THE JOURNAL OF NAVIGATION (2005), **58**, 459–470. © The Royal Institute of Navigation
doi:10.1017/S0373463305003310 Printed in the United Kingdom

High Spatial Variation Tropospheric Model for GPS-Data Simulation

Ashraf Farah, Terry Moore, Chris J Hill

(*University of Nottingham*)
(Email: ashraf_farah@yahoo.com)

Precise GPS simulated data requires accurate simulation of the two major sources of error in GPS measurements, namely the ionospheric and tropospheric delays. The ionospheric delay modelling has been handled in a previous work (Farah, 2002). In this paper the simulation of the tropospheric delay is discussed. The suggested model should be accurate in estimating the tropospheric delay as well as capable of simulating high spatial variations of the troposphere resulting in more realistic simulated GPS data. In this paper, the EGNOS tropospheric correction model is considered as a possible tool for simulating the tropospheric delay in order to obtain more realistic simulated GPS data. Comparing the total tropospheric zenith delays from the EGNOS model with the CODE-tropospheric product has allowed the quality of the EGNOS model to be assessed. Four IGS-tracking stations have been selected for this study. Data from four non-consecutive weeks in different seasons over a period of one year were tested to assess the seasonal variation of the weather conditions. It is shown that the EGNOS model agrees well with the CODE-estimations with a mean zenith delay difference of approximately 2 cm. The maximum zenith delay difference between the EGNOS model and the CODE-estimations was in the range of 5 cm to 16 cm, which agrees well with previous studies. A second study has investigated the behaviour of the EGNOS model with other established tropospheric models such as the Saastamoinen, the Hopfield, the Marini and the Magnet model for three IGS-stations. It can be concluded from this study that the EGNOS model shows better agreement with the IGS estimations than the Magnet model and compares well with other models. The major shortcoming in the EGNOS model is its inability to simulate the variations in the troposphere over small regions. This shortcoming could be overcome by using the theory of *Gaussian Random Fields*, which has been previously used to model real life phenomena such as surface roughness (Chan, 1999). This paper was first presented at ION GPS 2003, the 16th Technical Meeting of the Satellite Division of the Institute of Navigation held at Portland Oregon, USA.

KEY WORDS

1. GNSS. 2. Simulation. 3. Troposphere.

1. INTRODUCTION. The tropospheric delay error is typically the second most significant source of error for any satellite-based positioning system. The positioning error due to improper modelling of the tropospheric delay can be over 10 m, as the tropospheric delay itself can range from 2 m at the zenith to over 20 m at lower elevation angles (Dodson et al., 1999). Many attempts have been made to model the tropospheric delay. The most widely used formula for tropospheric

refractivity N is the Smith and Weintraub (1953) simplified two-term formula:

$$N = 77 \cdot 6 \frac{P}{T} + 3 \cdot 73 \times 10^5 \left(\frac{e}{T^2} \right) \tag{1}$$

where;
P: The total atmospheric pressure (mbar); T: temperature in Kelvin; e: Partial pressure of water vapour (mbar).

Two basic types of model exist, the first tries to relate the meteorological parameters in equation (1) to empirical surface meteorological measurements (*surface meteorological models*), and the second to global standard atmospheres (*global models*). These surface meteorological models are based on radiosonde profiles and relate the meteorological parameters of equation (1) to measurements taken at the ground surface. Typical examples include the Hopfield (1971) and Saastamoinen (1973) models. The global models avoid the use of surface meteorological data and assume that the atmosphere behaves in a certain manner depending on the expected variations of temperature, pressure, and humidity with height; a typical example being the Magnet model.

The European Geo-stationary Navigation Overlay Service (EGNOS) is the European contribution to the Global Navigation Satellite System-1 (GNSS-1). It comprises a number of ground Receiver Integrity Monitors (RIMs) and geostationary satellites. The RIMs provide the integrity and Wide Area Differential GPS (WADGPS) services for both the GPS and GLONASS systems. The geo-stationary satellites are used to broadcast the integrity and WADGPS corrections to the users and provide extra pseudo-range observations. To enable these corrections to be applied over a wide area the WADGPS service separates the measurement errors into different components; orbit, satellite clock, ionospheric delay and tropospheric delay. Due to the large variation in tropospheric delay with the different weather conditions that may prevail across the wide area considered, the tropospheric delay correction is not broadcast to the user. Instead, an estimate is generated locally by the user, based on a tropospheric model (Penna et al., 2001).

