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COMMENT ON "CAPM RISK ADJUSTMENT FOR EXACT AGGREGATION OVER FINANCIAL ASSETS," BY BARNETT, LIU, AND JENSEN

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The Divisia monetary index weights each monetary asset by its expenditure share, evaluated at its user cost.[1] In a world of perfect certainty, the user costs can be computed directly from asset return data, and so, the index is model-free. In the presence of uncertainty, however, the true user cost of a monetary asset depends on the covariance of the asset's return with the marginal utility of wealth. Intuitively, an asset's value depends both on its expected payoff and on the asset's ability to hedge wealth fluctuations. As a result, the true user cost cannot be computed without an explicit model of preferences. It follows that the exact Divisia monetary index in a stochastic economy is not model-free, but depends on the underlying preference assumptions.

Barnett et al. (1997) derive the exact risk-adjusted Divisia index in the context of a consumption-based model with time-separable preferences. In their model, money directly enters the utility function. [See their equation (1).] The user cost of the ith monetary asset (denoted Π_{it}) is given in their Theorem 1 as

$$\Pi_{it} = \pi_{it} + \psi_{it}, \tag{1}$$

where π_{it} is the certainty-equivalent user cost [defined in their equation (10)], and ψ_{it} is a risk adjustment [defined in their equation (11)] involving the conditional covariance between the marginal utility of wealth and the returns on both the ith monetary asset and the risk-free asset.

The time-separable consumption-based model is a natural starting point for this inquiry, because it is the dynamic equilibrium model most familiar to economists. However, the basic approach of this paper is far more general. Equation (1) is a special case of the following general characterization of Π_{it}:

$$\Pi_{it} \equiv 1 - E_t[MRS_{t+1} r_{i,t+1}], \tag{2}$$

where MRS_{t+1} denotes the intertemporal marginal rate of substitution in wealth of the representative agent between date t and date $t+1$, and $r_{i,t+1}$ denotes the

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gross real (pecuniary) return to the ith monetary asset.[2] In Barnett et al. (1997),

$$MRS_{t+1} = \rho\left(\frac{\partial V}{\partial c_{t+1}} \Big/ \frac{\partial V}{\partial c_t}\right), \tag{3}$$

where V denotes the implied utility function defined in Section 2.2 of their paper. If equation (3) is substituted into equation (2), one obtains (1). The intuitive interpretation for equation (3) is as follows: The price of asset i at date t equals 1 (a normalization because we are dealing with returns). The pecuniary payoff to asset i is $r_{i,t+1}$. For nonmonetary assets, the price must equal the risk-adjusted discounted pecuniary payoff,

$$E_t[MRS_{t+1} r_{i,t+1}] = 1,$$

where the appropriate risk adjustment is given by MRS_{t+1}. However, for monetary assets, the price exceeds this risk-adjusted discounted (pecuniary) payoff by the user cost Π_{it}, which is the discounted (pecuniary) return that is lost by holding money instead of illiquid assets. Presumably, agents receive some nonpecuniary or liquidity payoff from this monetary asset, which exactly compensates for its user cost.

In the money-in-the-utility-function framework of Barnett et al. (1997), the user cost of money, Π_t, equals $V_{m,t}/V_{c,t}$, the ratio of the marginal utility of real balances to the marginal utility of wealth. (See Definition 1, in Section 4.1 of their paper.) However, equation (2) holds for alternative representative-agent models of money. For example,

Transaction costs

Suppose money acquired in period t reduces cost of consumption transactions in period t.[3] If the real cost of purchasing c units of consumption when real balances equal m is given by the function $\phi(c, m)$, then

$$\Pi_t = -\phi_m(c_t, m_t),$$

the marginal transaction-cost savings from holding an additional unit of real balances.

Cash-in-advance transaction costs

Now suppose money acquired in period t reduces cost of consumption transactions in period $t + 1$.[4] The user cost is now given by

$$\Pi_t = E_t\{MRS_{t+1}[-\phi_m(c_{t+1}, m_{t+1})]\},$$

the expected marginal transaction-cost savings in $t + 1$, discounted back to date t by the marginal rate of substitution.

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Cash-in-advance constraint

In the familiar cash-in-advance model,[5] the user cost is given by

$$\Pi_t = E_t\left[MRS_{t+1}\frac{\mu_{t+1}}{\lambda_{t+1}}\right],$$

where $\mu_{t+1} \equiv$ multiplier on cash-in-advance constraint and $\lambda_{t+1} \equiv$ multiplier on budget constraint.

