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TESTING TIME-SERIES STATIONARITY AGAINST AN ALTERNATIVE WHOSE MEAN IS PERIODIC

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We develop a test of the null hypothesis that an observed time series is a realization of a strictly stationary random process. Our test is based on the result that the kth value of the discrete Fourier transform of a sample frame has a zero mean under the null hypothesis. The test that we develop will have considerable power against an important form of nonstationarity hitherto not considered in the mainstream econometric time-series literature, that is, where the mean of a time series is periodic with random variation in its periodic structure. The size and power properties of the test are investigated and its applicability to real-world problems is demonstrated by application to three economic data sets.

Keywords: Autoregressive (AR) Process, Discrete Fourier Transform, Randomly Modulated Periodic Process, Seasonality, Stationarity

1. INTRODUCTION

We present a test of the null hypothesis that an observed time series is a realization of a strictly stationary random process $\{x(t)\}$. The test statistic is a simple function of complex-valued Fourier transforms of non-overlapping sections of the observed time series. The statistical properties of the test statistic under the null are well behaved, assuming a set of basic properties for the stochastic process. The test statistic is designed to detect hidden periodicities in the data with random amplitude modulation. The alternative process is defined in Section 2.

To ensure the statistical properties we need, we limit attention to stationary general linear processes that include AR, MA, and ARMA models as special cases [Priestley (1981, p. 141)]. If the stationary general linear process is also invertible, we can subsequently model the process as a stable AR(p) process, which constitutes a large subset of all strictly stationary random processes that have absolutely summable covariances [Priestley (1981, p. 144)]. The invertiblility condition also ensures that there is a unique set of coefficients that correspond to any given form of autocovariance function [Priestley (1981, p. 145)].

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The definition of strict stationarity is that the joint distribution of $\{x(t_1), \ldots, x(t_n)\}$ for any set of times $(t_1, \ldots, t_n)$ is invariant to a shift of the time origin. The term nonstationarity has become equated with linear or polynomial trends, partially as a result of the great success of the time-series modeling strategy presented by Box and Jenkins (1970). Polynomial trends are simple forms of nonstationarity. A time series that is a polynomial trend plus stationary noise can be transformed into a stationary process by successive differencing. For example, if the trend is linear, the first difference renders the time series stationary. However, if the time series is a polynomial trend plus a general linear process, then the correct detrending method would involve modeling the trend by polynomial regression on time and then subtracting this estimate of the trend from the original time series. Differencing in this case is inappropriate because although it will render the time series stationary, it will also introduce unit roots into the MA part of the linear process, making the process noninvertible [Hamilton (1994, p. 444)]. It will also introduce spurious positive autocorrelations at the first few lags in the autocorrelation function of the residuals, thereby generating spurious periodicities in the power spectrum in the form of exaggerated power at low frequencies and attenuated power at high frequencies, thus leading to artificially dominant low-frequency cycles [see Chan et al. (1977, pp. 741–742), Nelson and Kang (1981, p. 742), and Nelson and Plosser (1982, p. 140)].

Another commonly held interpretation of nonstationarity is related to shifts in the mean and/or variance of a time series.[1] In the latter circumstance, a more appropriate spectral technique might be the concept of evolutionary (or time-varying) spectra introduced by Priestley (1965, 1981, 1988).[2]

In the econometrics literature, the majority of tests for stationarity are either tests for trend stationarity against an alternative of a unit root with or without drift [e.g., see Kwiatkowski et al. (1992), Bierens (1993), and Bierens and Guo (1993)] or, alternatively, tests of a unit root with or without drift against an alternative of trend stationarity [e.g., see Dickey and Fuller (1979, 1981), Phillips (1987), and Phillips and Perron (1988)]. The distribution theory for these tests is nonstandard and is based primarily upon continuous-time models of Brownian motion. Moreover, because unit root behavior implies long memory dependence, the absolute integrable condition required for the existence of the spectral density function is not satisfied [see Priestley (1981, pp. 213–214, 218–219) and Hidalgo (1996)].

In this paper, the alternative hypothesis we adopt is that the observed time series is a sum of a pure noise process (independent and identically distributed) and a periodic process with random variation in its amplitude, phase, and frequency. This type of process was defined by Hinich (1997) as a randomly modulated periodic process and may be created by some nonlinear physical or social mechanism that has a more or less stable inherent periodicity. The rationale for this type of process is the proposition that both nature and society do not generate perfectly periodic processes. There is always some variation in the periodic structure (waveform) over time, generating nonstationarity.

As an example, suppose that $x(t)$ represents a time series of aggregate monthly toy purchases in the United States, which has a seasonal of 12 months with the main peak occurring before Christmas. Although the calendar has no random variation, toy buying and other consumer behavior depend on the existing and expected economic conditions as well as weather conditions. The peak and troughs of the toy series will vary from year to year and some of that variation may not be well fitted using a covariate such as per capita disposable income.

Another example is the seasonal effect that weather has on crop price and output. The periodic structure will reflect seasonal influences attributable to the effect that variations in weather can exert upon crop sowing, growth, and harvesting, and through this, on crop output and price. Our test should have good power in detecting seasonal fluctuations that are likely to be present in such data. Our interest in detecting seasonality follows from the fact that it can be viewed as closely approximating the type of nonstationarity mentioned above; that is, it could be conceived as representing a periodic process with random variation.

The type of nonstationarity with which we are dealing is different in conception from the conventional types of nonstationarity mentioned above, and constitutes an additional type of nonstationarity. In particular, it is not related to unit root behavior because the distribution theory we use is standard and is predicated upon the existence of the spectral density function. This, in turn, means that the dependence structure is short memory and not long memory. Therefore, we are effectively assuming that any trend, whether deterministic or stochastic, has been removed before we apply our test.

The central issue addressed in this paper—that of testing for randomly varying periodic structure—is of fundamental importance to the question of determining whether it is *necessary* to employ a modeling framework that essentially fixes the periodic structure of the process on the one hand, or that permits the periodic structure to evolve over time on the other.[3] In economics, the above considerations extend quite generally to all economic processes with well-defined periodic structure and, in macroeconomic/econometric context, would include the modeling of macroeconomic phenomena that contain seasonal cycles.

The structure of this paper is as follows: The stationarity test is outlined in Section 2. Simulation results are presented in Section 3. We then demonstrate the test's applicability to *three* economic data sets. The first application is undertaken in Section 4 and entails applying the test to an AR model of United Kingdom (UK) average Wheat and Barley price indices for the period August 1965 to June 1995. Then, in Section 5, we then apply our test to the seasonally adjusted and unadjusted U.S. Currency Component of the Money Stock for the period January 1947 to November 1997.

2. TESTING THE STATIONARITY OF THE INNOVATIONS

Using a standard time-series notational convention, the time unit is set to 1 and t is an integer time index with the start of the sample set at $t = 0$. Let $x(0), \ldots, x(N-1)$ denote a sample from a time series $\{x(t)\}$. Then,

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$$X(k) = \sum_{t=0}^{N-1} x(t)\exp(-i2\pi f_k t)$$

is the complex value for frequency $f_k = k/N$ of the discrete Fourier transform of a sample frame of the process $\{x(t)\}$ for $t = 0, \ldots, N-1$.

The null hypothesis is that $\{x(t)\}$ is a stationary invertible general linear process. We can then represent the process as an unspecified AR(p) process whose innovations are pure noise [see Priestley (1981, pp. 141–147)]. Our test is based on the result that the expected value of $X(k)$ is zero under the null hypothesis [Brillinger (1981, p. 95)].

As mentioned in the introduction, the alternative hypothesis is that the observed time series is a sum of a pure noise process and a periodic process with random variation in its amplitude and phase. A formal definition of such a "varying" periodic process, called a *randomly modulated periodic process with period L* is presented by Hinich (1997), and can be defined as follows.

DEFINITION 1. *A process $\{w(t)\}$ is called a randomly modulated periodic process with period L if it has the form*

$$w(t) = K^{-1}\sum_{k=-K/2}^{K/2}[\mu_k + u_k(t)]\exp(i2\pi f_k t) \quad \text{for} \quad f_k = k/L, \tag{1}$$

where $\mu_{-k} = \mu_k^*$, $u_{-k}(t) = u_k^*(t)$, and $Eu_k(t) = 0$ for each k; E is the expectation operator; and the symbol "" denotes the complex conjugate. In terms of real-valued coefficients, $K_w(t)$ is of the form*

$$Kw(t) = \mu_0 + u_0(t) + \sum_{k=1}^{K/2}[\mathrm{Re}(\mu_k + u_k(t))\cos(2\pi f_k t) - \mathrm{Im}(\mu_k + u_k(t))\sin(2\pi f_k t)]. \tag{2}$$

The $K/2+1$ $\{u_k(t)\}$ are jointly dependent random processes with finite moments that satisfy two conditions:

(i) Periodic Block Stationarity. *The joint distribution of $\{u_{k_1}(t_1), \ldots, u_{k_r}(t_n)\}$ is the same as the joint density of $\{u_{k_1}(t_1+L), \ldots, u_{k_r}(t_n+L)\}$ for all $k_1, \ldots, k_r$ and $t_1, \ldots, t_n$ such that $0 < t_m < L$. Note that L is assumed to be the fundamental period of the process.*

(ii) Finite Dependence. *$\{u_{k_1}(s_1), \ldots, u_{k_r}(s_m)\}$ and $\{u_{k_1}(t_1), \ldots, u_{k_r}(t_n)\}$ are independent if $s_m + D < t_1$ for some D and any set of $k_1, \ldots, k_r$, $s_1 < \cdots < s_m$ and $t_1 < \cdots < t_n$.*

The process outlined in equation (1) can be written as $w(t) = s(t) + u(t)$, where

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$$s(t) = K^{-1} \sum_{k=-K/2}^{K/2} \mu_k \exp(i2\pi f_k t) \quad \text{and} \quad u(t) = K^{-1} \sum_{k=-K/2}^{K/2} u_k(t) \exp(i2\pi f_k t). \tag{3}$$

The periodic component $s(t)$ is the mean of $w(t)$. The zero-mean stochastic term $u(t)$ is a real-valued process that may be nonstationary.

