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MODELLING THE IMPACT OF REINSURANCE ON FINANCIAL STRENGTH

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ABSTRACT

This paper develops the Daykin *et al.* (1994) asset/liability model to examine specifically the effects of different reinsurance programmes on the capital of a direct insurance company. By modelling the gross premiums and claims separately from the impact of reinsurance on them, it is possible to examine directly the effects of different reinsurance programmes on a company's expected performance just as easily as changes in asset mix or business volumes. The paper goes on to discuss the necessary assumptions to be built into such a model, and then gives a worked example. The emphasis of the paper is on management reporting rather than on mathematical detail.

KEYWORDS

General Insurance; Reinsurance Programmes; Capital at Risk; Management Reports

1. INTRODUCTION

1.1 *Preliminaries*

1.1.1 This paper builds on the work published by several authors, in particular Coutts & Devitt (1986, 1988), Daykin *et al.* (1987) and Daykin *et al.* (1994), to describe a working stochastic asset/liability model, able to simulate the major types of reinsurance treaty. It goes on to show how the output from this model can be used to provide a mechanism for reviewing reinsurance retention levels and other potential strategic options available to management. The concept can then be further developed as a means of capital allocation by profit centre, taking fully into account the different risk profiles of different classes of business.

1.1.2 Whilst these previous papers, and also a paper presented at the 1996 General Insurance Convention (Christofides *et al.*, 1996), show the power of stochastic modelling as a technique for examining the overall risk being assumed by a non-life insurance company, there has been little published work on the impact of reinsurance on the company results, except for one paper recently published by Lowe & Stanard (1996), which looks explicitly at how a reinsurer assesses the financial effect of a property catastrophe claim on its book of business. Further, neither the United States Risk Based Capital (RBC) formulae (Actuarial Advisory Committee to the NAIC, 1992) nor the proposals for Lloyd's capital adequacy rules (Lloyd's of London, 1995) address explicitly the effect that different reinsurance structures or retention levels have on capital requirements or on the risk of ruin. The 'RBC formula' concept was explained in detail by Hooker *et al.* (1995).

1.1.3 The authors consider the lack of modelling reinsurance surprising, bearing in mind that reinsurance is often called a substitute for capital. This paper seeks to address this shortfall. We consider that a better understanding of the overall financial impact of reinsurance is of increasing importance because of the need to compare the relative merits of different reinsurance structures, both with each other and with the range of new capital market solutions being developed, which offer varying degrees of risk transfer.

1.1.4 We believe that, particularly in the light of the Cadbury Committee requirements on corporate governance (1992), Boards of Directors of insurance companies need a better understanding of the financial risks being assumed by their companies and how reinsurance arrangements reduce these to manageable proportions. Inevitably, the inter-relationships of all the different variables lead to highly complex mathematics, and we have, therefore, built our prototype to give graphical outputs as far as possible, in a form which can be readily understood by non-executive directors. This paper also tries to avoid mathematical formulae, to allow the underlying concepts to be better understood, and relies, instead, on worked examples.

1.1.5 Although such a modelling exercise cannot be carried out ignoring the regulatory regime under which a company operates, we concentrate, for modelling purposes, on the economic implications, thus emphasising 'capital at risk' rather than 'regulatory capital', and cash flows and net present values rather than reported earnings. However, the results presented to management will include both underwriting and revenue information.

1.2 *Structure of the Paper*

In Section 2 we set the scene, and, in particular, examine the importance of managing assets and liabilities, together with the concept that reinsurance is a substitute for capital. This leads naturally to a discussion of the RBC formula and the definition of probability of ruin. Section 3 describes the structure of the model, and Section 4 details the numerous assumptions required as input into the model. These inputs are based mainly on management data and reinsurance questionnaires. Section 4 also looks in detail at the assumptions required to analyse different reinsurance programmes. Section 5 provides examples of graphical output, and it goes on to examine how these techniques can be used to allocate capital by class of business, to assess how much capital reinsurance releases and what is the cost of this release. In Section 6 we present some examples. In Section 7 we discuss some of the practical issues of introducing this model. Finally, Section 8 very briefly explores possible future developments.

2. SETTING THE SCENE

2.1 *The Importance of Risk Management*

2.1.1 Insurance companies have, as their prime business, the accepting of unwanted risk on behalf of others. They accept different types of risk in the expectation of being able to generate an adequate return on capital from the premiums charged. The management of the risk so assumed is therefore of fundamental importance to the success of the operation.

2.1.2 Intuitively, an insurance company ought to be able to manage exposures of both liabilities and assets in such a way that it allocates its established 'risk tolerance' between underwriting activities and investment strategy to maximise its expected overall return on capital. The 'risk tolerance' level of an individual company is clearly a matter for its Board of Directors to establish, subject to regulatory minimum standards. (In Section 2.5 we use the concept of probability of ruin as a measure of this 'risk tolerance'.)

2.1.3 The risks to which the insurance company are subject can affect a company's balance sheet in different ways. The RBC formulae introduced recently in the U.S.A. are an attempt to quantify the overall effect of these risks and set appropriate minimum capital standards. Lloyd's is also working towards introducing something similar, and we believe that the trend is inevitably going to spread elsewhere, gradually refining in the light of experience and developments in actuarial science.

2.1.4 An RBC formula approach, along the lines of the U.S. model, will help strike a balance between risk and reward and between the risks carried by different parts of the operation. Thus, it will improve the ability of organisations to allocate capital properly, and establish methods of measuring corporate and/or profit centre performance, based on 'return on equity' considerations.

2.2 *RBC Formulae v Stochastic Asset/Liability Modelling*

2.2.1 Whilst recognising the RBC formula approach as a significant improvement on previous techniques, the authors see it as a regulatory tool. Since the rules are framed from a regulatory perspective, as a measure for consumer protection, they have a number of shortcomings for use inside a company.

2.2.2 The primary omissions are that:

- they look back at where the company has come from, rather than attempting to factor in future business plans; and
- reinsurance factors are based on past experience, and no explicit allowance is made for changing future reinsurance programmes.

2.2.3 Further, such an RBC formula approach involves the setting, for each class of business, of various parameters which tend to be based on market average data. In theory, it would be possible to adjust these market figures for internal management purposes and to assess the effect of different reinsurance

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arrangements. However, these adjusted parameters would need to be established and justified to management at both corporate and profit centre levels.

2.2.4 On the other hand, stochastic asset/liability modelling goes back to first principles to generate estimates of individual cash flows for each line of business. By modelling the gross premiums and claims separately from the impact of reinsurance on them, it is possible to examine directly the effects of different reinsurance programmes on a company's expected performance just as easily as changes in asset mix or business volumes.

2.2.5 Further, stochastic asset/liability models which simulate different reinsurance programmes give management a powerful tool for:
— allocating risk capital to each class of business;
— setting retentions at profit centre and corporate level;
— setting of profit targets; and
— comparing reinsurance against other forms of risk transfer.

(This subject is considered in more detail in a paper being prepared by the authors for the Casualty Actuarial Society (1997), provisionally titled 'Capital and risk and their relationship to reinsurance programmes'.)

2.2.6 Inevitably, such benefits can be obtained only at the cost of fitting far more complicated assumptions than are necessary to fit a model based on U.S. RBC formulae, but this paper shows that, for United Kingdom companies, most of the necessary data are available from:
— DTI returns; or
— best practice management reports; or
— reinsurance questionnaires; and,
— to supplement these, actuarial information systems.

2.2.7 Inevitably, management reports will vary considerably from one company to another, but will almost always include budgets and claims run-off triangles by line of business.

2.2.8 By using stochastic modelling to establish estimates of means/modes and variances, it is possible to take assumptions built up by underwriters, such as rate on line, using concepts with which they are familiar, and translate these into the language of investment portfolio management, such as return on capital. This reduces the gap in understanding across different disciplines at senior management level, and allows comparisons of reinsurance with other forms of risk transfer.

2.3 *Reinsurance as a Substitute for Capital*

2.3.1 Reinsurance has long been held to be a substitute for capital, as seen, for example, in the DTI solvency requirement. However, little work has been published as to how this can be measured. With the growing interest of capital markets in risk transfer products, this measurement will become critical, so that comparisons can be made into the cost-effectiveness of different instruments, including:
— traditional bond and equity finance;

— 'Act of God' bonds;
— reinsurance derivatives;
— financial or finite risk reinsurance;
— reinsurance captives; and
— traditional reinsurance.

2.3.2 A glossary of reinsurance terms, including these items, is shown in Appendix 1.

2.3.3 In particular, for management to assess the effect of a particular reinsurance contract as compared to alternative strategies, management needs to measure:
— how much capital is released by the reinsurance contract;
— how much it costs to service; and
— over what time-scale the capital has to be repaid.

2.3.4 The techniques we discuss in this paper show how these objectives can be achieved.

2.4 *Company Re-structuring*

2.4.1 In spite of several weaknesses, an RBC formula approach has led to management having a far greater incentive to look at risk management and capital allocation. Perhaps, therefore, the greatest contribution comes from forcing management to impose proper controls on capital allocation (just as in the early 1970s DTI regulations encouraged companies to recognise IBNR explicitly). Further, it has to be recognised that the present management structure, as shown in Figure 1, has to be altered.

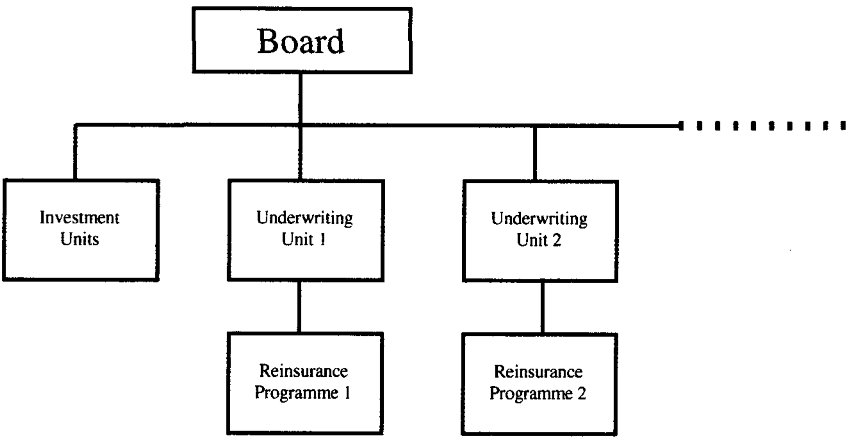

Figure 1. Present insurance company

2.4.2 In Figure 1, it is noticeable that each class of business has its own

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separate reinsurance arrangement. In fact this was mirrored in the reinsurance market in that the brokers until recently tended to be responsible for classes of business rather than reinsurance programmes for the whole company.

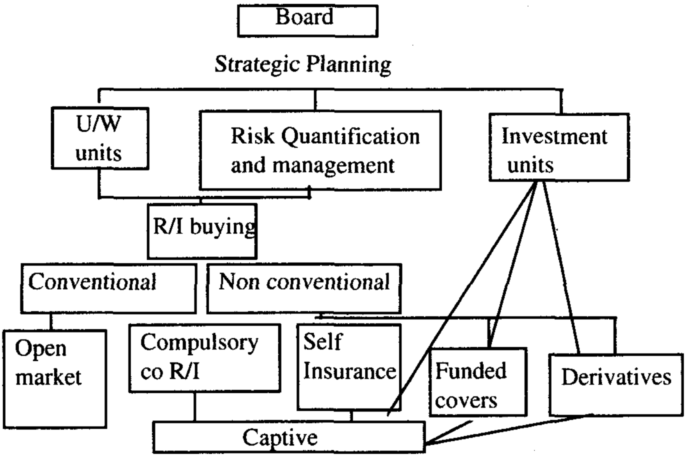

Figure 2. New insurance company

2.4.3 Figure 2 shows what we believe will be the structure of the new insurance company. The main change is the introduction of a cross-discipline committee at, or just below, board level, to examine and control the overall risk profile of the organisation. The reinsurance element has been promoted to be almost equal in importance to investments. Further, reinsurance requirements will be decided by looking at the corporate level as part of the overall risk management, rather than at a line of business level. Hence, the decision process between choosing reinsurance or capital becomes much closer in the management's thinking.

2.5 *Probability of Ruin*

2.5.1 In ¶2.1.2 we introduced the concept of 'risk tolerance'. By this we mean the management's willingness to live with unstable results in order to boost expected profitability. Such willingness can be expressed in terms of probability of ruin theory, which is well established. We define five measures of ruin, namely:

(1) management target;
(2) regulatory intervention level;
(3) net worth (assets minus liabilities) turning negative;
(4) exhaustion of cash and investments (ignoring liquidity issues); and
(5) inability to dispose of illiquid investments.

2.5.2 The above definitions are reasonably straightforward. In building our model, we considered definitions (1) to (4). However (5) has not been modelled, in that we assumed that all investments can be sold at prevailing market values.

2.6 *Data*

2.6.1 The authors were very conscious, because of their backgrounds, that, for the model to succeed, the input information had to be relatively straightforward to obtain. In the past, model attempts have failed because highly theoretical data sets were required, and therefore the models have not been pragmatic (Brown & Galitz, 1981). However, the work of Daykin *et al.* (1987) emphasised the importance of practical input data.

2.6.2 As mentioned in ¶2.2.6, the data which the authors used were easily available. Further, we are very aware of the importance of displaying the results in a way in which management can easily understand the significance of different sets of assumptions. With this in mind, we have emphasised the output in the form of diagrams rather than tables filled with figures.

3. Structure of the Model

3.1 *Overview*

3.1.1 The model is designed to simulate the development of both assets and liabilities of an insurance company. All cash flows are projected whilst this company accepts new business for a period of three years, before going into run-off. The projections then continue until all outstanding claims have been paid. The three-year planning horizon was set as a compromise between the desire to establish a medium-term view of the company's development and the difficulty of setting realistic input assumptions.

3.1.2 The assets are sub-divided by major categories such as government bonds, equities and property. The model projects cash flows until the claims have run off or the company is ruined.

3.1.3 The liabilities work on a class-by-class basis (see Section 3.2), modelling the claim payments gross of reinsurance, and their associated reinsurance recoveries and reinstatement premiums, after allowing for the effects of both financial and social inflation.

3.1.4 The reinsurance programme can comprise any combination of four main types: quota share; surplus; risk excess and catastrophe excess. The model can accommodate variable co-reinsurance of each cover, as well as factors such as event caps on proportional treaties. The catastrophe module allows information from GIS (Geographic Information Systems) models to be incorporated for storm, freeze, flood, earthquake and subsidence. This process is described more fully in ¶4.6.7, whilst a brief introduction to the design of such GIS models is shown in Bolton *et al.* (1995).

3.1.5 By combining the cash flows of assets and liabilities, the model

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produces the potential for profits or losses to emerge from the runoff of outstanding claims.

3.1.6 Each run consists of a user-specified number of Monte Carlo simulations, in each of which the variables are sampled from appropriate probability distributions, so that a probability distribution can be built up for the results of the company as a whole. The run can then be repeated with different assumptions, to examine the sensitivity of these results to changing circumstances.

3.2 *Class and Subclass Structure*

3.2.1 The model calculates gross of reinsurance transactions on a sub-class basis, whilst reinsurance transactions are at a class level (Figure 3).

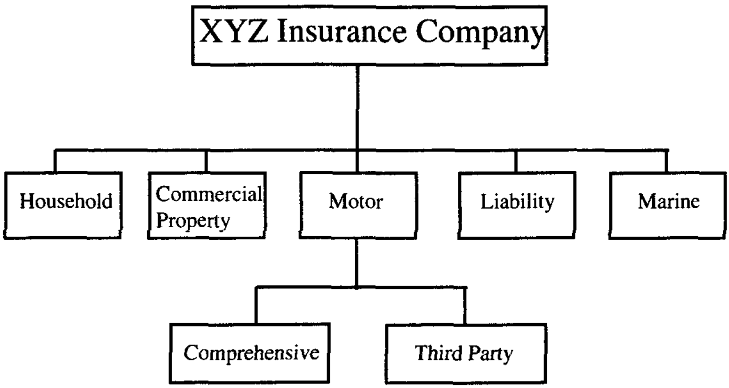

Figure 3. Tree structure for group, company, class and subclass

3.2.2 Figure 3 shows a typical division of class and subclass of an insurance company; the main classes of business being household, commercial property, motor, liability and marine, whilst motor can be divided into subclasses of comprehensive and third party. The amount of detail at subclass level is company dependent, for example if a company is writing only two classes of business, household and motor (say), it might be appropriate to have three or four subclasses for each class.

3.2.3 These are broadly parallel to the present DTI concept of accounting classes and risk groups. We believe that, in practice, the number of classes should be limited to six, and subclasses to no more than ten, so that the overall picture can still be seen without being lost in a mass of detail.

3.2.4 The class structure will vary from company to company, and it is essential to determine this before too much time is spent in trying to assemble input data.

3.3 *The Main Types of Reinsurance*

3.3.1 Reinsurance can be broken down into facultative (laying off parts of individual risks) and treaty (laying off a proportion of a block of business). Treaty reinsurance can be further analysed into proportional (principally quota share and surplus) and non-proportional (excess of loss on either a per risk or a per event basis, or stop loss). Fuller definitions of these, together with other reinsurance terms, are given in Appendix 1. To model reinsurances other than quota share treaties, it is necessary to generate both individual claims and event catastrophes (which is where claims aggregate across several policies to produce a potential recovery). Further, in the case of surplus treaties, commonly used to protect commercial property portfolios, it is necessary to determine the size of cession on each policy subject to a large claim, before a recovery can be calculated.

3.3.2 There is a bad debt risk involved in ceding business to any reinsurer, however much care is taken in selection, and this can never be entirely removed. Whilst management should not lose sight of this risk, we have ignored it in this paper for simplicity.

3.3.3 We expect a reinsurance programme for the classes of business in Figure 3 to resemble:

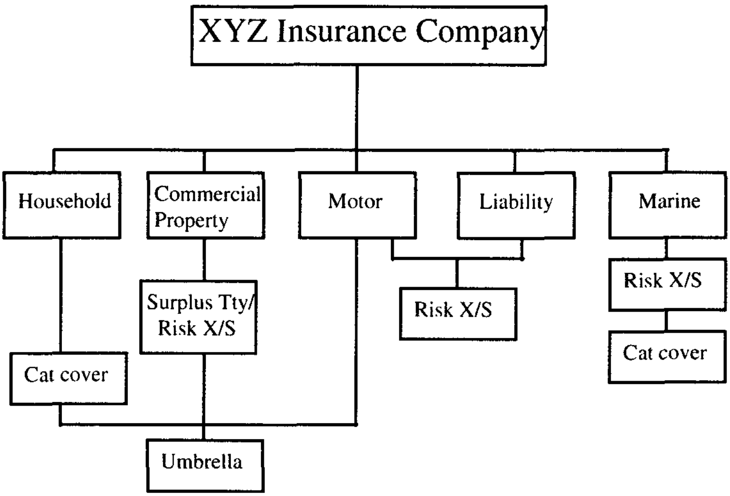

Figure 4. Simple reinsurance programme

3.3.4 Figure 4 shows that the household business is protected by a catastrophe treaty, whilst motor and liability are covered by risk excess of loss, and an umbrella whole account protection covers household, commercial property and motor. Typically, a marine account would be separately protected both for

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large individual losses and for storm accumulations. It may also have quota share or surplus reinsurance for 'capacity' reasons.

3.4 *Build up Cash Flows by Class of Business*

3.4.1 The concept of cash flow modelling is now well documented, for example Daykin & Hey (1990) and Daykin *et al.* (1994) (Chapter 1). In a simple diagram, Figure 5 illustrates the standard cash flows which have to be modelled.

