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ON DOMINANCE AND CONTEXT-DEPENDENCE IN DECISIONS INVOLVING MULTIPLE ATTRIBUTES

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In decision-making involving multiple criteria or attributes, a decision maker first identifies all relevant evaluative attributes in making decisions. Then, a dominance principle is often invoked whenever applicable: whenever an option x is better than an option y in terms of some attribute and no worse than y in terms of any other attributes, x is judged to be better than y. If, however, this dominance principle is not applicable, then the decision maker determines the relative importance between the identified evaluative attributes, consults with contextual features of the options under consideration, and makes a decision. It is shown that the combination of these principles runs into problems in the presence of rationality properties, such as transitivity, and a weak continuity requirement on decisions. The paper gives examples from welfare economics, and theories of individual and group decisions.

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1. INTRODUCTION

In this paper, we discuss a difficulty in decision-making when there are multiple evaluative attributes in terms of which the alternatives or options[1] are assessed.[2] We start with some examples from welfare economics, social choice theory and individual decision-making in the presence of moral conflicts.

Example 1 (Comparison of living standards). Suppose we have some real indicators of living standards, or, *functionings*, to use the terminology of Sen (Sen 1985, 1987), such as health, education, nutrition, leisure, etc. Using these functionings, all of which represent desirable evaluative attributes or criteria, we want to compare the well-being or living standards of a given individual in different situations and also the well-being or living standards of different individuals.[3] In this context, it has been suggested that, whenever an individual, p, is doing better than an individual, r, in terms of some of the functionings and is not doing worse in terms of any one of the functionings, p's overall living standard should be deemed to be at least as high as r's living standard, irrespective of the values and/or the social and cultural backgrounds of the two individuals. This is simply the *principle of dominance* as applied to standard of living comparisons, achievements in terms of the functionings being the evaluative criteria in terms of which dominance is defined. If, however, x and y are two functioning bundles one of which has more of some functionings and the other has more of some other functionings, then, in comparing a given individual's standard of living from the two functioning bundles, x and y, one can take a 'relativistic' position and allow the values of the individual under consideration and the norms and mores prevailing in her society to play some role in weighing the gains in terms of some functionings against the losses in terms of some other functionings. This is an example of what we call *context-dependence*, the values of the individual and the mores of her society being the contextual factors, which are invoked when neither functioning bundle dominates the other. Pattanaik and Xu (2007) called such context-dependence in standard of living comparisons 'minimal relativism'. As shown in Pattanaik and Xu (2007), if comparisons of living standards satisfy the principle of dominance and are context-dependent in the sense just explained, they cannot be both transitive and continuous. At the risk of emphasizing the obvious, note that the culture or social background of

[1] We use the terms 'alternatives' and 'options' interchangeably.

[2] In an unpublished paper, Weymark (2008) investigates similar issues, and, like us, he also discusses the commonality of the structure underlying several different decision problems. Weymark's paper and ours were written independently of each other, and we are grateful to Weymark for kindly letting us read his unpublished paper.

[3] We use the terms 'well-being' and 'standard of living' interchangeably.

a person is not a criterion in its own right. It does not admit the notion of 'more' or 'less'; nor does the appeal to culture in the decision process imply any judgement that one culture or social background is better or more desirable than another. It does, however, influence the trade-offs between the different evaluative attributes, i.e. the functionings, in our example.

Fleurbaey (2007*a*) proves a result closely related to that of Pattanaik and Xu (2007). Fleurbaey's Proposition 3 deals with the issue of ranking ordered pairs, (x, u), (y, v), . . . , where x, y, etc. denote alternative bundles of functionings or Rawlsian primary goods and u, v, etc. denote different utility functions. Thus, (x, u) refers to the 'situation' of an individual who has the bundle of functionings/primary goods x and the utility function u. In our terminology, the different functionings or primary goods are the evaluative criteria and the utility function constitutes the contextual feature. Fleurbaey's proposition demonstrates that, if the ranking of individual situations satisfies dominance with respect to the functionings or primary goods (i.e. with respect to the evaluative criteria) and continuity, then it must be completely insensitive to the contextual features (i.e. the utility functions).[4]

Example 2 (The Pareto principle and non-utility considerations). A basic value judgement, widely used in welfare economics, is the Pareto principle, which requires that, if a social state x offers more utility to some individuals in the society as compared to another social state y and no less utility to any individual as compared to y, then x is socially better than y. This is again the principle of dominance, the individuals' utilities being the evaluative attributes. If some individual, i, has higher utility in x than in y and some other individual, j, has higher utility in y than in x, then the Pareto principle does not apply. In these cases where the Pareto principle does not apply, it seems tempting to allow non-utility considerations to influence the issue of how one should weigh one individual's loss of utility against another individual's gain in utility. Thus, one can take the position that, if x and y differ only with respect to some aspect that relates to individual p's personal life and p's utility from x is higher than p's utility from y, then the consideration of p's utilities from x and y must take strict priority over the consideration of other individuals' utilities and x should be considered socially better than y; and one can take an analogous position with respect to the comparison of z and w, which differ only with respect to some aspect that relates to the personal life of another individual r (see Sen 1970*a*, *b*). This, of course, is an example of context-dependence. Again, note that the physical descriptions of the social states, which determine whether two social states differ solely with respect to

[4] See also Brun and Tungodden (2004), who, in the context of social choice, discuss implications of the principle of dominance.