Equation (2) can be further generalized by using results from Hansen and Richard (1987). These authors show that in any economy without arbitrage opportunities, there exists a portfolio payoff p^*, such that

$$E_t\left[p^*_{t+1}R_{t+1}\right] = 1 \tag{4}$$

for any return R_{t+1}. Equation (2) then generalizes to

$$\Pi_t \equiv 1 - E_t\left[p^*_{t+1}r_{i,t+1}\right]. \tag{5}$$

Equation (5) is consistent with the absence of arbitrage if the total return to the ith monetary asset includes both the pecuniary and nonpecuniary components, and the nonpecuniary component is set equal to Π_t/p^*_{t+1}. Equation (5) is more general than equation (2), because it does not require a well-defined representative agent, or even a well-defined equilibrium.

If markets are complete, equation (5) can be written in terms of the prices of state-contingent claims. Modifying the notation slightly, let S_{t+1} denote the set of possible states at date $t+1$, let $\tilde{s}_{t+1}$ be a random variable whose realization is the state at date $t+1$, and let f_t denote the conditional probability density function (conditional on date t information) associated with $\tilde{s}_{t+1}$.[6] Finally, let $p_t(s)$ denote the price at date t of one unit of consumption in date $t+1$ if the state at date $t+1$ is $s \in S_{t+1}$. In the framework of Hansen and Richard (1987), $p_t(s) = f_t(s)p^*_{t+1}(s)$, where $p^*_{t+1}(s)$ is the value of p^*_{t+1} that obtains when $\tilde{s}_{t+1} = s$. Equation (5) then implies

$$\Pi_{it} = 1 - \int_{s\in S_{t+1}} p_t(s)r_{t+1}(s)\,ds. \tag{6}$$

Equation (6) shows that there is no *conceptual* difference between the user cost under certainty and the user cost in the presence of risk. If the state prices $p_t(s)$ could be observed, then equation (6) would give the user costs Π_{it} as a function of observable variables, in the same way that the user cost is computed from observable returns in the traditional Divisia index. In practice, state prices are not observed; they must be inferred from a model of p^*_t. This is somewhat troubling. A strength of the traditional Divisia index is that it is truly model-free. In practice, the risk-adjusted Divisia index depends on a model of risk preferences. For this reason, it is essential to use a well-performing model of p^*_t, one that (approximately) satisfies (4) for most (nonmonetary) assets. The consumption-based model used by Barnett et al. (1997) is not the best model in practice. Time variation in user costs requires time variation in $\text{cov}_t(p^*_{t+1}, r_{t+1})$. It is difficult to get substantial time variation of this conditional covariance with consumption-based models at short horizons [see Daniel and Marshall (1997)]. Long horizons are difficult to justify for monetary assets.

Perhaps a better practical alternative would be to use an *ad hoc* model of p^*_{t+1}, such as the factor models of Bansal et al. (1993) and Farnsworth et al. (1995). For example, one could specify the universe of assets (including monetary assets), set p^*_t equal to the portfolio payoff that best fits (4) for the *nonmonetary* assets, and then use this p^*_t process to compute the user costs for the monetary assets according to (5). Although this procedure is not tightly derived from a fully specified equilibrium model, it probably would give a better representation of user costs in practice.

The danger with using (2) or (5) to measure the user cost of monetary assets is that the nonpecuniary return is not distinguished from model misspecification: Given a model of p^*_t, any failure of (4) could be attributed to the unobserved user cost Π_{it}. For example, the consumption-based asset pricing model with time-separable preferences predicts a counterfactually low mean return to nominally risk-free assets: Even with the coefficient of relative risk aversion set to 50, it is difficult to obtain a mean three-month nominal interest rate above 1% per year (as compared to the value of 5.4% in postwar U.S. data). Naive application of (2) would explain this discrepancy by positing a nonpecuniary return to three-month T-bills averaging around 4.4% per year. This is a bit too easy. To avoid this pitfall, one must impose some independent identifying restriction on Π_{it}, in addition to (2) or (5). For example, the Divisia aggregate implied by the estimated user costs Π_{it} should satisfy a stable money demand function. A priori restrictions on the behavior of this money demand function could act as additional identifying restrictions on the user costs Π_{it}.

NOTES

1. See Barnett et al. [1997, equations (5) and (6)].
2. I use the convention of subscripting returns by the date when they become known. This dating convention differs from that used by Barnett et al. (1997). Also, in the following, all returns are gross rather than net.
3. See, for example, Bekaert et al. (1997).
4. See Feenstra (1986) and Marshall (1992).
5. See Hodrick et al. (1991), and the references therein.
6. If the conditional probability distribution of $\tilde{s}_{t+1}$ does not have a density-function representation, the discussion in this section can be generalized in the obvious way.

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