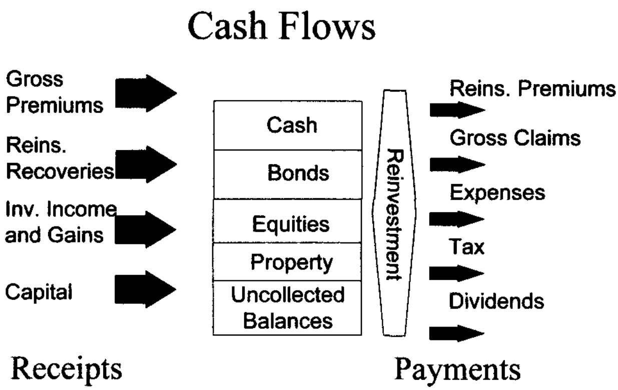

Figure 5. Cash flow

3.4.2 Alternatively, Figure 5 can be linked together in the Daykin *et al.* (1994) transition equation:

Assets(end of period) = Assets(beginning) +(Gross premium – Claims – Expenses – Reinsurance premiums + Reinsurance recoveries) + Investment income & gains – Taxation – Dividends + New capital [+ New borrowings].

3.4.3 With suitable adjustments for changes in provisions, or receivables, this equation can be interpreted on either a cash basis or an accounting accruals basis.

3.5 *Modular Approach gives Flexibility*

3.5.1 The cash flow computer programmes have to be designed very carefully, in particular the main problems are the inter-relationships between transactions, and that actuarial art in projecting forward is always improving. With this in mind, the model was built up as a series of linked modules, any of which can be enhanced separately, see Figure 6.

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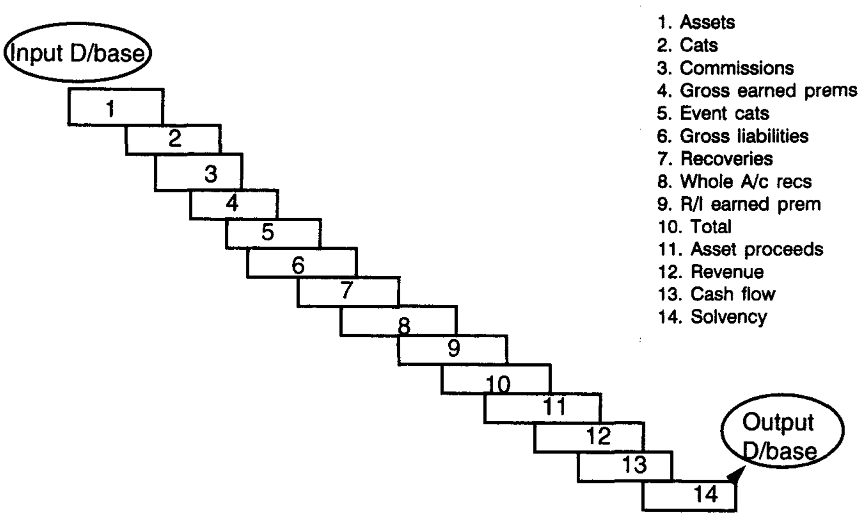

Figure 6. Modular structure

3.5.2 There are five distinct stages in building up the final output:

Stage 1: the data base input;

Stage 2: modules 1 and 2, which calculate inflation rates and investment returns (see Sections 4.2 and 4.3) and individual catastrophe losses (see ¶4.6.6);

Stage 3: modules 3, 4, 5, 6, which are defined for each sub-class, calculate cash flows and technical reserves gross of reinsurance (see Sections 4.4, 4.5, 4.6 and 4.7);

Stage 4 : modules 7, 8, 9, which are reinsurance recovery calculations (see Sections 4.8 and 4.9); and

Stage 5: modules 11,12,13,14, which are the basis for the outputs (see Section 5).

3.5.3 By building up the model in modules, as shown above, we have attempted to create a flexible structure which will enable changes in the computer program to be made with the minimum of effort. For example, these changes could take the form of a more sophisticated asset model, advances in actuarial techniques, the specification of a different family of claims curves, etc. This flexible approach has also been adopted in relation to links to other models — for example, we have not attempted to duplicate packages for reserving, or for turning claims data into probability distributions.

3.5.4 The above modular structure can alternatively be shown as a technical cash flow diagram (Figure 7).

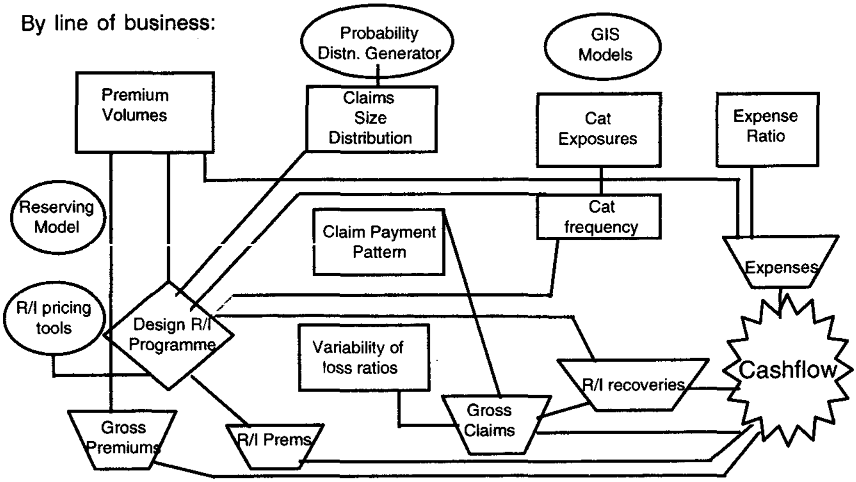

Figure 7. Technical structure of cash flow

3.5.5 The rectangles are the input assumptions (see Section 4) and the buckets are the outputs of the model. The ovals are tools outside the model, which may be necessary to convert company data into the required form. They include the GIS model (Bolton *et al.* (1995) and ¶4.6.7), reinsurance pricing tools (Sanders *et al.*, 1995) and actuarial models, such as the probability distributions of claims and reserving packages.

4. ASSUMPTIONS

4.1 *Overview*

The multi-disciplinary approach that the authors have adopted ensures that, as far as possible, the model reflects actuarial and accounting viewpoints as well as reinsurance market practice. The main assumptions are:

Section 4.2: economic (macro);
Section 4.3: investment (macro);
Section 4.4: company balance sheets (macro control);
Section 4.5: claims reserving (past claims, by sub-class);
Section 4.6: claims projecting (future claims, by sub-class);
Section 4.7: business plan;
Section 4.8: reinsurance programme (by class);
Section 4.9: reinsurance recoveries; and
Section 4.10: correlation between assets and liabilities.

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4.2 *Economic-Inflation*

The effect of inflation on an insurance company can be dramatic, particularly in long tail classes. We have assumed that interest rates on cash are at a fixed margin over financial inflation (i.e. Retail Price Index or similar), but that claims inflation is often at a higher rate, possibly because of linkage to average earnings or advances in medical science. We have, therefore, allowed for inflation by processing through three separate factors:

— financial inflation, determined for the company as a whole, and is the base from which all cash flows after year 1 are factored;
— social inflation on attrition claims — the amount by which attrition claims are expected to rise over and above financial inflation, whether through court awards, or medical advances, etc.; and
— social inflation for large individual claims, defined as those in excess of 50% of the deductible under the risk excess treaty, (if any) since we believe that large claims have a higher social inflation factor applicable to them than small claims.

4.3 *The Investment Model*

4.3.1 The model was designed primarily as a tool for examining different reinsurance programmes. Therefore, we deliberately selected simple models to represent both projections of financial inflation and asset investment, because we wanted to emphasise reinsurance rather than discuss general investment type models (for more detailed examples see Daykin *et al.* (1994) or Christofides *et al.* (1996)). However, the modular approach outlined above would allow more sophisticated financial and asset modules to be inserted if required.

4.3.2 Investments which the model can handle are divided into the following groups, namely:

— U.K. equities;
— foreign equities;
— property;
— fixed-interest bonds;
— index-linked bonds;
— cash; and
— net other assets.

4.3.3 The input parameters we require show an expected mean and standard deviation for:

— real return (net of financial inflation);
— running yield on each investment selected in ¶4.3.2; and
— financial inflation.

4.3.4 The financial inflation model and also the asset investment models are specified as normally distributed variables with mean and standard deviation as input. (Note that we have assumed that each year is independent, but see ¶4.10.)

4.3.5 Appendix 2 gives more details and definitions of both the financial inflation and asset investment models. However, we have the following overview comments to make:

— We require opening market values for each investment selected.
— Cash earns interest at a fixed margin above financial inflation.
— More than one asset can be specified in any group, for example short-term and long-term bonds, to allow for different volatilities in capital value.
— The model calculates both future income growth and future capital growth for each type of investment, and this enables investment and disinvestment to take place based on certain decision rules.

4.3.6 Rules are also required to determine priorities for investment and disinvestment of the year's net cash movements; for example, sell in proportion to the present distribution of assets. For investment income purposes, cash flows are assumed to take place on 30 June and investment/disinvestment on 31 December.

4.4 *Balance Sheet — Revenue Accounts*

4.4.1 We require, for overall control purposes, information from the balance sheet showing the:

— total gross opening outstanding claim reserves;
— unearned premium reserves; and
— current investment portfolio.

This is also required as a basis for the next year's balance sheet.

4.4.2 Tax is levied on the accounted profit, including investment gains, whether realised or not, after taking account of any unrelieved tax losses brought forward. Thus, an effective tax rate needs to be estimated, reflecting any unrealised gains or transfers to equalisation reserve. If desired, this could be enhanced in a relatively minor change to one calculation module.

4.4.3 Dividends are set as a proportion of profits.

4.5 *Claim Reserves — Past Claims*

4.5.1 The gross opening outstanding claims reserves are input, analysed by sub-class and accident year. These claims reserves can be adjusted to reflect management opinion that the book figures are weak or strong. These adjustments can be either on a deterministic or on a stochastic basis. Under the deterministic option, a table of adjustment factors is used to reflect management opinion that book figures are consistently overstated or understated, and the base reserves are multiplied by these factors before applying payment patterns. Alternatively, they can be subjected to randomisation (the stochastic option) based on a lognormal distribution.

4.5.2 A further decision has to be made as to whether the gross opening outstanding claims reserves include an allowance for future inflation or are in the balance sheet on a discounted basis. It is important to recognise if the reserves have been discounted or not, because future claim payments are inflated using the financial inflation parameters and social inflation, as mentioned in ¶4.2.

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4.5.3 These adjusted outstanding claims reserves are then broken down by year of payment, using the input payment patterns with their associated variations. These payments are then subject to inflation, both financial and social (as generated by the inflation model).

4.5.4 Outstanding claims reserves for each accident year are calculated at each year end, as the difference between opening adjusted outstanding losses and paid to date, until the total payments so established have been run off.

4.5.5 Appendix 3 contains a worked example.

4.6 *Claim Reserves — Future Claims*

4.6.1 *General approach*

4.6.1.1 Future claims are broken down into four different components per subclass, namely:

— attrition claims (ordinary claims well within the company's retention);
— large individual claims;
— event catastrophes (for example, a large fire affecting more than one policy); and
— catastrophes covering more than one class of business (for example, windstorm).

4.6.1.2 The concepts outlined in the following paragraphs are relatively simple to understand, but, in practice, it may prove difficult to obtain reasonable input assumptions.

4.6.2 *Attrition claims*

The attrition claims are generated as the product of earned premium and future expected loss ratios. The loss ratios are sampled from a normal distribution specified by mean and standard deviation as input variables.

4.6.3 *Large individual claims*

4.6.3.1 The method of generation of individual large loss figures, and thus the input data required, depends on whether or not there are surplus treaties for the particular class of business. We define large to be half the retention under the risk excess treaty.

4.6.3.2 The annual number of large losses is calculated by adjusting the base number of claims by the expected growth in the account, which is also an input value.

4.6.4 *Large individual claims — risk excess*

4.6.4.1 We first look at a sub-class of business which is affected only by risk excess protections, for example motor liability.

4.6.4.2 The model works by generating individual large claims, based on a probability distribution. This can be carried out in a straightforward way by sampling a specific member of a chosen family of distributions, as determined by input parameters. The model offers a choice of lognormal and Pareto, but this choice could be widened if appropriate. The parameters for the chosen

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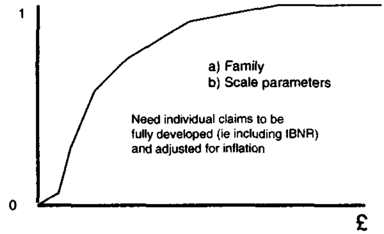
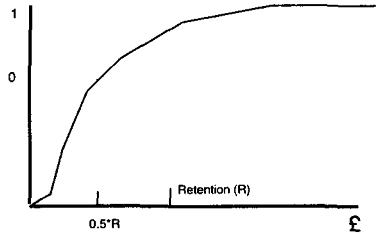
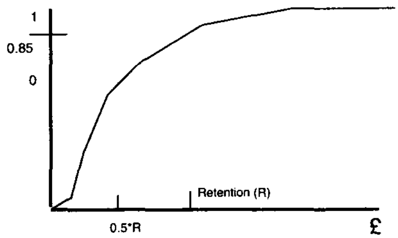
distribution need to be determined outside the model, possibly using a proprietary software package. The fitting of these distributions to large loss data, properly adjusted to reflect estimates for IBNR, and to eliminate the effects of past inflation, is fundamental to the reliability of the model in predicting the effects of changes in the reinsurance arrangements. Figure 8 shows the stages in the process.



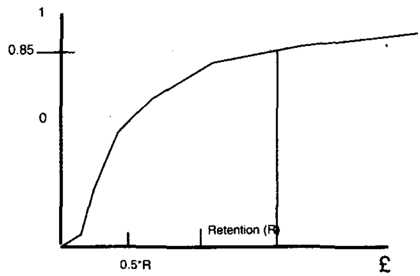
(a) Fit probability distribution to source loss data



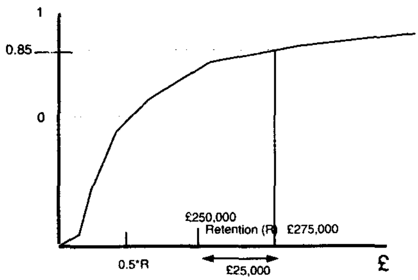
(b) Set cut-off point



(c) Select random number



(d) Read value off distribution function



(e) Calculate reinsurance recovery —excess loss

Figure 8. This shows the process of estimating risk excess

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4.6.4.3 Figure 8 shows, in cumulative distribution form, how the risk excess recoveries are calculated in the model. The importance of this process is emphasised when we discuss surplus treaties in the next section.

4.6.5 *Large individual claims — surplus treaty*

4.6.5.1 When there is a surplus treaty, it is necessary to estimate the size of the treaty cession as well as the size of loss. The model does this by using a bivariate distribution to determine both the gross claim amounts and the probable maximum loss (PML) of the relevant policy.

4.6.5.2 Figure 9 is an extension of Figure 8, and shows how for a one-line surplus treaty of £2 million how the reinsurance recovery is calculated (see Appendix 1 for glossary of reinsurance terms).

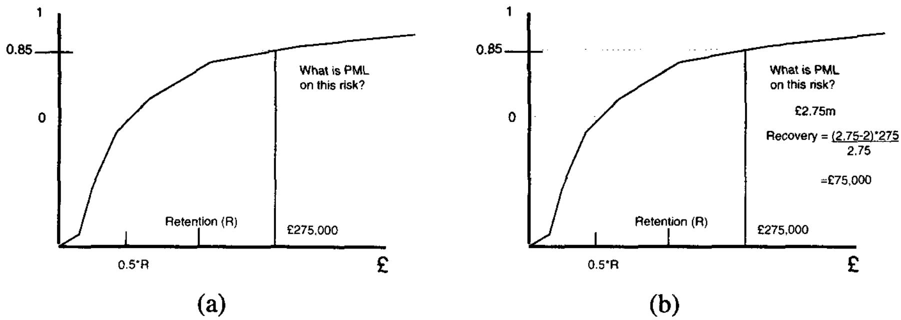

Figure 9. Surplus treaty calculation (1 line surplus treaty of £2m)

4.6.5.3 To calculate the size of recovery, it is necessary to know, for a particular claim, the associated PML of the individual policy. Without knowing this the recovery calculation is impossible. Figure 9 (a) shows the cumulative claims distribution. Figure 9 (b) shows how the recovery is calculated for a one line surplus treaty of £2m if the claim is made on a policy with a PML of £2.75m.

4.6.6 *Event catastrophe losses*

4.6.6.1 The next stage is to input parameters to produce figures for 'event cats' — losses affecting more than one policy, and therefore potentially subject to line-specific excess of loss recoveries. Such losses might include a conflagration loss affecting a commercial fire account, or a casualty clash loss, say in a professional indemnity class. They are estimated in two stages:

(1) the number of events is generated by sampling from a Poisson distribution; and
(2) the size of each claim, given that a claim has occurred, is generated from a user input discrete claim distribution.

4.6.6.2 For simplicity, we assumed that policies involved in 'event cats' are not subject to surplus treaty cessions (but such losses could be ceded to quota share treaties), so input data need to allow for the effect net of surplus cessions.

4.6.7 *Natural perils catastrophe losses*

4.6.7.1 Bolton *et al.* (1995) describes a GIS system which enables insurance companies to estimate PMLs associated with different levels of natural catastrophe events. In the U.K. there are three main natural perils, namely: flood; freeze and storm. In the first instance, we model these in the household account, with 'knock on' (see ¶4.6.8) claims into other classes and subclasses calculated by fixed, peril-dependent, percentages of the household loss. A catastrophe loss, itself, can either be input as a predetermined amount within a predetermined time, or generated in the same way as the event-cat, as described above, using separate assumptions for each peril.

4.6.7.2 If a stochastic approach is chosen, the model requires certain assumptions regarding the probability and potential size of each event for each peril to be input. These assumptions comprise:
- — the PML;
- — the probability that an event of at least one tenth this size will happen; and
- — a table setting out the relative probabilities of the size of the loss, given that one has happened. This table needs to be completed for each decimal of PML.

Table 1. Catastrophe return periods for storm and flood by size of loss

Return period			
Storm		Flood	
£m	Years	£m	Years
8	5	10	50
16	9	20	53
24	13	30	56
32	19	40	59
40	26	50	71
48	36	60	91
56	50	70	125
64	71	80	250
72	125	90	500
80	250	100	1,000

4.6.7.3 Table 1 shows the return periods for total PML losses up to £80m for storm and £100m for flood. These figures will be obtained either from a GIS type model or from general management views. For example, there is a 1 in 71 year chance of a storm occurring where the total loss will be at least £64m.

4.6.7.4 Individual catastrophe claims for each peril are then generated using the probabilities as defined in Table 1. The number of events for each peril is determined using a Poisson distribution, based on the assumed expected number

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of claims per year. The size of the loss is calculated using the discrete probability distribution generated from Table 1. Knock-on claims into other classes follow on a pro rata basis.

4.6.7.5 The model generates catastrophe losses using a discrete rather than a continuous distribution. This gives rise to two issues, namely those of PML failure and the practical disadvantages of simulation from a discrete distribution, which the GISMO paper (Christofides *et al.*, 1996) may go some way to eliminate. This is discussed more fully in Appendix 4.

4.6.8 *'Knock-ons'*

4.6.8.1 Catastrophe perils, such as storm and flood, affect other classes of business besides household. This 'knock-on' effect is modelled into other classes and subclasses, using fixed, peril-dependent percentages.

4.6.8.2 Unlike 'event cats', catastrophe losses and 'knock-ons' are calculated on a gross basis, and this gross loss is then allocated to both quota share and surplus treaties pro rata to aggregate exposure.

4.6.8.3 For example, a flood catastrophe primarily affecting household would also have a knock-on effect on both commercial property and motor (see Section 6.3).

