Macroeconomic Dynamics, **18**, 2014, 1383–1402. Printed in the United States of America.
doi:10.1017/S1365100512001009

ROOSEVELT AND PRESCOTT COME TO AN AGREEMENT

FRANK N. CALIENDO AND NICK L. GUO
Utah State University

Edward C. Prescott has argued that mandatory saving is socially desirable because it solves the problem of people intentionally free riding and becoming a welfare burden. Inspired by Prescott's argument, we develop a model in which rational individuals choose between saving and free riding. We find that free riding is a robust outcome for a significant share of the population and that everyone, including the free riders, benefits from the elimination of free riding through mandatory saving. Our results strengthen Prescott's position that free riding is a serious problem and that mandatory saving is socially desirable.

Keywords: Social Security, Welfare, Free Riding

> *We have tried to frame a law which gives some measure of protection to the average citizen and his family against the loss of a job and against poverty-ridden old age.*
> Franklin D. Roosevelt, August 14, 1935

> *Without mandatory savings accounts we will not solve the time inconsistency problem of people undersaving and becoming a welfare burden on their families and on the taxpayers.*
> Edward C. Prescott, November 11, 2004, *Wall Street Journal*

1. INTRODUCTION

Despite clear ideological differences, President Franklin D. Roosevelt and Nobel Laureate Edward C. Prescott agree for sure on one thing: mandatory saving for retirement.[1] For Roosevelt, mandatory saving insures against the risk of poverty during old age. For Prescott, it solves a time inconsistency problem of people *intentionally* undersaving (free riding), gambling that taxpayers will take care of them through some type of welfare program.[2] In other words, Prescott worries that without mandatory saving, the government will ultimately end up running a nonuniversal social security program that punishes savers and rewards nonsavers.[3]

We thank Zhaohu Nie for valuable assistance. We also thank Aspen Gorry, Makoto Nakajima, Jorge Alonso Ortiz, Scott Findley, Jason Smith, Nathalie Mathieu-Bolh, seminar participants at Utah State University, and especially an anonymous referee at *Macroeconomic Dynamics* for helpful comments. Address correspondence to: Department of Economics and Finance, 3565 Old Main Hill, Utah State University, Logan, UT 84322-3565, USA; e-mail: frank.caliendo@usu.edu.

 1365-1005/13

There has not been much formal work in macroeconomics on this issue, even though Prescott's argument appeared twice in the *Wall Street Journal* the year he won the Nobel Prize. In this paper we develop some of the formal macroeconomic theory that is needed to test Prescott's hypothesis.

We build a dynamic equilibrium model with perfectly rational households that differ according to the curvature of period utility as in Guvenen (2009). We compare two economies. The first economy has Roosevelt's pay-as-you-go social security system, and participation is mandatory. The second economy is counterfactual and is inspired by what Prescott envisions could happen in the absence of mandatory saving. Instead of social security, there is a tax-and-transfer welfare program that helps only retirees with no assets. Welfare taxes are levied on the wages of all workers (savers and nonsavers alike), and individuals may rationally choose to free ride and intentionally save nothing in order to qualify for welfare.[4] The size of the welfare transfer per recipient and the share of the population who choose to free ride are simultaneously determined in equilibrium.

We learn two main lessons from our theoretical model. First, if mandatory saving does not exist and instead the government operates a welfare program for nonsavers, then our model suggests that a significant portion of the population (even a majority) will tend to free ride. In fact, we prove that at least some free riding will always exist in equilibrium. Second, all individuals in the model, including those who free ride, typically benefit from eliminating the welfare program and replacing it with mandatory saving. We conclude that mandatory saving can be a *Pareto* solution to the free-rider problem. Mandatory saving resolves a coordination failure. Without it, rational individuals get stuck in a Nash equilibrium where many of them free ride and where *everyone* is worse off as a result. So in addition to the intuitive appeal of Prescott's hypothesis, it tests well in a formal, quantitative-theoretic model.

We emphasize that our baseline model is intentionally biased against social security, because we shut down every other channel through which social security is typically argued to enhance welfare. First, although individuals face mortality risk, they have access to competitive annuity markets, which eliminates any insurance role for social security. Second, individuals are perfectly rational and need no help saving for retirement. Third, everyone has the same income, so there is no redistributive role for social security. Thus, social security improves welfare in our model purely because it solves the free-rider problem.[5]

Other researchers have laid the foundation for thinking about these issues. Kotlikoff (1987, 1989) builds a model in which individuals are altruistic toward each other, creating an incentive to free ride and take advantage of the kindness of others. The government solves the externality problem through mandatory saving. By construction, individuals in Kotlikoff's model are identical ex ante and therefore everyone undersaves. In contrast, our model features heterogeneity in saving decisions and the share that free ride is endogenous. We accomplish this by allowing heterogeneity in the curvature of period utility.

https://doi.org/10.1017/S1365100512001009 Published online by Cambridge University Press

Like our paper, Homburg (2000) endogenizes the share of the population who rationally choose to free ride. The key to his analysis is heterogeneity in skill level, whereas we introduce heterogeneity in preferences. Modeling heterogeneity as we do allows us to go beyond Homburg's results to prove that at least some free riding will always exist and to establish formal conditions under which a majority will free ride.[6] We turn now to our theoretical model.

2. A DYNAMIC EQUILIBRIUM MODEL

2.1. Basic Notation and Review of Competitive Annuities

All individuals are perfectly rational in this model. Age is continuous and indexed by t. Individuals start work at age 0, retire exogenously at age $t = T$, and pass away and exit the model no later than $t = \bar{T}$. The unconditional probability of surviving to age t, from the perspective of age 0, is $S(t)$. Thus, $S(0) = 1$ and $S(\bar{T}) = 0$. Individuals receive wage income w during their working years.[7]

The flow of consumption at time t is $c(t)$. Period utility is $u(c) = c^{1-\sigma}/(1-\sigma)$, where σ is the inverse elasticity of intertemporal substitution. The key to our analysis is heterogeneity in σ across individuals, with density $f(\sigma)$ and support $[\sigma^-, \sigma^+]$. We disregard discounting above and beyond mortality risk.[8]

We consider a world where individuals save through annuities (insurance). This eliminates any gains from mandatory annuitization and biases the model against social security. Our treatment of annuities follows Sheshinski (2008). Let $a(t)$ be the quantity of annuities held by the individual at age t. Annuities can be bought and sold at unit price. Essentially, we are assuming a fully developed residual annuity market where annuities can be sold back to the originator at unit price. All individuals start and end the life cycle with no annuities, $a(0) = a(\bar{T}) = 0$. The holder of an annuity collects a flow of returns as long as he lives. The return $r(t)$ depends on his age. Deceased individuals surrender their annuities, and these annuities are given to survivors of the same age as the deceased. The annuity market is competitive—zero profits for the administrator of the annuities—because all surrendered annuities are distributed to survivors (none are retained by the administrator) and because annuities can be sold back to the administrator at unit price.