The EGNOS guidelines recommend that a user apply a correction for tropospheric delay that is compliant with the International Civil Aviation Organization (ICAO) Standard and Recommended Practices (SARPs) for Satellite-Based Augmentation Systems (SBAS) (RTCA, 1999). These guidelines also cover the USA Wide Area Augmentation System (WAAS) and the Japanese Multi-functional Transport Satellite, (MTSAT)-based Satellite Augmentation System (MSAS). The recommended SBAS model (EGNOS model) provides an estimate of the zenith tropospheric delay which is dependent on empirical estimates of five meteorological parameters at the receiver – namely, pressure, temperature, water vapour pressure, temperature lapse rate and water vapour lapse rate. These estimates of the meteorological parameters are dependent on the receiver's height, latitude and day-of-year, and are interpolated from reference values for the yearly averages of the parameters and their associated seasonal variations, derived primarily from North American meteorological data. The EGNOS guidelines then recommend mapping the zenith tropospheric delay estimate to the appropriate receiver-to-satellite elevation angle using an elevation angle-dependent mapping function.

The EGNOS model has been assessed in many studies such as Collins and Langley (1997, 1998), Dodson et al., (1999) and Penna et al., (2001), which all agree on their assessment of the ability of the model to describe effectively the mean tropospheric delay. This makes the EGNOS model a good choice for simulating the tropospheric delay in GPS-data simulation process.

Zenith tropospheric delay is one of the IGS-data products based on a global network of IGS-tracking stations. The Centre for Orbit Determination in Europe (CODE), one of the IGS-Analysis Centres has offered a product for the zenith tropospheric delay from an increasing number of stations since 1997. The consistency of the tropospheric estimates from two IGS-data centres, CODE and GFZ is about 5 mm (bias) and about 10 mm (rms) for single sites (Gendt and Beutler, 1995). Another study showed a high consistency in the tropospheric estimations between seven IGS analysis centres; CODE, EMR, ESA, GFZ, JPL, NGS and S10 (Gendt, 1996).

Four IGS-stations varying in latitude and height (BOR1, GALA, MAC1 and MAS1) were selected for this study. The tropospheric delays at these stations were estimated using the CODE-tropospheric product and then compared with the EGNOS tropospheric correction model to assess the accuracy of the model. To assess the effect of seasonal variations in weather conditions, data from four non-consecutive weeks, each in a different season were tested over a period of one year.

The behaviour of the EGNOS model has also been compared with other established tropospheric models. These models can be categorized into two groups; surface meteorological models such as the Saastamoinen, the Hopfield and the Marini and empirical global models such as the Magnet model. Tropospheric zenith delay data was compared for three IGS-tracking stations (HERS, BOR1 and BAHR) over a period of one year covering four non-consecutive weeks in different seasons. It can be concluded that the EGNOS model shows better agreement with the IGS Tropospheric estimations than the Magnet model does. Neither of these two models require real-time meteorological data. The EGNOS model also compares well with the surface meteorological models; however these models use real-time meteorological data to increase the accuracy of the tropospheric delay estimation and therefore are not ideal for data simulation purposes.

The major shortcoming of the EGNOS model is its inability to simulate the variations in the troposphere over small geographical regions. This shortcoming could be overcome by using the theory of *Gaussian Random Fields*. This approach enables the generation of controlled random surfaces over the basic EGNOS model, effectively resulting in a more realistic model of the tropospheric behaviour. A study considering two adjacent IGS-stations (HERS, NPLD) shows how the behaviour of the modified-EGNOS model follows typical real tropospheric variations. The geographical positions and coordinates of all IGS stations used in the tests described in this paper are in Figure 1 and Table 1.

2. EGNOS TROPOSPHERIC MODEL.

The algorithm of the EGNOS model is expressed in this section after Penna et al. (2001). The guidelines (RTCA, 1999) recommend users to model the total tropospheric delay for a receiver-to-satellite range at elevation angle α using:

$$d_{\alpha} = \left(d_{dry} + d_{wet}\right) \times MF(\alpha) \tag{2}$$

Table 1. Coordinates of the IGS Tracking Stations used in the various tests.