4.6.9 *Payment patterns*

Having determined the attrition losses, individual large losses and catastrophe loss figures as defined above, the model applies respective payment patterns and associated variations to these losses to turn the incurred claims into cash flows by individual development year, and calculates outstanding claim reserves at each year end. These cash flows are then adjusted for financial and social inflation. Note that social inflation is applied at differential rates for attrition and large losses. A worked example of this process is shown in Appendix 3.

4.7 *Business Planning*

4.7.1 The model requires, for a three-year period, projections of:

- gross premium volume;
- unearned premium reserve;
- policy count for assessing whether growth is derived from a change in volume or pricing levels;
- a central estimate of expected loss ratio (excluding catastrophe losses, which are handled separately); and
- the variability of this loss ratio.

4.7.2 Commission and expenses by class of business are assumed to be percentages of gross written premium, which can vary by class of business and projection year. There is also a provision for expenses in runoff.

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4.8 *Reinsurance Programme*

4.8.1 Data to specify the reinsurance programme for the next three years are also required. (Exposure during the runoff of unearned premium at the end of year 3 is assumed to benefit from the same protection as year 3 business, at a pro rata price.)

4.8.2 Usually for the first run of the model, assumptions for all years will mirror the programme currently in force, although it may be necessary to make a few modifications in order to live within the model's constraints. This will establish a base position against which to measure subsequent runs.

4.8.3 For each class of business, the model has the capacity to handle a quota share treaty and four surplus treaties, ten layers of risk excess, and ten layers of 'event cat' cover. Also a number of 'whole account' (umbrella) programmes can be specified, covering more than one class of business, which will protect any (natural peril) catastrophe loss burning through the line specific cover. All layers of the non-proportional covers can be specified with any number of reinstatements (paid or free), variable co-reinsurance percentages, and with aggregate deductibles, whilst all proportional treaties can be specified with an event limit.

4.8.4 The pricing of non-proportional covers is independent of assumed burning costs, thus allowing manual intervention to adjust theoretical rates to those achievable in current market conditions.

4.8.5 Whilst there is no specific facility for facultative reinsurance, this could be approximated by means of a dummy top layer surplus treaty.

4.9 *Recoveries*

4.9.1 Refer to Appendix 3 (Section 5) for a simple worked example of reinsurance excess of loss recovery.

4.9.2 *Run-off of recoveries on past claims*

Opening reinsurance recoveries for each subclass are specified as percentages of the gross outstandings for each accident year. For simplicity, receipts from reinsurance recoveries are established as this fixed percentage of the gross claims payments based on past reserves, but taking account of delays in collection. Outstanding recoveries are derived by summing the future receipts so established, and subtracting collections to date.

4.9.3. *Recoveries on future claims*

4.9.3.1 Recoveries in respect of future claims for each class of business are established in the following order:

— quota share / surplus;
— risk excess of loss; and
— 'event cats'.

4.9.3.2 The model assumes that proportional treaties do not benefit from non-

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proportional cover; where this is not the case, limits, deductibles and aggregates must be scaled down by the quota share and surplus percentage to establish an equivalent net position.

4.9.3.3 Once the recoveries from all line-specific reinsurances have been calculated, the remaining loss is aggregated across classes, as appropriate, to calculate recoveries from the whole account programme.

4.9.4 *Quota share/surplus*

4.9.4.1 Surplus treaty recoveries are processed for individual large losses, event catastrophes and knock-ons only, as described in Section 4.6. The recovery for each development year of each accident year comprises the following elements:

— *Attrition claims* are allocated to quota share treaties by multiplying gross attrition claims by the quota share percentage.
— *Large claims*: recoveries from surplus treaties are calculated using the bi-variate table outlined in Section 4.6.5. The balance is then shared with the quota share treaty as per attrition claims.
— *Event cats*: the gross claim for each event established above is multiplied by the quota share percentage, but restricted to the quota share event limit, if applicable, before being aggregated by accident year.
— *Catastrophes/knock-ons* have to be apportioned between the retention, quota share and surplus treaties, and then restricted to the event limit, if applicable (these apportionment percentages could be different for each peril).

4.9.4.2 These various components, as detailed in ¶4.9.4.1, are calculated on both a 'paid' and an 'incurred' basis, by reference to the gross loss development described in ¶4.6.9.1, the resulting recovery from treaty reinsurers being subjected to an appropriate payment delay.

4.9.5 *Risk excess*

Each individual large claim, suitably netted down for recoveries from quota share and surplus treaties, is then compared against the risk excess programme. Each claim is assumed to have the settlement pattern for the subclass from which it originated, and recoveries from each layer of risk excess are calculated annually. The calculation takes into account the cumulative payments on each claim, restricted, where applicable, by aggregate deductibles and co-reinsurance as well as the number of reinstatements. Then the reinstatement premium is calculated for each layer separately.

4.9.6 *Catastrophe excess of loss*

Recoveries and reinstatement premiums follow the same logic as risk excess.

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Note that it is assumed that none of a catastrophe claim is recoverable from risk excess.

4.9.7 *Whole account (umbrella)*

4.9.7.1 The whole account programme looks at natural peril claims for those classes which are covered by the whole account programme, net of recoveries from line-specific covers. (Note that it assumes that there are no gaps in coverage in the specific covers, and that all co-reinsurance in the specific layers is for net account.)

4.9.7.2 Recoveries also follow the same logic as risk excess above, and are apportioned to the cover in proportion to the losses, not otherwise recovered from the original classes. Whole account reinsurance premiums are allocated pro rata to the gross written premiums of the covered classes, and reinstatement premiums pro rata to recoveries.

4.9.8 *Reinstatement premiums*

Depending on the large claim simulation, the model will calculate when a reinstatement premium is due and will calculate its cost, using the input assumptions of number and cost of reinstatements for each layer.

4.10 *Correlation — between and within Assets and Liabilities*

We have deliberately ignored correlation issues in this paper. In particular, we have assumed that in the asset model there is no correlation between one year and the next and no correlation between assets, and further, in the liabilities that one class of business is not correlated to another. In addition, we have assumed that between assets and liabilities there is no correlation except indirectly via financial inflation. This is not to say that the authors consider that there are no correlations, as they certainly do believe that there are. However, the authors wanted this paper to address reinsurance issues, and believe that correlation issues deserve a paper on their own (Bulmer *et al.*, 1996).

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5. Model Output

5.1.1 In order to generate all the cash flows, the model effectively builds up, for each simulation in each run, a summary of the company's general ledger from last year-end until the run off of the last claim from business accepted in three year's time.

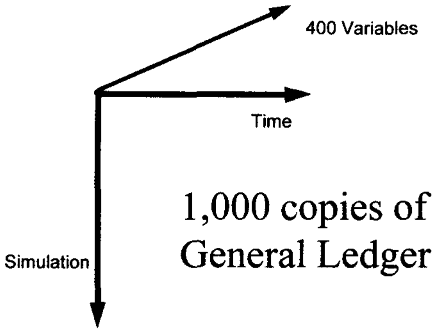

Figure 10. Outputs of each run

5.1.2 The output consists of values of a large number of variables (400), each of which is indexed by a simulation number and projection year. This produces an enormous amount of data, and we had to use a database package to manipulate it. The importance of keeping all the simulated data cannot be emphasised enough, because this allows the database to be interrogated to identify which particular simulation run is giving odd results and why.

5.2 *Conventional Output*

5.2.1 The conventional way of showing simulation output is shown in Figure 11.

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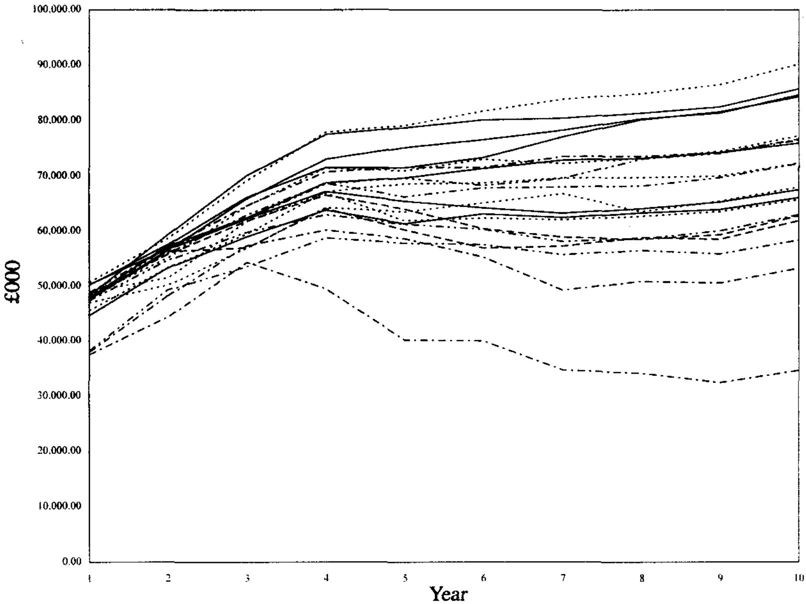

Figure 11. Net worth

5.2.2 The figure shows the net worth of a company with three years' new business. An alternative way of looking at Figure 11 is to look at the net worth at a point in time. This is done by plotting the probability distribution of the simulation output at a fixed time, for example at the end of three years, as shown in Figure 12.

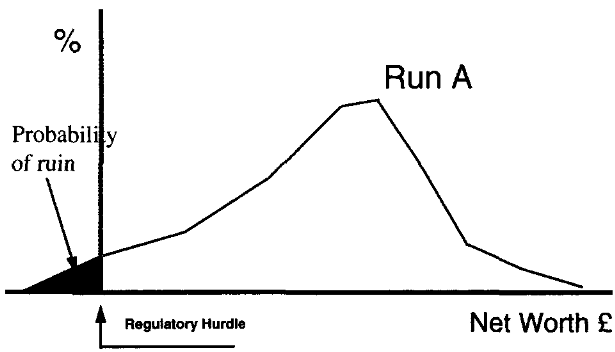

Figure 12. Probability distribution of net worth at the end of 3 years

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5.2.3 Figure 12 gives a very easily understood picture of the volatility of performance. The left hand side shows negative net worth and the right hand side shows positive net worth. Management should be trying to shift the graph as far to the right as possible, representing an increase in profits, whilst keeping it as peaked as possible, thus stabilising the profits. The 'regulatory hurdle' axis can be drawn in various places to indicate either internal or external requirements, whilst the 'probability of ruin' is the probability of failing to meet this yardstick at a fixed point of time.

5.2.4 One further problem is to know whether to measure the mean or the mode of the distribution. In our paper we have not identified which is the best measure, but we believe that it does not affect the underlying principles which we are establishing.

5.2.5 Finally, we have found, in practice, that sometimes it is more useful to show the distribution function as a cumulative distribution function such as Figure 13.

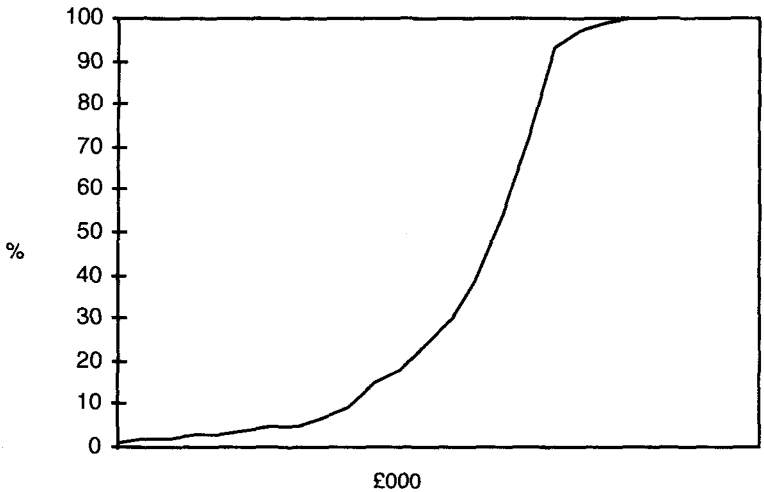

Figure 13. Cumulative probability distribution of net worth after three years

5.2.6 Figure 13 is identical to Figure 12, except that it is the cumulative value and is usually smoother than the distribution function. It is worth pointing out that the steeper the gradient of the curve the less the variability in the distribution.

5.3 *Capital Allocation by Line of Business*

5.3.1 Equity is expensive and cannot be wasted. Successful companies will be

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those which are efficient capital allocators. Their ongoing success will give them continuing access to capital markets.

5.3.2 A low-frequency, high severity business needs more capital than one writing homogeneous business to cope with the greater probability of ruin, and also needs a higher rate of return to compensate for the unstable earnings. The capital required will vary, depending on the state of the underwriting cycle, the strength of claims reserves and the degree of premium adequacy.

5.3.3 As mentioned in Section 2.2, we believe that any method of capital allocation has to be able to cope with these problems, and the RBC formulae are an attempt. More relevant results can be achieved with our model, since we are able to reflect explicitly the impact of changing reinsurance arrangements.

5.3.4 Management can achieve this by setting a maximum acceptable probability of ruin, possibly say one in 500 years for the company as a whole, and 1 in 100 years for any one account, to take note of the fact that it is unlikely that all classes of business will fail at the same time.

5.3.5 The model can now be run for a single profit centre within the company, to establish the capitalisation required to exceed the ruin probability hurdle.

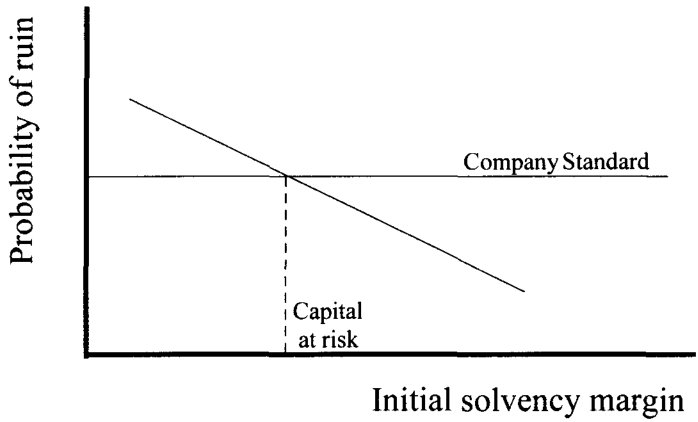

Figure 14. Set company standard for probability of ruin

5.3.6 Figure 12 showed the probability of ruin for a particular scenario. By altering the initial capital, leaving all other inputs unchanged, it is possible to build up a plot of the probability of ruin measured against the opening capital. Figure 14 shows this graph for a particular profit centre, and the capital at risk can be established by comparison against the company standard.

5.4 *How much Capital does a Reinsurance Contract Release?*

5.4.1 Once the first run has been completed (a major task comparable to, and probably done in conjunction with, the annual budgeting process), other runs (different reinsurance programmes or different asset-mixes, etc.) can be carried out (this is done by storing the original simulation results and then going through a different reinsurance programme).

5.4.2 One particular example of this process is to compare the effect of two different reinsurance programmes. We can plot probability of ruin against initial solvency margin for both runs. Applying the company minimum standard to these gives the following pictorial results for two different excess of loss reinsurance programmes:

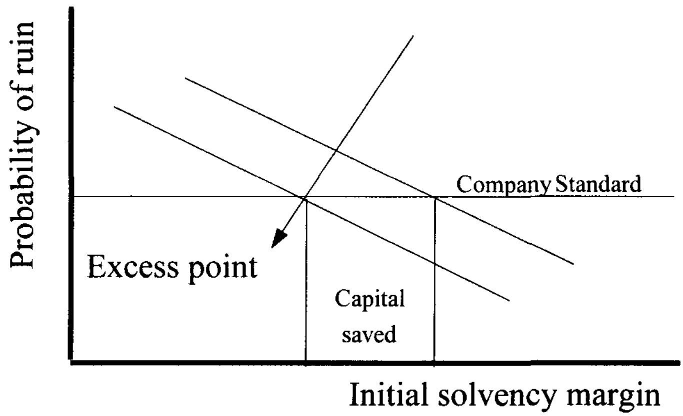

Figure 15. Measure capital saved

5.4.3 Figure 15 shows the capital saved by reducing the excess point at a predetermined probability of ruin.

5.5 *How much does this Cost to Service?*

5.5.1 For the same example as in Section 5.3, the cost difference between the two reinsurance programmes can be examined by looking at the probability distributions at the end of three years.

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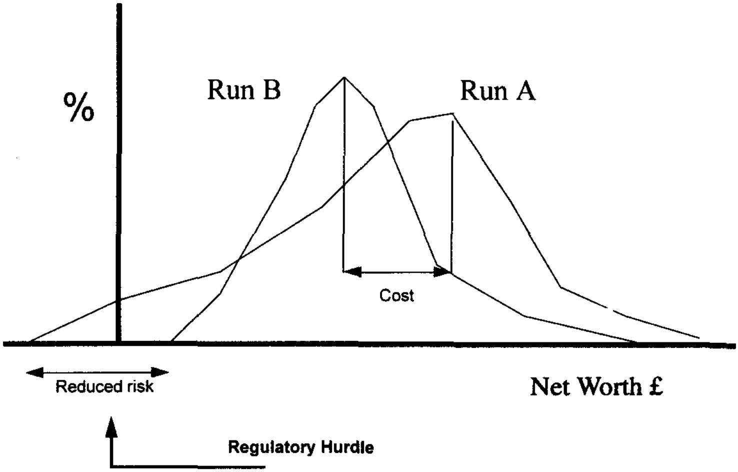

Figure 16. Expected servicing cost

5.5.2 Figure 16 shows the net worth of a company at the end of three years. Run A is the present reinsurance arrangement and Run B is a different one. The difference between Run A and Run B is that Run B is safer, but has a lower expected (mean or mode) return than Run A. However, the price the company pays under Run A is a higher probability of ruin.

5.5.3 This graph can also be used to measure different reinsurance programmes, but now it is used to measure the difference in expected cost.

6. Detailed Examples

6.1 *Simple Start-Up Company*

6.1.1.1 Let us consider a start-up company which has an initial premium of £100m and writes household business only. The initial capital is set at £50m, which represents a 2:1 solvency (premium to net worth) ratio. Ninety per cent of initial capital is invested in government bonds. Further, if there is any positive cash flow of premiums to claims and expenses, 50% is invested in government bonds and 50% is invested in equities, and further assets are bought and sold in proportion to the asset distribution. Detailed assumptions are listed in Appendix 4.

6.1.1.2 The underwriting results are:

	%
Premiums	100
Gross claims	(55)+/−2
Commissions	(23)
Office expenses	(5)

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6.1.1.3 The gross claims of 55%+/–2 represent claim costs excluding catastrophes at the current stage of the underwriting cycle. This information could be estimated from competitors' DTI returns.

6.1.2 *Reinsurance programme: Run 1*

6.1.2.1 Initially, the company wishes to consider a reinsurance programme which protects only against catastrophe. Prior to undertaking a full GIS study of catastrophe aggregate exposures and PMLs, the broker has suggested, given his knowledge of the market, the following:

Storm exposure 80% of gross premium
Flood exposure 100% of gross premium.

6.1.2.2 Management has come up with a market view of assumed return periods, which are shown in Table 2. (Note that these are for illustration only and are not intended to be realistic numbers.)

Table 2. Assumed return periods

Return period			
Storm		Flood	
£m	Years	£m	Years
8	5	10	50
16	9	20	53
24	13	30	56
32	19	40	59
40	26	50	71
48	36	60	91
56	50	70	125
64	71	80	250
72	125	90	500
80	250	100	1,000

6.1.2.3 Table 2 is the same as Table 1 in ¶4.6.7.2. The new company assumes that the probability of breaking an £80m PML in case of storm is negligible, whilst for flood it has assumed that it could have a PML of £100m.