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the private life of a given individual, do not constitute an independent criterion for judging the social states, though these physical descriptions may influence our judgement about whether we should give priority to one individual's gain (resp. loss) of utility over another individual's loss (resp. gain) in utility. As shown in Section 4, such context-dependence based on the notion of the private life of an individual and the dominance rule embodied in the Pareto principle cannot be both satisfied if the social ranking is transitive and continuous in the individual utilities. Kaplow and Shavell (2001) have a related result with similar but slightly different structural features.

Example 3 (Individual decision-making in the presence of moral conflicts). In the following example due to Hare (2007), an agent faces a moral problem.[5] There are several persons who are in distress and need help. The agent is faced with the problem of ranking, in terms of his ethical obligations, the options of helping these different individuals and also the option of helping nobody. There are three evaluative attributes: the urgency of a person's need for help (the more urgent the need, the greater the moral obligation of the agent to help); the extent to which the person in distress is responsible for his own situation (the less the person's responsibility, the greater the obligation of the agent to help); and the cost involved in helping the person in distress (the lower the cost, the greater the agent's moral obligation). In ranking the options of helping either of two individuals, the agent goes first by the dominance principle defined in terms of these three criteria whenever the dominance principle is applicable. If, however, these criteria conflict in some comparisons, then the agent needs to assess the relative importance of these different criteria. In doing so, he looks at contextual features such as whether the person in need is an adult member of his (i.e. the agent's) family or the person in need of help is a very young and inexperienced person who happens to be a stranger or the person is a very young and inexperienced family member, and so on. If the person happens to be young and inexperienced, the criterion of responsibility is given relatively less importance; on the other hand, if the person is a relation rather than a stranger, the criterion of cost is given relatively less importance. In considering these contextual features when the three moral criteria conflict, the agent does not use the contextual features as independent moral criteria; instead, he allows them to bear on the relative importance that he attaches to his evaluative attributes, the urgency of need of the person, the extent to which the person is responsible for his plight, and the cost involved in providing

[5] Hare (2007) also has another example, with a similar structure, of individual decision-making regarding non-ethical matters, and it comes within the scope of the formal results we present later. But, given our focus on ethical issues in this paper, we omit discussions of this other example of Hare.

help. In this case, again, the agent cannot have a transitive and continuous ranking over the options. Note that one of the evaluative attributes here, namely, the cost to the decision-maker, is, intuitively, an undesirable attribute, but, as we explain in Section 3.1, we can formally replace it by a suitably defined desirable evaluative attribute without losing any part of the underlying intuition.

While the three examples above deal with seemingly very different issues, they have the following common theme. In all of them, an agent is faced with the problem of ranking some (mutually exclusive) alternatives and he uses several evaluative criteria to rank them. The agent subscribes to the principle of dominance, which says that, if an alternative, x, is at least as good as another alternative, y, in terms of all the criteria and strictly better in terms of some criterion, then, overall, x is better than y. If, however, the criteria conflict so that the principle of dominance does not apply, then the agent sometimes depends on contextual circumstances to resolve the conflict and rank the two alternatives. The principle of dominance and the agent's context-dependence, together, imply that the agent's ranking violates either transitivity or a very weak version of continuity. In this paper, we develop an inclusive analytical framework, which allows us to highlight this structural unity of the examples given above. In the process, we derive some general results which imply as special cases various results in the literature (Sen 1970*a*, *b*; Kaplow and Shavell 2001; Fleurbaey 2007; Hare 2007; Pattanaik and Xu 2007).

The remainder of the paper is organized as follows. In Section 2, we develop a framework for our analysis. Section 3 introduces several properties of the ranking of the alternatives. We present our main results in Section 4, and, in Section 5, we comment on the properties figuring in our results. Section 6 contains some brief concluding remarks.