Station IGS-ID	Latitude (degree)	Longitude (degree)	Height (metre)
BOR1	52·276 N	17·073 E	124·358
GALA	0·742 S	89·696 W	7·441
MAC1	54·499 S	158·936 E	−6·763
MAS1	27·763 N	15·633 W	197·161
BAHR	26·209 N	50·608 E	−17·03
HERS	50·867 N	0·336 E	76·521
NPLD	51·421 N	0·338 W	72·719

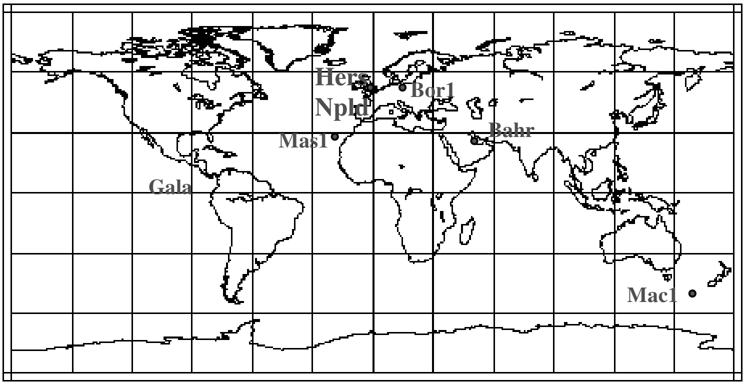

Figure 1. The geographical positions of the IGS Stations used in the various tests.

where: d_{dry} is the zenith dry delay; d_{wet} is the zenith wet delay; $MF(\alpha)$ is the mapping function to map the total zenith delay to the appropriate receiver-to-satellite elevation angle.

The model gives an estimate of the total zenith tropospheric delay based on five meteorological parameters, obtained from a look-up table of values given at discrete latitudes, with linear interpolation applied as necessary. Attempts to model the seasonal variation of the parameters is via a sinusoidal function of the day-of-year. The five meteorological parameters are the total pressure, temperature and water vapour pressure at mean sea level, and temperature and water vapour lapse rates, used to scale the pressures and temperatures to the user's height above sea level. The zenith dry and wet delays are computed using:

$$d_{dry} = z_{dry}\left[1 - \frac{\beta H}{T}\right]^{\frac{g}{R_d \beta}} \tag{3}$$

$$d_{wet} = z_{wet}\left[1 - \frac{\beta H}{T}\right]^{\frac{(\lambda + 1)g}{R_d \beta} - 1} \tag{4}$$

where: $g = 9{\cdot}80665\ \mathrm{m/s^2}$; H is the height of the receiver above mean sea level (m); T is the temperature at mean sea level (K); β is the temperature lapse rate (K/m);

Table 2. Average values and seasonal variation values of the five meteorological parameters used by the EGNOS model.

	Average				
Lat. (°)	P_0 (mbar)	T_0 (K)	e_0 (mbar)	β_0 (K/m)	λ_0
⩽15	1013·25	299·65	26·31	6·30e^{-3}	2·77
30	1017·25	294·15	21·79	6·05e^{-3}	3·15
45	1015·75	283·15	11·66	5·58e^{-3}	2·57
60	1011·75	272·15	6·78	5·39e^{-3}	1·81
⩾75	1013·00	263·65	4·11	4·53e^{-3}	1·55

	Seasonal Variation				
Lat. (°)	ΔP_0 (mbar)	ΔT_0 (K)	Δe_0 (mbar)	$\Delta\beta_0$ (K/m)	$\Delta\lambda_0$
⩽15	0·00	0·00	0·00	0·00e^{-3}	0·00
30	−3·75	7·00	8·85	0·25e^{-3}	0·33
45	−2·25	11·00	7·24	0·32e^{-3}	0·46
60	−1·75	15·00	5·36	0·81e^{-3}	0·74
⩾75	−0·50	14·50	3·39	0·62e^{-3}	0·30

$R_d = 287{\cdot}054$ J/kg/K; λ is the water vapour lapse rate (dimensionless); z_{dry} is the zenith dry delay at mean sea level given by:

$$z_{dry} = \frac{10^{-6}\,k_1\,R_d\,p}{g_m} \tag{5}$$

and z_{wet} is the zenith wet delay at mean sea level:

$$z_{wet} = \frac{10^{-6}\,k_2\,R_d}{g_m\,(\lambda+1)-\beta\,R_d} \times \frac{e}{T} \tag{6}$$

where $k_1 = 77{\cdot}604$ K/mbar; p is the pressure at mean sea level (mbar); $g_m = 9{\cdot}784$ m/s^2; $k_2 = 382000$ K^2/mbar; e is the water vapour pressure at mean sea level (mbar).