6.1.2.4 After performing our simulation, we can count how many storms and floods are observed for different PMLs, and these are displayed in Figures 17 and 18.

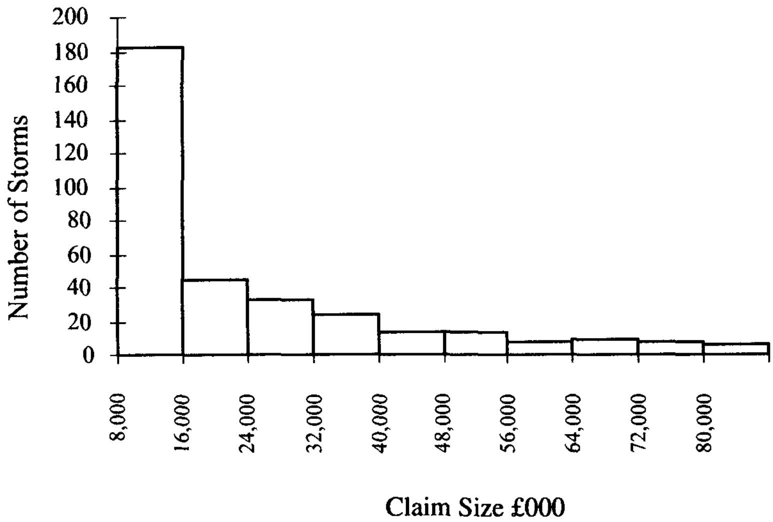

Figure 17. Storm losses from 500 simulations

6.1.2.5 Figure 17 shows that most (181) storms were under £10m PML, and that there was only one storm of £80 million. Altogether there were 337 storms simulated.

Figure 18. Flood losses from 500 simulations

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6.1.2.6 Figure 18 shows that the flood distribution is very different, and the reason is that if the flood defences hold then there will be no catastrophe, hence a smaller number of large claims.

6.1.2.7 Finally, Table 3 gives the proposed reinsurance programme which shows the layers, rate on line (ROL) and cost. It was also expected that reinsurers would require the company to keep 5% co-reinsurance. The £10m retention is believed to be slightly smaller than market averages, and the reinsurance purchased is sufficient to cover the whole storm exposure and almost all the flood.

Table 3. The proposed reinsurance programme

	ROL	£m
£10m xs £10m	20	2.0
£20m xs £20m	12	2.4
£20m xs £40m	8	1.6
£30m xs £60m	4	1.2
		7.2
less 5% co-reinsurance		£6.84m

6.1.2.8 The model is run with 1000 simulations (we show in our examples only 20, so that the reader can view what is happening). For the initial run, we have assumed that three years' new business is written at a stable volume and that the reinsurance programme is in place for three years without change. The assumptions can, of course, be changed in subsequent runs. The gross profits are shown in Figure 19. Note that each line is a simulation, and the variation inherent in the assumptions is displayed by the spread of the curves. This variation reduces dramatically after year 4, when the company is assumed to be in run-off.

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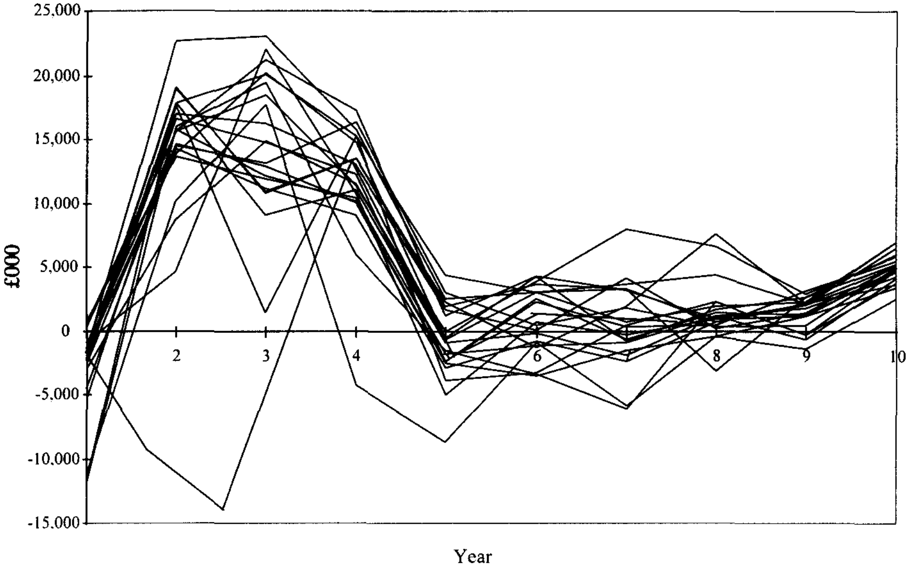

Figure 19. Gross profits

6.1.2.9 Figure 20 shows the net worth of the company. Notice that there is one simulation which indicates that a large part of the capital has been eliminated. By interrogating the database for this simulation, one finds that in year 1 the company suffered two catastrophe claims, the retentions and reinstatement premiums on which almost eliminated its capital base. (This loss scenario is, in fact, exactly what happened in 1987 in the U.K.—87A Freeze and 87J October Storm.)

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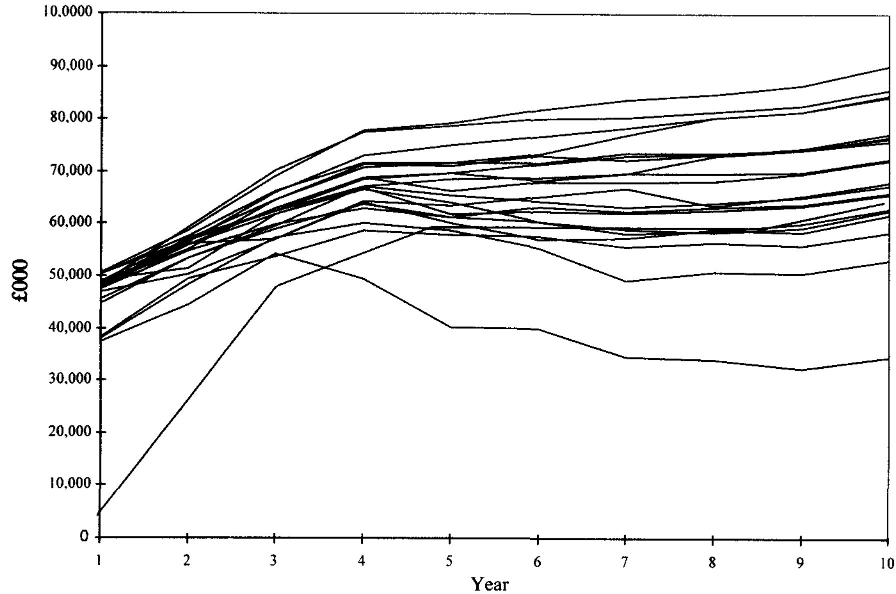

Figure 20. Net worth

6.1.2.10 An alternative way of looking at Figure 20 is to look at the range of net worth at a point in time. This is done by plotting a probability distribution as in Figure 21.

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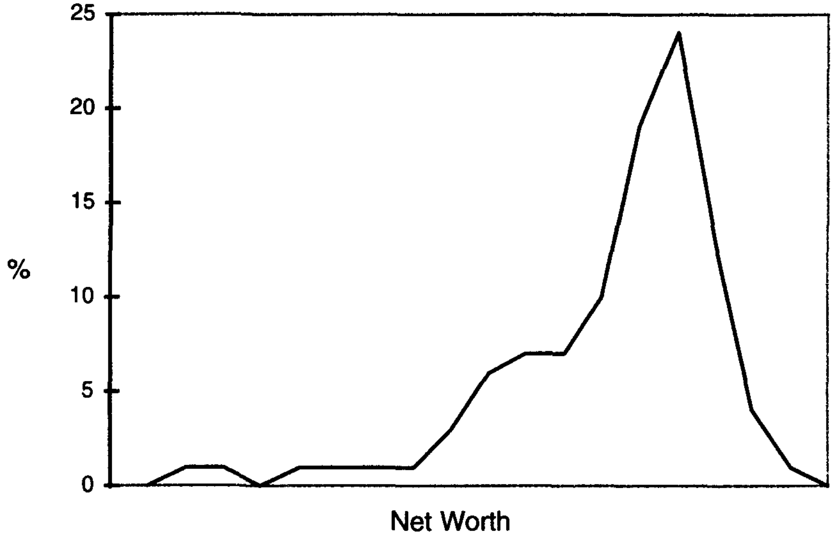

Figure 21. Probability density function of net worth at the end of three years

6.1.2.11 Figure 21 is rather spiky, and we believe that a better presentation is to plot it as a cumulative distribution function, as in Figure 22.

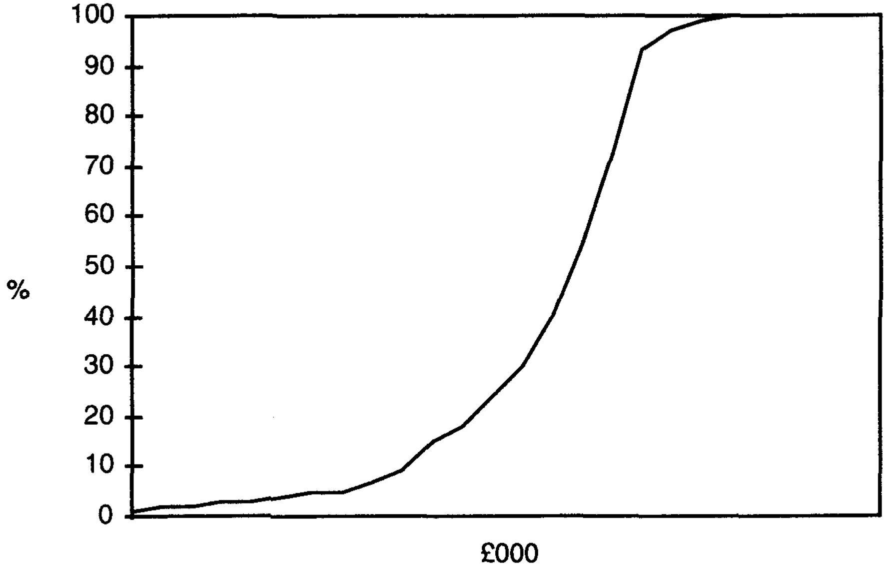

Figure 22. Cumulative probability distribution

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6.1.3 *Reinsurance programme: Run 2*

Since there seems to be a reasonable probability of having two catastrophes in one year, the company decided to try an alternative reinsurance programme, namely it adjusted the proposed reinsurance programme in Table 3 by including a first layer of £5m excess £5m. The effect is shown in Figure 23, namely that there is a reduction in expected net worth with a compensating reduction in risk.

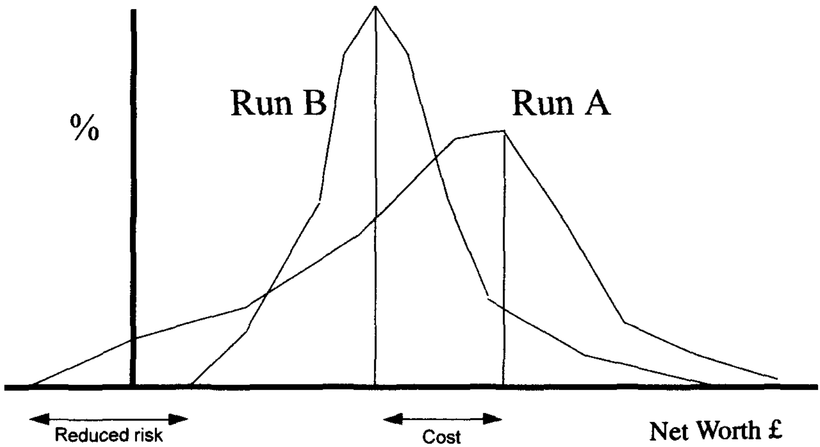

Figure 23. Risk/return relationship

6.1.4 *Reinsurance programme: Run 3*

6.1.4.1 As an alternative to Run 2, it was suggested to have the same reinsurance programme as Run 1 (Table 3), but to also have a 30% quota share. The results of this reinsurance arrangement are shown in Figure 24.

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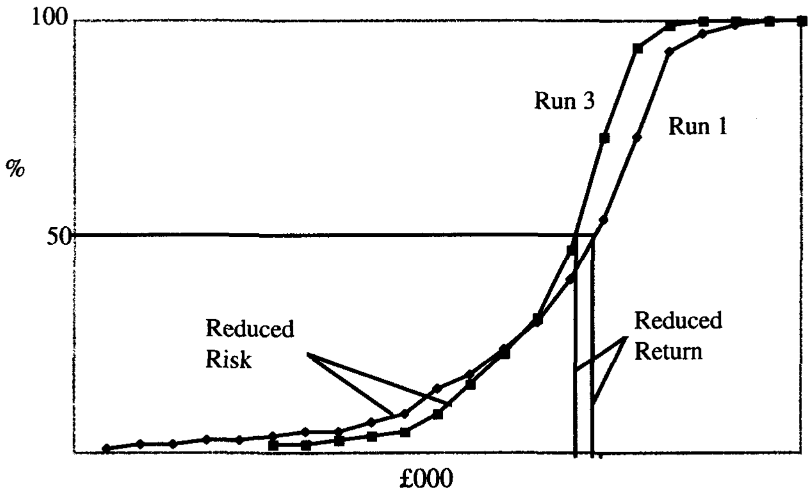

Figure 24. Risk/return relationship on a cumulative probability basis

6.1.4.2 In Figure 24 the reduction in variability is shown by the steeper curve in Run 3 compared with Run 1, but again there is a reduced risk with an offset of reduced return.

6.2 *Issues Raised*

We believe that it is necessary to examine why certain simulations give such poor profit performance. Also, it is necessary to question whether quota share is the best way of reducing the capital at risk, and whether an excess of loss programme could achieve the same benefit at lower costs.

6.3 *Complex Example*

6.3.1 We do not intend to review a detailed example in this paper. However, to show the power of the model, we illustrate in Figure 25 a more complex reinsurance programme which the model can analyse.

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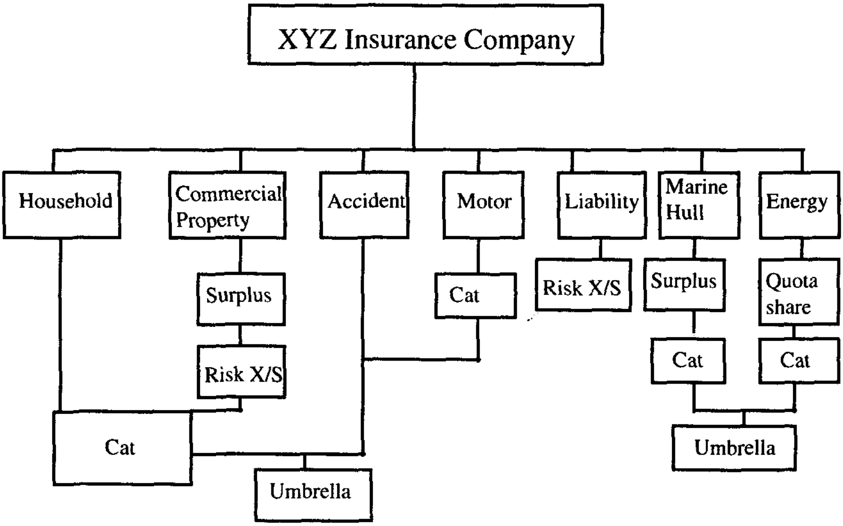

Figure 25. A complex reinsurance structure

6.3.2 Figure 25 shows a typical reinsurance programme for a multi-class general insurance company. Notice how the two umbrella protections each cover a number of classes of business.

6.3.3 Figure 26 shows the underwriting result for commercial property for the three years of new business plus run off.

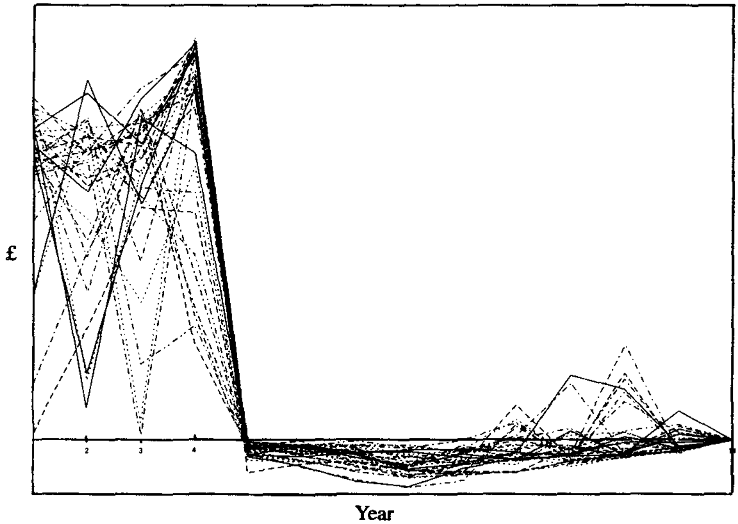

Figure 26. Underwriting result—commercial property

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6.3.4 The results show that the business is profitable, and this profitability is related to the present reinsurance programme.

6.3.5 Figure 27 shows the underwriting results for marine liability.

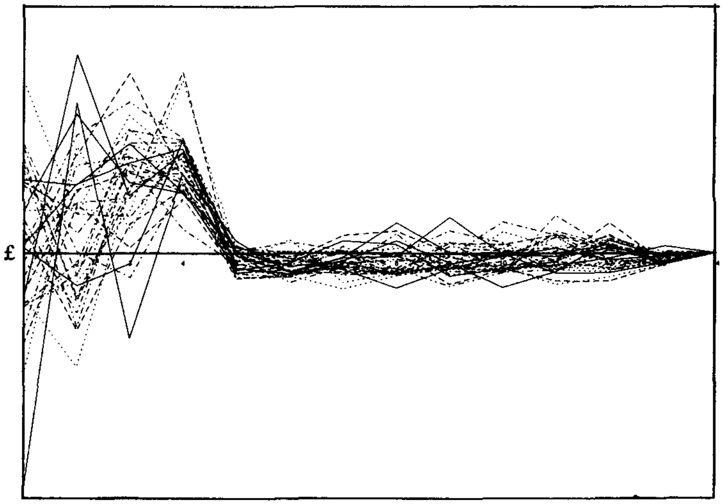

Figure 27. Underwriting result — marine liability

6.3.6 Here the results are more volatile, and the company may think of designing a different reinsurance programme. This would require a more detailed examination of the results of the different reinsurance treaties, and an assessment of why they had so little impact on stabilising the net account.

7. Practical Issues

7.1 *Integration with Management*

7.1.1 Because of the large number of detailed assumptions, covering each aspect of the company's business, required to make this model useful within management, we have found, in practice, that it is necessary that the decision to create such a model has to come from the chief executive. Ideally, it would be useful to have the model integrated with the business planning and budgeting process.

7.1.2 This cycle begins with the chief executive setting group objectives, taking into account the need to pay dividends, satisfy rating agencies, etc. The group's risk tolerance is set, benchmarking against market standards. Risk capacity is then allocated to individual business units, whose management is rewarded against targets.

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7.2 *Technical Issues*

7.2.1 Initially we developed the model to run on a PC. However, we found, in practice, that to test the computer program we needed to store every simulation for every variable. This slowed the results to an unacceptable level. We believe that, once the output from the model has been refined, then it will be possible to reduce the number of simulation points to be stored, and hence acceptable run times will be accomplished.

7.2.2 The importance of storing all the simulation points cannot be emphasised enough, since only by recording all the information can the user appreciate how the model is working.

7.2.3 Finally, the necessary external packages, such as graphics and executive information systems, have to be compatible with the computer environment.