Condition (i) implies that $c_u(t_1, t_2) = Eu(t_1 + L)u(t_2 + L) = Eu(t_1)u(t_2)$ if $|t_1 - t_2| < L$, but the equality does not necessarily hold when $|t_1 - t_2| > L$.

If the $u_k(t)$ are all covariance stationary, then $u(t)$ is stationary and the model simplifies to a periodic process in covariance stationary noise.

Condition (ii) ensures that $u(t)$ has finite dependence of gap length D. It then follows that all the joint cumulants of $u(t)$ are D dependent.

If we take the discrete Fourier transform of the process $w(t)$ defined in (1), we obtain

$$X(k) = \sum_{t=0}^{L-1} K^{-1} \sum_{k=-K/2}^{K/2} [\mu_k + u_k(t)] \exp(i2\pi f_k t) \exp(-i2\pi f_k t) = S(k) + U(k), \tag{4}$$

where

$$S(k) = \sum_{t=0}^{L-1} s(t) \exp(-i2\pi f_k t)$$

and

$$U(k) = \sum_{t=0}^{L-1} u(t) \exp(-i2\pi f_k t),$$

with $s(t)$ and $u(t)$ being defined in (3).

The character of the variation of $X(k)$ about its mean $S(k)$ will depend upon the variance of $U(k)$, termed $\sigma_u^2(k)$ for example. The explicit form of this variance is derived fully by Hinich (1997). Equation (4), in principle, permits the derivation of three types of processes. The first two can be regarded as *polar* cases. The first polar case is when $S(k)$ does not equal zero but $\sigma_u^2(k)$ does. Then, the kth Fourier component of the time series is a sine wave with fixed amplitude and phase. The second polar case follows when $S(k)$ is equal to zero but $\sigma_u^2(k)$ does not equal zero. If this case holds for all frequencies, the process is random with no periodic structure, which is the case for each component of a stationary random process satisfying any of the conventional mixing conditions [see Brillinger (1981, p. 95) and Hinich (1997)]. This process corresponds to the null hypothesis being employed in this paper.

The remaining category of processes corresponds to the definition of a randomly modulated periodic process mentioned earlier in the paper, which constitutes a more realistic alternative to the pure periodic plus noise model that is conventionally assumed. In this case, both $S(k)$ and $\sigma_u^2(k)$ do not equal zero and there is random variation in the kth component of the waveform over time. Furthermore, the larger the value of $\sigma_u^2(k)$, the larger will be the amount of random variation.

The test procedure involves *four* steps. The first step entails removing any trend present in the data, irrespective of whether it is a deterministic or stochastic trend. If a deterministic trend is present, one would use regression techniques involving a time trend to detrend the data. If a stochastic trend is present, one would have to determine the order of integration of the time series and then appropriately difference the time series.

The second step involves prewhitening the detrended data series. This is accomplished by fitting an AR(p) model using least squares or the Yule Walker equations. Assuming that N is much larger than p, the mean, covariances, and third- and fourth-order joint cumulants of the residuals $e(0), \dots, e(N-1)$ of a least-squares fit of the model will be approximately equal to the respective joint cumulants of the unobserved innovations with an approximation error $O(1/N)$. This error is assumed to be small enough that the residuals can be treated as if they are pure noise variates. Fitting an AR(p) model to the data without eliminating insignificant terms is a simple prewhitening operation that will yield pure noise residuals that are approximately identically distributed.

The third step involves centering the data to remove any mean periodic variation $S(k)$ that might be present. This is performed by dividing the residuals from the AR(p) fit in step 2 into P frames of length L. Discard the last partially filled frame if N is not divisible by L. The nth observation in the pth frame is $e(t_{pn})$, where $t_{pn} = (p-1)L + n$ for $n = 0, \dots, L-1$. The frame length L is chosen by the user to be the hypothetical period of the periodic component with random variation that the investigator believes to be the most probable alternative to the null hypothesis. If the frame length used is not an integer multiple of the true period, then the test will lose power. Cycles in economic time series are either related to calendar-based seasonal variation or are cycles “detected” by looking at a plot of the time series.

To center the data, compute the mean $\overline{e}(t_{pn})$ of the P values of $e(t_{pn})$ for each $n = 0, \dots, L-1$. Then subtract $\overline{e}(t_{pn})$ from $e(t_{pn})$ yielding a residual that we denote by $y(t_{pn})$. If the periodicity is purely deterministic—that is, if $S(k)$ does not equal zero but $\sigma_u^2(k)$ equals zero—then there will be no periodicity left in the residuals after the centering operation. Note that in the context of a seasonal periodicity the centering operation would have the same effect as time-domain seasonal adjustment methods. Specifically, if the generating process is a deterministic seasonal plus stationary and ergodic noise, then the Fourier transform of the seasonal component will have a fixed amplitude and phase. The centering operation will remove the deterministic periodicity (with fixed amplitude and phase) completely, leaving residuals that are pure noise innovations. If the generating process is a randomly modulated periodic process, on the other hand, then the centering operation will purge the series of the mean periodic variation but some periodic structure will remain in the residuals. This situation arises because $\sigma_u^2(k)$ does not equal zero, which means that some variation in the periodic structure about $S(k)$ will remain, reflecting variation in the phase and amplitude of the spectral density at frequency k.

The final step is to compute and apply the test statistic that is now presented. For each k, compute the average $\bar{Y}(k)$ of the kth discrete Fourier transform $Y_p(k)$ for the P frames,[4] where

$$Y_p(k) = \frac{1}{\sqrt{L}} \sum_{n=0}^{L-1} y(t_{pn}) \exp\left(\frac{-i2\pi kn}{L}\right), \qquad k = 1, \ldots, L/2. \tag{5}$$

The test statistic is

$$S = P \sum_{k=1}^{L/2} |\bar{Y}(k)|^2. \tag{6}$$

Under the null hypothesis $E[Y_p(k)] = 0$ for each $k = 1, \ldots, L/2$ and p implying that $E[\bar{Y}_p(k)] = 0$. It is shown in the Appendix (cf. Theorem 1) that under the null the asymptotic distribution of $\{\sqrt{P}\bar{Y}(1), \ldots, \sqrt{P}\bar{Y}(L/2)\}$ is complex normal N(0,1) as P goes to infinity with L fixed. Thus, given the null hypothesis, $P|\bar{Y}(k)|^2$ is approximately chi square with 2 degrees of freedom for large P, implying that under the null hypothesis, the distribution of S is approximately chi square with L degrees of freedom for large P.

Rather than using chi-square tables, the statistic S is transformed to a uniform variable under the null by computing $F(S)$, where F is the cumulative distribution function (c.d.f.) of a chi-square distribution with $2M$ degrees of freedom, where $M = L/2$ is the upper point of the discrete sum in (6).

The key implication of the null hypothesis is that it is *impossible* to obtain any periodic structure from applying a linear filter to a pure noise process. This follows from the results of Fourier Analysis applied to stationary random processes, which can be defined as processes fulfilling Theorem 4.4.1 of Brillinger (1981). As the frame length grows and the resolution bandwidth shrinks, the $Y_p(k)$ representation of any stationary random process becomes independent, with the resulting implication that the real and imaginary parts of the Fourier transform also become independent and normally distributed with a mean of zero and variance equal to half of the spectrum. This means, in turn, that the phase, which is defined as arctan(Im/Re) of the Fourier transform is uniformly distributed in the interval $(-\pi, \pi)$ for frequency k [see Fuller (1976, pp. 315–316)].

The key implication of this result is that it is impossible to distinguish either the time origin or relative time—confirming the reason why the process is defined as stationary. This is the reason why, under the null, a least-squares AR fit of the data will preserve the underlying stationarity of the process, producing no periodic structure. Under the alternative, an AR fit will change the amplitude and phase of the periodic components, which exist by definition. Therefore, we cannot generate the alternative model from random noise even if we apply a linear filter to the noise input, as is the case with the AR fit.

To understand when this test has power, it is important to consider a realistic alternative to the null. We use the following alternative model to demonstrate that our test has power against a randomly modulated periodic process. The alternative is, for each frame,

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$$w(t) = \sum_{m=1}^{M}(\alpha_m + u_{1m}) \sin 2\pi\left(\frac{k_m t}{L} + \phi_m + u_{2m}\right) + \phi(t)$$
$$= \sum_{m=1}^{M}(\alpha_m + u_{1m})\left[\cos(\phi_m + u_{2m}) \sin 2\pi\left(\frac{k_m t}{L}\right)\right.$$
$$\left. + \sin(\phi_m + u_{2m}) \cos 2\pi\left(\frac{k_m t}{L}\right)\right], \quad (7)$$

where $\phi_1, \ldots, \phi_M$ are a set of phases that have random errors $u_{2m}, \alpha_1, \ldots, \alpha_M$ are amplitudes of the sinusoids that have random errors u_{1m}, and the $\phi(t)$ variates satisfy an AR(p) model. We suppress the subscript p to simplify notation. The sum of sinusoids in expression (7) will shift the mean of $Y_p(k)$ from zero and will increase its variance.