The average values and seasonal variations for the five meteorological parameters are given in Table 2. Using these values each meteorological parameter value (ξ) may then be computed using the following equation,

$$\xi(\phi, D) = \xi_0(\phi) - \Delta\xi(\phi) \times \cos\left[\frac{2\pi(D - D_{min})}{365{\cdot}25}\right] \tag{7}$$

where: ϕ is the receiver's latitude; is the day-of-year (starting with 1 January); $D_{min} = 28$ for northern latitudes and 211 for southern latitudes; ξ_0 and $\Delta\xi$ are the average and seasonal variation respectively for the particular parameter at the receiver's latitude.

The mapping function $MF(\alpha)$ is expressed as:

$$MF(\alpha) = \frac{1{\cdot}001}{\sqrt{0{\cdot}002001 + sin^2\alpha}} \tag{8}$$

The mapping function is not valid for elevation angles of less than 5 degrees (RTCA, 1999).

Table 3. Dates of data samples.

GPS week	1097	1110	1123	1136
Date	14/1/01–20/1/01	15/4/01–21/4/01	15/7/01–21/7/01	14/10/01–20/10/01

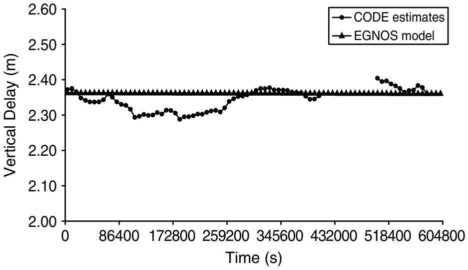

Figure 2. The total tropospheric zenith delays in GPS week 1123 [MAC1] station.

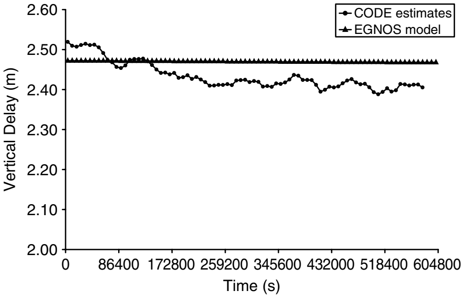

Figure 3. The total tropospheric zenith delays in GPS week 1136 [MAS1] station.

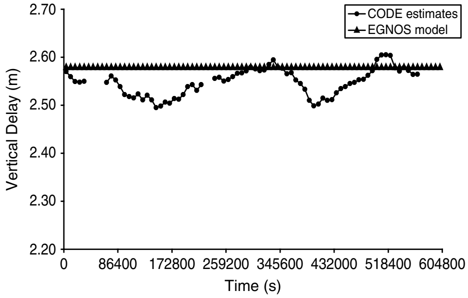

Figure 4. The total tropospheric zenith delays in GPS week 1123 [GALA] station.

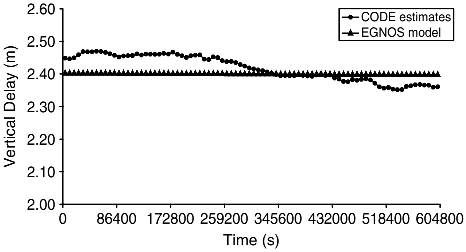

Figure 5. The total tropospheric zenith delays in GPS week 1136 [BOR1] station.

3. ASSESSING THE EGNOS TROPOSPHERIC MODEL

3.1. *First Test.* The first comparison study was conducted between the EGNOS model and the CODE tropospheric product. CODE produces zenithal tropospheric delay products from about 114 IGS-tracking stations every two hours with a 10-degree cut-off elevation angle using the Niell (1996) (wet) mapping function. Four stations varying in latitude and height (BOR1, GALA, MAC1 and MAS1) were selected for the study and tropospheric zenith delay data from four non-consecutive weeks in 2001, each in different seasons; was used to assess the seasonal variation of the weather conditions. The dates of data samples are shown in Table 3. The estimation of the total tropospheric delay from the CODE-Tropospheric products, the differences in total zenith delay between the EGNOS model and the CODE-Troposphere estimation gives an indication of the quality of the EGNOS model and assesses its adequacy for GPS-data simulation.