8. Future Applications

The model took eighteen months to specify and develop. As the model evolved, the authors found that they wanted to enhance the structure almost on a daily basis. A decision was made that regular enhancements would bring about a collapse of the project; so it was decided to work on an 80-20 principle, namely having 80% of the model working. In fact, we have 45 enhancements which we wish to introduce to the existing model. The following list identifies some of the areas:

— correlation between asset and liability variability;
— more complex asset models;
— equalisation reserves;
— financial reinsurance;
— facultative reinsurance;
— PML failure;
— more complicated reinsurance arrangements, including stop loss;
— derivatives, both financial and reinsurance;
— currency exposure;
— reinsurance failure;
— appraisal values;
— Ogden tables; and
— underwriting cycles.

9. Conclusion

9.1 *The Requirement*

9.1.1 Asset risk theory is well developed, and highly sophisticated analyses of the various components of risk in an investment portfolio are now carried out as a matter of routine. Even in a fixed-interest account, items such as maturity, shape of yield curve and quality differentials are studied and predicted. This

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analysis has led to a whole range of derivative instruments, swaps, options, forward rate agreements, etc., based on standardised products, thus allowing investment decisions to be made to take account of each individual component of risk separately.

9.1.2 Finance directors will look for similar flexibility in managing the risk profile of their liabilities, and this can be expected to produce a similar spread of standardised risk management products, of which market loss franchise covers, Chicago Board of Trade cat options, etc. can be considered the front runners, subject, of course, to the impact of regulatory constraints. However, a necessary first step is to formulate and standardise the identification and measurement of insurance risk, including the effect of different reinsurance arrangements. In this way, underwriting risk and expected return on capital can be compared against opportunities on the asset side of the balance sheet. In turn, this will lead to demands for yet more sophistication in actuarial and risk modelling techniques, and for an infrastructure where the outputs of these techniques can be understood and acted upon.

9.1.3 The present generation of stochastic models which include explicit reinsurance are in their infancy, and will grow in importance. They need to be driven from the top of an organisation as an integral part of the planning process, and require constant amendment and refinement.

9.2 *The Effort*

Does information generated by the model justify the effort of data collection and the large set of assumptions to input? We believe the answer must be 'yes', because of the growing sophistication of the investment and reinsurance markets.

9.3 *Health Warning*

It is important to appreciate that the model is a simplification of real life, and that the answers it gives are totally dependent on the quality of the input assumptions. Nevertheless, we believe that it gives added insight into the inter-relationships of all the variables and the overall risk profile of the company's activities. Further, what we have discussed in this paper is a strategic planning tool, and not a tactical one.

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The opinions of this paper are the authors' personal opinions, and they are also responsible for any errors and omissions or other deficiencies in the paper.

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APPENDIX 1

GLOSSARY OF REINSURANCE TERMS

A. *Types of Reinsurance Treaty*

A treaty is the reinsurance of a block of business or whole account in accordance with the terms of a contract between the cedant and the reinsurer. It may be either proportional or non-proportional.

Proportional Treaty

This is a term used to classify surplus and quota share reinsurance, whereby the original insurer and the reinsurers share losses in the same proportions as they share premiums.

Non-proportional Treaty

This is a term used to classify treaties, such as excess of loss or stop loss, in which the sharing of losses between the original insurer and reinsurer is not proportional to the share of premiums. These contracts are normally subject to a predetermined number of compulsory reinstatements in the event of loss, and the reinstatement premium is expressed as a multiple of the original rate.

Quota Share

This is a proportional form of reinsurance. A fixed proportion (usually expressed as a percentage) of all risks accepted by the primary insurer are ceded to the reinsurer. The reinsurer receives the same proportion of all the original premiums, less commission, and pays the same proportion of all the losses.

Surplus

This is an alternative form of proportional reinsurance in which the reinsurer also receives an agreed proportion of the original premium, less commission, and pays the same proportion of all losses. However, in this instance, only amounts accepted by the primary insurer above its own retention are ceded to the reinsurer. The treaty is usually expressed as a number of lines, with one line being equal to the reinsured's retention.

Excess of Loss

This is described as non-proportional reinsurance. In return for an agreed premium, the reinsurer accepts liability for losses incurred by the reinsured in excess of an agreed amount, subject to an upper limit, although certain classes of business can be written on an unlimited basis. The most common forms of excess of loss reinsurance are:

— risk excess of loss;
— occurrence or catastrophe excess of loss;
— stop loss; and

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— aggregate excess of loss.

Risk Excess Of Loss

Risk excess of loss reinsurance is a form of non-proportional reinsurance in which the reinsurer pays any loss on an individual risk in excess of a predetermined amount. Losses of a catastrophic nature, involving two or more risks, are usually protected by other forms of reinsurance such as catastrophe excess of loss.

Catastrophe Excess of Loss

This is a non-proportional treaty, structured in the same way as the risk excess treaty above, but protecting either one line of business or several different lines (umbrella cover), rather than just one risk.

Stop Loss

This is a reinsurance contract in which the cedant's losses in a particular year for a particular class of business are capped, either at a monetary limit or at an agreed percentage of net premium. Such policies usually have a finite limit to the amount the reinsurer is liable to pay.

Market Loss Franchise (or Original Loss Warranty)

This is a type of catastrophe excess of loss treaty, which is triggered only if the loss to the whole insurance market (as defined in the contract—normally by reference to an independent source) is larger than a stated size, which would normally be in excess of £1 billion or more.

Second Event Cover

This type of reinsurance is an excess of loss treaty, where the amount otherwise recoverable is subject to an aggregate deductible high enough to eliminate the first loss occurrence and respond only to the second. This should have the benefit of keeping the reinsured's cost down.

Back-up Cover

These covers provide additional reinstatements to a layer of non-proportional reinsurance in the event that the backed-up layer becomes exhausted.

Financial Reinsurance, Finite Risk or Funded Covers

These are forms of alternative risk transfer, usually arranged on a multiple-year basis, to cater for specific circumstances of the cedant. They involve the creation of a pool or fund into which premium and investment income are placed, and from which losses, expenses and profit commission are drawn. The resulting balance of the fund at the end of the contract is shared in pre-agreed proportions between the reinsured and the reinsurer. The reinsurer's liability is normally capped over the life of the contract. There is considerable debate over the degree of risk transfer in such products, and the appropriate accounting treatment.

B. *Other Reinsurance Terms*

Act of God Bond

This is a type of bond issued by insurance companies, which pays a high rate of interest if there is no defined catastrophe event during its life, but where lenders suffer a loss of either interest or principal, or both, if there is such an event.

Aggregate Deductibles

These are the total losses otherwise recoverable from a contract, that must be borne by the reinsured (in addition to the normal retention) before they can make recoveries.

CBoT Derivative

A 'Contract of Difference' traded on the Chicago Board of Trade, whereby the seller of the option is liable to pay the buyer a predetermined monetary amount if the ratio of catastrophe losses across either the whole U.S.A., or a designated region, is in excess of the contract minimum. Neither the buyer nor the seller need be an insurance company.

Co-reinsurance

This occurs where a particular reinsurance policy is not placed 100%. This can be either a voluntary retention by the cedant or a compulsory retention insisted upon by reinsurers.

Event Limit (or Caps)

Event limits are monetary limits imposed upon either risk excess of loss or proportional treaties to limit the amount a reinsured can recover from that reinsurance with regard to all losses arising from one event, such as, but not limited to, windstorm, flood, earthquake and riot.

Rate on Line

This is the premium payable on an excess of loss policy, expressed as a percentage of the sum insured or indemnity.

Reinsurance Captive

A major development of the post-War period has been the formation by major industrial and commercial companies of so-called captive insurance companies to handle the insurance business of their parent company and fellow subsidiaries. Many of these captives are established in tax havens such as Bermuda or Guernsey.

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This structure has also been adopted by many international insurers as a tool to manage the gap between a prudent retention in individual subsidiaries and that which can be absorbed by the group as a whole.

APPENDIX 2

THE ASSET MODEL

1. *Introduction*

This appendix considers in a little more detail the asset model described in this paper.

2. *Type of Model*

The choice of a simple investment model has reflected the nature of the paper, i.e. modelling the impact of reinsurance on the financial strength of a general insurance operation.

We have chosen a model based on the Normal distribution with the features described below:

— The investment categories modelled are U.K. equities, overseas equities, U.K. fixed-interest bonds, U.K. index-linked bonds, property and cash.

— For each investment category (i) the following input parameters are required:
  - — the expected (or mean) total real return—*RealReturn* (i);
  - — the initial running yield—*RunYld* (i);
  - — the volatility of the capital growth—*CapVol* (i); and
  - — the volatility of the income growth—*IncGrVol* (i).

— Other parameters required are:
  - — a correlation matrix giving the correlation $R(i,j)$ between the capital growth rates for investment categories i and j;
  - — an inflation vector $Inf(j)$ giving the expected retail price inflation rate for each year j of the model projection term (n); and
  - — an inflation volatility scalar, i.e. *InfVol*.

From these inputs the asset model generates three items of output for use in the cash flow model, namely:

— a capital growth matrix $CapGr(i,j)$ being the growth rate of asset i in year j;
— an income yield matrix $IncYld(i,j)$ being the yield of asset i in year j; and
— an inflation vector $RPI(j)$ giving the retail price inflation rate in year j.

3. *Some Formulae*

(A) The geometric mean of the values of inflation is given by:

$$\left[\prod_{j=1}^{n}\{1+Inf(j)\}\right]^{1/n}-1.$$

We call this average inflation (*Avginf*).

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(B) The implied annual nominal total return *INTR(i)* for each asset class (*i*) is obtained as:

$$INTR(i) = \{1 + Real\ Return(i)\} \times (1 + Avg\ inf) - 1.$$

Note: *INTR(i)* is not required in the model for cash and is set to zero.

(C) The implied annual income growth rate *IIG(i)* for each asset class (*i*) is given by:

$$IIG(i) = \frac{\{1 + INTR(i)\}}{\{1 + RunYld(i)\}} - 1.$$

Note: *IIG(i)* is zero for fixed interest and is not required in the model for cash. For index-linked assets *IIG(i)* is set to the inflation rate.

(D) The implied annual capital return *ICR(i)* for each asset class is given by:

$$ICR(i) = \begin{cases} IIG(i) & \text{for assets where income grows, e.g. U.K. equities, overseas equities and property} \\ \dfrac{\{1 + INTR(i)\}}{\{1 + RunYld(i)\}} - 1 & \text{for bonds} \\ 0 & \text{for cash.} \end{cases}$$

(E) The covariance matrix *CoVar(i,j)* is obtained for each element (*i,j*) as:

$$CoVar(i, j) = R(i, j) \times CapVol(i) \times CapVol(j).$$

(F) Inflation is assumed to be normally distributed with mean and variance given by the inputs *Inf(i)* and *InfVol* and the rates in different years are assumed to be independent. Thus, we obtain:

$$RPI(i) = Inf(i) + InfVol \times \alpha \qquad \text{where } \alpha \sim \mathrm{N}(0,\ 1).$$

(G) The capital growth achieved each year (*j*) for each asset class (*i*) is obtained using a Multivariate Normal distribution. If *CapGr(j)* denotes the *j*th column of the matrix *CapGr* viewed as a vector with length equal to the number of asset categories, then:

$$CapGr(j) \sim \mathrm{MN}(\boldsymbol{ICR},\ \boldsymbol{CoVar}).$$

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(H) The income growth rates *IncGr(i,j)* for each asset (*i*) for each future year (*j*) depend on the asset type. For U.K. equities, overseas equities and property the income growth in each year is assumed to be normally distributed and independent. There is (by definition) no income growth in respect of fixed-interest bonds and cash. For index-linked bonds the income growth is equal to the RPI increase (see (F)). We thus obtain:

$$IncGr(i,j) = \begin{cases} ICR(i) + IncGrVol(i) \times \beta_i, & \text{where } \beta_i \sim \text{N}(0,1) \text{ for U.K. equities, overseas equities and property} \\ RPI(j) & \text{for index-linked bonds} \\ 0 & \text{for cash and fixed-interest bonds.} \end{cases}$$

(I) The income *INC(i,j)* generated by each asset type (*i*) in each future year (*j*) for an initial investment of 100 is given by:

$$INC(i,j) = \begin{cases} INC(i,j-1) \times \{1 + IncGr(i,j-1)\} & \text{if } j > 1 \\ RunYld(i) & \text{if } j = 1. \end{cases}$$

Note:

$$INC(i,j) = \{1 + RPI(j)\} \times \{1 + Real\ Return(i)\} - 1$$

for cash, i.e. compounding the inflation rate for the year and the input real return on cash.

(J) The capital value *CapVal(i,j)* of each asset category (*i*) at the start of each year (*j*) is the cumulative growth given by:

$$CapVal(i,j) = \begin{cases} 1 & \text{if } j = 1 \\ CapVal(i,j-1) \times \{1 + CapGr(i,j-1)\} & \text{if } j > 1. \end{cases}$$

(K) The income yield *IncYld(i,j)* for each asset category (*i*) for each year (*j*) is the income in that year divided by the capital value at the start of that year, i.e.:

$$IncYld(i,j) = \frac{INC(i,j)}{CapVal(i,j)}.$$

4. *Simplification*

For the purposes of this paper, we have assumed that *R(i,j)* is an identity matrix.

APPENDIX 3

WORKED EXAMPLE OF INFLATION, CLAIMS RUN OFF AND REINSURANCE RECOVERY

This example shows how claims are modelled taking into account the claim inflation implicitly allowed for in the loss ratios, and the inflation stochastically generated by the model. For simplicity, it shows the run off for past claims, plus two future years, ignoring claims from event losses and catastrophes, and ignoring social inflation, for a class of business with a four-year run off pattern.

The loss ratios allow implicitly for claim inflation at 5%, whereas the rates generated stochastically by the model have a mean of 5%, but the actual values generated in successive years are 7%, 4%, 6%, 5%, 3%.

The claim settlement pattern is also generated stochastically by the model, and for simplicity we assume that the following pattern has been produced in each year (note that each year's run off pattern is actually generated independently): 20%, 60%, 15%, 5%.

1. *Past Claims*

For simplicity, it is assumed in this example that all past claims are from accident year zero.

Opening reserves 1,000			
Development year	2	3	4
Projection year	1	2	3
Claims paid, assuming that implicit inflation is equal to actual inflation	60%*1000/80%	15%*1000/80%	5%*1000/80%
equals	750	187.5	62.5
Discount by implicit inflation:	$1.05^{1/2}$	$1.05^{3/2}$	$1.05^{5/2}$
equals	731.93	174.27	55.32
Reflate by actual inflation	$1.07^{1/2}$	$1.07*1.04^{1/2}$	$1.07*1.04*1.06^{1/2}$
Actual claims paid	757.11	190.16	63.38

2. *Claims Incurred in First Year of Projection*

Assume that the model has generated a value for incurred claims of 100.

Development year	1	2	3	4
Projection Year	1	2	3	4
Claims paid, assuming that implicit inflation is equal to actual inflation	20%*100	60%*100	15%*100	5%*100
equals	20	60	15	5
Discount by implicit inflation	19.52	55.77	13.28	4.22
Reflate by actual inflation	20.19	60.86	15.21	5.10

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3. *Claims Incurred in Second Year of Projection.*

Assume that the model has generated a value for incurred claims of 110.

In exactly the same way as in Section 2 of this appendix, the model will generate payments of 22.30, 66.93, 16.81, 5.56 in each year.

4. *Revenue Account*

These payments, together with the movements in outstanding loss reserves, are then drawn together as incurred claim movements for incorporation into the revenue accounts as follows:

	Projection year					
	1	2	3	4	5	Total
Past claims						
Paid in year	757.11	190.16	63.38			1,010.65
Outstandings brought forward	(1,000.00)	(242.89)	(52.73)	0.00		(1,000.00)
Outstandings carried forward	242.89	52.73	0.00			
Incurred	0.00	0.00	10.65	0.00	0.00	10.65
Accident year 1						
Paid in year	20.19	60.86	15.21	5.10		101.36
Outstandings brought forward		(79.81)	(18.95)	(3.74)		
Outstandings carried forward	79.81	18.95	3.74			
Incurred	100.00	(0.00)	0.00	1.36	0.00	101.36
Accident year 2						
Paid in year		22.30	66.93	16.81	5.56	111.60
Outstandings brought forward			(87.70)	(20.77)	(3.96)	
Outstandings carried forward		87.70	20.77	3.96		
Incurred	0.00	110.00	0.00	(0.00)	1.60	111.60
Total						
Paid in year	777.30	273.32	145.52	21.91	5.56	1,223.61
Outstandings brought forward	(1,000.00)	(322.70)	(159.38)	(24.51)	(3.96)	(1,000.00)
Outstandings carried forward	322.70	159.38	24.51	3.96	0.00	
Incurred	100.00	110.00	10.65	1.36	1.60	223.61

5. *Reinsurance Recovery—Detailed Working*

As described in Sections 4.6.3 to 4.6.8, each individual large claim (or large event) is calculated separately, and the resulting incurred claim is then subjected to a randomised payment pattern and adjusted for inflation to produce a series of cash flows.

The incurred claim and resulting cash payments are then compared to the reinsurance programme to calculate potential recoveries, and reinstatement premiums if applicable. In the case of recoveries from risk or event excess of loss covers, this process can be shown by taking the claims payments derived in Section 2 of this appendix, and treating them as if they were all attributable to a single loss, protected by the following reinsurance programme:

	Layer 1	Layer 2
Indemnity	25	25
Retention	50	75
Rate on line	20%	10%
Reinstatement	1@100%	1@100%
Co-reinsurance	10%	nil

The cumulative recoveries before co-reinsurance are derived:

Development year	Gross paid	Gross incurred	Layer 1 Paid	Layer 1 Incurred	Layer 1 Reinstatement premium	Layer 2 Paid	Layer 2 Incurred	Layer 2 Reinstatement premium
1	20.19	100.00	0.00	25.00	0.00	0.00	25.00	0.00
2	81.05	100.00	25.00	25.00	5.00	6.05	25.00	0.61
3	96.26	100.00	25.00	25.00	5.00	21.26	25.00	2.13
4	101.36	101.36	25.00	25.00	5.00	25.00	25.00	2.50

We can now apply the co-reinsurance total across the layers, and turn the cumulative paid and incurred positions into a series of cash flows and outstandings:

Development year	Cumulative paid	This period	Incurred	Outstanding	Reinstatement premium cumulative	Reinstatement premium this period
1	0.00	0.00	47.50	47.50	0.00	0.00
2	28.55	28.55	47.50	18.95	5.11	5.11
3	43.76	15.21	47.50	3.74	6.63	1.52
4	47.50	3.74	47.50	0.00	7.00	0.37

These resulting cash flows can then be delayed as appropriate, to reflect the time taken to collect the money from reinsurers.

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APPENDIX 4

DETAILED ASSUMPTIONS FOR THE SIMPLE START-UP COMPANY

A detailed list of the parameters (others than those previously mentioned) used in the simulations of the simple start-up company are given below:

— *Opening investment portfolio*

Bonds	£45m
Cash	£3m
Working capital	£2m

— *Buying rules.* If there is any positive cash flow of premiums to claims and expenses, 50% is invested in government bonds and 50% is invested in equities, and further assets are bought and sold in proportion to the asset distribution.

— *Selling rules.* There are two alternative strategies for how a negative cash flow will affect disinvestment: firstly, to disinvest in proportion to asset holdings at the start of the year; or secondly, the assets are ordered and the asset with the highest priority is sold first. For the start-up company we use the first method.

— *The investment assumptions* are as follows:

	Cash %	Equities %	Bonds %
Mean real return	1	5	3
Running yield	4.5	3	6.5
Volatility of capital growth		20	10
Volatility of income growth		5	1

— *The minimum acceptable solvency ratio.* For the start-up company this is taken as 30% of premium net of reinsurance.