3. ASSESSING THE SIZE AND POWER OF THE TEST USING SIMULATIONS

The use of central limit theory to prove asymptotic normality does not answer the question of how large in this case the number of frames must be for the approximation to be good enough to be applied to data. We set $\alpha_m = \alpha$, $u_{1m} = 0$, and $\phi_m = 0$ for each m in the simulations for the power of the test.[5]

The model defined by (7) was used to generate time series to estimate the power of the test. The AR variates $\phi(t)$ satisfied an AR(2) model

$$w(t) = a_1 w(t-1) + a_2 w(t-2) + e(t),$$

where the innovations $e(t)$ were either independently distributed normal N(0,1), double-tailed exponential or uniform pseudorandom variates with zero means and unit variances. The AR parameters were generated so that the AR(2) model would have two stable conjugate root pairs, $z = r\exp(i2\pi\theta)$ and $z = r\exp(-i2\pi\theta)$, where $0 < r < 1$ and $0 < \theta < 90$ deg. Thus, $a_1 = 2r\cos(2\pi\theta/180)$ and $a_2 = -r^2$. In the simulations, r was set equal to either 0.2 or 0.9, and θ was set equal to 10. The errors u_{2m} were uniform in the support set $-\beta < u_{2m} < \beta$, where β is a small jitter parameter. A typical setting used for β is $\beta = 0.05$. Therefore, parameter u_2, through parameter β, captures phase jitter.

Note that our prime focus in this section is on investigating the size properties of the test statistic. This will enable us to assess how well the test's size properties in finite samples match the asymptotic results. We include the power simulation as an example of an alternative—in this case, an alternative containing phase variation in the periodic structure. A more thorough investigation of the power properties of the test statistic is the subject of ongoing research.

Several values for N, L, P, M, α, and β were used. The signal amplitude parameter α was set either equal to zero, giving size results, or equal to 0.5, giving power results. For a set of parameter values, 6,000 replications were generated. A least-squares AR(2) fit was made for each replication of the time series and the residuals were standardized by subtracting the sample mean and dividing by the sample standard deviation for that replication. The test statistic was computed using the standardized residuals.

The large sample approximation from the asymptotic theory was used to set the threshold for the test statistic at both the 0.05 and 0.01 levels of significance. Two broad types of simulations were performed. The first type was implemented to investigate the potential importance of frame averaging. This was undertaken by fixing the frame length and then increasing the sample size, thus increasing the number of frames and frame averaging involved in the simulation. The detail and results of these simulations are reported in Tables 1 and 2 for a frame length (L) of 5, and in Tables 3 and 4 for a frame length of 10 observations.

The second type of simulation involved examining the importance of frame length in assessing the size and power of the test. This type of simulation was implemented by fixing the number of frames and varying the sample size, thus permitting the block length to increase with sample size. These simulations are reported in Tables 5 and 6 for the number of frames (P) set equal to 5, and in Tables 7 and 8 for P set equal to 10.

The results of the simulations indicate two important findings. First, the approximations appear to be conservative for small values of the frame length (L). This is evident in both the size and power results reported in Tables 1, 2, 3, and 4. When we increase the frame length to 10 observations, the estimated size results are consistent with the theoretical size levels. Power results are shown in Tables 2 and 4. Frame averaging over at least 10 frames is needed to obtain good power levels. This means that for the two frame lengths of 5 and 10 observations, respectively, we need samples of 50 and 100 observations to ensure good levels of power.

To examine the question of the trade-off between the number of frames and the number of observations per frame, we performed the second type of simulation, which entailed setting the number of frames and varying the frame length. In the actual simulations performed, the number of frames was set to 5 and 10. The size results listed in Table 5 correspond to simulations involving averaging over five frames. It is evident from inspection of this table that the estimated size results are similar to those documented above. Power results are documented in Table 6, which indicate that to obtain good power levels, an effective lower bound must be placed on the frame length. This is borne out by the requirement that to obtain good power, we would need to employ a frame length that is greater than 10 observations.

This latter conclusion is reinforced by the results from simulations involving a larger number of frames, namely, frame averaging over 10 frames. The size results are reported in Table 7.

TABLE 1. Size test results for stationarity test: Frame length = 5

					Type of pure noise input, %		
Simulation	N^a	P^b	r^c	Size, %	Gaussian	Exponential	Uniform
A	30	6	0.2	5	2.0	1.6	2.2
				1	0.1	0.1	0.1
B	30	6	0.9	5	0.1	0.1	0.1
				1	0.0	0.0	0.0
C	50	10	0.2	5	2.8	2.6	3.3
				1	0.2	0.2	0.3
D	50	10	0.9	5	0.1	0.2	0.2
				1	0.0	0.0	0.0
E	100	20	0.2	5	4.2	3.8	4.2
				1	0.6	0.5	0.5
F	100	20	0.9	5	0.6	0.4	0.5
				1	0.1	0.0	0.1
G	200	40	0.2	5	4.4	4.5	4.3
				1	0.9	0.8	0.7
H	200	40	0.9	5	1.8	1.6	1.7
				1	0.2	0.2	0.2
I	400	80	0.2	5	4.6	4.8	4.8
				1	0.9	0.8	0.9
J	400	80	0.9	5	2.9	3.1	3.0
				1	0.4	0.3	0.4

[a] Sample size.
[b] Number of frames.
[c] Modulus of AR(2) roots.

The power results listed in Table 8 were determined from simulations containing the same parameter setting used in the simulations reported in Table 7 (except for the signal amplitude parameter, which was set equal to 0.5). The most important finding to emerge from Table 8 is that with the increase in the number of frames, good power can be achieved for a frame length of 10 observations. This represents a reduction in the underlying frame-length requirements from the results reported in Table 6. Recall in that particular case, that for averaging conducted over five frames, we needed a frame length of 20 observations to secure good levels of power.

Finally, note that the findings associated with both Tables 6 and 8 also reinforce the previous observation that frame averaging over at least 10 frames seems to be necessary to secure good levels of power. This would appear to be a useful working guide, although it may be possible to secure good power levels for frame averaging involving fewer than 10 frames. This latter situation would certainly require the use of larger frame lengths.

Table 2. Power test results for stationarity test: Frame length = 5

					Type of pure noise input, %		
Simulation	N^a	P^b	r^c	Power	Gaussian	Exponential	Uniform
A	30	6	0.2	0.05	20.3	22.5	19.0
				0.01	1.7	1.5	1.8
B	30	6	0.9	0.05	18.3	19.5	17.0
				0.01	1.5	1.8	1.2
C	50	10	0.2	0.05	57.5	56.9	54.2
				0.01	18.9	19.8	16.8
D	50	10	0.9	0.05	75.2	73.6	73.9
				0.01	39.8	39.4	39.2
E	100	20	0.2	0.05	95.3	94.8	95.9
				0.01	79.6	80.6	80.8
F	100	20	0.9	0.05	100.0	100.0	100.0
				0.01	99.8	99.6	99.8
G	200	40	0.2	0.05	100.0	99.9	100.0
				0.01	99.9	99.8	99.9
H	200	40	0.9	0.05	100.0	100.0	100.0
				0.01	100.0	100.0	100.0
I	400	80	0.2	0.05	100.0	100.0	100.0
				0.01	100.0	100.0	100.0
J	400	80	0.9	0.05	100.0	100.0	100.0
				0.01	100.0	100.0	100.0

[a] Sample size.
[b] Number of frames.
[c] Modulus of AR(2) roots.

4. APPLICATION TO UK WHEAT AND BARLEY PRICES FOR AUGUST 1965 TO JUNE 1995

To demonstrate the applicability of our method, we applied the test to the residuals from an AR prewhitening fit of monthly growth rates of wheat and barley prices in the United Kingdom for the period August 1965 to June 1995. The time series used were the weighted average market prices for wheat and barley in England and Wales, measured in pounds (£) per tonne, which were purchased from growers in prescribed areas in England and Wales, in accordance with the Corn Returns Act of 1882. The data were compiled and supplied by the Ministry of Agriculture, Fisheries, and Food.