Figures 2 to 5 show samples of the total zenith delay estimates from both the EGNOS model and the CODE-Tropospheric delay estimates resulting from our

Table 4. Total zenith delay difference between EGNOS model and CODE estimates.

GPS week	Station	Mean (cm)	RMS (cm)	Max. (cm)
1097	BOR1	4·4	4·6	8·3
	GALA	1·4	1·95	4·9
	MAC1	−10·3	11·2	−16·7
	MAS1	−3·4	3·8	−7·7
1110	BOR1	−3·6	3·95	−7·1
	GALA	6·2	6·3	9·1
	MAC1	−8·6	9·2	−13·5
	MAS1	−10·1	10·5	−12·8
1123	BOR1	4·7	5·8	10·9
	GALA	−3·0	4·1	−8·4
	MAC1	−1·8	3·6	−6·9
	MAS1	−11·96	12·1	−15·2
1136	BOR1	1·8	4·2	6·7
	GALA	−9·9	10·3	−15·96
	MAC1	−11·2	11·3	−14·3
	MAS1	−3·4	4·8	−7·98

study. Table 4 shows the total zenith delay differences between the EGNOS model and the CODE-Tropospheric delay estimation.

3.2. *Second Test.* A second study compared the EGNOS model with other established tropospheric models. These models have been selected in two categories:

- Surface meteorological models, which need surface meteorological input data; such as the Saastamoinen, Marini and Hopfield models.
- Empirical global models which do not need surface meteorological data; such as the Magnet model.

The study involved three IGS-stations (HERS, BOR1 and BAHR). Figures 6 to 8 show the total tropospheric zenith delays which resulted from IGS-tropospheric estimations (combined tropospheric estimates from all IGS analysis centres) as a reference and the other five models. Real time IGS meteorological data was used as input data for the surface meteorological models.

3.3. *Discussion.* The EGNOS model is an empirical seasonal model that does not have the ability to model sub-seasonal variations. However it can be stated from previous studies and our current study that the model can describe the mean tropospheric delay reasonably well as the model agrees closely with the average seasonal trends of the CODE-estimates. The mean differences between the EGNOS and the mean total zenith delay from CODE-estimations are within a range of 1 to 4 cm with rms in the range of 2 to 10 cm. However in some cases the mean difference reached 11 cm. This may be attributed to the fact that the EGNOS model cannot accommodate changes in the tropospheric delay due to rapid weather changes. The maximum zenith delay difference between the model and CODE estimates over the four weeks, at the four stations, are 5 cm to 16 cm. These results agree well with related work by Dodson et al. (1999).

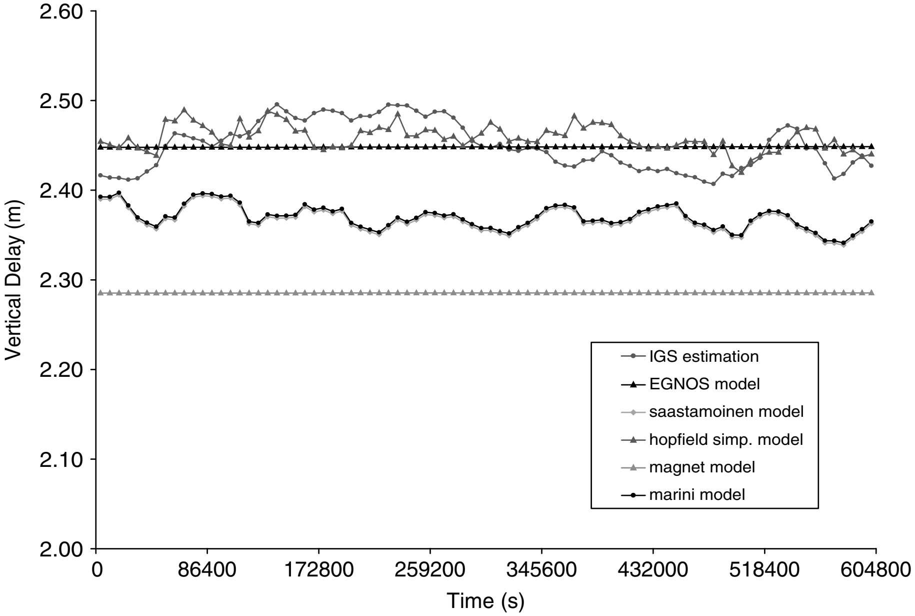

Figure 6. The total tropospheric zenith delays in GPS week 1175 [HERS] station.