At each moment a new cohort is born. Each cohort contains a mass of infinitely divisible individuals (which can be normalized to 1). Although there is heterogeneity in σ, as we show in the following, individuals who choose to hold annuities will hold the same amount regardless of σ. Let N be the mass of individuals who hold annuities in a given cohort. The survival probability $S(t)$ is also the actual percentage of a given cohort that is alive at age t. And $-dS(t)/dt$ is the fraction of a cohort who die at age t. Therefore, $-N\,(dS(t)/dt)\,a(t)$ is the quantity of annuities surrendered by those who die at age t, and $NS(t)$ is the mass of annuity holders who survive to age t. Because of zero profits in the annuity market, we can divide the quantity of annuities surrendered by the surviving population to get

https://doi.org/10.1017/S1365100512001009 Published online by Cambridge University Press

annuities received per survivor, which is akin to interest income from a savings account, $[-dS(t)/dt]a(t)/S(t)$. Hence

$$r(t) = \frac{-dS(t)/dt}{S(t)} = -\frac{d\ln S(t)}{dt}. \tag{1}$$

2.2. Regime 1: Roosevelt's Pay-As-You-Go Social Security

The government's only function is to administer a mandatory social security program.[9] Taxes are collected on wage income at a rate τ in exchange for social security benefits b during retirement. Individuals behave according to

$$\max : \int_0^{\bar{T}} S(t)u(c(t))dt, \tag{2}$$

subject to

$$\frac{da(t)}{dt} = r(t)a(t) + y(t) - c(t), \tag{3}$$

$$y(t) = (1-\tau)w, \text{ for } t \in [0, T], \tag{4}$$

$$y(t) = b, \text{ for } t \in [T, \bar{T}], \tag{5}$$

$$a(0) = 0, \quad a(\bar{T}) = 0. \tag{6}$$

Regardless of the curvature parameter σ, all individuals will choose the same consumption profile (the solution to the preceding problem):

$$c(t) = c = \left[\int_0^{\bar{T}} S(t)y(t)dt\right] \times \left[\int_0^{\bar{T}} S(t)dt\right]^{-1}. \tag{7}$$

The social security budget will balance if $b = \tau w R$, where R is the ratio of workers to retirees, $R \equiv \int_0^T S(t)dt / \int_T^{\bar{T}} S(t)dt$. But this implies that

$$c(t) = c = w\alpha, \tag{8}$$

where $\alpha \equiv [\int_0^T S(t)dt] \times [\int_0^{\bar{T}} S(t)dt]^{-1} < 1$ is the share of the population who work. Notice that social security does not factor into (8) and therefore does not affect the welfare of the individual. This is a standard result. If the returns on annuities are priced competitively, then the decentralized life-cycle consumption allocation will be the same as the first-best allocation [Sheshinski (2008)]. In such a world there is no role for social security.

Hence, it is tempting to conclude that social security is unnecessary if competitive annuities exist. But as Prescott has (basically) argued, this is not the relevant comparison, because the absence of social security could give rise to a world like the one we model next.

https://doi.org/10.1017/S1365100512001009 Published online by Cambridge University Press

2.3. Regime 2: Prescott's World with Free Riding and Welfare

This is a counterfactual world with no social security program. Instead, the government operates a welfare program that pays benefits to just those retirees who have no income at all (i.e., those who fail to save anything).[10] The welfare program is financed by taxes at a rate ϕ, levied on wages, and pays out a transfer Φ for those who qualify.

Rational individuals must now choose between two options. One option is to save according to a control problem, recognizing that they will not qualify for welfare,

$$\max : \int_0^{\bar{T}} S(t)u(c(t))dt, \tag{9}$$

subject to

$$\frac{da(t)}{dt} = r(t)a(t) + y(t) - c(t), \tag{10}$$

$$y(t) = (1-\phi)w, \text{ for } t \in [0, T], \tag{11}$$

$$y(t) = 0, \text{ for } t \in [T, \bar{T}], \tag{12}$$

$$a(0) = 0, \quad a(\bar{T}) = 0. \tag{13}$$

The solution to the optimal control problem is

$$c(t) = c = (1-\phi)w\alpha. \tag{14}$$

The other option is to free ride and save nothing, in which case we denote consumption with a circumflex to distinguish from the optimal control:

$$\hat{c}(t) = (1-\phi)w, \text{ for } t \in [0, T], \tag{15}$$

$$\hat{c}(t) = \Phi, \text{ for } t \in [T, \bar{T}]. \tag{16}$$

Thus, the rational individual living in Regime 2 makes the following choice, taking ϕ and Φ as given:

$$\max_{\{c, \hat{c}(t)\}} : \left\{ \int_0^{\bar{T}} S(t)u(c)dt, \int_0^{\bar{T}} S(t)u(\hat{c}(t))dt \right\}. \tag{17}$$

We define a threshold value of σ, call it σ^*, for which the individual free rides when $\sigma < \sigma^*$ and follows the consumption-smoothing path when $\sigma > \sigma^*$ (he is indifferent when $\sigma = \sigma^*$). Free riding, which maximizes lifetime *income*, is more appealing as period utility becomes closer and closer to linear. One must accept an uneven consumption profile to qualify for the extra income that comes from free riding. This is a trade-off that has everything to do with the curvature of period utility. We now develop this concept formally.