— *Tax* is 33% on net profits.

— *Dividend* is 50% of net profits.

— *Financial inflation* is assumed to be normally distributed with a mean of 3.5% and a standard deviation of 0.5%.

— *The average rate of financial inflation* assumed in calculating the mean value of future loss ratios is also assumed to be 3.5% (for example see Appendix 3).

— *Social inflation* is applied at differential rates for attrition and large losses, but is ignored for the start-up company.

— *The delay (in months) between making gross payments in respect of past and future claims and receiving the recovery payments.* For the start-up company these values are taken as three months for quota share and one month for excess of loss.

— *The percentage level to which expenses reduce once business is no longer being written.* This is taken as 1% of year 3 gross written premium.

— *The unearned premium carried forward at the end of each year* is assumed to be 40%.

— *The business plan* assumes that, in each of the three years of the modelling period, the gross premium is £100m, and that losses other than catastrophe ones are normally distributed with a mean loss ratio of 55% and standard deviation of 2%. Commissions and office expenses are assumed to be 28% of premiums.

— *Natural perils catastrophe losses.* Whilst it would have been possible to specify any combination of individual catastrophe losses during the modelling period, we ran the model under the stochastic option. Under this, the *model* requires certain assumptions regarding the probability and potential size of each event for each peril to be input. These assumptions, obtained either from a GIS type model or from general management views, comprise the probable maximum loss, the probability of an event of at least one tenth this size happening, and a table setting out the relative probabilities of the size of the loss, given that one has happened. This table needs to be completed for each decimal of PML. For the start-up company the tables are as follows:

	Claim size up to £000	Probability
Storm	8,000	0.42
PML £80,000,000	16,000	0.20
Probability 20%	24,000	0.12
	32,000	0.07
	40,000	0.05
	48,000	0.04
	56,000	0.03
	64,000	0.03
	72,000	0.02
	80,000	0.02
Flood	10,000	0.05
PML £100,000,000	20,000	0.05
Probability 2%	30,000	0.05
	40,000	0.15
	50,000	0.15
	60,000	0.15
	70,000	0.20
	80,000	0.10
	90,000	0.05
	100,000	0.05

These probabilities combine to produce the table of assumed return periods in Table 2. For example, there is a probability of 20% that a storm will occur in any one year, and, given that a storm has occurred, it has 2% probability of being greater than £72m, but less than or equal to £80m (a 250-year return period). Paragraph 4.6.7.5 referred to two issues. Firstly, it can be seen that these inputs allow for zero probability of a loss exceeding the PMLs of £80m and £100m respectively. Secondly, the use of a discrete distribution forces, for example, all storm losses falling in the band size greater than £56m and up to

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£64m to be given the same value, namely £64m. The results of this can be seen in Figures 17 and 18.

— Because the account is not subject to any wide fluctuations in size of sum insured, no reinsurance of individual risks is necessary, and therefore this run of the model did not need to generate individual large losses other than for catastrophes.

— *Claims run-off patterns*. The values of the mean proportion and the standard deviation of a claim paid in year *i* of development of the claim are needed for past, future and catastrophe knock-on claims. These are assumed to follow the same pattern:

Year	Run off pattern	Standard deviation
1	64	5
2	28	3
3	4	3
4	2	2
5	1	2
6	1	2

ABSTRACT OF THE DISCUSSION

Mr T. R. H. Thomas (introducing the paper): The model we describe in this paper owes its origins to various discussions between Dr Coutts and myself, whilst I was in charge of security vetting for a major insurance broker. We tried to build a model, using publicly available data, which could be used to test the capital adequacy of various markets, both in the United Kingdom and overseas.

Based on my previous experience as the finance director of a multinational insurance sub-group, which included the establishment of a reinsurance captive, we soon reached the conclusion that, if we could amplify the published information on reinsurance, the tool that we were building would be useful in helping our clients structure their reinsurance programmes. At this stage we widened the project team to include several reinsurance brokers, each specialising in a different class of business. Our aim was to model the operations in a practical way, using information which would be readily available to almost any company. As the project developed further, it became apparent that we were building a tool for managing the company's capital more effectively, rather than its reinsurance programme in isolation. Our paper represents the outcome of this multi-disciplinary approach to the problem facing all insurance companies; determining how reinsurance programmes affect the capital at risk, and, therefore, whether they represent good value for money.

Our multi-disciplinary approach mirrors the skills available in the management team of a typical insurance company, and we believe that it is vitally important that the whole management team understands and supports the basic concepts behind any capital allocation process, expressed in terms which reflect its daily decision making. It, as well as the Board of Directors, also needs to understand and accept the outputs, but not necessarily the detailed mathematics involved. We have, therefore, aimed at producing the outputs as a series of easily interpreted graphs, rather than tables of figures.

This technique allows the different languages used in different parts of the organisation to be more easily interpreted; for example, a reinsurance buyer or underwriter will think in terms of balancing the account, whereas the investment manager will talk of protecting the return on capital — they are both saying the same thing in their own way.

In order to manage 'capital at risk' and plan an adequate return at group level, it is necessary to divide up both capital and profit targets by individual company and profit centre, in a way which fairly reflects the different risks involved.

Reinsurance has traditionally been bought to stabilise both profits and capital of an insurance company. Therefore, it has a potentially major impact on the risk capital requirements of both the company, as a whole, and of each individual profit centre. If we are to be able to manage risk capital, we have to be able to understand how reinsurance affects it. Whilst risk-based capital models, as used in the United States of America, are a significant step forward in focusing on the concept of risk capital, they cannot do this. As soon as we move away from straight quota share covers, as is inevitable in any commercial lines account, we see no alternative to stochastic modelling, in a way which allows individual large claims to be generated at random from an appropriate probability distribution, and then compared with the proposed reinsurance programme.

It is the overall structure of the model, as shown in Figure 6, which is important, rather than the detailed modelling of any section. For example, we deliberately used a very simple asset model, since our experience is that most major companies have developed, or are developing, their own sophisticated ones, all of which are different. It would be relatively easy to slot out our asset modules and replace them with a more complicated version. Equally, extra modules could be inserted to process alternative reinsurance products, where necessary.

Our experience, in fitting this model to a major Scandinavian insurer, demonstrated that the input assumptions require very careful assessment, but are, in most cases, the standardisation and reduction to mathematical form of the thought processes already undertaken by line managers. Further, although there may well be practical difficulties in producing workable assumptions, just trying to complete the exercise yields significant benefits to management in the understanding of the business.

We should like to emphasise that such a model as this is an enhancement to existing management information, rather than a replacement of it, and only one tool in the armoury of a successful company. Certainly, it is no replacement for common sense.

We believe that the general insurance industry is some ten years or so behind the banks in recognising the importance of the concept of 'capital at risk' and return on that capital. Just as the banks have built highly sophisticated models of their business, so will we.

Mrs K. A. Morgan, F.I.A. (opening the discussion): The paper adds to the work done by Daykin & Hey in their 1990 paper, 'Managing Uncertainty in a General Insurance Company' (*J.I.A.* **117**, 173-278), and looks at one aspect of managing a general insurance company; namely, the impact of reinsurance. In the process of doing this the authors touch on some important points relating to the future management of general insurance companies — the trend in the non-life actuarial profession to develop and use complex simulation models, and how management information is best presented. Also, perhaps most importantly, they propose a restructuring within general insurance companies around the management of the risks facing the company as a whole, rather than around each class of business.

The paper provides a framework for assessing the impact of different reinsurance programmes, based on a general insurance company's perception of the risks it is covering. This point is important — attrition claims, which will give rise to no recoveries, are the easiest to predict. A company will have much information about these claims, as they are the most numerous. Large individual claims are less easy to model. The authors, in ¶4.6.4.2, state correctly that "the fitting of these distributions to large loss data... is fundamental to the reliability of the model in predicting the effects of changes in the reinsurance arrangements." Unfortunately, this is not easy to do.

Event catastrophe claims are obtained from a 'user input discrete claim distribution'. Catastrophe losses from natural perils, which are, by their nature, rare, are modelled using a distribution based on management views. The authors state, in ¶4.6.7.3, that the return periods could come from a GIS-type model or from management views, but, as management will adopt or disagree with the GIS-type model, the final model will reflect management views.

The claims models are the cornerstone of the model, and are also one of the areas of greatest uncertainty. Because of this, it might be useful to run the model with different perceptions of claims costs in order to gauge just how sensitive it is to these assumptions.

Is reinsurance value for money? This is a question that insurance companies ask each year at renewal time. The answer will depend on the individual circumstances of a company, as reinsurance brings more than just protection against large losses, for example in the smoothing of profits and in the release of capital. The importance of these benefits will be different between a quoted company with not much capital and an unquoted company with large free reserves. The model can help with assessing both these benefits and the estimated true cost of a reinsurance programme, as shown in Section 5. Analysing possible reinsurance programmes, using the model, will highlight the differences in perception of risk between the insurer and the reinsurer. This will probably boil down to different assumptions in the probability distributions for large claims and catastrophe losses, and I fear that the two sides may never agree. One outcome of using the model may be that all insurance companies decide that reinsurance is very overpriced, and decide to retain all risks in house!

Looking at the insurance company as a whole, when assessing the impact of a programme, will give a picture of the estimated true cost of underwriters' 'sleepeasy' covers. Such an exercise should avoid wasting money on what could turn out to be unnecessary reinsurance.

Clearly, any model used during the renewal season should be practical, with easy-to-understand output and inputs. It needs to be flexible and quick to run. The model, as proposed, will involve, say, 1,000 simulations for each reinsurance programme being tested, to assess the true cost of the programmes and the impact on smoothing of profits. Many more simulations will be needed to assess how much capital is released, as the initial solvency margin will need to be changed for each run. This could be a time-consuming process. There will be many outputs to look at and assess for reasonableness.

The paper sets out another model for a general insurance company. It seems that any problem involving *one* aspect of an insurance company is being addressed by modelling the whole company. This involves a multitude of assumptions about the different parts of a company's operations, all of which act in different ways. Can we learn anything from such models that we did not know already? I am not surprised that buying more reinsurance releases capital, but reduces net worth. The model gives me an estimate of how much capital is saved, but how reliable is this? Could an experienced company actuary come up with a first order estimate that was close?

Developing one's own model of a general insurance company will give insights into how different aspects of the company interact, although I am not convinced that the same insights could be achieved using a model developed by someone else. Much of the value of a model is in thinking through the way that things work oneself. Using someone else's model may lead to incorrect conclusions being drawn through not understanding the construction of the model. This will depend on how 'black boxed' the model is. Ideally, all assumptions underlying the model should be obvious or clearly stated.

The modular approach taken in the paper could be used to enable quick development of in-house models, by replacing what are perceived as key modules with in-house ones, but using the other modules for less key areas.

Are non-life actuaries, by developing these models and advising that companies should restructure to put the models at their heart, recommending that insurance company managers should use these models to assess the possible consequences of their judgement, or should we be saying that anyone can run a complex model once it has been written, but that actuaries have the ideal skills to interpret the output, and so run insurance companies? We should avoid endowing with 'actuarial mystique' the models we have developed, or the profession could become stuck in a modelling spiral, with actuaries spending all their time analysing possible scenarios, rather than dealing with the real world.

Do models give a false sense of security? We may have looked at all the model outcomes of a set of decisions, and so we feel that we fully understand an issue. Disasters come from the unexpected, however, and so will not be incorporated into the model. Will using models mean that there is a danger that commonsense will be regarded as unnecessary? As a profession, we should seek to build on skills present in the industry, and demonstrate that we can complement these skills. We should aim to replace only those skills which do not work.

A strength of the model approach is that sensitivity to assumptions can be readily tested. This can give a much improved understanding as to what to concentrate on in terms of managing a company. The paper shows a prime example — the outwards reinsurance programme can have a significant effect on a company's profits and net worth, and so reinsurance is an area for management to concentrate on.

The paper recommends graphical output as easy to understand. I endorse the aim of easy-to-understand reports. Before I joined the actuarial profession, I read that it was one where it was necessary to express complex ideas in simple language. I believe that, for us to progress as a profession, we have to achieve this aim, as far as possible, in all our work.

The *raison d'être* of general insurance companies is to diversify risks. Thus, it makes sense to organise a company by giving highest priority to quantifying and managing risk. However, the management of the company, although focused on insurance risk, will still be the same people who may have misread the market or who employed a rogue underwriter. Refocusing the company will give due importance to the insurance risks, but may not remove the normal risks of running a business. In addition, these normal risks can have a much more dire impact on a general insurance company than on other types of business, as the nature of insurance contracts makes the downside potentially enormous.

It seems sensible to treat reinsurance as equally important to investment, as reinsurance can be thought of as an asset that matches the liabilities by type and term — in fact, it is the only available asset that does this. The cost and security of this asset are important issues, and cost is addressed in the paper.

While I have indicated that there are limitations to the value of complex models, this paper can only add to the debate on the use of models in general insurance companies. It gives a practical

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approach, and should make actuaries advising on reinsurance arrangements look at a different method for evaluating different programmes.

Mr J. P. Ryan, F.I.A.: When first seeing the title of this paper, I asked myself what was the point of writing a paper on the evaluation of a reinsurance programme in today's markets? The way that prices have been cut, if offered reinsurance with a secure reinsurer, buy it.

This paper is extremely important, particularly at this time in the development of the profession's activities in the general insurance field. It demonstrates the power of cash flow modelling. This is one of the essential skills that we, in the profession, can bring to the general insurance table. It is, perhaps, the major contribution that actuaries can make in this area, and it differentiates us from some of the other financial advisers. I have experience of cash flow modelling, and I believe that it can provide practical solutions — and, perhaps, even more importantly, insights — into quite complex issues. Modelling allows one to explore the dynamics of a particular situation, and what variables need to be evaluated. The authors, perhaps, overstate some practical problems in this type of modelling. In Section 7.1 they indicate the need for the chief executive's approval to set up a full-scale asset/liability model. If one is setting up a full dynamic financial analysis — to use North American jargon — or a full asset/liability model, then that view is probably correct, because, as the authors again point out in the paper, there is a very large number of variables that need to be modelled if one is to evaluate and model a company or a Lloyd's syndicate properly. However, many situations, and much of the evaluation of reinsurance programmes, can be analysed without going into such detail. One of the potential problems restricting the use of actuaries in the non-life area, apart from scarcity of resource, is cost. Therefore, the design of this programme is important, bearing in mind the need for simplicity and low cost.

It is, therefore, important, when undertaking this type of exercise, to understand exactly what question one is trying to answer, and how these types of modelling techniques may be used to provide the answer to those questions. The authors do not lay down precise criteria for this, and run into a certain amount of confusion in different parts of the paper.

Here are some examples.

In ¶2.2.1 the authors write about risk-based capital (RBC), and that it is not based on these types of techniques. In general, that is not the case. Much of the U.S. work was based on these types of analytic techniques, but they were then simplified to put them into a regulatory framework. Much of the Lloyd's RBC work, in which I had some involvement, was also based on these types of analyses, and, therefore, was not fundamentally different from the techniques used by the authors. The actual mechanics, as publicised, was somewhat different, because both the North American and the Lloyd's RBC models were designed essentially as regulatory tools, although they have some uses similar to the approaches suggested in the paper. This is illustrated by Figures 1 and 2 on page 641.

Figure 1 shows the amount of capital required for a mixture of two classes of business only, taking whatever capital criterion one uses to determine that capital requirement, and that, in itself, is a non-trivial question. Using the modelling techniques outlined in the paper on the liability side, it is possible to identify the variability of the outcome of those particular liabilities, and then to derive the amount of capital required for a given volume of business. By doing the simulations for differing proportions of the two classes of business that are illustrated, you can get a curve shaped as indicated in the figure. In this case one class is a relatively low-risk class and the other is a high-risk class. No matter how risky the business, provided that the two lines of business are not correlated, you will get some reduction in the amount of capital required as a proportion of the premium written by mixing two lines of business. One will get a curve such as is illustrated, and the dot on the left-hand side of the figure shows the situation where you might minimise that capital requirement. If both were equally profitable lines, then that is, perhaps, the optimal level at which you would wish to write the business. However, if the more risky line is more profitable, then you would probably move somewhere over towards the right, as indicated by the dot on the right-hand side of the figure.

The opener said that these models are needed to allocate capital between subsidiaries. The problem with this is that, by mixing lines of business, the capital requirements can be reduced, and, as is in the paper, it is a myth that higher risk lines of business require higher returns. That might be a

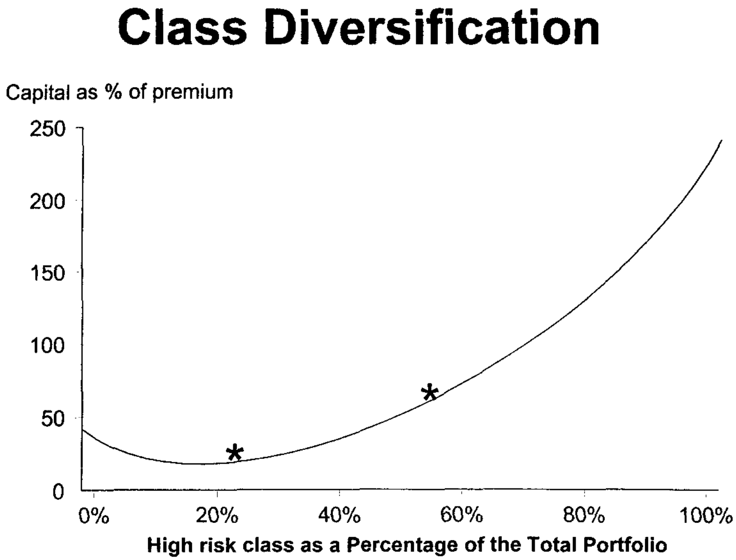

Figure 1.

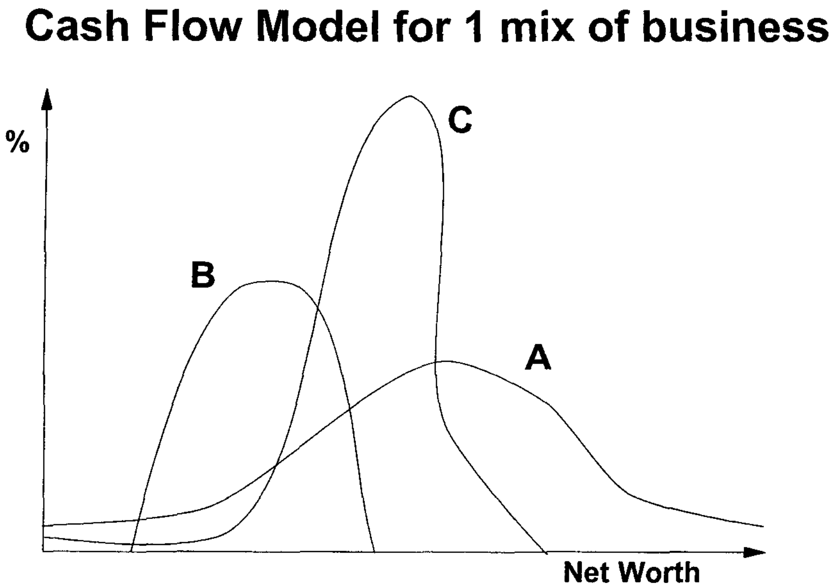

Figure 2.

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desirable thing from an overall strategy point of view, but, in general, unless there is a shortage of capacity in that market, the market will not allow you to obtain that, because somebody in the market will discover that they can reduce their risk by writing more of the high-risk line, which will drive the rate down to the return at the left-hand dot. This can be observed in the property catastrophe market. Here the business for the Bermuda re-insurers was able to be written at very high rates when there was a shortage of capacity. Now, with everybody getting in on the act, to diversify and reduce their risks, you can see people moving down the left-hand side. You can also see one of the reasons why the big property catastrophe specialists are doing their best to diversify.