2. THE FRAMEWORK

Let $C = \{1, 2, \ldots, m\}$ be the set of all evaluative attributes or criteria, where m is an integer greater than 1. The interpretation of these evaluative attributes will be different in different contexts as seen from the examples given in Section 1. For every evaluative attribute j in C, let X_j be the set of all possible values (or 'levels') of attribute j; we assume that, for every j in C, X_j has at least two elements. For some attributes, such as the amount of leisure enjoyed by a person in the problem considered in Example 1, the values or levels may be real numbers. On the other hand, for some attributes, such as the health of a person in Example 1, the possible values or levels may be indicated by qualitative epithets (e.g. 'excellent', 'good', 'poor' and 'very poor'); in these cases, the qualitative epithets are assumed

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to be such that no two distinct epithets indicate the same value or level of the attribute.

Assumption 1. For every j in C, there exists a linear ordering[6] R_j defined over X_j. Further, for some attribute j in C, X_j is a real interval $[\beta_j, \alpha_j]$, such that $-\infty \leqslant \beta_j < \alpha_j \leq \infty$ and, for all s, t in $[\beta_j, \alpha_j]$, sR_jt if and only if $s \geqslant t$.

For every evaluative attribute j, it makes sense to speak of more or less of the attribute j, and the ordering R_j formally captures this notion of more or less. The ordering R_j stands for the binary relation 'represents at least as high a level of evaluative attribute j as'. For every j in C, and for all x_j, x'_j in X_j, $x_jR_jx'_j$ denotes that x_j represents at least as high a level of attribute j as x'_j. Given this interpretation and given our interpretation of X_j as the set of all possible different levels or values of attribute j, it seems natural to assume that the ordering R_j is linear (i.e. no distinct x_j and x'_j in X_j can represent the same level of attribute j). Let P_j denote the antisymmetric factor of R_j (so that, intuitively, P_j stands for 'represents a higher level of attribute j than').

Let D denote the set of those evaluative attributes j for which X_j is an interval $[\beta_j, \alpha_j]$, such that $-\infty \leq \beta_j < \alpha_j \leq \infty$ and for all s, t in $[\beta_j, \alpha_j]$, sR_jt if and only if $s \geqslant t$. It may be noted that, given our Assumption 1, the set D is non-empty. Under Assumption 1, it is possible to have evaluative attributes, such as health, which are ordinally measurable but which are not cardinally measurable.

In Example 1, the set of all evaluative criteria is the set of all functionings. It is often assumed that every functioning is measurable on a ratio scale along a non-degenerate real interval, so that $D = C$. In Example 2, the utilities are the evaluative attributes in C, and in Example 3, the evaluative criteria are: the urgency of a person's need for help, the extent to which the person in distress is responsible for his own situation, and the cost involved in helping the person in distress.

Let X_{m+1} be the non-empty set of all alternative specifications of the contextual features in the problem under consideration. We assume that X_{m+1} contains at least two elements. Thus, in our Example 1 in Section 1, the values and the cultural and social background of the individual whose standard of living is under consideration constitute the contextual feature. Similarly, in Example 2 there, the physical description of the social state constitutes the contextual feature, and in Example 3, 'being a member of the family', 'being young', etc., constitute the contextual features.

[6] Recall that R_j is an ordering over X_j iff R_j is reflexive, transitive, complete, and the ordering R_j is linear if and only if, for all distinct $x_j, x'_j \in X_j$, not$[x_jR_jx'_j$ and $x'_jR_jx_j]$.

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Let $X \equiv X_1 \times X_2 \times \cdots \times X_{m+1}$ be the set of all conceivable alternatives. Let $\succeq$ be a binary relation defined over X, the asymmetric and symmetric parts of $\succeq$ being denoted by $\succ$ and $\sim$, respectively. In the context of Example 1, for all x, y in X, $x \succeq y$ is to be interpreted as denoting that the standard of living corresponding to x is at least as high as the standard of living corresponding to y. Analogous interpretations of $\succeq$ can be given in the context of Examples 2 and 3.

We say that $\succ$ is *acyclic* if and only if there does not exist a $\succ$-cycle in X, i.e. there do not exist $x^1, x^2, \ldots, x^h$ in X, such that $[x^1 \succ x^2$ and $x^2 \succ x^3$ and ... and $x^{h-1} \succ x^h$ and $x^h \succ x^1]$. It is well-known that acyclicity of $\succ$ is a much weaker requirement than transitivity of $\succeq$.

3. ALTERNATIVE NOTIONS OF DOMINANCE, CONTEXT-DEPENDENCE AND CONTINUITY

3.1 Weak dominance and dominance

We introduce two alternative formulations of the dominance principle.

> $\succeq$ satisfies *dominance*(resp.*weak dominance)* iff, for all x and y in X and for all $j \in C$, if $x_i = y_i$ for all $i \in C - \{j\}$ and $x_j P_j y_j$, then $x \succ y$ (resp. $x \succeq y$).