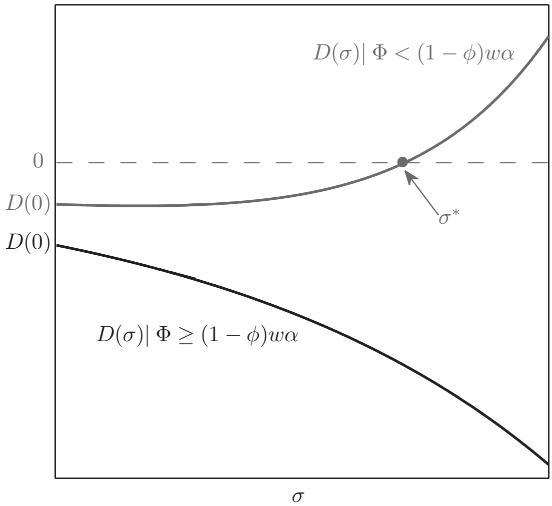

FIGURE 1. Distance function from Proposition 1 and Proof of Proposition 1. This figure illustrates the condition under which a threshold σ^* exists and is unique, as well as the condition for nonexistence.

DEFINITION 1. *Define σ^* as the solution to $D(\sigma) = 0$, where*

$$D(\sigma) \equiv \frac{[(1-\phi)w\alpha]^{1-\sigma}}{1-\sigma} - \alpha\frac{[(1-\phi)w]^{1-\sigma}}{1-\sigma} - (1-\alpha)\frac{\Phi^{1-\sigma}}{1-\sigma} \quad \textbf{(18)}$$

is lifetime utility from saving minus lifetime utility from free riding, normalized by the size of the population $\int_0^T S(t)dt$.

PROPOSITION 1. *If $\Phi \geq (1-\phi)w\alpha$, then σ^* does not exist, $D(\sigma) < 0$, and everyone free rides. On the other hand, if $0 < \Phi < (1-\phi)w\alpha$, then σ^* exists and will be unique. In the latter case, $\sigma < \sigma^*$ implies free riding ($D(\sigma) < 0$) and $\sigma > \sigma^*$ implies saving ($D(\sigma) > 0$). (The proof of this proposition is illustrated in Figure 1.)*

Proof. First, consider $\Phi \geq (1-\phi)w\alpha$,

$$D(\sigma) \leq \frac{[(1-\phi)w\alpha]^{1-\sigma}}{1-\sigma} - \alpha\frac{[(1-\phi)w]^{1-\sigma}}{1-\sigma} - (1-\alpha)\frac{[(1-\phi)w\alpha]^{1-\sigma}}{1-\sigma}, \quad \textbf{(19)}$$

which implies that

$$D(\sigma) \leq \alpha\frac{[(1-\phi)w\alpha]^{1-\sigma}}{1-\sigma} - \alpha\frac{[(1-\phi)w]^{1-\sigma}}{1-\sigma} < 0, \quad \textbf{(20)}$$

for all $\sigma \in [0, \infty)$, in which case σ^* does not exist and everyone free rides.

https://doi.org/10.1017/S1365100512001009 Published online by Cambridge University Press

Next, consider the second case, $0 < \Phi < (1-\phi)w\alpha$. We focus first on the existence of σ^*. Under linear utility ($\sigma = 0$),

$$D(0) = -(1-\alpha)\Phi < 0. \tag{21}$$

Meanwhile, at the other extreme, we have

$$\lim_{\sigma\to\infty} D(\sigma) = \lim_{\sigma\to\infty}\left[\left(\frac{(1-\phi)w\alpha}{\Phi}\right)^{1-\sigma} - \alpha\left(\frac{(1-\phi)w}{\Phi}\right)^{1-\sigma} - (1-\alpha)\right] \times \lim_{\sigma\to\infty}\frac{\Phi^{1-\sigma}}{1-\sigma}. \tag{22}$$

Using the condition $0 < 1 < (1-\phi)w\alpha/\Phi$, rewrite (22) as

$$\lim_{\sigma\to\infty} D(\sigma) = -(1-\alpha) \times \lim_{\sigma\to\infty}\frac{\Phi^{1-\sigma}}{1-\sigma}. \tag{23}$$

Without loss of generality, we envision normalizing the model so that $w = 1$, in which case the condition of interest $0 < \Phi < (1-\phi)w\alpha$ becomes $0 < \Phi < (1-\phi)\alpha < 1$. Hence the limit on the right-hand side of (23) is the indeterminate form $-\infty/\infty$, so we use L'Hôpital's rule,

$$\lim_{\sigma\to\infty}\frac{\Phi^{1-\sigma}}{1-\sigma} = \lim_{\sigma\to\infty}\Phi^{1-\sigma}\ln\Phi = -\infty \Rightarrow \lim_{\sigma\to\infty} D(\sigma) = \infty. \tag{24}$$

Considering (21) and (24) together, it must be the case that $D(\sigma) = 0$ at least once. This implies existence of σ^*.

Next we show uniqueness. Compute

$$\frac{dD(\sigma)}{d\sigma} = \frac{1}{1-\sigma}D(\sigma) + \frac{1}{1-\sigma}\left\{\begin{array}{c} -\left[(1-\phi)w\alpha\right]^{1-\sigma}\ln\left[(1-\phi)w\alpha\right] \\ +\alpha\left[(1-\phi)w\right]^{1-\sigma}\ln\left[(1-\phi)w\right] \\ +(1-\alpha)\Phi^{1-\sigma}\ln\Phi \end{array}\right\}. \tag{25}$$

If $\sigma = \sigma^*$, then by definition $D(\sigma^*) = 0$, which in turn implies that

$$\Phi = (1-\phi)w\alpha\left\{\frac{1-\alpha^{\sigma^*}}{1-\alpha}\right\}^{1/(1-\sigma^*)}. \tag{26}$$

Combine (25) and (26) and simplify:

$$\frac{dD(\sigma^*)}{d\sigma} = \frac{1}{1-\sigma^*}\left\{\begin{array}{c} -\alpha\left((1-\phi)w\right)^{1-\sigma^*}\ln\alpha \\ +\left((1-\phi)w\alpha\right)^{1-\sigma^*}\frac{1}{1-\sigma^*}\ln\left(\frac{1-\alpha^{\sigma^*}}{1-\alpha}\right) \\ -\alpha\left((1-\phi)w\right)^{1-\sigma^*}\frac{1}{1-\sigma^*}\ln\left(\frac{1-\alpha^{\sigma^*}}{1-\alpha}\right) \end{array}\right\}. \tag{27}$$

https://doi.org/10.1017/S1365100512001009 Published online by Cambridge University Press

Further simplify and use the notation $t \equiv \alpha^{\sigma^*}$ and $\sigma^* = \ln t / \ln \alpha$:

$$\frac{dD(\sigma^*)}{d\sigma} = \frac{1}{(1-\sigma^*)^2} \left((1-\phi)w\alpha\right)^{1-\sigma^*} h(t), \tag{28}$$

where

$$h(t) \equiv (1-t)(\ln(1-t) - \ln(1-\alpha)) - t(\ln\alpha - \ln t). \tag{29}$$

From the mean value theorem, there exist ξ and η between α and t such that

$$h(t) = (\alpha - t)\left[\frac{1-t}{1-\xi} - \frac{t}{\eta}\right]. \tag{30}$$

If $\alpha > t$, then $\alpha > \xi, \eta > t, 1-\alpha < 1-\xi, 1-\eta < 1-t$, and

$$\frac{1-t}{1-\xi} - \frac{t}{\eta} > 0. \tag{31}$$

Similarly for the case $\alpha < t$. Thus $h(t) > 0$ and therefore $dD(\sigma^*)/d\sigma > 0$, which ensures that $D(\sigma)$ cannot be zero more than once. This completes the proof of uniqueness. Q.E.D.