In Figure 2 we can put some of this in the authors' terminology. The authors compare two particular strategies, A and B, and they rightly point out that, in general, strategy A is more attractive than strategy B. By combining them in some form, as indicated by Figure 1, you get a result on the lines of C, which ties in with the authors' objective of making the graph more vertical, and moving over towards the right. The authors' methodology is a good way of comparing two strategies. In order to define the optimal solution, it is necessary to go through a whole range of As and Bs. The original graph helps to show underwriters an easier way of determining where they want to be.

The two approaches are not inconsistent. The RBC approach or the criteria used by Lloyd's may not be appropriate for a particular company, but it is possible to derive that type of information as peculiar for particular companies. The authors quite rightly make the point about the Lloyd's RBC, that it did not take into account the specific reinsurance programme. One reason is the extensive amount of work required on a syndicate-by-syndicate basis. In deference to Lloyd's, they actually tackle the problem by individual visits to syndicates, and making *ad hoc* adjustments accordingly.

With all such models, the input of the individual assumptions is extremely important. The devil is in the detail. The use of the wrong catastrophe model can make these types of techniques fatal. On the other hand it provides many insights. Looking at all the simulations, as the authors correctly specify, is important, and can provide considerable insights into model mis-specification. The authors point out that they do not look at the correlations between assets and liabilities. In determining reinsurance programmes, I think that that is right, unless you are considering the correlations with asset hedges or something of that nature. On the other hand, it can be very important if you are looking at full scale capital requirements. This is why I set up the Asset Liability Working Party on correlations, chaired by Mr Bulmer, which is an extremely important aspect in helping our understanding in this area.

The authors, by laying things out in this detail, have done a great service to the profession. The paper demonstrates the powers that actuaries can have in providing solutions to many complex problems in this area. These types of techniques can help provide solutions and insights into many of these complex problems. Even LMX business can be examined using these techniques, and there is now no excuse for underwriters just recording policy limits or PMLs and not undertaking a full-scale analysis in this area. Underwriters must utilise these techniques and get actuaries involved. Those who do not are going to find themselves the pariahs of the next downturn of the underwriting cycle, in the same way as the backwoodsmen of the LMX markets did in the last.

Mr S. Christofides: In ¶9.3 the authors state: "It is important to appreciate that the model is a simplification of real life, and that the answers it gives are totally dependent on the quality of the input assumptions". This 'health warning' needs to be expanded to remind the reader that only some models will turn out to be useful in practice. Furthermore, the warning should appear prominently at the beginning of this paper. The authors make a number of sweeping assumptions and statements, both about the nature of insurance risk and about the management of this risk. They then infer that their particular model represents a significant step forward in our understanding, quantification and management of this risk.

I question whether the impact of reinsurance on financial strength is worth quantifying using an asset/liability model. I also question whether values from the extremes of the distributions derived from such an asset/liability model are usable in the way that the authors propose, given the considerable opportunity for error at these extremes. This has significant implications for any attempt to measure the impact of reinsurance on financial strength, as well as any attempt to allocate capital back to classes of business, based on such values.

The authors' motivation for this paper is from the commonly-held belief that 'reinsurance is an alternative to capital'. They then set out to measure this relationship. There is no attempt to examine this assertion and its significance in relation to other sources of risk that this capital may be exposed to, such as the risk from asset or currency movements. With the exception of catastrophe reinsurance, reinsurance has only a second order impact on the financial strength of insurance companies compared to the impact from movements in asset values and changes in underwriting profitability as a result of underwriting cycles. The impact on financial strength from reinsurance is primarily the cost of this reinsurance. For most insurance companies the main benefit of reinsurance is short-term protection of the profit and loss account. Even with catastrophe reinsurance, any benefit that may arise following a catastrophe loss is soon clawed back by the increased costs of reinsurance protection, including payback.

In the case of catastrophe reinsurance, it is very easy to develop a simple relationship between the cost of the reinsurance and the amount of protection that it provides at different levels of comfort. If we then choose to interpret this as a 'release of capital', we can derive the 'risk-return' relationship at various comfort levels and for different levels of reinsurance. This is neither complicated nor new. All the details can be found in my article on 'Exposure and Risk in Non-Life Insurance' which appeared in the January/February 1995 edition of *The Actuary*. Although a simple simulation model is described in this article, all the results can be derived, with a reasonable degree of accuracy, by use of a calculator with square root and logarithmic functions. An asset/liability model is not necessary to do this sort of calculation.

Insurance managers need to understand the risk associated with each class of business, and what this implies for the overall risk profile of the company. The starting point is to determine the economic underwriting performance for the class by discounting all the associated cash flows at an appropriate risk-free rate. The 'riskiness' of the particular class can then be described by the distribution of these economic returns over time, and this will be influenced by market competitiveness or the pricing cycle for the class, its catastrophe exposure and any inherent uncertainty in the measure of this performance due to delays in claims settlements.

The underwriting model described in this paper measures, at best, only a fraction of this 'riskiness', primarily from stochastic variation, whilst ignoring parameter and specification errors. The authors compound this serious limitation by choosing to use values at the extremes of the derived distributions for their decision making. For example, they suggest use of 'ruin probabilities' of 1 in 100 or 1 in 500 years to quantify the impact of the reinsurance on financial strength and to allocate capital to classes of business. I would like to see an explanation as to how this model produces values, at these extreme probability levels, which are sufficiently reliable for such use. Stochastic variation in claims costs is unlikely to be the cause of insolvency in a reasonably managed company.

The model presented includes an asset module, which appears to project values around an assumed trend, based on the normal distribution. Although the authors allow for correlations between asset classes, there is no attempt to justify or verify that this model can replicate any of the behaviour that it is supposed to model. I would be very interested to hear which financial market exhibits these characteristics.

These limitations of the model raise serious doubts on the reliability of the output and any inferences derived from this output. In particular, any allocation of capital to classes of business, as indicated, will be haphazard and potentially dangerous. It is haphazard because there is no unique, technically consistent and justified basis for determining how this should be accomplished. It is dangerous if we actually believe these answers and act on them, as the authors seem to imply that we should. In ¶5.3.1, they write: "Successful companies will be those which are efficient capital allocators". I could not find an explanation of what they mean by 'efficient' in this context. In the absence of a definition, followed by a justification that the approach proposed satisfies the necessary conditions, the use of any such results for allocating capital or for adjusting performance measures for 'risk' may simply be a way for diluting the quality of the existing information rather than enhancing it.

In ¶1.1.4 the authors note "that, particularly in the light of the Cadbury Committee requirements on corporate governance (1992), Boards of Directors of insurance companies need a better understanding of the financial risks being assumed by their companies". Clearly we would all agree with this statement. In order to make progress in such a task, we will need to research and develop appropriate asset/liability models that help us evaluate the risks faced by general insurers and measure the impact from changes in our strategies, whether these are to do with the management of the assets or of the liabilities.

The model described by the authors, and all the previous models they mention, including the GISMO, are intended to further our knowledge and understanding of this complex process, and any progress has to be welcome. We will, however, need to be extremely careful in the claims that we make about these models, to ensure that we do not mislead those that may be users of them, but may not be in a position to verify that the particular model is appropriate for their purposes.

Mr C. T. Pettengell, F.I.A.: It is useful for the profession, as a whole, to make steps down the road to modelling the overall finances of a company. I fear, however, that we are still a long way off getting some really tangible results out of such modelling. The data available are not currently sufficient, and probably will not be for another 10 years.

The reinsurance programme that is modelled is fairly simple. I have spent a long time trying to build a program that models what happens in practice, and have been frustrated. Nor does the model allow for parameter risk, and if you are assessing excess of loss versus proportional reinsurance, I would argue that you do not make a valid comparison if you exclude parameter risk.

Issues are raised about the future of the reinsurance industry. There seems to be a trend towards group risk management, that I believe is desirable. A holistic view of a corporation's finances, considering assets and liabilities at the same time, can help management think about where they should be spending their time.

Will this holistic risk management lead to one-stop shopping? Will the group risk manager buy protection from an overall risk management operation which looks at investment risks and reinsurance risks at the same time? There are already signs that the investment and reinsurance markets are colliding. Securitisations of reinsurance risk have been performed. We are at the start of a very exciting period in the reinsurance industry. To my knowledge, most insurance companies, when thinking about the bonds that they buy, would not entertain bonds that were, say, below an A grade. Although there is a slight trend to move out on the risk curve as general yields have decreased, there is a definite preference for high grade securities on the investment side. This may be one of the reasons why we are seeing a flight to top grade security in the reinsurance market.

On the ability to charge for risk, one line versus another, it is important to understand the overall diversifications that there are between different lines of business. That affects the desired business mix when it comes to thinking of what level of variability can be delivered to the shareholders, that is if you are a proprietary company. However, you still have to decide whether to write an individual risk for line A, which is 5 times more volatile than the average, and I think of as step number one. If one line is 5 times more risky than another line, should I be accepting the same loss ratio for that risk? To me the answer is no. It may be that the market will not allow me to write that business, but should I want to write it?

Mr G. G. Wells, F.I.A.: In life assurance, reinsurance has tended to be modelled as a second order effect, primarily because it tends to be swamped by financial related activities, in particular investment returns and expenses. In general, the modelling of life reinsurance is focused on financing and/or surplus relief treaties, with risk items being assumed to be largely cost neutral. Such an approach cannot easily be adopted in the general insurance market, where the risk element of the cover is fundamental and, as past experience has demonstrated, can manifest itself in a catastrophic manner. It is, therefore, surprising that the modelling of reinsurance has not, to date, been given greater significance in the general (and life) insurance arena. This paper goes a long way to addressing this gap in insurance modelling.

The asset model used alongside the modelling of reinsurance has to be tailored to the situation. The asset model used by the authors is, by their admission, a simple one for illustrative purposes. A more complicated, and perhaps a more appropriate, asset model, could, therefore, easily be incorporated into the overall modelling process, depending upon the preference of the user and the use to which the model is to be put, without detracting, in any way, from the contribution that this paper makes to the subject of reinsurance modelling.

Mr C. D. Daykin, C.B., F.I.A. (in a written contribution that was read to the meeting): Dr Coutts was influential in the development of the thinking of the Solvency Working Party of the General Insurance Study Group towards cash flow models and stochastic simulation techniques. This enabled assets and liabilities to be modelled in a consistent manner and some of the complex interactions between the variables to be explored in a way which was not possible using classical analytical risk theory.

We were very conscious of the importance of reinsurance for many general insurers, but decided to leave consideration of the reinsurance issues for subsequent development. In the model which G. B. Hey and I presented to the Institute in 1990 ('Managing Uncertainty in a General Insurance Company', *J.I.A.* **117**, 173-277), we modelled premiums net of reinsurance cessions, and claims net of reinsurance recoveries. One reason for this was that it enabled us to keep the simulation of the claims experience at a relatively simple aggregate level, rather than modelling separately the number and size of claims.

The corresponding simulation model developed by the Finnish Solvency Working Party, under the chairmanship of Professor Teivo Pentikäinen, took the more disaggregated approach of modelling claim numbers and claim size and simulating the compound distributions which form an important part of classical risk theory. Having dealt with claims in this way, it is not difficult, in principle, to model the impact of a reinsurance programme.

I am glad that the authors have concentrated on the reinsurance problem, exploring some of the particular issues which need to be addressed. If the simulation of reinsurance is to be at all realistic, it needs to consider, not only variability in individual claim sizes and claim frequency, but also the impact of a series of catastrophe claims, aggregations of risk across a portfolio, events having an impact on more than one class of business and correlations between risks.

The authors have made an excellent start on many of these topics, although they have deliberately ignored questions of correlation, believing that this would merit a paper of its own. Another key omission concerns the bad debt risk for reinsurance recoveries. Whilst accepting that this also might merit a study in its own right, it is worth registering the fact that the methodology presented here provides a good vehicle for modelling the possible impact of reinsurance failure, subject to the necessity of developing a methodology for modelling probabilities of failure of reinsurance on the different components of the programme. Taking this aspect further is likely to become important for the profession in the near future, in order to be able to respond to demands for actuarial opinions encompassing the evaluation of the bad debt risk. Another application of this type of model might be to explore the extent to which solvency margin or risk-based capital requirements could be reduced to allow for reinsurance.

However, the authors rightly focus on the value to the general insurance company of analysing the impact of different reinsurance programmes using the model. An important problem in classical risk theory is to construct optimal reinsurance arrangements. The results are, inevitably, somewhat stylised, and a number of simplifying assumptions have to be made. The authors' model provides the means to generalise such results to more complex reinsurance programmes.

In Canada, the Appointed Actuary of a general insurance company is required to use dynamic financial adequacy testing in order to prepare a financial condition report. Such reports should include consideration of the reinsurance programme and the extent to which this is designed to minimise risk. The authors' model will be a valuable tool for carrying out such responsibilities, although, in this context, it would probably be necessary for the actuary to assume five further years of new business, rather than the three years mentioned in the paper.

I look forward to seeing further extensions of the methodology to support full dynamic financial adequacy testing in general insurance companies.

Mr A. D. Smith: We are increasingly seeing capital market alternatives to reinsurance and various hybrid products, the most recent being the Winterthur hailstorm bond. In addition, it intrigues me why reinsurance programs are so complex — it is not clear whether, in any fundamental sense, the kind of pretty scatters we see coming out of our reinsurance projection system are better than a single rectangular block half way up the left hand side. It seems to me that some form of stochastic model office would be the preferred route for evaluating such issues. Thus far, I agree with the sense of what the authors propose.

Sadly, it is hard to establish whether this paper gets us any further, because points of detail are lacking. For example, in the context of surplus treaties, one needs to generate PMLs as well as gross losses. Paragraph 4.6.5.1 states that the authors' model does this by using a 'bivariate distribution' — that is, two random variables at once. That is all we are told — the choice of distribution is not discussed. There is similar vagueness in relation to loss development on future claims.

Another area that irritated me was the use of zero correlations, both between asset classes and also between assets and liabilities. Some of these correlation assumptions are just silly — for example, the notion that equity capital values move independently of dividends. There also seems to be nothing in the model to make gilts mature at 100 — the capital value just does a random walk.

The various capital market alternatives to reinsurance rely on a sensible appraisal of correlation effects. In addition, investment markets will affect future new business premium scales, because of the effect of historic profits or losses on current capacity, currency effects on the definition of reinsurance layers and also market interest rates on choice of required rates of return. Furthermore, reinsurance rates are known to react to large losses, making future market rates of cover even more unpredictable. The model proposed here fails to capture these nasty features of the reinsurance market, and so would tend to favour conventional covers in preference to some of the more innovative structures now available to exploit the capital market capacity. The ability of the model to make such comparisons in a meaningful fashion is supposedly one of its selling points. It seems more like a flaw.

The allocation of capital by line of business is treated in Section 5.3. This is a topical subject, which invites the application of models of the type proposed in this paper. However, Figures 14, 15, and 16 do not seem to be model output; they are the kind of picture drawn to explain a concept. The authors' description of how a model could be used for this purpose is vague — I wonder whether they have actually applied it for this purpose.

Section 8 provides a list of future applications to which the model does not yet apply. Having worked on the GISMO with Mr Christofides, I was interested to see that some of the most time consuming and intellectually challenging aspects of GISMO have been left by the current authors as the '20% tail' for further development. For example, when we considered correlations between assets and liabilities, it became clear to us that this had such profound implications that we needed to rebuild both asset and liability modules from scratch. The exclusions in Section 8 are so wide as to cover most of the questions we might want to ask in practice. Models which take claims experience and apply a specified structure of reinsurance treaties already exist in the market, and it seems pointless to reinvent them. When it comes to the significant aspects of generating the assumptions in the first place, the authors seem to abandon science in favour of guesswork.

Mr M. Arnold, F.I.A.: I consider this paper from an historical perspective. When the first papers on simulation modelling were produced, reinsurance was relegated to a minor consideration for a number of reasons. First, it was relatively cheap; the market had forgotten how serious some events could be, and did not factor the potential cost of catastrophes into the pricing process. Also, a reinsurer could find another reinsurer to retrocede the risk at a discount on the original rate (the spiral market).

The catastrophic events of the late 1980s and early 1990s have laid bare the folly of this approach, resulting in reinsurance insolvencies and a radical upturn in pricing. Partly because of this increase in pricing, the net retention that a direct insurer will currently hold is often 10 times the amount held in the late 1980s, when even a minor freeze would create a claim on the excess of loss treaty. However, there has never been a satisfactory method for measuring whether the reinsurance is really expensive or not. Reinsurance was purchased "so that the management could sleep at night". Today the management still needs to sleep, but the apparently cheap option of the 1980s is no longer available, and insomnia is often the order of the day.

As the authors correctly identify, reinsurance is a substitute for capital, and the repricing of this capital means that managers of an insurance company have to consider their options. The matter is not a simple accounting exercise, but needs a sophisticated model and a clear understanding of the conflicting issues.

When Monte Carlo simulation methods were first considered in the 1940s, they were seen as the philosopher's stone for a number of intractable problems. However, as the methodology was developed, it was found often to give false hopes, and even, on occasions, to look more like fool's gold rather than a philosopher's stone; and other methods have been developed alongside. Similarly, the methods discussed in the paper should, at best, be one of a number of tools used to assess the position.

For the results to be of real use, it is vital that the correct exposures are measured and input into the model. The better the exposure analysis, the better the understanding of the risks involved, the better the model and the more useful the results. It is also vital to check 'loss scenarios' to assess the likely worst case scenario. As Mr Daykin wrote in his contribution, multiple events also need to be considered — for example a windstorm coinciding with a market collapse, such as occurred in October 1987, or an earthquake triggering a market collapse, such as Kobe in 1995, and so on. The active management of the true 'net' exposures of a company is vital, and reinsurance models, such as that described in the paper, can realistically help in such assessment.

It is particularly significant that the paper has been developed by an actuary and a reinsurance broker jointly. As the profession is currently attempting to expand into wider fields, I believe that the joint work undertaken in producing this paper sets a fine example of how we might best achieve our goals in areas of work where the actuary can bring expertise and add value to a 'team' effort.

Mr K. P. W. Larner, F.I.A.: This is a cash flow approach; it is a distributional approach; and the addition of anything of this nature to our literature should be welcomed. It fits an approach which is being enhanced and generally brought through because of the growth in technology. We could not have attempted to build these models 20 years ago.

I do not like capital allocation methods. Allocation methodologies sit in the context of evaluating return on capital, effectively looking at whether a class of business is viable, when we should really be evaluating the 'cost of capital.' Capital need not necessarily be in the form of cash or visible assets; a guarantee is just as appropriate. Thus, I do not like capital allocation as a concept, as represented in the paper. It is merely a device to calculate the 'cost of capital'.

The capital allocation, as portrayed to calculate the 'cost of capital', is based upon risks of ruin of 1 in 500, or 1 in 100, in the paper. Certainly, in my experience, this approach is not right. I am not sure that it fits with management's requirements for capital. Typically, earnings volatility in a narrower range is more useful. The 'risk of ruin' approach drives part of the paper in looking at modelling net worth. I prefer a model based on volatility of earnings. I would not trust this model to model year-on-year earnings, because I believe that it will give the wrong results. Has there been any testing of the volatility of the year-on-year earnings values, and whether they fit reality for an insurance company in any way?

When looking at volatility of earnings, we look at issues driving the normal levels of volatility. We are not talking about insurance companies at risk of insolvency, We are looking more towards marginal costs of capital. Typically, we are trying to answer the question of where to invest the next dollar or pound to maximise the return. In particular, I do not see how this model will fit with models of other businesses in which the organisation is involved. It is unusual to find an insurance operation, at the moment, that is not linked in with other businesses. We need some mechanic here, even if only conceptual, linking to other businesses. At a minimum, the methodology must deal with life assurance.

