, when it is possible, because they avoid mechanism lock and/or high internal load occurrence in presence of geometric errors due to the manufacturing process.

Independent constraint mechanisms with three dof and equal legs must use legs leaving five dof to the platform motion relative to the base.[12]

Karouia and Hervé[12] showed that a lot of parallel architectures with independent constraints and equal legs can allow the platform to accomplish infinitesimal (elementary) spherical motion when a few of geometric conditions are matched. The ability to perform elementary spherical motion is a necessary but not sufficient condition to conclude that the platform is constrained to accomplish finite spherical motion, i.e. that the mechanism is a spherical parallel manipulator (parallel wrist). Later, this author[13,14] showed that the 3-UPU architecture matching the Karouia and Hervé geometric conditions (3-UPU wrist) is a parallel wrist[13] and presented the static analysis and the singularity locus[14] of that wrist.

The 3-UPU architecture (Figure 1) is a parallel mechanism where platform and base are connected by three legs of

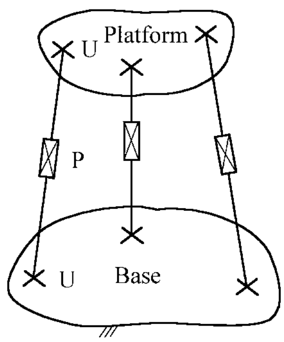

Fig. 1. 3-UPU mechanism.

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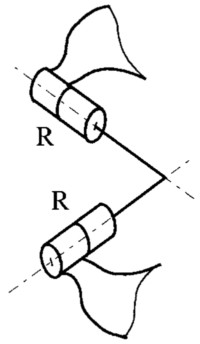

Fig. 2. Universal joint (R stands for revolute pair).

type UPU (U and P stand for universal joint and prismatic pair respectively). It can be used to obtain both pure translation manipulators[2] and spherical manipulators.[13] Since a universal joint consists of two revolute pairs in series, whose axes are incident and perpendicular to one another (Figure 2), UPU kinematic chains are special cases of RRPRR (R stands for revolute pair) kinematic chains and 3-UPU mechanisms are special cases of 3-RRPRR mechanisms (Figure 3).

3-RRPRR mechanisms need fewer geometric constraints than 3-UPU ones. Therefore, they are easier to manufacture.

In the literature, 3-RRPRR architectures were proposed to obtain pure translation manipulators.[4] Moreover, Karouia and Hervé[12] enunciated the geometric conditions which 3-RRPRR architectures must match in order to make the platform perform an elementary spherical motion.

This paper demonstrates that 3-RRPRR architectures, matching the geometric conditions enunciated by Karouia and Hervé,[12] make the platform accomplish finite spherical motion, i.e. they are parallel wrists (3-RRPRR wrist), provided that some singular configurations, named translation singularities, are not reached.

In addition, it will show that 3-RRPRR wrists belong to a family of parallel wrists which share the same analytic expression of the constraints that the legs impose on the platform.

Finally, the condition which identify all the translation singularities of the mechanisms of this family will be determined and geometrically interpreted. The result of this analysis is that the translation singularity locus can be represented by a surface (singularity surface) in the configuration space of the mechanism. Drawing the singularity surface by exploiting the given condition is useful during the design of these wrists.

2. GEOMETRIC CONDITIONS FOR SPHERICAL MOTION IN 3-RRPRR MECHANISMS

Figure 4 shows a 3-RRPRR mechanism meeting the following geometric conditions:

(i) the axes of the revolute pairs adjacent to the platform converge towards a unique point, P, of the platform (manufacturing condition);
(ii) the axes of the revolute pairs adjacent to the base converge towards a unique point, P′, of the base (manufacturing condition);
(iii) the mechanism is assembled so that the platform point P coincides with the base point P′ (mounting condition);
(iv) in each leg the axes of the two intermediate revolute pairs are parallel to one another and perpendicular to the sliding direction of the prismatic pair (mounting and manufacturing condition).