Turning to the balanced budget requirement, transfers per recipient must equal

$$\Phi = \begin{cases} \phi w R \left(\int_{\sigma^-}^{\sigma^*} f(\sigma)d\sigma\right)^{-1}, & \text{if } \sigma^* \in (\sigma^-, \sigma^+), \\ \phi w R, & \text{if } \sigma^* \geq \sigma^+, \\ \infty, & \text{if } \sigma^* \leq \sigma^-. \end{cases} \tag{32}$$

Note the inherent simultaneity: Φ depends on σ^*, but then σ^* depends on Φ. To compute an equilibrium we guess a value of Φ, compute the threshold value σ^* that derives from the guess, compute the value of Φ that derives from σ^* from the previous step, and then iterate until the guess and feedback values of Φ converge. As a guide for our following quantitative work, the following analysis provides a sufficiency condition that guarantees the existence and uniqueness of an interior equilibrium.

DEFINITION 2. *An interior equilibrium is characterized by a pair* (Φ, σ^*) *for which* $\sigma^* \in (\sigma^-, \sigma^+)$, *the budget balances as in (32), and all consumers maximize expected utility, taking taxes and transfers as given. In words, an interior equilibrium means some but not all individuals will free ride.*

PROPOSITION 2. *An interior equilibrium will exist and will be unique if and only if*

$$\phi < \left(\frac{1-\alpha^{\sigma^+}}{1-\alpha}\right)^{1/(1-\sigma^+)} \times \left\{\left(\frac{1-\alpha^{\sigma^+}}{1-\alpha}\right)^{1/(1-\sigma^+)} + \frac{1}{1-\alpha}\right\}^{-1} \equiv \bar{\phi}. \tag{33}$$

(The proof of this proposition is illustrated in Figure 2.)

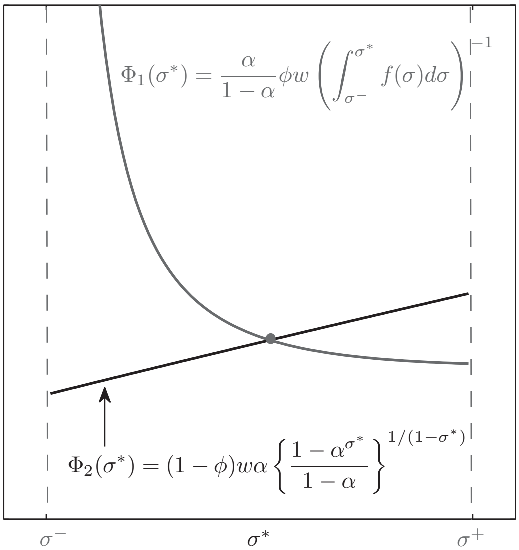

FIGURE 2. Existence and uniqueness of an interior equilibrium. This figure accompanies Proposition 2 and the Proof of Proposition 2.

Proof. We begin with existence. An interior equilibrium, if it exists, is a solution to a nonlinear system of two equations and two unknowns (Φ and σ^*). Subscripts "1" and "2" are used to denote the two equations. The first equation is the balanced budget equation,

$$\Phi_1(\sigma^*) = \frac{\alpha}{1-\alpha}\phi w \left(\int_{\sigma^-}^{\sigma^*} f(\sigma)d\sigma\right)^{-1}, \qquad (34)$$

and the second equation comes from setting $\sigma = \sigma^*$ in (18) to find the point of indifference,

$$[(1-\phi)w\alpha]^{1-\sigma^*} = [(1-\phi)w]^{1-\sigma^*}\alpha + \Phi^{1-\sigma^*}(1-\alpha), \qquad (35)$$

or, rewritten compactly,

$$\Phi_2(\sigma^*) = (1-\phi)w\alpha \left\{\frac{1-\alpha^{\sigma^*}}{1-\alpha}\right\}^{1/(1-\sigma^*)}. \qquad (36)$$

From (34), $\Phi_1(\sigma^+) = \phi w \frac{\alpha}{1-\alpha}$, $\lim_{\sigma^* \to \sigma^-} \Phi_1(\sigma^*) = \infty$, and $d\Phi_1(\sigma^*)/d\sigma^* < 0$ for all $\sigma^* \in (\sigma^-, \sigma^+]$. From (36), note that $\Phi_2(\sigma^-)$ is finite. Taking all this together, an intersection of the curves $\Phi_1(\sigma^*)$ and $\Phi_2(\sigma^*)$ is guaranteed if

$$\Phi_2(\sigma^+) > \Phi_1(\sigma^+) = \frac{\alpha}{1-\alpha}\phi w. \qquad (37)$$

https://doi.org/10.1017/S1365100512001009 Published online by Cambridge University Press

Evaluate (36) at $\sigma^* = \sigma^+$ and then insert into (37) to obtain (33). This completes the proof of existence.