The underlying intuition of dominance and weak dominance is familiar. We would, however, like to note two points. First, dominance and weak dominance implicitly assume that all evaluative attributes are 'desirable'. This is true for most of the examples that we discussed in Section 1. What if some of the evaluative attributes in C are, like the decision-maker's cost in Example 3 in Section 1, undesirable? This, however, is not a serious problem. Consider the evaluative attribute of cost to the decision-maker. Suppose this cost can be anything between 0 dollars and 5000 dollars. Then, while modelling the problem, we can replace the evaluative attribute of cost by a formally specified evaluative attribute g (one can call it the decision-maker's monetary benefit), which can take any value in the interval [-5000, 0]. An increase in g will then be intuitively equivalent to a decrease in the cost and will make the option more desirable. In general, without any loss of intuition, an undesirable evaluative attribute, j, with the set of possible values, X_j, can be replaced by a desirable attribute, j^*,with a set of possible values, X_j^*, such that, for some one-to-one function f from X_j to X_j^* and for some linear ordering R_{j^*}over X_j^*, we have [for all $x_j, x_j' \in X_j$,$x_j R_j x_j'$ iff $f(x_j') R_{j^*} f(x_j)$]; intuitively, j^* can be thought of as the 'opposite' of attribute j.

3.2 Context-dependence

The basic intuitive idea of context-dependence is that, though contextual features do not constitute evaluative criteria on their own, they may,

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nevertheless, influence the ranking of options by affecting the relative importance to be attached to the different evaluative criteria in the absence of dominance in terms of these criteria. One can, however, think of different alternative formulations of this basic idea. To deal with the examples that we started with in Section 1, we need the following two versions of our intuitive notion of context-dependence.

$\succeq$ satisfies *type 1 context-dependence* iff there exist a, b, c, d in X, such that [for all j in C, $a_j = c_j$ and $b_j = d_j$], [for some $k \in D, a_k \neq b_k$], and [$a \succ b$ and $d \succ c$].

$\succeq$ satisfies *type 2 context-dependence* iff there exist $x, y \in X$, such that ($x_j = y_j$ for all $j \in C$) and $x \succ y$.

Type 1 context-dependence stipulates the existence of four alternatives a, b, c and d, such that a and b are different with respect to some attribute in D, a and c are indistinguishable in terms of the evaluative attributes and so are b and d, but, overall, a ranks higher than b and d ranks higher than c. Clearly, this happens because the contextual features in a are different from the contextual features in c or the contextual features in b are different from the contextual features in d. The context-dependence alluded to in Examples 1, 2, and 3 is type 1 context-dependence. Consider, for instance, Example 2. Here $C = D$ can be interpreted as the set of individual utilities and X_{m+1} can be interpreted as the set of physical descriptions of the social state. Then Sen's (Sen 1970*a*, *b*) minimal liberalism would correspond to type 1 context-dependence insofar as minimal liberalism ensures the existence of alternatives a, b, c and d, such that a and b differ only with respect to individual i's personal matters (which figure in a_{m+1} and b_{m+1}), c and d differ only with respect to individual j's personal matters (which figure in c_{m+1} and d_{m+1}); $a_i = c_i > b_i = d_i$; $b_j = d_j > a_j = c_j$; a is ranked higher than b socially (given $a_i > b_i$); and d is ranked higher than c socially (given $d_j > c_j$).[7]

Type 2 context-dependence is not logically comparable with type 1 context-dependence. It stipulates the existence of two alternatives x and y, such that x and y are indistinguishable in terms of the evaluative attributes, but, overall, x ranks higher than y. Apparently, the higher rank of x as compared to y is due to differences between the contextual features in x and those in y. This is the type of context-dependence used in Kaplow and Shavell (2001).

To avoid possible misunderstanding, we would like to note briefly that our notion of context-dependence is different from the phenomenon of menu-dependence (i.e. the phenomenon of choice that depends on

[7] Strictly speaking, it is the combination of Sen's minimal liberalism and his assumption of unrestricted domain, which ensures the existence of alternatives a, b, c and d fulfilling the conditions stipulated here.

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the presence or absence of 'irrelevant' alternatives in the set of feasible options) discussed by many writers. Contributions on menu-dependence are too diverse for us to consider here all the different strands in those contributions. We would, however, like to comment on the type of menu-dependence highlighted in certain examples of Luce and Raiffa (1957) and Sen (1993), which were intended to demonstrate how 'reasonable' agents might violate the properties of rational choice familiar in economics, and which have given rise to a sizable literature. Essentially, the phenomenon of menu-dependence highlighted by the examples of Luce and Raiffa (1957) and Sen (1993) arises either from the existence of menu-dependent evaluative criteria which refer to aspects not captured in the description of the options or from the existence of menu-dependent information, which is relevant for the agent's decisions but which does not constitute a part of the description of the options. Menu-dependence in this sense has little to do with context-dependence as we define it here. The basic intuition of our notion of context dependence refers to the fact that, while an agent may use a given set of evaluative criteria to evaluate the options, the way he may resolve a possible conflict among these evaluative criteria may depend on aspects ('contextual features') of the descriptions of options, which do not constitute the basis of any evaluative criterion.