Hereafter, a 3-RRPRR mechanism, matching the above-listed geometric conditions, will be called 3-RRPRR wrist. In the following paragraphs, a 3-RRPRR mechanism encountering the 3-RRPRR wrist geometric conditions will be shown to be able to make the platform accomplish finite spherical motion, provided that some singular configurations (translation singularities) are avoided.

Figure 5 shows the i-th leg for i = 1, 2, 3 of the 3-RRPRR wrist and the notations that will be used. With reference to Figure 5, $\mathbf{w}_{ji}$ and θ_{ji} for j = 1, . . . , 4 and i = 1, 2, 3 are the unit vector of the axis and the joint coordinate, respectively, of the j-th revolute pair in the i-th leg (the revolute pairs are numbered with the j index that increases from the base to the platform). A_i is one base point lying on the axis of the first revolute pair; B_i is one point lying on the axis of the second revolute pair; C_i is the foot of the perpendicular through B_i to the axis of the third revolute pair; D_i is one platform point lying on the axis of the fourth revolute pair. h_i is the length of the segment B_iC_i; $\mathbf{u}_i$ is the unit vector $(\mathbf{B}_i - \mathbf{C}_i)/h_i$ and is parallel to the sliding direction of the prismatic pair. Since the axes of the second and the third revolute pairs are parallel (geometric condition (iv)), without loss of generality, $\mathbf{w}_{3i}$ will be chosen so that

$$\mathbf{w}_{3i} \equiv \mathbf{w}_{2i}, \qquad i = 1, 2, 3 \tag{1}$$

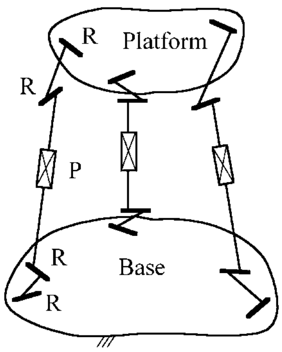

Fig. 3. 3-RRPRR mechanism.

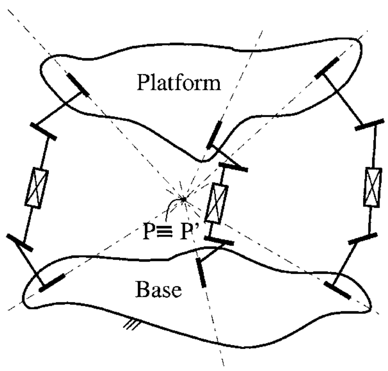

Fig. 4. 3-RRPRR wrist.

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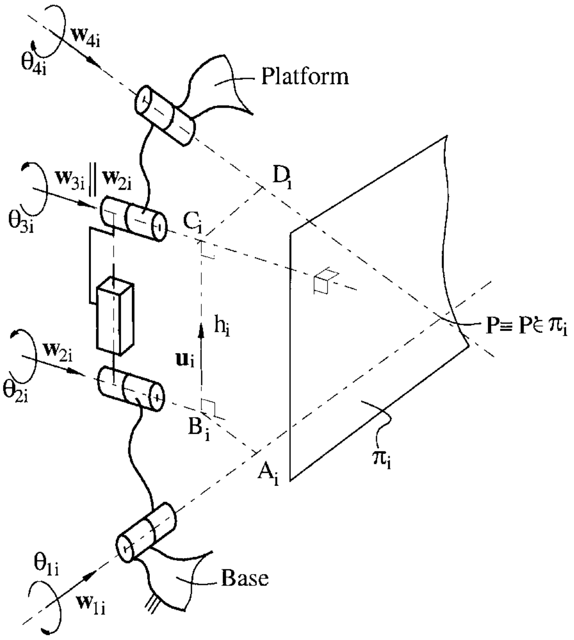

Fig. 5. i-th leg of the 3-RRPRR wrist.