Our a priori expectation is that our test should have good power in detecting seasonal fluctuations, which are likely to be present in such data. Our interest in detecting seasonality follows from the fact that it can be viewed as closely approximating the type of nonstationarity mentioned in the introduction of this paper; that is, it could be conceived as representing a periodic process with random variation. In the context of the price series being considered in the paper, the

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TABLE 3. Size test results for stationarity test: Frame length = 10

					Type of pure noise input, %		
Simulation	N^a	P^b	r^c	Size, %	Gaussian	Exponential	Uniform
A	30	3	0.2	5	1.3	1.2	1.5
				1	0.0	0.0	0.1
B	30	3	0.9	5	0.4	0.3	0.6
				1	0.0	0.0	0.1
C	50	5	0.2	5	2.9	2.3	2.9
				1	0.2	0.1	0.2
D	50	5	0.9	5	0.3	0.5	0.6
				1	0.0	0.0	0.1
E	100	10	0.2	5	4.1	3.9	4.2
				1	0.4	0.5	0.6
F	100	10	0.9	5	0.9	0.8	0.9
				1	0.1	0.2	0.2
G	200	20	0.2	5	5.0	4.4	4.7
				1	0.9	0.8	1.0
H	200	20	0.9	5	2.0	1.7	1.7
				1	0.3	0.2	0.2
I	400	40	0.2	5	4.7	4.5	5.3
				1	1.0	0.7	0.9
J	400	40	0.9	5	2.6	3.0	3.0
				1	0.4	0.5	0.4

[a] Sample size.
[b] Number of frames.
[c] Modulus of AR(2) roots.

periodic structure will reflect seasonal influences attributable to the effect that variations in weather can exert upon crop sowing, growth, and harvesting, and through this, on crop output and price. Because of the dependence of cereal prices on cereal yields, which, in turn, will depend upon the growth cycle of the cereals and weather conditions prevailing relative to the growth cycle, it is likely that the cereal price series will exhibit a natural form of seasonality.[6] Furthermore, because of dependence on variation in weather conditions, this seasonality is unlikely to be purely deterministic in character. In these circumstances, our test should have good power in detecting and confirming the postulated randomly modulated seasonal variation.

Unit root tests activated in the PC Give 8 econometric package [Doornik and Hendry (1994)] were applied to the wheat- and barley-level time series. In neither cases could we reject the null hypothesis of a unit root using both the Dickey Fuller (DF) and Augmented Dickey Fuller (ADF) tests with the latter test having 20 lags. This conclusion was also robust when a constant, a trend, and seasonal dummies were included in the regression; see the results listed in Table 9A and 9B, respectively. Unit root tests conducted on the first differences of both series

Table 4. Power test results for stationarity test: Frame length = 10

					Type of pure noise input, %		
Simulation	N^a	P^b	r^c	Power	Gaussian	Exponential	Uniform
A	30	3	0.2	0.05	8.8	9.4	8.7
				0.01	0.4	0.5	0.4
B	30	3	0.9	0.05	5.4	5.9	6.0
				0.01	0.4	0.4	0.3
C	50	5	0.2	0.05	34.7	35.5	32.9
				0.01	7.0	7.8	6.9
D	50	5	0.9	0.05	41.8	42.5	42.0
				0.01	12.0	12.4	11.4
E	100	10	0.2	0.05	86.1	85.6	86.3
				0.01	57.8	58.6	56.8
F	100	10	0.9	0.05	99.7	99.6	99.8
				0.01	97.8	97.5	98.1
G	200	20	0.2	0.05	99.9	99.9	100.0
				0.01	98.9	98.6	99.1
H	200	20	0.9	0.05	100.0	100.0	100.0
				0.01	100.0	100.0	100.0
I	400	40	0.2	0.05	100.0	100.0	100.0
				0.01	100.0	100.0	100.0
J	400	40	0.9	0.05	100.0	100.0	100.0
				0.01	100.0	100.0	100.0

[a] Sample size.
[b] Number of frames.
[c] Modulus of AR(2) roots.

led to the strong rejection of the null of a unit root at the 0.01 level of significance by all of the above tests; see Table 9A and 9B. This indicates that both wheat- and barley-level series are $I(1)$. However, the preferred transformation by the authors involves taking the natural logarithm of the time series for both levels and then first differencing. This transforms the time series for both levels to growth rates, which is more meaningful from the perspective of explanation than is the differenced series.[7] Unit root tests conducted on the growth-rate data also indicated strong rejection of the null of a unit root at the 0.01 level of significance, thus also indicating that both series are $I(0)$; see Table 9A and 9B.

To investigate this problem, we fit AR(18) models to the growth rates of the data. We stress that the AR fitting is employed purely as a prewhitening operation. We are not attempting to obtain a model of best fit. From the perspective of applying our test, we do not require a model of best fit; we only require that the data have been whitened.

The residuals from the AR model adopted are then standardized, and the test is used to see if the complex amplitude of the discrete Fourier transform of the residuals for a 12-month period is significantly different from zero.[8] Summary

TABLE 5. Size test results for stationarity test: Number of frames = 5

Simulation	N^a	P^b	L^c	r^d	Size, %	Type of pure noise input, % Gaussian	Exponential	Uniform
A	30	5	6	0.2	5	2.0	1.7	2.2
					1	0.1	0.1	0.1
B	30	5	6	0.9	5	0.2	0.1	0.3
					1	0.0	0.0	0.0
C	50	5	10	0.2	5	2.9	2.3	2.9
					1	0.2	0.1	0.2
D	50	5	10	0.9	5	0.3	0.5	0.6
					1	0.0	0.0	0.1
E	100	5	20	0.2	5	2.7	2.8	2.8
					1	0.3	0.3	0.2
F	100	5	20	0.9	5	0.2	0.2	0.2
					1	0.0	0.0	0.0
G	200	5	40	0.2	5	3.0	2.9	2.7
					1	0.4	0.4	0.4
H	200	5	40	0.9	5	0.2	0.2	0.3
					1	0.0	0.0	0.1
I	400	5	80	0.2	5	3.1	2.9	2.9
					1	0.3	0.5	0.4
J	400	5	80	0.9	5	0.7	0.5	0.6
					1	0.1	0.0	0.1

[a] Sample size.
[b] Number of frames.
[c] Frame length.
[d] Modulus of AR(2) roots.

statistics associated with the AR(18) fits are listed in Tables 10A and 11A. The adjusted R^2 values for the wheat and barley models are 0.277 and 0.307, respectively. Although these results appear low, this is not uncommon for specifications based on growth-rate data. The standard error of the AR fits are 0.0334 and 0.0389, respectively. Both sets of residuals do not "trip" the Hinich Portmentau C test for autocorrelation [see Hinich (1996)]. The p values of 0.994 and 0.989 indicate that the null hypothesis of pure white noise cannot be rejected at the 0.05 level of significance. The descriptive statistics of the residuals from the AR(18) fits are documented in Tables 10B and 11B.

The "whiteness" of the residuals of the AR(18) fits for wheat and barley also can be seen in the power spectra of the residuals. Plots of the power spectra of the residuals are outlined in Figures 1 and 2. These spectral values were obtained by adopting a frame length (and resolution bandwidth) of 24 observations. Tables 10 C and 11C contain the parameter values and test statistic results associated with the application of the test statistic. The sample size was 358 observations, and with the adopted frame length (L) of 24 observations, generated 14 frames (P). Because we

Table 6. Power test results for stationarity test: Number of frames = 5

						Type of pure noise input, %		
Simulation	N^a	P^b	L^c	r^d	Power	Gaussian	Exponential	Uniform
A	30	5	6	0.2	0.05	21.2	23.2	20.2
					0.01	2.2	2.4	2.0
B	30	5	6	0.9	0.05	16.9	18.8	16.3
					0.01	2.0	2.3	1.7
C	50	5	10	0.2	0.05	34.7	35.5	32.9
					0.01	7.0	7.8	6.9
D	50	5	10	0.9	0.05	41.8	42.5	42.0
					0.01	12.0	12.4	11.4
E	100	5	20	0.2	0.05	52.4	50.7	50.3
					0.01	15.9	16.0	15.2
F	100	5	20	0.9	0.05	93.9	92.3	93.4
					0.01	78.2	76.3	76.7
G	200	5	40	0.2	0.05	78.9	77.7	78.1
					0.01	41.0	40.6	40.7
H	200	5	40	0.9	0.05	100.0	99.9	100.0
					0.01	99.8	99.6	99.9
I	400	5	80	0.2	0.05	97.5	97.2	97.5
					0.01	83.6	83.0	83.7
J	400	5	80	0.9	0.05	100.0	100.0	100.0
					0.01	100.0	100.0	100.0

[a] Sample size.
[b] Number of frames.
[c] Frame length.
[d] Modulus of AR(2) roots.

are using monthly data, these parameter settings permit us to estimate the power spectral density at the 2-year cycle and its subharmonics, which include the annual (12-month) cycle. The large sample standard error is 1.161.

In the results reported, we are able to detect seasonal variation at the annual cycle and its harmonics even when the periodicity is subject to random variation. For both wheat and barley, the null hypothesis of stationarity is strongly rejected—with p values of 0.0000 and 0.0000, respectively, for wheat and barley. As such, we can conclude that the residuals are not stationary.

These results point to statistically significant seasonal variation in the residuals of the AR fits, which generate correlation between the periodic structure of the residuals and sinusoids at the annual frequency and its harmonic frequencies. This fundamental seasonality can also be clearly discerned from inspection of the seasonal patterns, which are evident in the plots of the two respective growth-rate data series; consult Figures 3 and 4. Finally, note from the results listed in Table 9A and 9B that the unit root tests could not detect this structure, even in the case in which the regressions did not contain seasonal dummies. Recall that,

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TABLE 7. Size test results for stationarity test: Number of frames = 10

Simulation	N^a	P^b	L^c	r^d	Size, %	Type of pure noise input, % Gaussian	Exponential	Uniform
A	28	7	4	0.2	5	1.7	1.4	2.0
					1	0.0	0.1	0.1
B	28	7	4	0.9	5	0.2	0.1	0.2
					1	0.0	0.0	0.0
C	50	10	5	0.2	5	2.8	2.6	3.3
					1	0.2	0.2	0.3
D	50	10	5	0.9	5	0.1	0.2	0.2
					1	0.0	0.0	0.0
E	100	10	10	0.2	5	4.1	3.9	4.2
					1	0.4	0.5	0.6
F	100	10	10	0.9	5	0.9	0.8	0.9
					1	0.1	0.2	0.2
G	200	10	20	0.2	5	4.6	3.6	3.9
					1	0.7	0.6	0.6
H	200	10	20	0.9	5	0.9	0.9	1.1
					1	0.1	0.1	0.1
I	400	10	40	0.2	5	4.2	3.9	3.8
					1	0.6	0.7	0.5
J	400	10	40	0.9	5	1.2	1.4	1.4
					1	0.1	0.2	0.1

[a] Sample size.
[b] Number of frames.
[c] Frame length.
[d] Modulus of AR(2) roots.

for both cereals, the conclusions from applying the battery of unit root tests to the growth-rate data was that both series were $I(0)$ and therefore stationary.