3.3 Continuity in attributes in D

In microeconomics, continuity of a preference relation is often invoked. In this paper, we consider the following two continuity properties.

> $\succeq$ satisfies *weak continuity in attributes in* D iff, for all x and y in X, and, for all $j \in D$, ([$x \succ y$ and $x_j > \beta_j$] implies [for some x' in X, $x'_j < x_j$, $x_k = x'_k$ for all $k \in C - \{j\}$, and $x' \succ y$]) and ([$x \succ y$ and $y_j < \alpha_j$] implies [for some y' in X, $y'_j > y_j$, $y'_k = y_k$ for all $k \in C - \{j\}$, and $x \succ y'$]).

> $\succeq$ satisfies *minimal continuity in attributes in* D iff, for all x and y in X, and, for all $j \in D$, ([$x \succ y$, $x_j > \beta_j$, and $x_j \neq y_j$] implies [for some x' in X, $x'_j < x_j$, $x_k = x'_k$ for all $k \in C - \{j\}$, and $x' \succ y$]) and ([$x \succ y$, $y_j < \alpha_j$, and $x_j \neq y_j$] implies [for some y' in X, $y'_j > y_j$, $y'_k = y_k$ for all $k \in C - \{j\}$, and $x \succ y'$]).

It may be noted that minimal continuity is weaker than weak continuity. It is easy to check that weak continuity rules out standard lexicographic orderings (defined with reference to attributes in D) over X, though such lexicographic orderings satisfy minimal continuity. In this paper, we mostly use the property of minimal continuity rather than the somewhat stronger property of weak continuity. This allows us to bring within the scope of our results certain types of intuition, discussed by several writers, which would be excluded by the use of weak continuity. Many writers have suggested a hierarchy of human needs, at least over certain ranges

of values of the attributes (see, for example, Maslow 1943, 1954; see also Streeten and Burki 1978, Streeten *et al.* 1981, and other contributions to the literature on basic needs in development economics). When we seek to compare living standards (see Example 1 above), weak continuity, by itself, will rule out such hierarchy; the existence of such hierarchy, however, is consistent with minimal continuity.

4. POSSIBILITY AND IMPOSSIBILITY

We now state and discuss some propositions which highlight the common analytical structure underlying our Examples 1,2, and 3; the proofs of these propositions are to be found in Appendix *A*.

Proposition 1. *No $\succeq$ can simultaneously satisfy transitivity (resp. acyclicity of $\succ$), weak dominance (resp. dominance), type 1 context-dependence and minimal continuity in attributes in D.*

Pattanaik and Xu's (2007) result (see Example 1), Fleurbaey's Proposition 1 (see again Example 1), and Hare's (2007) analysis (see Example 3) constitute special cases of our Proposition 1; more specifically, they are special cases of the part of Proposition 1 that uses weak dominance and transitivity of $\succeq$. The part of Proposition 1 that uses dominance and acyclicity of $\succ$ implies a variant of Sen's (1970*a*, *b*) famous impossibility of the Paretian liberal. In this variant, the dominance principle appears in the form of the Pareto criterion, type 1 context-dependence manifests itself as Sen's condition of minimal liberalism, minimal continuity of the society's ranking of options replaces Sen's original condition that the set of all possible profiles of individual orderings over the options be the domain of the social decision rule, and the social strict preference relation is assumed to be acyclic.

The implications of type 2 context-dependence in conjunction with other properties are less straightforward: depending on the specific requirement of continuity, we can have either a possibility result (Proposition 2) or an impossibility result (Proposition 3).

Proposition 2. *There exists a transitive $\succeq$ that simultaneously satisfies dominance, type 2 context-dependence and minimal continuity in attributes in D.*

Proposition 3. *There does not exist any $\succeq$ that simultaneously satisfies weak dominance, type 2 context-dependence and weak continuity in attributes in D.*

Proposition 3 constitutes a generalization of the result of Kaplow and Shavell (2001). Given Proposition 2, it is clear that Kaplow and Shavell's result is sensitive to the continuity property that one imposes on the ranking $\succeq$.

For each of the two parts[8] in Proposition 1, it is possible to demonstrate that the properties figuring in that part are independent in the sense that, for each of these properties, one can construct a binary relation $\succeq$ over X, which violates the specific property under consideration while satisfying the rest of the properties. Similarly, the properties figuring in Proposition 3 are also independent.