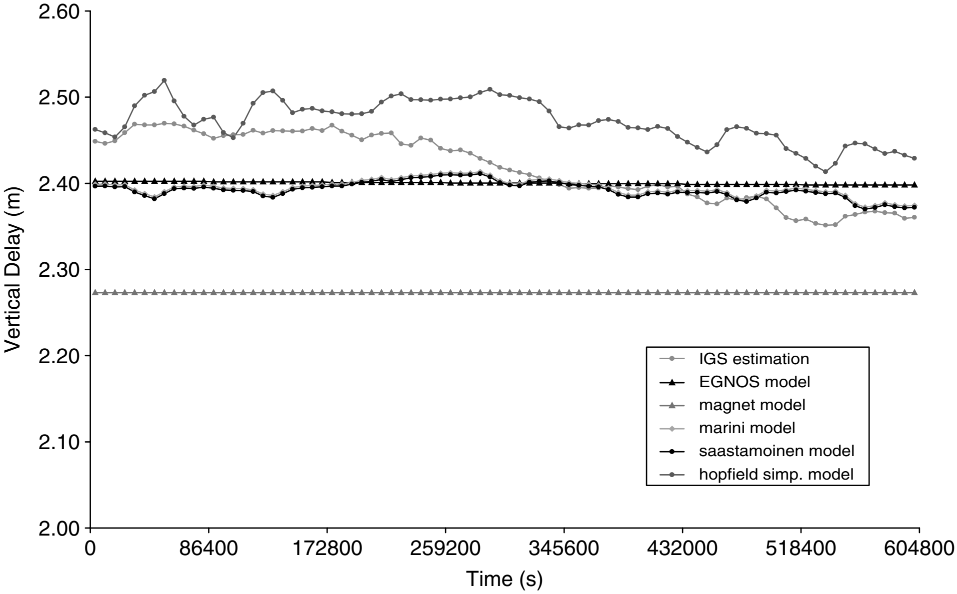

Figure 7. The total tropospheric zenith delays in GPS week 1136 [BOR1] station.

The EGNOS model follows the IGS estimated delays more closely than the Magnet model does. The Magnet model was previously considered as a probable model for tropospheric delay in GPS data simulation process, because it is an empirical model which does not require meteorological data as input. The EGNOS

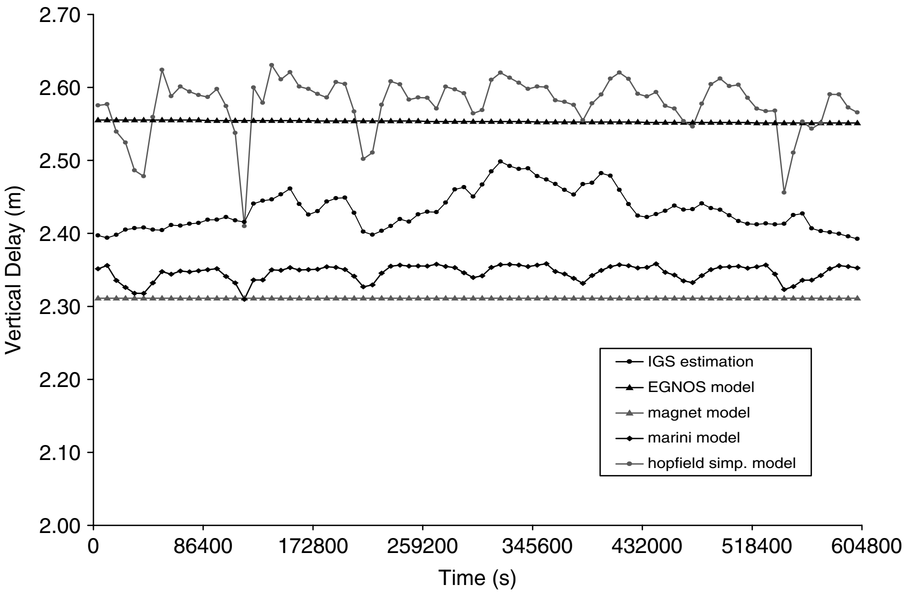

Figure 8. The total tropospheric zenith delays in GPS week 1188 [BAHR] station.