With these notations, the velocity, $\dot{\mathbf{P}}$, of the platform point P can be written in the following three different ways

$$\dot{\mathbf{P}}=\dot{\mathbf{C}}_i+[\dot{\theta}_{1i}\mathbf{w}_{1i}+(\dot{\theta}_{2i}+\dot{\theta}_{3i})\mathbf{w}_{2i}]\times(\mathbf{P}-\mathbf{C}_i), \qquad i=1,2,3 \quad (2)$$

where $\dot{\mathbf{C}}_i$ is the velocity of point C_i and $\dot{\theta}_{ji}$ ($j=1, \ldots, 4$; $i=1, 2, 3$) is the rate of the joint coordinate θ_{ji}.

The analysis of Figure 5 reveals that $\dot{\mathbf{C}}_i$ velocity can be written as follows

$$\dot{\mathbf{C}}_i=\dot{h}_i\mathbf{u}_i+\dot{\mathbf{B}}_i+(\dot{\theta}_{1i}\mathbf{w}_{1i}+\dot{\theta}_{2i}\mathbf{w}_{2i})\times(\mathbf{C}_i-\mathbf{B}_i), \qquad i=1,2,3 \quad (3)$$

with

$$\dot{\mathbf{B}}_i=\dot{\theta}_{1i}\mathbf{w}_{1i}\times(\mathbf{B}_i-\mathbf{A}_i), \qquad i=1,2,3 \quad (4)$$

where $\dot{\mathbf{B}}_i$ is the velocity of point B_i and $\dot{h}_i$ is the rate of the joint coordinate h_i.

The introduction of relationships (3) and (4) into relationships (2) and the rearrangement of the resulting expressions yield

$$\dot{\mathbf{P}}=\dot{h}_i\mathbf{u}_i+\dot{\theta}_{1i}\mathbf{w}_{1i}\times(\mathbf{P}-\mathbf{A}_i)+\dot{\theta}_{2i}\mathbf{w}_{2i}\times(\mathbf{C}_i-\mathbf{B}_i)+(\dot{\theta}_{2i}+\dot{\theta}_{3i})\mathbf{w}_{2i}$$
$$\times(\mathbf{P}-\mathbf{C}_i), \qquad i=1,2,3 \quad (5)$$

Since geometric condition (iii) holds, point P lies on the axis of any leg's first revolute pair (see Figure 5) and all the cross-products $\mathbf{w}_{1i}\times(\mathbf{P}-\mathbf{A}_i)$, $i=1, 2, 3$, are null vectors. As a consequence, expressions (5) become

$$\dot{\mathbf{P}}=\dot{h}_i\mathbf{u}_i+\dot{\theta}_{2i}\mathbf{w}_{2i}\times(\mathbf{C}_i-\mathbf{B}_i)+(\dot{\theta}_{2i}+\dot{\theta}_{3i})\mathbf{w}_{2i}\times(\mathbf{P}-\mathbf{C}_i),$$
$$i=1,2,3 \quad (6)$$

Reminding that unit vector $\mathbf{u}_i$ (Figure 5) is perpendicular to unit vector $\mathbf{w}_{2i}$ (geometric condition (iv)), the dot-products of the i-th relationship (6) and $\mathbf{w}_{2i}$ for $i=1, 2, 3$ give the following three scalar equations

$$\dot{\mathbf{P}}\cdot\mathbf{w}_{2i}=0, \qquad i=1,2,3 \quad (7)$$

Time differentiation of Eqs. (7) yields

$$\dot{\mathbf{P}}\cdot\dot{\mathbf{w}}_{2i}+\ddot{\mathbf{P}}\cdot\mathbf{w}_{2i}=0, \qquad i=1,2,3 \quad (8)$$

where $\ddot{\mathbf{P}}$ is the acceleration of platform point P and $\dot{\mathbf{w}}_{2i}$ is the time derivative of $\mathbf{w}_{2i}$.