Before concluding this section, we would like to note one possible extension of the results stated above. It may be argued that, though weak dominance and dominance are plausible properties, they have one restrictive feature. They are based on the implicit assumption that, in our intuitive description of the problem, every evaluative attribute is either desirable over the entire range of its values, or is undesirable over the entire range of its values (in this later case, the evaluative attribute under consideration can be replaced by a suitably defined evaluative attribute, which is exactly the 'opposite' of the original evaluative attribute and is desirable over the entire range of its values). While this is true of all the examples that we discussed in Section 1 and we do not know of any example in the existing literature that violates this intuitive assumption, one can nevertheless think of examples where this may not be true. Suppose we have an analytical framework where the well-being of a person depends on the level of her calorie consumption among other things. It is possible that, intuitively, an increase in the level of calorie consumption is considered desirable until it reaches the level of 2500 calories, but, any further increase beyond that is undesirable.[9] In this case, dominance and weak dominance, as we have defined these properties earlier, would not be plausible.[10] It is, however, possible to weaken our properties of weak dominance and dominance considerably to permit such cases. Consider, for example, the following property: for every evaluative attribute, j, if $j \in D$, then, starting with any x in X, either all small increases in the amount of j make the option more attractive or all small increases in the amount of j make the option less attractive; and similarly for small decreases in the amount of j.[11] It can be shown that this property is again incompatible with context dependence in the presence of transitivity and minimal continuity.

[8] The first of the two parts, which have been merged in a single statement, uses transitivity of $\succeq$ and weak dominance, while the second uses acyclicity of $\succ$ and dominance.

[9] We are grateful to Sudhir Anand for this example.

[10] Note that no such intuitive problems regarding weak dominance or dominance arise in Sen's functioning approach to living standards (see Example 1 in Section 1) since functionings are defined in such a way that they are all desirable attributes. Thus, calories would not be a functioning in Sen's sense though being well nourished would be so.

[11] Note that this property does not impose any restriction on the desirability or undesirability of attributes in $C - D$.

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5. DISCUSSION OF THE RESULTS

Given the conflict between weak dominance and context-dependence in the presence of transitivity and minimal continuity, the question naturally arises whether one should relax or discard any of these conditions. We feel that minimal continuity is an extremely weak continuity property. The typical objection to continuity has its origin in the intuition that there may be a lexicographic hierarchy among attributes, but minimal continuity is consistent with such lexicographic hierarchy. It is not, therefore, clear that there is much intuitive justification for discarding minimal continuity. As Proposition 1 shows, we do not gain much by relaxing transitivity of $\succeq$ to the much weaker property of acyclicity of $\succ$ if we are prepared to accept dominance, a property that is almost as compelling as weak dominance. Thus, effectively, we have to choose between dropping the dominance principle and dropping context-dependence.

First, note that if we discard either the dominance principle or context-dependence, then we can easily find binary relations $\succeq$, which satisfy all the rest of the properties. For example, suppose we give up weak dominance. Then, we can find a reflexive and transitive binary relation that satisfies type 1 context dependence and minimal continuity. To see this and for ease of presentation, consider the following. Let $C = D = \{1, 2, \cdots, m\}$, $X_1 = X_2 = \cdots = [0, \infty)$ and $X_{m+1} = \{p, r\}$. Let u be a real valued function defined over X such that, for all $x \in X$, [if $x_{m+1} = p$, then $u(x) = 3x_1 + 7x_2 + x_3 + \cdots + x_m$] and [if $x_{m+1} = r$, then $u(x) = 9x_1 + x_2 + x_3 + \cdots + x_m$]. Let $\succeq_*$ be a binary relation over X, such that, for all $x, y \in X$, $x \succeq_* y$ iff $u(x) \geqslant u(y)$. It can be checked that, in addition to satisfying transitivity, type 1 context-dependence, and minimal continuity, $\succeq_*$ also satisfies the following weaker version of the dominance principle defined for any binary relation $\succeq$:

> for all x and y in X and for all $j \in C$, if $x_i = y_i$ for all $i \in C - \{j\}$, $x_{m+1} = y_{m+1}$ and $x_j > y_j$, then $x \succ y$.

But $\succeq_*$violates weak dominance. To see that $\succeq_*$ violates weak dominance, consider a, $b \in X$, such that $a_1 = 10, a_2 = 4, a_3 = \cdots = a_m = 10, a_{m+1} = p, b_1 = 9, b_2 = 3, b_3 = \cdots = b_m = 10$, and $b_{m+1} = r$. Then a dominates b in terms of the evaluative criteria, but $b \succ_* a$.

Similarly, it can be easily checked that, if we drop context-dependence, we can find reflexive and transitive binary relations satisfying weak dominance and minimal continuity.