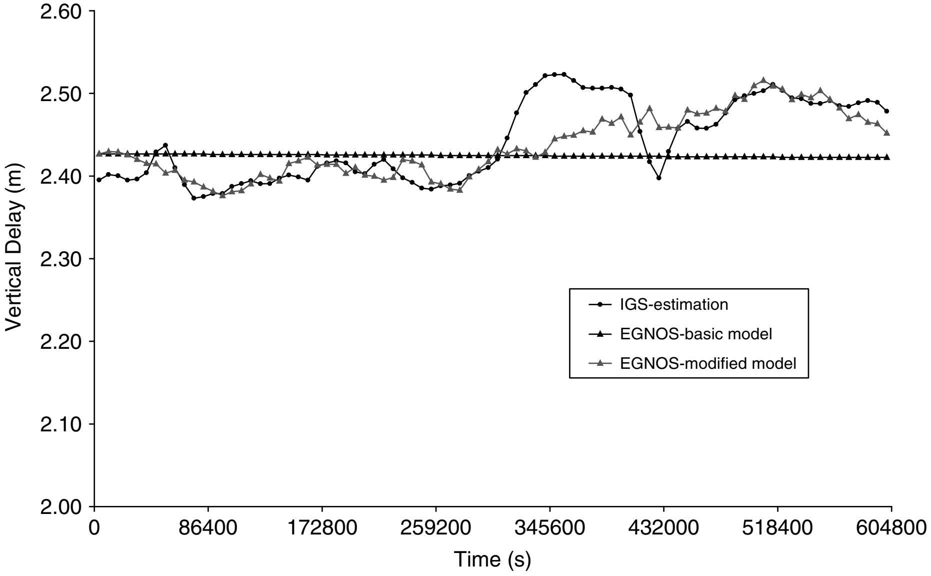

Figure 9. The total tropospheric zenith delays in GPS week 1135 [HERS] station.

model also compares well with other surface meteorological models, which need meteorological data as input (are so are not particularly appropriate for GPS data simulation process). The Saastamoinen and the Marini models gave almost identical results, and so only one line is shown in Figure 8.

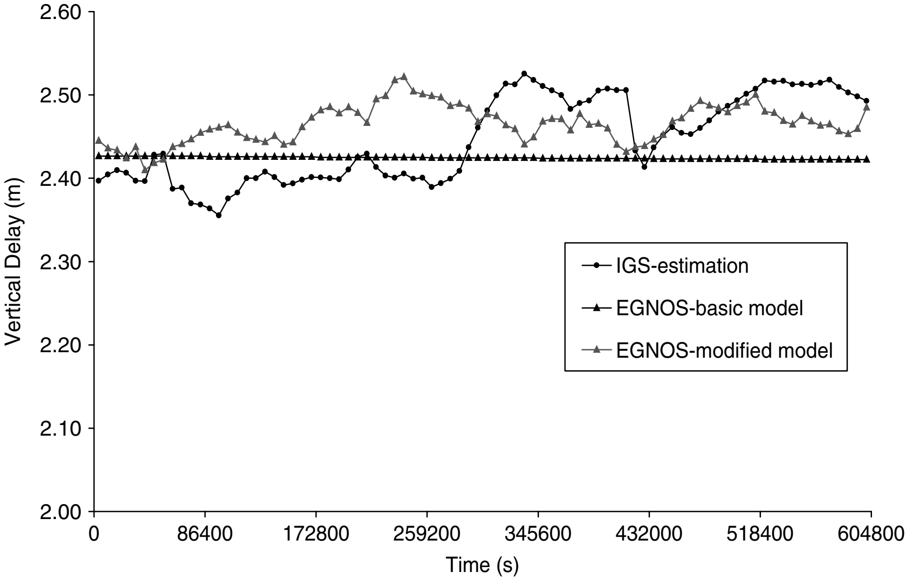

Figure 10. The total tropospheric zenith delays in GPS week 1135 [NPLD] station.