To prove uniqueness, it is sufficient to prove that $d\Phi_2(\sigma^*)/d\sigma^* > 0$, which ensures at most one intersection of the curves $\Phi_1(\sigma^*)$ and $\Phi_2(\sigma^*)$. Define an auxiliary function $t(\sigma^*) \equiv \alpha^{\sigma^*}$, and making use of $\ln t(\sigma^*) = \sigma^* \ln \alpha$, rewrite (36) as

$$\Phi_2(\sigma^*) = (1-\phi)w\alpha \left\{ \frac{1-t(\sigma^*)}{1-\alpha} \right\}^{1/(1-\ln t(\sigma^*)/\ln \alpha)}. \tag{38}$$

With some algebra

$$\Phi_2(\sigma^*) = (1-\phi)w\alpha \left[\exp\left(-\ln \alpha \frac{\ln(1-t(\sigma^*)) - \ln(1-\alpha)}{\ln t(\sigma^*) - \ln \alpha} \right) \right]. \tag{39}$$

Define another auxiliary function,

$$g(t(\sigma^*)) \equiv \frac{\ln(1-t(\sigma^*)) - \ln(1-\alpha)}{\ln t(\sigma^*) - \ln \alpha}, \tag{40}$$

and hence

$$\Phi_2(\sigma^*) = (1-\phi)w\alpha[\exp(-g(t(\sigma^*))\ln \alpha)], \tag{41}$$

$$\frac{d\Phi_2(\sigma^*)}{d\sigma^*} = -(1-\phi)w\alpha[\exp(-g(t(\sigma^*))\ln \alpha)]g'(t(\sigma^*))\frac{dt(\sigma^*)}{d\sigma^*}\ln \alpha. \tag{42}$$

Recall that $\alpha \in (0,1)$; thus $\ln \alpha < 0$, $t(\sigma^*) \in (0,1)$, and $dt(\sigma^*)/d\sigma^* < 0$ and hence $d\Phi_2(\sigma^*)/d\sigma^* > 0$ if and only if $g'(t(\sigma^*)) < 0$. Proving this last inequality is all that remains. Thus

$$g'(t) = \frac{\frac{-1}{1-t}(\ln t - \ln \alpha) - \frac{1}{t}(\ln(1-t) - \ln(1-\alpha))}{(\ln t - \ln \alpha)^2}. \tag{43}$$

Recalling the mean value theorem, there exist ξ and η between α and t such that

$$g'(t) = \frac{\frac{-1}{1-t}\frac{1}{\xi}(t-\alpha) - \frac{1}{t}\frac{-1}{1-\eta}(t-\alpha)}{(\ln t - \ln \alpha)^2} \tag{44}$$

$$= \frac{(t-\alpha)\left(\frac{1}{t}\frac{1}{1-\eta} - \frac{1}{1-t}\frac{1}{\xi}\right)}{(\ln t - \ln \alpha)^2}. \tag{45}$$

If $t > \alpha$, then $\alpha < \xi, \eta < t$ and $1-\alpha > 1-\xi, 1-\eta > 1-t$, so

$$\frac{1}{t}\frac{1}{1-\eta} - \frac{1}{1-t}\frac{1}{\xi} < 0, \tag{46}$$

and hence $g'(t) < 0$. Similarly when $t < \alpha$. This completes the proof of uniqueness. Q.E.D.

COROLLARY 1. *Given $\phi > 0$, it is never the case that everyone saves in equilibrium.*

https://doi.org/10.1017/S1365100512001009 Published online by Cambridge University Press

Proof. Suppose everyone saves. Then by Proposition 1, $\sigma^* \leq \sigma^-$, and from (32) we have $\Phi = \infty$. But, also from Proposition 1, if $\Phi \geq (1-\phi)w\alpha$, then σ^* does not exist and everyone free rides, which is a contradiction. Q.E.D.

PROPOSITION 3. *A majority of the population will free ride if and only if*

$$\phi > \underline{\phi}, \tag{47}$$

where

$$\underline{\phi} \equiv \left(\frac{1-\alpha^{\sigma^m}}{1-\alpha}\right)^{1/(1-\sigma^m)} \times \left\{\left(\frac{1-\alpha^{\sigma^m}}{1-\alpha}\right)^{1/(1-\sigma^m)} + \frac{2}{1-\alpha}\right\}^{-1}, \tag{48}$$

and σ^m *is defined as the solution to*

$$\int_{\sigma^-}^{\sigma^m} f(\sigma)d\sigma = \frac{1}{2}. \tag{49}$$

Proof. A majority will free ride if and only if $\sigma^* > \sigma^m$. Recall that $\lim_{\sigma^* \to \sigma^-} \Phi_1(\sigma^*) = \infty > \Phi_2(\sigma^-)$; thus $\sigma^* > \sigma^m$ is guaranteed if $\Phi_1(\sigma^m) > \Phi_2(\sigma^m)$, i.e.,

$$\frac{\alpha}{1-\alpha}\phi w \left(\int_{\sigma^-}^{\sigma^m} f(\sigma)d\sigma\right)^{-1} > (1-\phi)w\alpha \left\{\frac{1-\alpha^{\sigma^m}}{1-\alpha}\right\}^{1/(1-\sigma^m)}. \tag{50}$$

Combining (49) and (50) gives (48). Q.E.D.

PROPOSITION 4. *The fraction of the population who choose to free ride at equilibrium is increasing in the tax rate* ϕ.

Proof. From (34) and (36), an interior equilibrium satisfies the equations

$$J_1(\sigma^*, \Phi, \phi) \equiv \Phi - \frac{\alpha}{1-\alpha}\phi w \left(\int_{\sigma^-}^{\sigma^*} f(\sigma)d\sigma\right)^{-1} = 0, \tag{51}$$

$$J_2(\sigma^*, \Phi, \phi) \equiv \Phi - (1-\phi)w\alpha \left\{\frac{1-\alpha^{\sigma^*}}{1-\alpha}\right\}^{1/(1-\sigma^*)} = 0, \tag{52}$$

where ϕ is an exogenous variable and σ^* and Φ are endogenous variables. Differentiating (51) and (52) with respect to ϕ obtains

$$\begin{bmatrix} \partial J_1/\partial\sigma^* & \partial J_1/\partial\Phi \\ \partial J_2/\partial\sigma^* & \partial J_2/\partial\Phi \end{bmatrix} \begin{bmatrix} d\sigma^*/d\phi \\ d\Phi/d\phi \end{bmatrix} = \begin{bmatrix} -\partial J_1/\partial\phi \\ -\partial J_2/\partial\phi \end{bmatrix}, \tag{53}$$

where

$$\frac{\partial J_1}{\partial\phi} = -\frac{\alpha}{1-\alpha} w \left(\int_{\sigma^-}^{\sigma^*} f(\sigma)d\sigma\right)^{-1} < 0, \tag{54}$$

$$\frac{\partial J_1}{\partial \sigma^*} = -\frac{d\Phi_1(\sigma^*)}{d\sigma^*} > 0, \tag{55}$$