The important issue seems to be whether there is any intuitive justification for dropping either weak dominance or context-dependence. It is difficult to assess this outside a specific context: it may be possible to argue persuasively for rejecting the dominance principle or context-dependence for one class of problems, though for a different class of

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problems the same property may make perfect sense. For example, Fleurbaey (2007*a*, *b*) has made interesting proposals for constructing social preferences based on the fairness approach where the dominance principle is constrained to be satisfied on a narrowly defined domain. In such context, Fleurbaey shows that the fairness approach is the only possibility for constructing reasonable social preferences and presents several illuminating applications (Fleurbaey 2007*b*).

Our personal inclination is to reject the dominance principle, while retaining context-dependence, in each of the three examples discussed in the introduction. For the sake of brevity, we focus on the first example, which relates to the measurement of living standards, but similar arguments can be given for Example 3 also. So far as Example 2 is concerned, the reader may like to refer to Sen's (1976) persuasive reasons for constraining the application of the dominance principle as reflected in the Pareto criterion while retaining context-dependence as reflected in Sen's (1970*a*, *b*) condition of minimal liberalism.

In Example 1, we start with the position that a person's living standards are to be judged by her achieved functioning bundle. Given this position, it would seem compelling to let the standard of living comparisons for an individual to be influenced, at least in some cases, by the relative importance that the individual herself or her society or culture attaches to the different functionings. To deny this would amount to a form of rigid universalism that rules out altogether the possibility that the individuality of the person may play some role in the assessment of her own standard of living. But, once we permit any difference between the evaluation of some functioning bundles in terms of the living standards of an individual, p, and the evaluation of the same functioning bundles in terms of the living standards of another individual, r, it is not clear that a dominance relation between the functioning bundle of p and that of r provides an intuitively firm basis for making interpersonal comparison of living standards of p and r. An analogy with the standard utility-based approach in economics to well-being or living standards may be helpful. In this approach, we identify the well-being of an individual with the utility of an individual, utility being interpreted either as preference satisfaction or as happiness.[12] It is also often assumed that an individual's utility exclusively depends on her consumption bundle and that a consumer's utility is increasing in the quantities consumed of the different commodities. But, even with these assumptions, few people would be prepared to say that, in the utility-based framework, if individual p has a bigger consumption bundle than individual r, then p's well-being is higher than that of

[12] The interpretation of utility as preference satisfaction is more widely accepted in welfare economics.

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r. In the functioning approach to the notion of personal well-being, if we permit the evaluation of some functioning bundles in terms of one individual's well-being to differ from the evaluation of the same functioning bundles in terms of another individual's well-being, then it is not clear to us that the dominance relation that may exist between the two individual's functioning bundles provides any firmer intuitive basis for interpersonal comparison of the two individuals' well-being levels than the domination relation between two individuals' consumption bundles provides for interpersonal comparison of well-being in the utility-based approach to well-being. In general, the basic point is this. Suppose we have a set of evaluative criteria for ranking different alternatives or situations. If these criteria do not capture all the considerations that are considered relevant for the evaluation of the alternatives and there are considerations ('contextual' circumstances) outside these criteria which sometimes influence the ranking of the alternatives, then the dominance principle defined in terms of the initial set of evaluative criteria seems to lose much of its appeal unless these contextual circumstances happen to be the same in the two alternatives. This preliminary intuition, however, needs further study.

One suggestion that we have received is to retain dominance but to enlarge the set of evaluative criteria by incorporating in the set of criteria the contextual factors after transforming them into 'objective' criteria. This is an interesting suggestion, but, in its general form, it needs careful and detailed investigation. Such investigation is beyond the scope of the present paper, the main purpose of which is to bring out the underlying unity of several problems discussed in very diverse intuitive contexts.

6. CONCLUDING REMARKS

In this paper we have sought to highlight how several contributions dealing with seemingly very different problems have a common underlying formal structure. All of them involve: (i) evaluation of certain alternatives in a multi-criterial setting; (ii) the dominance principle; (iii) dependence on contextual factors in ranking the alternatives when the criteria under consideration conflict so that the dominance principle does not apply; and, finally, (iv) a tension between the dominance principle and context-dependence in the presence of transitivity and a very weak continuity property of evaluations. While the intuitive bases of the dominance principle and context-dependence need much further investigation, our preliminary inclination is to reject the dominance principle, while retaining context-dependence, in the specific problems discussed by the contributions under consideration.

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APPENDIX A

Proof of Proposition 1

Step I. Suppose a binary relation $\succeq$ satisfies transitivity, weak dominance, type 1 context-dependence, and minimal continuity in attributes in D. We shall show that this supposition leads to a contradiction.

By type 1 context-dependence,

$$\text{there exist } a, b, c, d \in X, \text{ such that (for all } j \in C, a_j = c_j \text{ and } b_j = d_j),$$

(1) $$[\text{for some } k \in D, a_k \neq b_k], \text{ and } (a \succ b \text{ and } d \succ c)....$$

In the rest of this proof, we shall treat k, referred to in (1), as fixed.