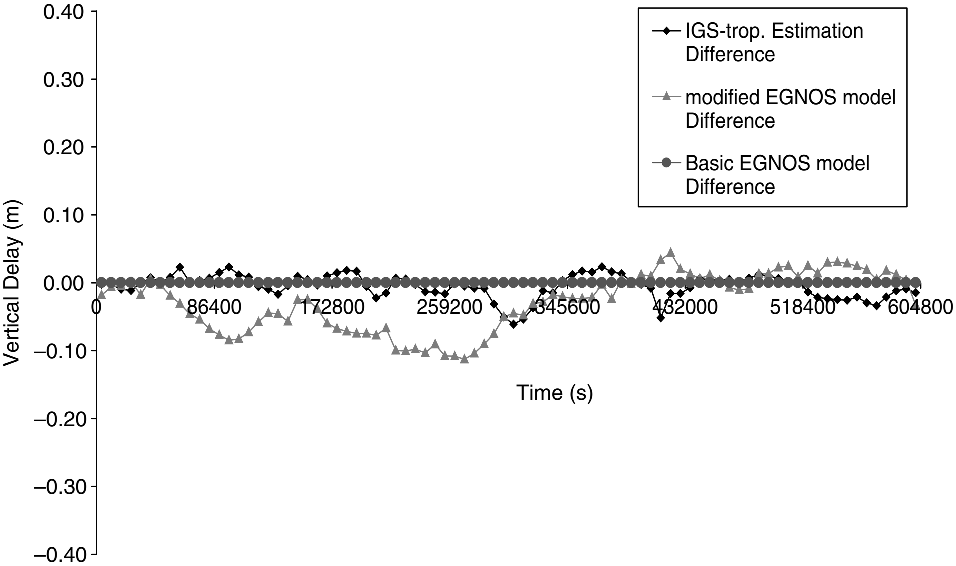

Figure 11. The total tropospheric zenith delays difference in GPS week 1135 [HERS & NPLD] stations.

The two studies have demonstrated that the EGNOS model is highly suitable for the simulation of the tropospheric delay in the GPS data simulation process, as the model is computationally simple and gives good accuracy in simulating the average tropospheric behaviour.

4. IMPROVING THE EGNOS MODEL.

This section will deal with a proposed method to improve the EGNOS tropospheric model. As has been shown above the EGNOS model is a suitable method of simulating the tropospheric delay in the GPS-data simulation process. Yet there is deficiency in the EGNOS model in that it is unable to simulate the variations in the tropospheric delay over small geographical regions. Use of a mathematical technique called *Gaussian Random Fields* (Chan, 1999) can add a controlled *random surface* variation over the basic values of the EGNOS model to enable it to simulate those regional variations. This *random surface* is tailored to match typical regional variations of the troposphere. The troposphere could face a change of 5 cm to 10 cm in zenith wet delay for spatial scale of 10 km to 100 km (Bock et al., 2001).

A test study was conducted involving two adjacent IGS stations in the United Kingdom (HERS, NPLD) with a separation of about 77 km. The total zenithal tropospheric delay was plotted for both stations based on the IGS-trop estimations, the EGNOS basic model and the EGNOS modified model. Those plots are shown in Figures 9 and 10. The difference between the total tropospheric zenith delay estimates at the two stations is shown in Figure 11.

Figures 9 and 10 clearly show the impact of the new *Gaussian Random Field* algorithm on the behaviour of the basic EGNOS tropospheric model. They clearly demonstrate that the new modified model can accurately simulate the typical behaviour of the real troposphere. Figure 11 shows that the total tropospheric zenith delays difference for the two stations resulting from IGS-tropospheric estimations and the modified EGNOS model agree well, and that the basic EGNOS model is incapable of delivering these variations. The new modified model is capable of accurately simulating the characteristic regional variations of the troposphere, which would not be possible with the basic EGNOS model.

5. CONCLUSIONS.

The EGNOS tropospheric correction model has shown acceptable level of accuracy in describing the average tropospheric delay model as it agrees reasonably well with the CODE-tropospheric products based on GPS measurements. The mean difference in total zenith delay between the EGNOS model and the precise CODE-estimations of the tropospheric delay is about 2 cm with maximum difference of 5 to 16 cm. The EGNOS model shows a better level of agreement with the IGS estimates than do other empirical tropospheric models; such as the Magnet model. The EGNOS model also shows a good level of agreement with surface meteorological tropospheric models. However, these need real time meteorological input data to estimate the tropospheric delay, and so are not ideal for GPS data simulation.

The modified EGNOS model using the *Gaussian Random Fields* theory has been shown to simulate typical regional variations in the troposphere, and so can provide an effective tool for simulating characteristic tropospheric behaviour. This modification has resulted in a significant improvement on the basic model. The modified EGNOS model is considered appropriate for GPS-data simulation as it fulfils many requirements;

- Computationally simple
- Good description of mean tropospheric delay

- Simulates regional tropospheric variations
- Behaviour may be improved significantly by adding real time surface meteorological data.

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