Equations (7) and (8) constitute a linear and homogeneous system of six equations in six unknowns: the components of $\dot{\mathbf{P}}$ and $\ddot{\mathbf{P}}$. The matrix form of such a system is

$$\mathbf{M}\begin{Bmatrix}\dot{\mathbf{P}}\\ \ddot{\mathbf{P}}\end{Bmatrix}=0 \quad (9)$$

where

$$\mathbf{m}=\begin{bmatrix}\mathbf{H} & \mathbf{0}\\ \dot{\mathbf{H}} & \mathbf{H}\end{bmatrix} \quad (10.1)$$

$$\mathbf{H}=[\mathbf{w}_{21}, \mathbf{w}_{22}, \mathbf{w}_{23}] \quad (10.2)$$

and $\mathbf{0}$ is the 3×3 null matrix.

Since unit vectors $\mathbf{w}_{2i}$, $i=1, 2, 3$, depend only on the configuration assumed by the manipulator, matrix $\mathbf{M}$ also depends only on that configuration. If the manipulator does not assume a configuration (translation singularity) that makes matrix $\mathbf{M}$ singular, the unique solution of system (9) will be

$$\dot{\mathbf{P}}=0 \quad (11.1)$$

$$\ddot{\mathbf{P}}=0 \quad (11.2)$$

Results (11) justify the following statement:

Statement 1: If a 3-RRPRR mechanism performs elementary motion starting from a configuration which is not a translation singularity and matches the 3-RRPRR wrist geometric conditions, then it will reach a new configuration where platform point P (see Figure 4) still is in the initial position (relationship (11.1)) and at rest (relationship (11.2)), i.e. the new configuration still matches the 3-RRPRR wrist geometric conditions.

Statement 1 brings about the following corollary: the platform of a 3-RRPRR wrist is constrained to accomplish sequences of elementary spherical movements with center P, i.e. finite spherical motion with center P, as long as the mechanism is out of translation singularities. In other words, the 3-RRPRR wrist is a spherical parallel manipulator provided that translation singularities are not met during motion.

The i-th Eq. (7) analytically expresses the mobility constraint which the i-th leg of a 3-RRPRR wrist imposes on the platform. It lends itself to the following kinematic interpretation: one leg of type RRPRR, which satisfies geometric condition (iv), and whose two ending-revolute-pair axes are neither skew nor parallel (Figure 5), forbids the translation of the platform point, instantaneously coinciding

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with the intersection of its ending-revolute-pair axes, along the direction of its intermediate-revolute-pair axes (direction of $\mathbf{w}_{2i}$ in Figure 5).

It is worth noting that, when the ending-revolute-pair axes are parallel the mobility constraint due to the i-th RRPRR leg cannot be expressed by the i-th Eq. (7) any longer. Instead, the equations reported in the literature[4,15] for the i-th leg of translational 3-RRPRR mechanisms must be used. Those equations show that, in this case, the movement forbidden to the platform by the i-th RRPRR leg is a rotation around an axis perpendicular to all the revolute pair axes of the leg.[15]

3. A NEW FAMILY OF SPHERICAL PARALLEL MANIPULATORS

Results (11), which bring about the conclusion that 3-RRPRR wrists are spherical parallel manipulators, derive directly from relationships (7). Therefore, if leg topology changes, because kinematic pair sequences change and/or one kinematic pair is substituted by another one, and the new legs give a mobility constraint on the platform that still is expressible through relationships (7), then the parallel manipulator with the new type of legs will still be a spherical parallel manipulator. Hence, the demonstration that a parallel manipulator is spherical can be limited to the demonstration that the mobility constraints on the platform due to the legs are expressible through relationships (7). Hereafter, this criterion will be used to find new architectures of spherical parallel manipulators.

3.1. Spherical parallel manipulator 3-(5R)

Manipulators of type 3-(5R) are three-dof parallel manipulators having three legs of type RRRRR. Figure 6 shows a 3-(5R) manipulator that satisfies geometric conditions (i), (ii) and (iii) and the following additional geometric condition:

(iv.1) in each leg the axes of the three intermediate revolute pairs are parallel (manufacturing condition).