5. DETECTION OF SEASONALITY IN THE U.S. CURRENCY COMPONENT OF THE MONEY STOCK: JANUARY 1947–NOVEMBER 1997

In this section, we demonstrate that the test can be used to detect whether there is any seasonal structure remaining in a seasonally adjusted macroeconomic time series that has been generated by a randomly modulated seasonal periodicity. Recall that this embedded structure would reflect instability in the phase, frequency, and amplitude of the time series at the annual frequency and possibly its subharmonics. The time series we use are the seasonally adjusted and unadjusted U.S. Currency Component of the Money Stock Figures, for the period January 1947 to November 1997. This data can be freely obtained from the internet by accessing the FRED database of the St. Louis Federal Reserve Bank.[9]

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TABLE 8. Power test results for stationarity test: Number of frames = 10

Simulation	N^a	P^b	L^c	r^d	Power	Type of pure noise input, %		
						Gaussian	Exponential	Uniform
A	28	7	4	0.2	0.05	13.0	14.8	12.8
					0.01	0.6	0.7	0.4
B	28	7	4	0.9	0.05	17.6	20.2	16.6
					0.01	3.6	4.6	2.8
C	50	10	5	0.2	0.05	57.5	56.9	54.2
					0.01	18.9	19.8	16.8
D	50	10	5	0.9	0.05	75.2	73.6	73.9
					0.01	39.8	39.4	39.2
E	100	10	10	0.2	0.05	86.1	85.6	86.3
					0.01	57.8	58.6	56.8
F	100	10	10	0.9	0.05	99.7	99.6	99.8
					0.01	97.8	97.5	98.1
G	200	10	20	0.2	0.05	98.1	98.2	98.4
					0.01	87.2	88.1	88.1
H	200	10	20	0.9	0.05	100.0	100.0	100.0
					0.01	100.0	100.0	100.0
I	400	10	40	0.2	0.05	100.0	100.0	100.0
					0.01	99.7	99.6	99.8
J	400	10	40	0.9	0.05	100.0	100.0	100.0
					0.01	100.0	100.0	100.0

[a] Sample size.
[b] Number of frames.
[c] Frame length.
[d] Modulus of AR(2) roots.

Our approach is to test for seasonal structure by applying the stationarity test statistic to both the unadjusted and seasonally adjusted time series. The application of the test to both of the above series will confirm if there is any randomly modulated seasonal structure in the unadjusted series and, if so, whether the seasonal filtering algorithm employed by the FRB removes this seasonal structure from the time series in question.

Once again, because we are using monthly data and are interested in the annual (12-month) cycle and its subharmonics, we adopt a frame length and resolution bandwidth corresponding to 24 observations. The sample size for the level series is 611 observations. However, there is an obvious trend in the data (see Figure 5). Unit root tests were conducted on the levels of the two time series, which were found to be $I(2)$, although the Dickey Fuller test, by itself, provided support for the proposition that the first differences and growth rates were $I(0)$, indicating that the levels were $I(1)$.[10] The broad conclusion, however, is that the level series differenced twice or change in growth rates are $I(0)$. Moreover, these results are robust to the inclusion of a constant, trend and seasonal dummies in the regression. Details of these test results are documented in Tables 12A and 12B, respectively.

Table 9. Unit root test results

	Levels				First differences				Growth rates			
Test details	Calc.	5% Crit.	1% Crit.	Concl.	Calc.	5% Crit.	1% Crit.	Concl.	Calc.	5% Crit.	1% Crit.	Concl.
A. Wheat Time Series												
DF	1.097	−1.94	−2.571	$I(1)$	−12.05	−1.94	−2.571	$I(0)$	−12.55	−1.94	−2.571	$I(0)$
ADF(20)	1.263	−1.94	−2.572	$I(1)$	−4.752	−1.94	−2.572	$I(0)$	−4.498	−1.94	−2.572	$I(0)$
ADF(20) with constant	−1.482	−2.87	−3.452	$I(1)$	−5.275	−2.87	−3.452	$I(0)$	−5.069	−2.87	−3.452	$I(0)$
ADF(20) with constant and trend	−0.504	−3.425	−3.989	$I(1)$	−5.506	−3.425	−3.989	$I(0)$	−5.493	−3.425	−3.989	$I(0)$
ADF(20) with constant and seasonals	−1.491	−2.87	−3.452	$I(1)$	−4.748	−2.87	−3.452	$I(0)$	−4.63	−2.87	−3.452	$I(0)$
ADF(20) with constant, trend, and seasonals	−0.572	−3.425	−3.989	$I(1)$	−4.962	−3.425	−3.989	$I(0)$	−5.039	−3.425	−3.989	$I(0)$
B. Barley Time Series												
DF	0.554	−1.94	−2.571	$I(1)$	−15.91	−1.94	−2.571	$I(0)$	−14.42	−1.94	−2.571	$I(0)$
ADF(20)	1.362	−1.94	−2.572	$I(1)$	−4.601	−1.94	−2.572	$I(0)$	−4.228	−1.94	−2.572	$I(0)$
ADF(20) with constant	−1.513	−2.87	−3.452	$I(1)$	−5.162	−2.87	−3.452	$I(0)$	−5.782	−2.87	−3.452	$I(0)$
ADF(20) with constant and trend	−0.322	−3.425	−3.989	$I(1)$	−5.402	−3.425	−3.989	$I(0)$	−5.205	−3.425	−3.989	$I(0)$
ADF(20) with constant and seasonals	−1.469	−2.87	−3.452	$I(1)$	−4.924	−2.87	−3.452	$I(0)$	−4.497	−2.87	−3.452	$I(0)$
ADF(20) with constant, trend, and seasonals	−0.345	−3.425	−3.989	$I(1)$	−5.141	−3.425	−3.989	$I(0)$	−4.866	−3.425	−3.989	$I(0)$

TABLE 10. AR(18) fit of wheat growth rate model ($N = 358$)

A. Summary Statistics		
Lag	Coefficient	t values
1	0.45	8.22
2	−0.16	−2.66
3	0.12	1.93
4	−0.20	−3.28
5	0.02	0.29
6	−0.08	−1.35
7	0.16	2.66
8	0.00	0.03
9	−0.09	−1.41
10	0.06	1.01
11	−0.03	−0.45
12	0.20	3.29
13	−0.08	−1.29
14	−0.09	−1.38
15	0.04	0.62
16	−0.03	−0.45
17	0.06	1.03
18	−0.15	−2.79
Adjusted R^2		0.277
Standard error of AR fit		0.0334
Hinich Portmentau C statistic test for autocorrelation P value		0.994

B. Descriptive Statistics of Residuals	
Mean	−0.00003
Standard deviation	0.0325
Skewness	0.502
Kurtosis	3.24
Maximum value	0.144
Minimum value	−0.116

C. Spectral Properties of Residuals	
Sampling interval	1.00 month
Frame size	24
Resolution bandwidth	24.00 month
No. of frames	14
Passband	(24.00 2.00) months
Stationarity test p value	0.0000
No. of frequencies in band	12
Large sample standard error	1.161

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TABLE 11. Summary statistics of AR(18) fit of barley growth rate model ($N = 358$)

A. Summary Statistics		
Lag	Coefficient	t values
1	0.37	6.67
2	−0.19	−3.25
3	0.04	0.67
4	−0.07	−1.25
5	−0.05	−0.84
6	0.03	0.46
7	0.05	0.95
8	−0.04	−0.69
9	0.00	0.06
10	−0.09	−1.62
11	0.00	0.02
12	0.33	5.92
13	−0.25	−4.37
14	−0.07	−1.13
15	0.02	0.31
16	0.00	0.03
17	−0.03	−0.54
18	−0.09	−1.59
Adjusted R^2		0.307
Standard error of AR fit		0.0389
Hinich Portmentau C statistic test for autocorrelation P value		0.989

B. Descriptive Statistics of Residuals	
Mean	−0.0002
Standard deviation	0.0378
Skewness	0.566
Kurtosis	2.19
Maximum value	0.162
Minimum value	−0.115

C. Spectral Properties of Residuals	
Sampling interval	1.00 month
Frame size	24
Resolution bandwidth	24.00 months
No. of frames	14
Passband	(24.00 2.00) months
Stationarity test p value	0.0000
No. of frequencies in band	12
Large sample standard error	1.161

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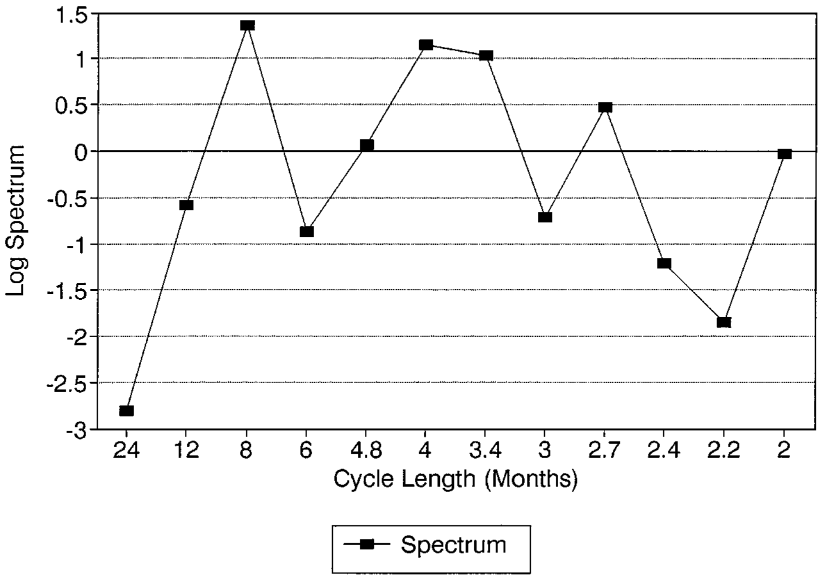

FIGURE 1. Power spectrum of residuals of AR(18) wheat growth rate model.