$$\frac{\partial J_1}{\partial \Phi} = \frac{\partial J_2}{\partial \Phi} = 1, \tag{56}$$

$$\frac{\partial J_2}{\partial \phi} = w\alpha \left\{ \frac{1-\alpha^{\sigma^*}}{1-\alpha} \right\}^{1/(1-\sigma^*)} > 0, \tag{57}$$

$$\frac{\partial J_2}{\partial \sigma^*} = -\frac{d\Phi_2(\sigma^*)}{d\sigma^*} < 0, \tag{58}$$

where the inequalities in (55) and (58) come from the proof of Proposition 2. Cramer's Rule gives

$$\frac{d\sigma^*}{d\phi} = \frac{\det \begin{bmatrix} -\partial J_1/\partial \phi & \partial J_1/\partial \Phi \\ -\partial J_2/\partial \phi & \partial J_2/\partial \Phi \end{bmatrix}}{\det \begin{bmatrix} \partial J_1/\partial \sigma^* & \partial J_1/\partial \Phi \\ \partial J_2/\partial \sigma^* & \partial J_2/\partial \Phi \end{bmatrix}} > 0. \tag{59}$$

Thus, equilibrium σ^* is increasing in ϕ, and hence the share of the population who choose to free ride at equilibrium is increasing in ϕ. Q.E.D.

3. NUMERICAL EXAMPLES

We set the survival probability $S(t) = 1-(t/\bar{T})^s$, with $s = 3.2$ and $\bar{T} = 75$, which is a reasonable approximation for a typical U.S. household and implies a reasonable ratio of workers to retirees ($R = 2.1$). Imagining a 40-year working period, we set $T = 40$. We normalize $w = 1$. As an example, we set the counterfactual welfare tax to $\phi = 10\%$. Other tax rates can be used to illustrate the same point.

We consider the simple case where $f(\sigma)$ is a general, quasi-normal function,

$$f(\sigma) = f_{\max} \exp\left[-\mu(\gamma\sigma - 1)^2\right], \quad \text{where } \mu, \gamma \in \mathbf{R}^+. \tag{60}$$

The thickness of this function is controlled by μ, the mode is γ^{-1}, and the extremum is $f_{\max}$. This density has one peak, is truncated, and need not be symmetric. The parameter $f_{\max}$ can be normalized to ensure that $f(\sigma)$ is a proper density with unit area under the curve,

$$f_{\max} = \left[\int_{\sigma^-}^{\sigma^+} \exp\left[-\mu(\gamma\sigma - 1)^2\right] d\sigma \right]^{-1}. \tag{61}$$

Setting $\sigma^- = 1.01$ and $\sigma^+ = 1.99$ creates a range of variation in the curvature of period utility to ensure an interior solution for σ^*. The results are not oversensitive to the particular support. Though we will experiment with the remaining parameters, μ and γ, we choose the baseline values $\mu = 10$ and $\gamma = 2/3$, which gives a symmetric, truncated bell curve, as shown in Figure 3. Values of σ near 1.5, which

https://doi.org/10.1017/S1365100512001009 Published online by Cambridge University Press

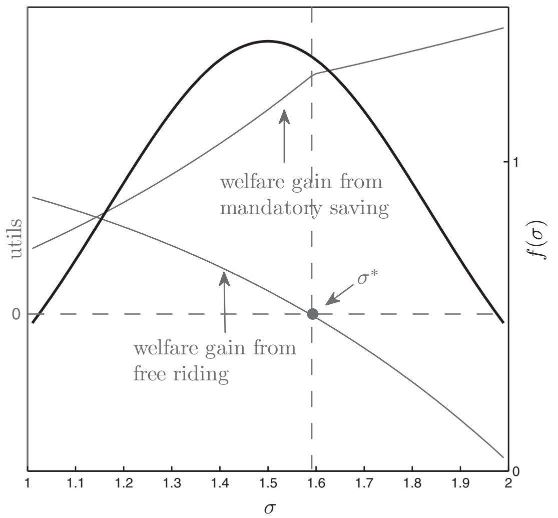

FIGURE 3. Welfare gain from free riding and welfare gain from mandatory saving. The baseline density function is also pictured.

is the mean and mode of the baseline density, are common in macroeconomic studies.

Finally, from (33), we can anticipate that an interior equilibrium exists if $\phi < \bar{\phi} = 16.1\%$, and from (47) and (48) we can anticipate that a majority will free ride if $\phi > \underline{\phi} = 7.9\%$. Our chosen tax rate ($\phi = 10\%$) meets both conditions. If we choose a tax that is less than $\underline{\phi}$ we will still have free riding, but the majority will save. The equilibrium for the Prescott economy with all of these parameters is $\Phi = 33.3\%$, $\sigma^* = 1.59$. This implies that 62.8% of the population choose to free ride.

Figure 3 illustrates the main point. Two welfare graphs are pictured. One is labeled "welfare gain from free riding," which corresponds to the equilibrium in the Prescott economy. This is expected lifetime utility from free riding minus expected lifetime utility from following the consumption path from the optimal control problem. The other graph is labeled "welfare gain from mandatory saving," which is expected utility in FDR's social security economy, minus the maximum of the utility from free riding or following the control solution from the Prescott economy. Notice that *everyone* gains from mandatory saving in the baseline calibration.

The key to these results is the assumption of heterogeneity in period utility across individuals. Free riding generates extra income, but the individual must subject himself to an uneven consumption profile in order to qualify for the extra income. Thus, those with period utility functions that are closest to linear will be the most likely to free ride, because an uneven consumption profile causes relatively minor losses of lifetime utility. Those with the most curvature will reject free riding even if the extra income is significant. With heterogeneity in period utility, there can be an equilibrium in which part of the population free ride and another part save for retirement.

We emphasize that even those who rationally choose to free ride (62.8%) would be better off if free riding were disallowed. Free riding is rational conditional on the existence of the tax-and-transfer welfare program. As long as they must pay a tax on their wages, and as long as welfare benefits are available to just those without any income, rational individuals will intentionally save nothing. But, if it were in their power, they would instead choose to live in a different world with mandatory saving rather than tax-and-transfer welfare.

Why? This seems like a contradiction at first, but the answer is intuitive. Consider the choice between saving and free riding, given the existence of a welfare program. The cost of free riding is the uneven consumption profile and the gain is the extra income during retirement. Welfare taxes do not count as a cost of free riding because taxes must be paid whether the individual free rides or saves. However, if the individual could choose whether or not to have a welfare program in the first place, then he would count the taxes as a cost of free riding. In this case, free riding carries two costs: the taxes paid and the uneven consumption profile. It is therefore possible that individuals could rationally choose to free ride, conditional on the existence of welfare, and at the same time prefer that the welfare program be eliminated altogether.