Henceforth, a 3-(5R) mechanism matching geometric conditions (i), (ii), (iii) and (iv.1) will be called 3-(5R) wrist and geometric conditions (i), (ii), (iii) and (iv.1) will be called 3-(5R) wrist geometric conditions.

Figure 7 shows the i-th leg of a 3-(5R) wrist and the notations that will be used. The i-th leg of a 3-(5R) wrist

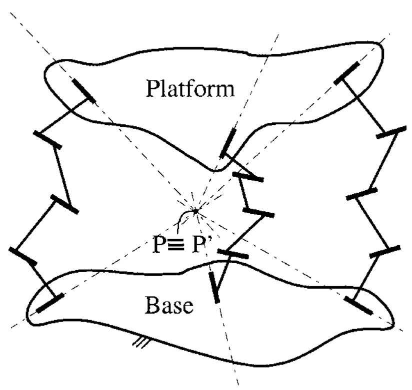

Fig. 6. 3-(5R) wrist.

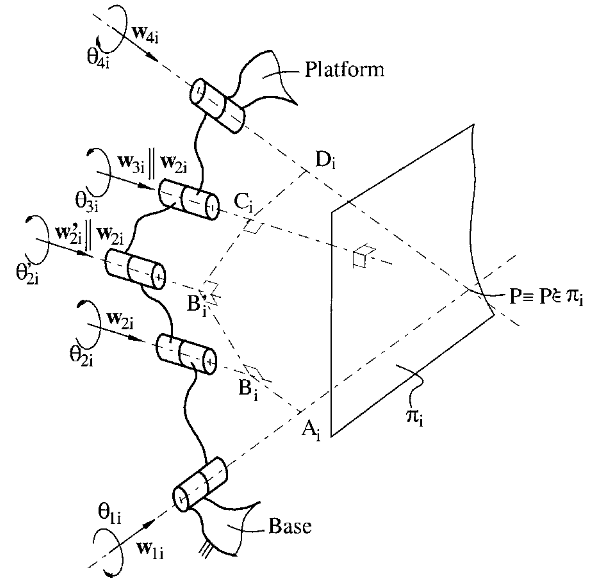

Fig. 7. i-th leg of the 3-(5R) wrist.

(Figure 7) is obtained from the i-th leg of a 3-RRPRR wrist (Figure 5) by replacing the prismatic pair with a revolute pair whose axis is parallel to the axes of the two adjacent revolute pairs. With reference to Figure 7, $\mathbf{w}_{ji}$ $(j=1, \ldots, 4)$, θ_{ji} $(j=1, \ldots, 4)$, A_i, B_i, C_i and D_i coincide with the homonymous quantities defined in Figure 5 for the i-th leg of the 3-RRPRR wrist; $\mathbf{w}'_{2i}$ and θ'_{2i} are the unit vector of the axis and the joint coordinate, respectively, of the revolute pair that replaces the prismatic pair; B'_i is the foot of the perpendicular through B_i to the axis of the revolute pair that replaces the prismatic pair. Without loss of generality, $\mathbf{w}'_{2i}$ will be chosen so that

$$\mathbf{w}'_{2i} \equiv \mathbf{w}_{2i}, \qquad i=1, 2, 3 \tag{12}$$

With these notations, the velocity, $\dot{\mathbf{P}}$, of the platform point P can be written in the following three different ways

$$\dot{\mathbf{P}} = \dot{\mathbf{C}}_i + [\dot{\theta}_{1i}\mathbf{w}_{1i} + (\dot{\theta}_{2i} + \dot{\theta}'_{2i} + \dot{\theta}_{3i})\mathbf{w}_{2i}] \times (\mathbf{P} - \mathbf{C}_i), \qquad i=1, 2, 3 \tag{13}$$

where $\dot{\theta}'_{2i}$ is the rate of the joint coordinate θ'_{2i}.