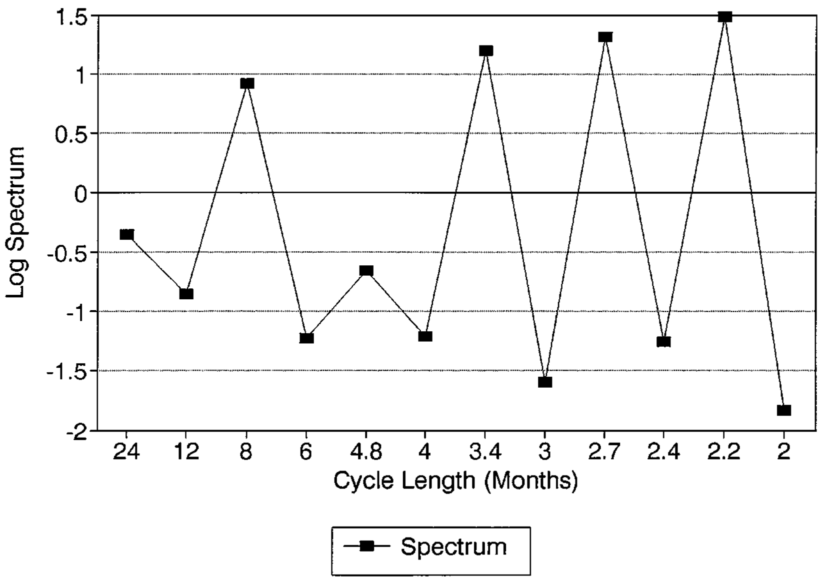

FIGURE 2. Power spectrum of residuals of AR(18) barley growth rate model.

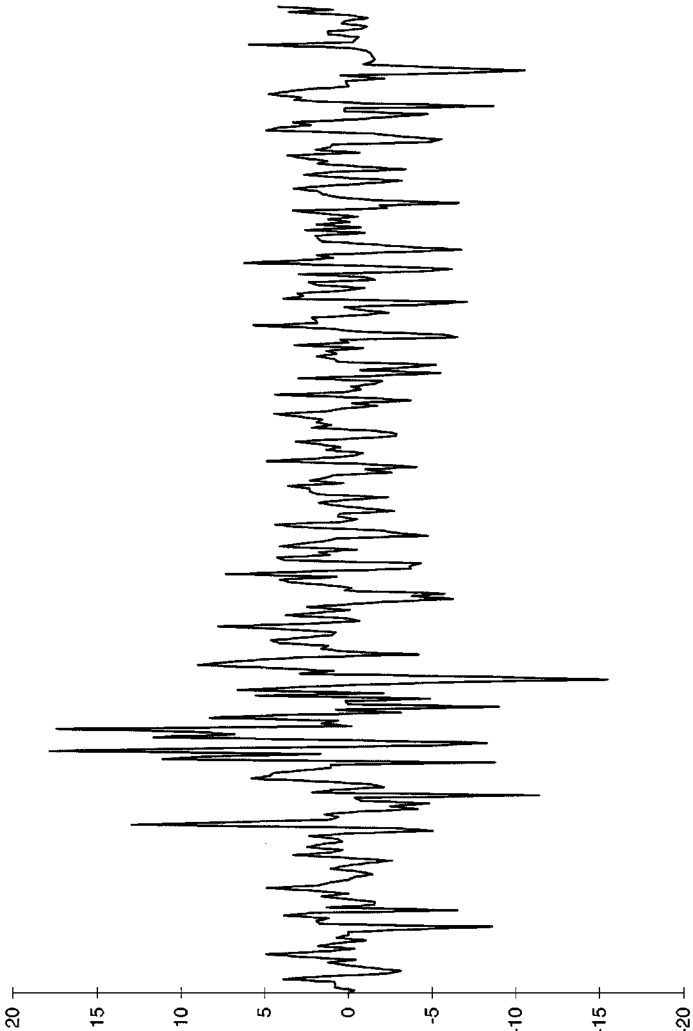

FIGURE 3. Percentage growth of average wheat prices: August 1965 to June 1995.

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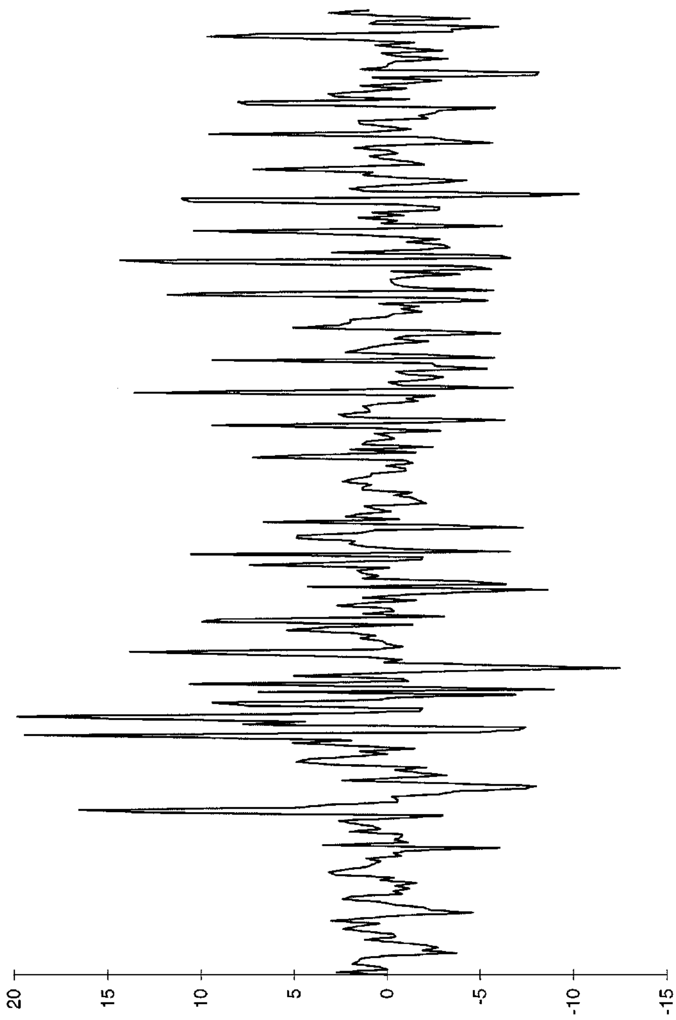

Figure 4. Percentage growth of average barley prices: August 1965 to June 1995.

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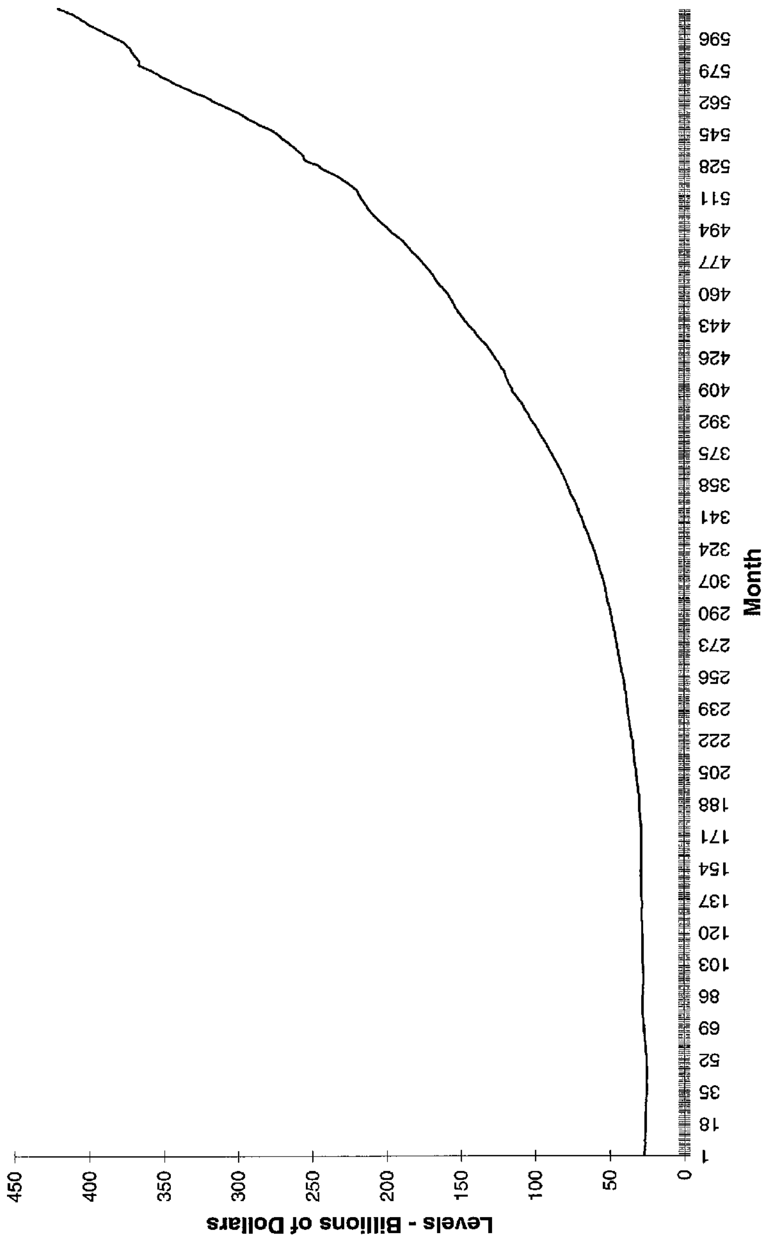