The welfare results shown in Figure 3 are very robust to changes in both the dispersion and the skewness of the baseline density function $f(\sigma)$. After redoing the analysis with a number of alternative density functions, we find no material difference in the results.

The most obvious way to weaken our results is to select a tax that falls below the range that we know from Proposition 3 is required to generate majority free riding. For example, if $\phi = 5\%$, then in equilibrium 31.5% will choose to free ride and 100% would be better off with mandatory saving. Pushing further toward the extreme, if say $\phi = 2\%$, then 13.5% will free ride in equilibrium and 97.9% would be better off with mandatory saving. Although a very low tax breaks the result that a majority will free ride, this is not particularly damaging to Prescott's hypothesis, because, after all, we would expect that as the size of the welfare program shrinks to nothing, its effect is minimized.

4. ROBUSTNESS

In the preceding two sections, we studied and compared two economies: one with a pay-as-you-go social security program and the other with a tax-and-transfer welfare program. The studies were conducted under two assumptions: (1) factor prices (wage and interest rate) are exogenous, and (2) competitive annuity markets exist. These two assumptions helped us to isolate the role of social security as a solution to the free-rider problem, and they also enabled us to derive clean analytical results.

In this section, we relax these two assumptions and test whether our main results can still hold in a more realistic environment. We reexamine the same two economies with the new assumptions that (1) factor prices are endogenous and (2) annuity markets are missing. Our new findings show that the main results from the preceding two sections still hold. In the remainder of this section, we first characterize the equilibria in the two economies with new assumptions, and then provide a numerical comparison of the equilibria.

4.1. Regime 1: Roosevelt's Pay-As-You-Go Social Security

In the economy with a pay-as-you-go social security program, all households pay taxes during the working years and all households receive benefits during retirement. Households can save in a savings account, $k(t)$, and the savings from the deceased are redistributed to all survivors as bequest income. Households take bequest income B, social security tax rate τ and retirement benefits b, and factor prices w and r as given, and they solve the problem

$$\max : \int_0^{\bar{T}} S(t)u(c(t))dt, \tag{62}$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + y(t) - c(t), \tag{63}$$

$$y(t) = (1-\tau)w + B, \quad \text{for} \quad t \in [0, T], \tag{64}$$

$$y(t) = b + B, \quad \text{for} \quad t \in [T, \bar{T}], \tag{65}$$

$$k(0) = k(\bar{T}) = 0. \tag{66}$$

At the aggregate level, labor and capital markets clear:

$$\int_0^T S(t)dt = L, \tag{67}$$

$$\int_{\sigma^-}^{\sigma^+} \int_0^{\bar{T}} kS(t)f(\sigma)dtd\sigma = K. \tag{68}$$

The factor prices, wage w and interest rate r, are determined by competitive firms, which use a constant-returns-to-scale technology $Y = K^\theta L^{1-\theta}$,

$$w = (1-\theta)\left(\frac{K}{L}\right)^\theta, \tag{69}$$

$$r = \theta\left(\frac{K}{L}\right)^{\theta-1} - \delta, \tag{70}$$

https://doi.org/10.1017/S1365100512001009 Published online by Cambridge University Press

where δ is the depreciation rate of capital. The government's budget is balanced:

$$\tau w \int_0^T S(t)dt = b \int_T^{\bar{T}} S(t)dt. \quad \textbf{(71)}$$

Finally, accidental bequest income is determined by

$$\int_{\sigma^-}^{\sigma^+} \int_0^{\bar{T}} \left(-\frac{dS(t)}{dt}\right) kf(\sigma)dtd\sigma = B \int_0^{\bar{T}} S(t)dt. \quad \textbf{(72)}$$

A stationary equilibrium for any given tax rate τ in this economy is characterized by household allocations (k, c), bequest income B, social security benefits b, aggregate capital and labor K, L, and factor prices w, r such that (i) given bequest income B, social security benefits b, and factor prices w, r, household allocations solve the household maximization problem (62)–(66); (ii) factor markets clear; (iii) the government's balanced budget condition (71) is satisfied; and (iv) the aggregate bequest condition (72) is satisfied.

To numerically find the stationary equilibrium for any given tax rate τ, we guess the aggregate capital K and bequest income B. Based on the guesses, we calculate the households' decisions and then find aggregate bequest income and aggregate capital. The initial guess is then updated until the conditions on aggregate bequest income and aggregate capital are satisfied.

4.2. Regime 2: Prescott's World with Free Riding and Welfare

In the economy with a welfare program, all households pay taxes during the working years, but only those that fail to save for retirement will receive benefits. The households take bequest income B, tax rate ϕ, welfare benefits Φ, and factor prices w and r as given, and they choose to save or free ride depending on which strategy delivers higher lifetime utility. If households save throughout the lifetime and thus do not qualify for the welfare program, their problem is

$$\max : \int_0^{\bar{T}} S(t)u(c(t))dt, \quad \textbf{(73)}$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + y(t) - c(t), \quad \textbf{(74)}$$

$$y(t) = (1-\phi)w + B, \quad \text{for} \quad t \in [0, T], \quad \textbf{(75)}$$

$$y(t) = B, \quad \text{for} \quad t \in [T, \bar{T}], \quad \textbf{(76)}$$

$$k(0) = k(\bar{T}) = 0. \quad \textbf{(77)}$$

If households choose not to save for retirement and thus become qualified for welfare benefits, they will aim at having a zero balance in the savings account

https://doi.org/10.1017/S1365100512001009 Published online by Cambridge University Press

at retirement age and on. Thus, income and consumption during the retirement period for free riders are the same and equal to the sum of welfare benefits and bequest income. To sum up, the constraints that free riders face are

$$\frac{dk(t)}{dt} = rk(t) + y(t) - c(t), \tag{78}$$

$$y(t) = (1-\phi)w + B, \quad \text{for} \quad t \in [0, T], \tag{79}$$

$$y(t) = c(t) = \Phi + B, \quad \text{for} \quad t \in [T, \bar{T}], \tag{80}$$

$$k(0) = k(t) = 0, \quad \text{for} \quad t \in [T, \bar{T}]. \tag{81}$$

The market clearing conditions (67)–(68), the factor price conditions (69)–(70), and the condition that bequest income satisfies (72) are the same as in the previous economy. But here the government runs a welfare program for nonsavers,

$$\int_{\sigma^-}^{\sigma^*} \int_T^{\bar{T}} f(\sigma) S(t) \Phi dt d\sigma = \int_0^T S(t)\phi w dt, \tag{82}$$

or

$$\Phi = \phi w R \left(\int_{\sigma^-}^{\sigma^*} f(\sigma) d\sigma \right)^{-1}, \tag{83}$$

where households of type σ^- to σ^* are free riders. For simplicity and without loss of generality, we consider only the case when $\sigma^* \in (\sigma^-, \sigma^+)$.