The analysis of Figure 7 reveals that the following relationships can be written

$$\dot{\mathbf{C}}_i = \dot{\mathbf{B}}'_i + [\dot{\theta}_{1i}\mathbf{w}_{1i} + (\dot{\theta}_{2i} + \dot{\theta}'_{2i})\mathbf{w}_{2i}] \times (\mathbf{C}_i - \mathbf{B}'_i), \qquad i=1, 2, 3 \tag{14}$$

$$\dot{\mathbf{B}}'_i = \dot{\mathbf{B}}_i + (\dot{\theta}_{1i}\mathbf{w}_{1i} + \dot{\theta}_{2i}\mathbf{w}_{2i}) \times (\mathbf{B}'_i - \mathbf{B}_i), \qquad i=1, 2, 3 \tag{15}$$

$$\dot{\mathbf{B}}_i = \dot{\theta}_{1i}\mathbf{w}_{1i} \times (\mathbf{B}_i - \mathbf{A}_i), \qquad i=1, 2, 3 \tag{16}$$

where $\dot{\mathbf{B}}'_i$ is the velocity of point B'_i.

The introduction of relationships (14), (15) and (16) into (13) yields

$$\dot{\mathbf{P}} = \dot{\theta}'_{2i}\mathbf{w}_{2i} \times (\mathbf{P} - \mathbf{B}'_i) + \dot{\theta}_{2i}\mathbf{w}_{2i} \times (\mathbf{C}_i - \mathbf{B}_i) + (\dot{\theta}_{2i} + \dot{\theta}_{3i})\mathbf{w}_{2i} \times (\mathbf{P} - \mathbf{C}_i), \qquad i=1, 2, 3 \tag{17}$$

The dot-products of the i-th relationship (17) and $\mathbf{w}_{2i}$ for $i=1, 2, 3$ give the three scalar Equations (7). Therefore, the 3-(5R) wrist is a spherical parallel manipulator provided that translation singularities are not met during motion.

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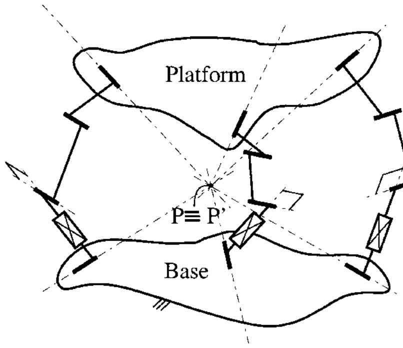

Fig. 8. 3-RPRRR wrist.

3.2. Spherical parallel manipulators 3-RPRRR and 3-RRRPR

Manipulators 3-RPRRR are three-dof parallel manipulators having three legs of type RPRRR. If the role of platform and base is interchanged by a kinematic inversion, manipulators 3-RPRRR will become manipulators 3-RRRPR whose legs are kinematic chains of type RRRPR. As a consequence, all the results regarding the kinematic behavior of a 3-RPRRR manipulator also hold for the 3-RRRPR manipulator coming from the 3-RPRRR mechanism by interchanging the role of the platform and the base. In the remainder of this subsection, demonstrations will be referred to a 3-RPRRR manipulator and the results will be extended to 3-RRRPR manipulators via kinematic inversion.

Figure 8 shows a 3-RPRRR manipulator that satisfies geometric conditions (i), (ii) and (iii) and the following additional geometric conditions:

(iv.2) in each leg the axes of the two intermediate revolute pairs are parallel (manufacturing condition);
(v) in each leg the sliding direction of the prismatic pair is perpendicular to the axis of the intermediate revolute pair adjacent to the prismatic pair (manufacturing condition).

Hereafter, a 3-RPRRR mechanism meeting geometric conditions (i), (ii), (iii), (iv.2) and (v) will be called 3-RPRRR wrist and geometric conditions (i), (ii), (iii), (iv.2) and (v) will be called 3-RPRRR wrist geometric conditions.