FIGURE 5. U.S. seasonally adjusted currency component: January 1947–November 1997.

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Therefore, we transformed each level series into *change in growth rates* by taking first differences of the natural logarithm of the original level series, and then first differencing these transformed data series. With this transformation, we lose two observations; hence, the sample size is 609 observations.

In this section, the data are prewhitened by employing an AR(22) fit, which was adequate to prewhiten the data. Summary statistics associated with the AR(22) fits are listed in Tables 13A and 14A. The adjusted R^2 values for the unadjusted and seasonally adjusted models are 0.919 and 0.466, respectively. The standard error of the AR fits are 0.0034 and 0.0022, respectively. Neither set of residuals "trips" the Hinich Portmentau C test for autocorrelation [see Hinich (1996)]. The p values of 0.867 and 0.982 indicate that the null hypothesis of pure white noise cannot be rejected at the 0.05 level of significance. The descriptive statistics of the residuals from the AR(22) fits are documented in Tables 13B and 14B.

Recall that because we are using monthly data and are interested in the annual (12-month) cycle and its subharmonics, we adopt a frame length and resolution bandwidth corresponding to 24 observations. Tables 13C and 14C contain the parameter values and test statistic results associated with the application of the test statistic. The sample size was 609 observations, and with the adopted frame length (L) of 24 observations, generated 25 frames (P).

The results from applying the test to these transformed series indicate evidence of significant structure for the transformed unadjusted series: The stationarity p value is 0.0000, indicating strong rejection of the null hypothesis of stationarity (see Table 13C). In contrast, the results of the test applied to the transformed seasonally adjusted series indicate that there is no significant randomly modulated seasonal structure. The stationarity test p value was 0.9250, which means that we cannot reject the null hypothesis of stationarity (see Table 14C).

The above results indicate that the seasonal adjustment procedure employed by the FRB removes any nonstationarity associated with random variation in the phase and amplitude of the annual cycle and its subharmonics. As such, the seasonal adjustment procedure is removing more than just the deterministic seasonal component of the time series in question. It is also apparent that it must be the seasonal adjustment techniques employed by the FRB that are removing any randomly modulated periodic structure because randomly modulated variation is still evident in the seasonally unadjusted series. Furthermore, the removal of any randomly modulated periodic structure in the seasonally adjusted series was not an artefact of any filtering operation performed in activating our test because these operations, notably the AR prewhitening fit and centering operation to remove any mean periodicity, did not remove the randomly modulated structure from the seasonally unadjusted series. Finally, note that the unit root tests did not collectively account for the presence of randomly modulated periodic structure in the seasonally unadjusted series and lack of such structure in the seasonally adjusted series. In both cases, the source time series were found to be $I(0)$ and hence stationary.

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Table 12A. Unit root test results for seasonally unadjusted U.S. currency time series

	Levels				First differences				Second differences			
Test details	Calc.	5% Crit.	1% Crit.	Concl.	Calc.	5% Crit.	1% Crit.	Concl.	Calc.	5% Crit.	1% Crit.	Concl.
DF	23.73	−1.94	−2.569	$I(2)$	−14.01	−1.94	−2.569	$I(0)$	−35.04	−1.94	−2.569	$I(0)$
ADF(25)	3.725	−1.94	−2.569	$I(2)$	−0.863	−1.94	−2.569	$I(1)$	−6.928	−1.94	−2.569	$I(0)$
ADF(25) with constant	3.592	−2.867	−3.444	$I(2)$	−0.099	−2.867	−3.444	$I(1)$	−7.083	−2.867	−3.444	$I(0)$
ADF(25) with constant and trend	3.436	−3.42	−3.978	$I(2)$	−2.492	−3.42	−3.978	$I(1)$	−7.154	−3.42	−3.978	$I(0)$
ADF(25) with constant and seasonals	3.924	−2.867	−3.444	$I(2)$	−0.088	−2.867	−3.444	$I(1)$	−6.49	−2.867	−3.444	$I(0)$
ADF(25) with constant, trend, and seasonals	3.768	−3.42	−3.978	$I(2)$	−2.559	−3.42	−3.978	$I(1)$	−6.564	−3.42	−3.978	$I(0)$

	Growth rates				Change in growth rates			
Test details	Calc.	5% Crit.	1% Crit.	Concl.	Calc.	5% Crit.	1% Crit.	Concl.
DF	−17.61	−1.94	−2.569	$I(0)$	−33.7	−1.94	−2.569	$I(0)$
ADF(25)	−0.501	−1.94	−2.569	$I(1)$	−7.187	−1.94	−2.569	$I(0)$
ADF(25) with constant	−2.351	−2.867	−3.444	$I(1)$	−7.224	−2.867	−3.444	$I(0)$
ADF(25) with constant and trend	−2.545	−3.42	−3.978	$I(1)$	−7.286	−3.42	−3.978	$I(0)$
ADF(25) with constant and seasonals	−2.3	−2.867	−3.444	$I(1)$	−6.978	−2.867	−3.444	$I(0)$
ADF(25) with constant, trend, and seasonals	−2.474	−3.42	−3.978	$I(1)$	−7.039	−3.42	−3.978	$I(0)$

TABLE 12B. Unit root test results for seasonally adjusted U.S. currency time series

	Levels				First differences				Second differences			
Test details	Calc.	5% Crit.	1% Crit.	Concl.	Calc.	5% Crit.	1% Crit.	Concl.	Calc.	5% Crit.	1% Crit	Concl.
DF	57.69	−1.94	−2.569	$I(2)$	−3.888	−1.94	−2.569	$I(0)$	−34.81	−1.94	−2.569	$I(0)$
ADF(25)	5.154	−1.94	−2.569	$I(2)$	1.009	−1.94	−2.569	$I(1)$	−4.827	−1.94	−2.569	$I(0)$
ADF(25) with constant	6.117	−2.867	−3.444	$I(2)$	−0.119	−2.867	−3.444	$I(1)$	−5.003	−2.867	−3.444	$I(0)$
ADF(25) with constant and trend	5.217	−3.42	−3.978	$I(2)$	−2.453	−3.42	−3.978	$I(1)$	−5.101	−3.42	−3.978	$I(0)$
ADF(25) with constant and seasonals	5.272	−2.867	−3.444	$I(2)$	−0.115	−2.867	−3.444	$I(1)$	−4.966	−2.867	−3.444	$I(0)$
ADF(25) with constant, trend, and seasonals	5.182	−3.42	−3.978	$I(2)$	−2.422	−3.42	−3.978	$I(1)$	−5.064	−3.42	−3.978	$I(0)$

	Growth rates				Change in growth rates			
Test details	Calc.	5% Crit.	1% Crit.	Concl.	Calc.	5% Crit.	1% Crit.	Concl.
DF	−6.297	−1.94	−2.569	$I(0)$	−40.79	−1.94	−2.569	$I(0)$
ADF(25)	−0.501	−1.94	−2.569	$I(1)$	−6.429	−1.94	−2.569	$I(0)$
ADF(25) with constant	−2.358	−2.867	−3.444	$I(1)$	−6.478	−2.867	−3.444	$I(0)$
ADF(25) with constant and trend	−2.961	−3.42	−3.978	$I(1)$	−6.529	−3.42	−3.978	$I(0)$
ADF(25) with constant and seasonals	−2.336	−2.867	−3.444	$I(1)$	−6.42	−2.867	−3.444	$I(0)$
ADF(25) with constant, trend, and seasonals	−2.933	−3.42	−3.978	$I(1)$	−6.471	−3.42	−3.978	$I(0)$

Table 13. AR(22) fit of seasonally unadjusted U.S. currency model: Change in growth rates ($N = 609$)