The level for considering normal ranges for the volatility of earnings means that we are working, not in a range of 1 in 100 or 1 in 500, but we are working, in terms of volatility, in the 1 in 5 or 1 in 10 range, and there are completely different answers when looking at the marginal cost of capital on that basis. Thus, it is extremely important to separate and answer the correct question. The paper's focus on extreme risks of ruin may, therefore, give the wrong answer to the question of where to invest.

The paper gives the impression that the suggested approach is a way of optimising a reinsurance programme. Again, it is not the way that I have seen it done in practice. The approach is generally to evaluate basic structures for the reinsurance programme, and in that the paper is useful. It certainly does not optimise a particular structure. Mr Christofides explained that a reinsurance programme could be optimised layer by layer, dealing with marginal issues, which is a much simpler approach than that suggested in the paper, and requires less detailed modelling.

Mr P. D. Smith, F.I.A.: I am pleased that the paper concentrates on insurance and reinsurance and does not go into the complex mathematics. In this I differ from previous speakers. Had the authors tried to develop a model that dotted every 'i' and crossed every 't', it would have been too unwieldy to be of practical use, and would only have been of interest to a relatively small handful of people as a debating point. In particular, I agree with the authors' stress on the importance of the presentation of results.

Some of the required inputs to the model are hard to measure and assess. However, they are essential to the management of an insurance company. I have made assumptions about nearly all the inputs for business planning purposes or for work that I have been doing on the costs and benefits of reinsurance purchases. For a flood PML, assessing a figure for a 1 in 1,000-year return period is an almost unanswerable question. By definition, we have no data, but only theories about the extent to which changes in building, changes in sea defences, etc., will make the future different from the past. Every company is making some assumption, be it explicit or implicit, when they decide how high a top layer of catastrophe reinsurance to buy. Whether they pick a 1 in 500 years event, 1 in 1,000 years or 1 in 2,000 years, is not only unanswerable, but is actually of secondary importance, as a model like this can test the sensitivity of the assumption made.

The management of any company may not be totally comfortable with the way that cost benefit type results are presented. As the authors stressed, good presentations are important and can overcome this. For instance, managements already accept the risks of making incorrect assumptions about the claims process — that is, getting their basic premium rates wrong. Similarly, they accept the risks involved in getting their expense projections wrong and their business volumes wrong. They may feel very unhappy about the risks implicit in moving a catastrophe retention from £5m to £15m; but the strength of a model of this form is that it enables the risk of moving the catastrophe retention to be compared with the risks that the insurance company management are already happy to bear.

Mr D. I. Tomlinson, F.I.A.: General insurance covers a very wide field. There is a lot of difference between a marine account and a household account, or some kind of London Market reinsurance account. Statements made in this discussion have sometimes concentrated on one class of business, without thinking that they might want to say something different if they had other classes in mind.

Attritional losses can, in some cases, can be predicted fairly closely; although not always the marine account. There you might have the possibility of quite a wide variation, and you might be exposed to making quite hefty losses.

It is not just the lines of business that are different, but also the companies. A small London Market company does one sort of business; catastrophe protections are extremely important, because, if you get them wrong, you go bust. For a large quoted composite insurer, things look very different. It has much more capital and much more stability. In theory. it can drive the price down, because, for it, the extra risk is not material.

I agree with the criticism of the use of ruin probabilities. We are talking about the *tails* of distributions that we have only guessed at. I made this same comment to one of the authors of the original Solvency Working Party paper, and waited for him to tell me where I was wrong. He said, "No, I quite agree with you. I do not believe them either. I look at the ones that fail and decide if that is a scenario that concerns me. If it is not, I forget about it. If it is, I say that it is one we should take into account."

I agree with the authors when they say that they want to keep the results of the simulations, so that they can go back and look at them. I am a scenario man, not a stochastic man.

On the subject of communication with the outside world, I have had trouble explaining myself to non-actuaries — I am sure that we all have. I have even had trouble explaining myself to actuaries. Those I have worked with have had great trouble explaining things to me! However, we should not make presentations pretty and glossy, and wrong, merely so that they can be understood. At some time we have to accept that things are complex, and have to try to get the real message over, difficult though it may be, even if it brings us into slight disrepute as those who cannot say 'yes' or 'no' or '3%'.

Mr D. H. Craighead, F.I.A.: I think that the paper is valuable, but I cannot help but be aware of the very great difficulties which are involved in developing the type of concept portrayed. Where one would like it most is in the reinsurance market and in Lloyd's. At the present such an application is almost impossible — at least in those areas where it would be most valuable.

Take, for example, a high-risk syndicate which has written, in the course of a year, 4,000 separate risks of a very disparate nature, all of them either facultative reinsurance of individual risks or treaties of reinsurance, either proportional or non-proportional.

The 4,000 risks may cover 200 or 300 different territories, with 1, 2, 3 or 4 different types of high loss possible in each of those territories. To assign to each of those several hundred different groupings a possible scenario of loss frequency would be quite impractical. Yet that is what is needed in the type of analysis envisaged in the paper.

As to marine insurance or reinsurance, a marine loss may consist of: the loss of a ship which is covered in one risk or one treaty; the loss including the liability of the crew and any passengers there might be; and the loss of cargo which might arise from 5, 10 or 15 different treaties of reinsurance, all of which have to be amalgamated. To allow for any frequency or quantum of such a loss is very difficult. Furthermore, the reinsurance might be for total loss only, or for part of a proportional loss, or an excess of loss treaty.

Is the purchase of low level reinsurance worthwhile? When I was examining a number of syndicates and reinsurance companies in the early 1950s, it was quite common to find horizontal exhaustion in the lower layers. One might query, at first sight, whether there is a point in reinsuring if there is going to be horizontal exhaustion. Since the premiums were not 100% — they were a good deal lower — there would be a profit in each of those cases where there was horizontal exhaustion, since it was only the excess over and above all treaties written that gave a net loss.

For the reasons I have mentioned, I feel that this paper could be applied only to a large general insurance direct writer, a personal lines type of writer, and it is certainly valuable in that regard.

The background of the reinsurance will largely be determined by the reinsurance rates offered in the market. There was a crazy period in the late 1980s where the rates for the high layers of reinsurance were so low that underwriters who were very foolish started thinking that there was no risk of huge losses ever occurring, and wrote very large volumes of business. We know only too well what happened in those cases.

Concerning the actual method of calculation used in this model, for the expense ratio the model is for run off after 3 years, but once you have a run-off situation, the expense ratios start increasing very rapidly. I presume that the assumption is that the run-off will be in the general framework of a continuing office, which means that the expense ratios will stay more or less constant.

Mr D. M. Hart, F.I.A. (closing the discussion): Reinsurance is currently at a crossroads. By that I mean that increasing consideration is being given by reinsurance purchasers to the costs and benefits of purchasing full risk-bearing reinsurance programmes, compared with the alternatives, such as finite risk reinsurance or the use of the capital markets.

Such considerations are focusing particularly on the most appropriate means of protecting capital against exceptional losses, whether they be from individual losses or accumulations of smaller ones. I tend to agree with the authors that the general insurance industry has a degree of catching up to do in this area compared with the banks, but I do not agree with those who maintain that reinsurance is a second-order effect and not significant. It depends on the market that you are dealing with. It may be true of the major composite insurers, but certainly not of some of the smaller London Market writers.

Turning to the model, I was pleased that the authors did not attempt to consider reinsurance protections in isolation from other aspects of insurance business. I think that such an attempt would have been doomed to failure, in view of the inter-relations which exist between different aspects of the business.

Nevertheless, the paper focuses on reinsurance, and treats the other aspects of the business in less detail. I believe that this is a satisfactory compromise, because it is not possible to deal with all aspects, all parts of the account, in the same detail. Particularly, I agree with the opener regarding the dangers of using a model as a black box. This could give a false sense of security if the underlying assumptions are not fully understood by the user of the model.

I now consider whether or not this is a good model. First of all, we need a definition. The context of the paper suggests that it is required to be applicable to a range of different insurance entities, hence we need robustness in the model. Also, I believe that any model needs to produce a practical balance of simplicity of use and realism, and the ability to provide a greater insight into the workings and dependencies within the organisation. Against these criteria, does this model pass muster, and is it better than others put forward recently in the general insurance actuarial arena? I think that there have been fairly wide variations in the views expressed.

Unfortunately the paper is rather lacking in the detail necessary to address these questions. In this I agree with Mr Ryan, Mr Christofides and others, the devil is actually in the detail. To the extent that the details are somewhat lacking, I agree with Mr Andrew Smith's comments.

The impression given by the paper is that the authors tend to err on the side of simplicity of use for the sake of some loss in realism. Some good examples of this are the assumption that there is no correlation within the asset model between different years and the assumption within the liability model that different classes are not correlated. In these respects, I agree with Mr Daykin and Mr Arnold. Given the lack of consideration to such matters within the model, I was surprised that the authors went to the trouble of introducing differential rates of social inflation for large and attritional losses. It seemed to me that this was a rather spurious adjustment.

Does the model give greater insight into the workings of the organisation? Here I agree with a number of speakers, including Mr Ryan, who believed that this is so, and that it is likely to prove of assistance in understanding the main features driving the variability of an insurer's results. In this respect, I applaud the ability within the model, as explained in ¶5.1.2, to interrogate the database to identify the assumptions behind particular outlying results. Here I particularly agree with Mr Tomlinson. I was glad that somebody shared my view on that particular point.

The dependency that we are talking about may be obvious in some cases, but I envisage that some interesting causes and effects may arise by investigating further such outlying results. Also, in this context, I applaud the widespread use of graphical output in the results.

On the question of robustness, the most important factor appears to be the appropriateness of the assumptions underlying the model. Inevitably, a model of this complexity will require a vast array of assumptions, both within and between modules. A number of these are highlighted by the authors. In my view, a complete list, including those not currently stated, would be a useful preparation for the application of the model to a specific insurer. In this way the appropriateness of the underlying assumptions could be assessed.

So, which model should we use? Currently there is a vogue for producing stochastic models of general insurance offices. Apparently every good actuary should have one. Apart from the ones developed for commercial benefit on an unpublished basis, two sets of developments were aired at the 1996 General Insurance Convention. Clearly the author's model addresses the reinsurance elements in greater detail, but is the basic asset/liability structure as robust as GISMO? After the contributions of certain of the authors of that paper, I suspect not. It would be helpful in assessing such issues if parallel runs of the different models could be carried out and the results compared. Thus features of the different models could become more obvious, and then a rational view of the advantages and disadvantages could be taken. At present, we need to fall back on a term more familiar to Faculty members, 'not proven'. I agree with Mr Christofides that we should not hold up such models as giving the ultimate answer to the problem. Clearly they have to have a health warning, as does any model.

Turning to the reinsurance module, three points attracted my attention. First, PMLs clearly affect the proportions ceded to a surplus treaty, but I had considered that they became irrelevant once the reinsurance proportion had been determined. Thereafter, I would have used the ceded proportion. In Section 4.6.5 the authors approach the amount recoverable via the PML. I believe that this may have benefits in this type of context. A similar approach, however, was not used for attritional losses recoverable from a surplus treaty, so, unfortunately, no overall recoveries from the surplus treaty are estimated in the model. Next, I was pleased that Mr Daykin addressed the practical issue of bad debts. The third point relates to the apparent assumption that the state of the reinsurance market is capable of prediction in advance. This underlies the assumption, in Section 4.8, that data will be available to specify the reinsurance programme for three years in advance. The practicality of this is very questionable. Mitigation of the potential for sudden changes in the reinsurance market is one reason why reinsurance programmes are often bought with varying renewal dates. It is not clear whether the model even caters for this.

Although I have made a number of criticisms of the authors' model, I believe that the general thrust of their approach is appropriate to developing a better understanding of the impact of reinsurance on capital requirements. I was particularly pleased that the paper is a joint effort between actuarial and non-actuarial professionals, bringing their different expertise and skills to bear.

The President (Mr D. G. R. Ferguson, F.I.A.): It is a great pleasure to thank the authors for preparing this stimulating paper. We have had a wide-ranging and interesting debate. It is particularly pleasing, in view of our intention to thrust out into the wider fields, that this paper has been jointly produced by a member and a non-member in the field of reinsurance broking. General insurance is the fastest growing area of actuarial work, and we have an enormous amount of talent working in the area, and some very interesting research being produced. Still, there is a great deal more to be done. It is a young section of the profession, and that was mirrored in the average age of the members who participated in the 1996 General Insurance Convention. We shall be having many more papers on the subject, and I am sure that Dr Coutts and Mr Thomas are not claiming to have given us the final word on the subject. We all join to thank the authors.

Dr S. M. Coutts, F.I.A. (replying): I first address the comment made that we might have put the cart before the horse — namely, that we did not know what question to ask, but we decided that this would be a nice answer. We did have a question to answer. That was the question which invariably comes from management. I have sat on management tables many times when the reinsurance manager has said, "We have taken on this reinsurance programme". The question asked was, "What was the cost of that in relation to the capital that I have got?" Time and time again I never heard an answer. I believe that Mr Thomas and I have not heard an answer from the professionals in this area.

The actuary has to accept that insurance has been around for a long time, and that these models, quite correctly, carry a health warning, in the sense that they do not solve problems. However, actuaries have to add something so that they can justify themselves being asked for consultancy or advice as employees of insurance companies. Mr Thomas and I believe that, for many years, we have not addressed the problem of reinsurance.

Parameter risk was criticised by a number of speakers — quite rightly. If an actuary is advising a company on a model, whether it is a model for AIDS or car insurance premium ratings, for which everybody here would be quite happy to use a generalised linear model, I expect the actuary to say, "I have estimated my parameter risk outside this model". Then he puts it in, either as a variable or as part of the standard error. The idea of our model was to modularise a concept which has not been modularised in public before. I added a rider to that comment, that the GISMO model goes some way towards it.

We have been quite rightly criticised that we lacked detail in our model. For example, we made some brief comments about bivariate distributions. If we had put in every single comment about how we did this and how we did that, the paper would have been 180 pages long, and no-one would have known what the structure was.

I agree with the comment about a probability of 1 in 100. Statisticians talk about the famous bell shaped curve, and they have a probability of 95% of getting it correct. To most insurance companies that 5% is death or life; a 95% probability does not help insurance companies at all. That is also a tail. All that somebody has to do when they run this model is to define what tail they want. They do not have to go to extremes of 1 in 100. They can go to 1 in 10, but then try and sell it to the management!

To be criticised that our data requirements are too advanced, and that we may have to wait another ten years, strikes me as attacking the real root of our profession. We have to go to management and say, "These are the data we require to run an insurance company." If there is a cost, we have to tell them what that cost is. To go back and say, "We cannot get it because management feel that this is too much", is an unfair criticism.

I take issue with a number of speakers who consider reinsurance to be a secondary risk. If a large, well-known composite has £800m reinsurance premiums, and that is secondary, I must find out what primary is!

The asset risk is great, and the GISMO paper with Mr Andrew Smith's methodology (Christofides *et al.*, 1996) and the other models around, go a long way towards solving that problem. It is odd that the people we have spoken to have said to us that they have their own asset models and are not interested in ours. Had we got a good asset model, I would not be here talking.

We mentioned risk-based capital in our paper only because it is a topical subject. Had we not mentioned it, we would have been criticised for that.

We have also been told that the timing is bad. As Mr Ryan quite rightly said, at the moment the market is so cheap for reinsurance, why bother with all this? The reason for bothering is that it will take anybody here two years to build, and by then the market will have changed.

Mr Craighead was quite right about expenses and bad debts. We started this model with bad debts in mind. We were going to develop this model for the Insolvency Group to measure the life of a particular company going insolvent.

WRITTEN CONTRIBUTIONS

Dr A. Punter: As a non-actuary, but a financial statistician consulting in the general insurance and reinsurance market, I add some, non-correlated, observations:

(1) From a financial viewpoint, I welcome the exposure and discussion that this paper has generated. As was mentioned by several speakers, buyers are increasingly seeking better financial justification of the level and price of reinsurance. They are also seeking to put the reinsurance purchase decision in a broader corporate context, as exemplified by the authors' approach. This is not to say that brokers, consultants and cedants have not already developed financial models of their own (I developed one whose primary output was graphical, some years ago), but, admittedly, most have been scenario-based rather than stochastic. The offering documents for some of the recently formed property catastrophe reinsurance companies have included distributions of internal rates of return based on stochastic loss scenarios, and have incorporated GIS studies for catastrophe exposures.

(2) Turning to a statistical perspective, mention has been made of 400 variables for this model, and 1,000 simulations were performed for the results presented in the paper. This seems far too few to provide any degree of credible convergence in the results — no mention is made in the paper of stability of results. Although, theoretically, there are no problems running more simulations, say 10,000 plus, the suggested approach, of storing all simulations, will ultimately run into real practical problems of data storage and results presentation (only 20 were used for the purposes of some of the graphics in the paper).

(3) Finally, a practical consulting observation. The (relatively) simple problem is to build a model to project results for a limited portfolio, containing several specific classes of business,

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particularly for a new or simple start-up insurance company. The more difficult, but realistic, problem, in my experience, is to combine this within the same model as the run off of past and current business, to represent the full ongoing operations of an insurance company. Issues such as capital adequacy, solvency and return on capital cannot really be examined without also including all aspects of the current (and past) book of business.

The authors subsequently wrote: We deliberately took the view, not that asset modelling is unimportant, but that it had already been well documented. Similarly, the potential correlation between various factors is a hugely complex subject, worthy of considerable study in its own right, and we referred interested readers to the Bulmer *et al.* (1996), paper for a fuller explanation. If the correlations are known, and the parameters can be estimated, they can, of course, be added into the model.

We deliberately concentrated on reinsurance — probably the largest or second largest expense item, after claims, for most general insurance companies. As Mr Daykin pointed out in his written comments, such an approach needs a fundamental redesign of the model to simulate the number and size of individual claims, rather than just their aggregate total.

This leads directly to the need to sample these claims from a probability distribution. Many speakers commented that the better the underlying data from which to produce these assumptions, the more reliable the results will be. We totally concur with this view, but, if the data are not available, management (or their actuary) will need to estimate the inputs using their own experience, which, to a large extent, they are already doing, even if only implicitly — as Mr P. Smith pointed out. If management overrule the data, as was suggested as a possibility by the opener, they do so at their peril, but modelling will help give an indication of the risk involved.

Both Mr Daykin and the closer referred to reinsurance failure. This can take the form, either of the failure of an individual reinsurer, or the losing of a coverage dispute. We felt that a full analysis of this problem, including the potential for 'domino effect' losses, as one reinsurance failure leads to others, would detract from the clarity that we were seeking to achieve, and was worthy of a paper in its own right. It would, however, be possible to model the credit risk on a more simplistic approach. This would take the form of scaling down, by appropriate factors, recoveries from each class of business, accident year and reinsurance treaty. Of course, the factors themselves would also need to be determined.

Several speakers disagreed with the concept of attempting to allocate capital by line of business, although one contradicted himself by saying that, in a conglomerate, the insurance activity needed to compete for resources with other businesses. Of course, this is an identical mathematical problem to that of allocating capital to one business line or another within an insurance group. Mr Ryan's exposition of the benefits of diversification is perfectly valid, but managers of a business still need to be able to allocate and prioritise resources, and to set realistic profit targets to individual areas. The actuarial profession needs to be able to help square this circle, and we believe that this cannot be done realistically without looking particularly at retention levels — which, in turn, involves looking at reinsurance arrangements.

We are aware that there are a number of proprietary packages to optimise the reinsurance arrangements for a single line of business from a tactical perspective. What we tried to achieve was to show how reinsurance can, and must, be incorporated into a strategic overview of the whole company.