Figure 9 shows the i-th leg of a 3-RPRRR wrist and the notations that will be used. The i-th leg of a 3-RPRRR wrist (Figure 9) is obtained from the i-th leg of a 3-RRPRR wrist (Figure 5) by means of the elimination of the prismatic pair between the intermediate revolute pairs and the introduction, between the first and the second revolute pair, of a prismatic pair whose sliding direction is perpendicular to the axis of the second revolute pair (geometric condition (v)). With reference to Figure 9, $\mathbf{w}_{ji}$ $(j=1,\ldots,4)$, θ_{ji} $(j=1,\ldots,4)$, A_i, B_i, C_i and D_i coincide with the homonymous quantities defined in Figure 5 for the i-th leg of the 3-RRPRR wrist; $\mathbf{v}_i$ and s_i are the unit vector of the sliding direction and the joint coordinate, respectively, of the prismatic pair.

With these notations, the velocity, of the platform point P can be written in the following three different ways

$$\dot{\mathbf{P}}=\dot{\mathbf{C}}_i+[\dot{\theta}_{1i}\mathbf{w}_{1i}+(\dot{\theta}_{2i}+\dot{\theta}_{3i})\mathbf{w}_{2i}]\times(\mathbf{P}-\mathbf{C}_i),\qquad i=1,2,3 \quad (18)$$

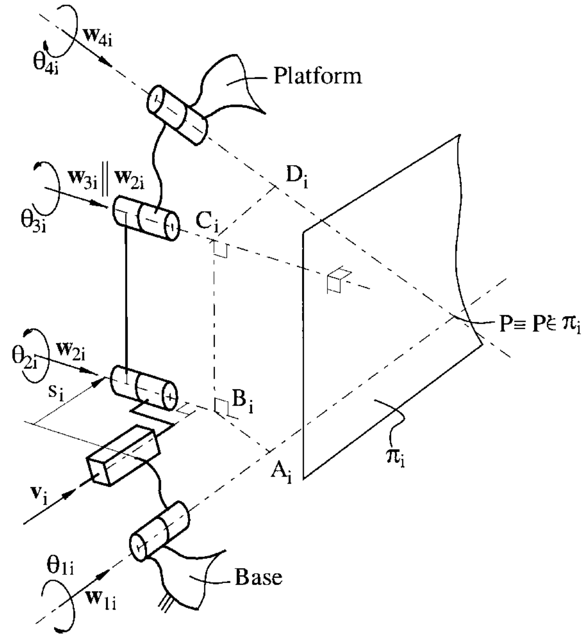

Fig. 9. i-th leg of the 3-RPRRR wrist.

The analysis of Figure 9 reveals that the following relationships can be written

$$\dot{\mathbf{C}}_i=\dot{\mathbf{B}}_i+(\dot{\theta}_{1i}\mathbf{w}_{1i}+\dot{\theta}_{2i}\mathbf{w}_{2i})\times(\mathbf{C}_i-\mathbf{B}_i),\qquad i=1,2,3 \quad (19)$$

$$\dot{\mathbf{B}}_i=\dot{s}_i\mathbf{v}_i+\dot{\theta}_{1i}\mathbf{w}_{1i}\times(\mathbf{B}_i-\mathbf{A}_i),\qquad i=1,2,3 \quad (20)$$

where $\dot{s}_i$ is the rate of the joint coordinate s_i.

The introduction of relationships (19) and (20) into relationship (18) yields

$$\dot{\mathbf{P}}=\dot{s}_i\mathbf{v}_i+\dot{\theta}_{2i}\mathbf{w}_{2i}\times(\mathbf{C}_i-\mathbf{B}_i)+(\dot{\theta}_{2i}+\dot{\theta}_{3i})\mathbf{w}_{2i}\times(\mathbf{P}-\mathbf{C}_i),\qquad i=1,2,3 \quad (21)$$

The dot-products of the i-th relationship (21) and $\mathbf{w}_{2i}$ for $i=1,2,3$ give the three scalar Equations (7). Therefore, 3-RPRRR wrists are spherical parallel manipulators provided translation singularities are not met during motion. If the mechanisms obtained from 3-RPRRR wrists via kinematic inversion of base and platform are named 3-RRRPR wrists, then 3-RPRRR wrists will also be spherical parallel manipulators provided that translation singularities are not met during motion.