A. Summary Statistics		
Lag	Coefficient	t values
1	−0.79	−20.76
2	−0.65	−13.62
3	−0.52	−9.61
4	−0.56	−9.67
5	−0.41	−6.77
6	−0.40	−6.32
7	−0.45	−6.99
8	−0.44	−6.57
9	−0.30	−4.30
10	−0.44	−6.36
11	−0.47	−6.69
12	0.43	6.13
13	0.24	3.39
14	0.09	1.37
15	−0.03	−0.50
16	0.01	0.09
17	−0.09	−1.48
18	−0.11	−1.79
19	−0.03	−0.45
20	−0.02	−0.33
21	−0.16	−3.37
22	−0.05	−1.27
Adjusted R^2		0.919
Standard error of AR fit		0.0034
Hinich Portmentau C statistic test for autocorrelation P value		0.867
B. Descriptive Statistics of Residuals		
Mean		−0.0001
Standard deviation		0.0033
Skewness		0.042
Kurtosis		0.417
Maximum value		0.011
Minimum value		−0.012
C. Spectral Properties of Residuals		
Sampling interval		1.00 month
Frame size		24
Resolution bandwidth		24.00 months
No. of frames		25
Passband		(24.00 2.00) month
Stationarity test p value		0.0000
No. of frequencies in band		12
Large sample standard error		0.8686

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Table 14. AR(22) fit of seasonally adjusted U.S. currency model: Change in growth rates ($N = 609$)

A. Summary Statistics		
Lag	Coefficient	t values
1	−0.76	−19.16
2	−0.62	−12.38
3	−0.42	−7.58
4	−0.41	−6.97
5	−0.26	−4.33
6	−0.21	−3.38
7	−0.20	−3.19
8	−0.15	−2.38
9	−0.06	−1.03
10	−0.07	−1.10
11	−0.07	−1.23
12	−0.18	−3.01
13	−0.13	−2.20
14	−0.12	−1.97
15	−0.18	−2.95
16	−0.22	−3.56
17	−0.15	−2.53
18	−0.09	−1.45
19	−0.07	−1.23
20	−0.02	−0.45
21	−0.03	−0.51
22	−0.09	−2.21
Adjusted R^2		0.466
Standar error of AR fit		0.0022
Hinich Portmentau C statistic test for autocorrelation P value		0.982

B. Descriptive Statistics of Residuals	
Mean	−0.000004
Standard deviation	0.0022
Skewness	−0.114
Kurtosis	1.33
Maximum value	0.0097
Minimum value	−0.0092

C. Spectral Properties of Residuals	
Sampling interval	1.00 month
Frame size	24
Resolution bandwidth	24.00 months
No. of frames	25
Passband	(24.00 2.00) months
Stationarity test p value	0.9250
No. of frequencies in band	12
Large sample standard error	0.8686

6. CONCLUSIONS

In this paper, a stationarity test was developed that can be applied to residuals from AR fits to test whether the residuals are white noise. In theoretical terms, the test makes use of the fact that the mean of the complex amplitude of the discrete Fourier transform is zero for all frequencies. The only assumptions we made about the innovations of the AR process was that they are pure noise and have finite moments.

The test that we developed will have considerable power against an important form of nonstationarity not considered so far in the mainstream time-series literature—namely, where the time series has a mean that is periodic with random variation in its waveform. The importance of testing for this type of nonstationarity reflects two key issues. First, this type of model is based on the proposition that both nature and society rarely generate periodic processes that are perfectly periodic. There is usually some random variation in the structure of these periodic processes. Second, from a modeling perspective, the test will help to establish whether it is legitimate to employ a model that fixes the periodic structure of the process or whether one has to employ a model that allows the periodic structure of the process to evolve over time.

To assess size and power of the proposed test statistic, an AR(2) model was generated in a simulation experiment that had two stable conjugate root pairs. The innovations used in the simulations were either independently distributed normal $N(0,1)$, double-tailed exponential, or uniform pseudorandom variates.

The relevance of the test to actual economic application was demonstrated by testing the stationarity of residuals from an AR fit of monthly time-series data on average wheat and barley prices in the United Kingdom for the period August 1965 to June 1995. It was argued that these time series were likely to contain a natural form of seasonality because of the effect that variation in weather conditions could exert upon crop size and quality. Unit root tests conducted on both level series indicated that the series were $I(1)$. We transformed the level series into growth rates by taking natural logarithms and then first differencing. For both average cereal price series, AR(18) fits were adopted. The evidence we obtained from applying the test to the residuals of the AR prewhitening fits suggest that the seasonal patterns are significant. This means that the residuals contain significant periodic structure, which is not consistent with the null hypothesis of white-noise residuals.

We also applied our test to the seasonally unadjusted and adjusted U.S. currency component of the money stock figures, for the period January 1947 to November 1997. Unit root tests indicated that both level series were $I(2)$. Hence, we adopted change in growth rate specifications. We also employed AR(22) fits to prewhiten the data. The results of our investigation indicate that there is embedded seasonal structure in the seasonally unadjusted series but this structure is effectively removed by the seasonal adjustment filter employed by the FRB. Hence, the seasonal adjustment techniques are clearly removing more than deterministic seasonal structure.

The key implication therefore from the test is that there is embedded structure in the residuals, pointing to the existence of unexplained seasonality. In a forecasting context, this information will be important if all available information about the process is to be used in forecasting. In a modeling context, it is evident that one would have to use a model that allows the periodic (seasonal) structure to vary over time. This raises the issue of how best to model such evolving processes. One possibility would be to adopt model frameworks alluded to by Harvey (1989) and Priestley (1981). Another possible approach relates to the explicit use of the information obtained from the test developed in this paper in a spectral regression framework. Research on this latter approach, together with a fuller treatment of the power properties of the test statistic, is being undertaken.

NOTES

1. Consult Priestley (1988, p. 174) for a survey of this literature.
2. Also consult Cohen (1989), Artis et al. (1992), and Foster and Wild (1995).
3. The latter category of model would include the evolving models outlined by Harvey (1989, pp. 39, 42–43) and Priestley (1981, p. 600; 1988), for example. Also consult Harvey (1997, p. 198).
4. An alternative prewhitening method to the fitting of an AR(p) model is to divide $Y_p(k)$ by the square root of the frame-averaged spectra at frequency k [see Hinich (1982) and Hinich and Rothman (1998)]. Simulation results indicate that the size and power properties of the test statistic reported in Section 3 are robust to the type of prewhitening operation adopted.
5. A Fortran 77 simulation program and a program to compute the test are available from the authors on request.
6. For details on factors affecting the growth cycle of cereals in the United Kingdom, see Colman (1972, pp. 27–31).
7. See Hamilton (1994, p. 438) for a discussion of the economic rationale for adopting a growth-rate specification.
8. A copy of the output files that list all of the reported results is available from the authors on request.
9. The World Wide Web address for the FRED database is http://www.stls.frb.org/fred/.
10. A lag of 25 was employed in the ADF tests.

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APPENDIX

THEOREM 1. *Assume that* $\{y(n)\}$ *is a pure white-noise process where* $Ey(n) = 0$ *and* $Ey^2(n) = 1$. $\langle A_y(k)\rangle$ *is the average of P complex amplitudes*

$$A_y\left(\frac{k}{p}\right) = \frac{1}{\sqrt{L}} \sum_n y(n + (p-1)L) \exp\left(\frac{-i2\pi kn}{L}\right).$$

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The asymptotic distribution of $\{\sqrt{P}\langle A_y(1)\rangle, \ldots, \sqrt{P}\langle A_y(L/2)\rangle\}$ *is a complex normal N*(0,1) *distribution as* $P \to \infty$ *with* L *fixed.*

Proof. The expected values of Re $A_y(k/p)$, the real part of $A_y(k/p)$ and the imaginary part Im $A_y(k/p)$ are zero. Their covariance is the sum for $n = 0, \ldots, L-1$ of $\sin(2\pi kn/L)$ $\cos(2\pi kn/L) = \sin(4\pi kn/L)/2$ since the y's are independent and have unit variance. The sum of $n = 0, \ldots, L-1$ of $\sin(2\pi jn/L)$ [and $\cos(2\pi jn/L)$] are zero for any integer j and thus Re $A_y(k/p)$ and Im $A_y(k/p)$ are uncorrelated. The variances of Re $A_y(k/p)$ and Im $A_y(k/p)$ are equal to $1/2$ since $\cos^2(2\pi kn/L) = [1 + \cos(4\pi kn/L)]/2$ and $\sin^2(2\pi kn/L) = [1 - \cos(4\pi kn/L)]/2$.

Re $A_y(k_1/p)$ and Re $A_y(k_2/p)$ are uncorrelated since $\cos(2\pi k_1 n/L)\cos(2\pi k_2 n/L) = [\cos(2\pi(k_1 - k_2)n/L) + \cos(2\pi(k_1 + k_2)n/L)]/2$ and similarly for Im $A_y(k/p)$ and Im $A_y(k/p)$.

Since the frames are independent, so are the $A_y(k/p)$ for $p = 1, \ldots, P$. Thus, by the central limit theorem [Billingsley (1986, Theorem 27.5)], $\{\sqrt{P}\langle \text{Re } A_y(1)\rangle, \ldots, \sqrt{P}\langle \text{Re } A_y(L/2)\rangle\}$ are asymptotically independent normal $N(0,1/2)$ variates as $P \to \infty$, and similarly for $\{\sqrt{P}\langle \text{Im } A_y(1)\rangle, \ldots, \sqrt{P}\langle \text{Im } A_y(L/2)\rangle\}$. In addition, the real and imaginary components are asymptotically independent.