We consider the following cases:

(2) $$a_k = c_k = \beta_k, b_k = d_k > \beta_k; ...$$

(3) $$a_k = c_k > \beta_k, b_k = d_k = \beta_k; ...$$

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and

$$a_k = c_k > \beta_k, b_k = d_k > \beta_k, a_k \neq b_k.... \tag{4}$$

Suppose (2) holds. Then, by minimal continuity in attributes in D and $(d \succ c)$, there exists $d' \in X$, such that, $d'_k < d_k = b_k$, $(d'_j = d_j$ for all $j \in C - \{k\})$ and $d' \succ c$. By minimal continuity in attributes in D and $(d' \succ c)$, there exists $c' \in X$, such that $c'_k > c_k = a_k$ and $(c'_j = c_j$ for all $j \in C - \{k\})$ and $d' \succ c'$. By weak dominance, $b \succeq d'$ follows from $b_k > d'_k$ and $(b_j = d'_j$ for all $j \in C - \{k\})$, and $c' \succeq a$ follows from $c'_k > a_k$ and $(c'_j = a_j$ for all $j \in C - \{k\})$. Consequently, we obtain: $a \succ b$, $b \succeq d'$, $d' \succ c'$ and $c' \succeq a$, a contradiction of the transitivity of $\succeq$.

The proof for the case where (3) holds is similar to the proof for the case where (2) holds, and we omit it.

Suppose (4) holds. Then, by minimal continuity in attributes in D and $(a \succ b)$, there exists $a' \in X$, such that $a'_k < a_k = c_k$, $(a'_j = a_j$ for all $j \in C - \{k\})$ and $a' \succ b$; and by minimal continuity in attributes in D and $(d \succ c)$, there exists $d' \in X$, such that $d'_k < d_k = b_h$, $(d'_j = d_j$ for all $j \in C - \{k\})$ and $d' \succ c$. Noting that $(c_k > a'_k$ and $c_j = a'_j$ for all $j \in C - \{k\})$ and $(b_k > d'_k$ and $b_j = d'_j$ for all $j \in C - \{k\})$, by weak dominance, we obtain $c \succeq a'$ and $b \succeq d'$. Therefore, we have: $a' \succ b$, $b \succeq d'$, $d' \succ c$ and $c \succeq a'$, which contradicts the transitivity of $\succeq$.

Step II. The proof that no $\succeq$ can simultaneously satisfies acyclicity of $\succ$, dominance, type 1 context-dependence, and minimal continuity is exactly similar to the proof in step I above. Therefore, we omit it. ■

Proof of Proposition 2

Let $x', y' \in X$ be two given distinct alternatives such that $x'_j = y'_j$ for all $j \in C$. Now consider a binary relation $\succeq^*$defined over X, such that, for all $x, y \in X$, $x \succeq^* y$ if and only if $[x = y]$ or $[x_j R_j y_j$ for all $j \in C$ and $x_j P_j y_j$ for some $j \in C]$ or $[x = x'$ and $y = y']$. It can be checked that $\succeq^*$ satisfies transitivity, dominance, type 2 context-dependence, and minimal continuity in attributes in D. ■

Proof of Proposition 3

Suppose some $\succeq$ satisfies weak dominance, type 2 context-dependence, and weak continuity in attributes in D. We shall show that this leads to a contradiction.

Given type 2 context-dependence, there exist $a, b \in X$, such that (for all $j \in C, a_j = b_j$) and $a \succ b$.

Let $g \in D$. We distinguish three cases: (i) $a_g = b_g = \beta_g$; (ii) $a_g = b_g = \alpha_g$; and (iii) $\beta_g < a_g = b_g < \alpha_g$. In case (i), by $a \succ b$ and weak continuity in attributes in D, we have $a \succ b'$ for some $b' \in X$ with $b'_g > b_g = \beta_g$ and $b'_j = b_j$ for all $j \in C - \{g\}$. By weak dominance and noting that $b'_g > a_g = \beta_g$ and $b'_j = a_j$ for all $j \in C - \{g\}$, we have $b' \succeq a$, an immediate contradiction of $a \succ b'$. In case (ii), by $a \succ b$ and weak continuity in attributes in D, we have $a' \succ b$ for some $a' \in X$ with $a'_g < a_g = b_g = \alpha_g$ and $a'_j = a_j$ for all $j \in C - \{g\}$. By weak dominance, it follows that $b \succeq a'$, an immediate contradiction of $a' \succ b$. In case (iii), by $a \succ b$ and by a straightforward application of weak continuity in attributes in D, we have $a'' \succ b$ for some $a'' \in X$ with $a''_g < a_g = b_g$ and $a''_j = a_j$ for all $j \in C - \{g\}$. Then, by weak dominance, $b \succeq a''$, a direct contradiction of $a'' \succ b$. ■

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