4. TRANSLATION SINGULARITIES

Equation systems (7), (8) and (9), derived for 3-RRPRR wrists, also hold for 3-(5R) wrists, 3-RPRRR wrists and 3-RRRPR wrists. All these new spherical parallel manipulators work properly only if relationships (11) give the only solution of system (9). This condition occurs if and only if coefficient matrix **M** of system (9) is not singular. Translation singularities are mechanism configurations making matrix **M** singular. Matrix **M** is singular when its determinant, det(**M**), vanishes, that is the following singularity condition is satisfied

$$\det(\mathbf{M})=0 \quad (22)$$

When singularity condition (22) is matched, both $\dot{\mathbf{P}}$ and $\ddot{\mathbf{P}}$ are not determined and can be different from zero, i.e. the

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manipulator cannot make the platform accomplish spherical motion any longer. This remark justifies the name "translation singularity" given to a mechanism configuration meeting condition (22).

Definition (10.1) of matrix **M** allows the following relationship to be written

$$\det(\mathbf{M}) = [\det(\mathbf{H})]^2 \quad (23)$$

Therefore, singularity condition (22) has the same solutions as the following simplified singularity condition

$$\det(\mathbf{H}) = 0 \quad (24)$$

Moreover, definition (10.2) of matrix **H** allows the following relationship to be written

$$\det(\mathbf{H}) = \mathbf{w}_{21} \cdot \mathbf{w}_{22} \times \mathbf{w}_{23} \quad (25)$$

Relationships (24) and (25) bring about the conclusion that translation singularities are the mechanism configurations which satisfy the following condition

$$\mathbf{w}_{21} \cdot \mathbf{w}_{22} \times \mathbf{w}_{23} = 0 \quad (26)$$

From a geometric point of view, condition (26) is matched when the three unit vectors $\mathbf{w}_{2i}$, i = 1, 2, 3, are parallel to a unique plane. If a plane, π_i, associated to the i-th leg (see Figures 5, 7 and 9), is defined as the plane perpendicular to the unit vector $\mathbf{w}_{2i}$ and passing through platform point P, condition (26) can be expressed as follows: translation singularities are characterized by the fact that the three planes π_i, i = 1, 2, 3, have a straight line passing through P as common intersection. The same geometric condition can be specularly expressed by saying that the three planes π_i, i = 1, 2, 3, have only point P as common intersection out of translation singularities.

From an analytic point of view, condition (26) is a scalar equation containing the geometric parameters which define manipulator's geometry and the three generalized coordinates which define manipulator's configuration. Therefore, when manipulator's geometry is fixed, condition (26) becomes a scalar equation in three unknowns: the three generalized coordinates. Such an equation is the analytic expression of a surface (translation-singularity surface) of manipulator's configuration space (Cartesian space whose coordinates are the generalized coordinates of the manipulator). The translation-singularity surface is the geometric locus locating all the translation singularities in the configuration space and can be drawn by solving Eq. (26).

5. CONCLUSION

A new family of spherical parallel manipulators has been presented. The new family contains four different architectures. One out of these architectures contains the 3-UPU wrist, already presented in the literature, as a particular geometry.

All the manipulators of this family have independent constraints and three equal legs. Furthermore, they have less manufacturing constraints than the 3-UPU wrist.

The condition which identifies the translation singularities of all the manipulators of these family has been found and its geometric interpretation has been given. This condition can be used as a practical tool to find a surface (translation-singularity surface) which is the geometric locus locating all the translation singularities in the configuration space of the manipulator (Cartesian space whose coordinates are the generalized coordinates of the manipulator).

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