A stationary equilibrium for any given tax rate ϕ in this economy is characterized by household allocations (k, c), bequest income B, tax-and-transfer welfare program benefits Φ, aggregate capital and labor K, L, and factor prices w, r such that (i) given bequest income B, government benefits Φ, and factor prices w, r, household allocations maximize the household problem; (ii) factor markets clear; (iii) the government's budget is balanced; and (iv) the aggregate bequest condition is satisfied.

To numerically find the stationary equilibrium for every given tax rate ϕ, we first guess a vector of four variables: the aggregate capital K, bequest income B, welfare program benefits Φ, and the cutoff σ^* at which households are indifferent between free riding and saving. Based on the guesses of these variables, we calculate the households' decisions and then find the aggregate bequest, aggregate capital, welfare benefits, and the cutoff σ^*. The initial guesses will then be updated until convergence.

4.3. Numerical Examples

In the calibration exercises for both of the economies we use the same values of the parameters for survival probabilities, utility functions and the distribution of σ, and retirement age as in the numerical analysis in Section 3. There are two new parameters, capital's share θ and the depreciation rate δ, and we pick $\theta = 0.35$

https://doi.org/10.1017/S1365100512001009 Published online by Cambridge University Press

TABLE 1. Comparison of insurance economies with annuities to production economies with capital: The role of mandatory saving

Tax rate	Capital K	Output Y	% who free ride	% who prefer SS
(a) Baseline: Insurance economies with exogenous factor prices and with competitive annuities				
$\phi = 10\%$			62.80%	100%
$\phi = 5\%$			31.50%	100%
$\phi = 2\%$			13.50%	97.9%
(b) Robustness: Production economies with endogenous factor prices and without annuities				
$\phi = 10\%$	282.70	77.46	61.01%	100%
$\phi = 5\%$	341.98	83.00	41.71%	100%
$\phi = 2\%$	403.28	84.94	22.62%	100%

and $\delta = 0.08$, which are commonly used in the literature. We first consider the economy with a social security program with $\tau = 10.6\%$, which is the current social security tax rate in the United States, and we then consider the economy with a welfare program with $\phi = 10\%, 5\%$, or 2%.

We report the numerical results in Table 1. For completeness, part (a) of Table 1 recalls Section 3: it lists the percentage of the population who free ride and the percentage of the population who prefer social security when factor prices are exogenous and when annuity markets exist. Part (b) of Table 1 summarizes the equilibria found when factor prices are endogenous and when annuity markets are missing. The table shows that our results in the preceding two sections still hold: there is always a significant portion of the population who free ride in an economy with a tax-and-transfer welfare program; the ratio of free riders increases as the tax rate rises; and the social security program Pareto-dominates the welfare program because the entire population, including free riders, are better off with mandatory saving.

5. CONCLUDING REMARKS

In this paper we develop some of the theoretical tools that are needed to test Prescott's hypothesis. We build a dynamic equilibrium model with perfectly rational households that differ according to the curvature of period utility as in Guvenen (2009). We learn two main lessons from our theoretical model. First, if mandatory saving does not exist and instead the government operates a welfare program for nonsavers, then our model suggests that a significant portion of the population, even a majority, will tend to free ride. Second, all individuals in the model, including those who free ride, typically benefit from eliminating the welfare program altogether. So in addition to the intuitive appeal of Prescott's hypothesis, it tests well in a formal, quantitative-theoretic model.

There is still much work to be done. We view this paper as an initial attempt to study Prescott's hypothesis. One interesting extension would be to endogenize the labor choice (both intensive and extensive margins). Another would be to consider the *political* feasibility of the type of welfare program that we model. Future work could expand our analysis in these directions.

NOTES

1. Although Prescott advocates the replacement of Roosevelt's pay-as-you-go program with a system of individual accounts, he shares Roosevelt's view that saving should be mandatory. Our focus is on the mandatory saving aspect of social security rather than on how it is financed.

2. When Prescott uses the term "time inconsistency problem," he is not referring to irrational saving behavior; he is referring to the problem of the government suddenly inventing some type of welfare program (even though it said it would not) to care for those who fail to save adequately. Indeed, Prescott's whole point is that we need mandatory saving because people are *perfectly rational*, not because they are irrational [see Prescott (2004a)].

3. See Prescott (2004a, 2004b). Of course, Prescott is not alone in his view. Other economists have considered the same issue and called it either a "saving moral hazard" problem or a "Samaritan's dilemma." For example, see the citations at the end of our Introduction. Hayek (1960) is perhaps the earliest citation.

4. Although free riders pay into the welfare system through a tax on wages (just like savers), they enjoy welfare benefits that are only partly financed by themselves. This is why we use the term "free rider."

5. The literature on the three roles of social security is huge. We will cite just a few key studies. İmrohoroğlu et al. (1995) is an important contribution on the insurance aspect of social security, and Hosseini (2010) pushes the frontier forward in the same area. Feldstein (1985) is a classic paper on the undersaving (myopia) role, which triggered a batch of follow-up pieces, beginning with İmrohoroğlu et al. (2003) and followed by Cremer et al. (2008), Andersen and Bhattacharya (2011), Bucciol (2011), Caliendo (2011), and many others. The role of redistribution is studied carefully in Cremer et al. (2008) and others.

6. Emre (2007) constructs a model that is similar to Homburg's, in which skill heterogeneity is key and a portion of the population free ride. Emre does not study heterogeneity in preferences. Finally, Homburg (2006) adds endogenous work effort to the analysis, but as in the other papers, preferences are uniform.

7. Throughout the baseline model, we assume the wage rate is exogenous and constant. Holding factor prices fixed sets a lower bound on the welfare gains from mandatory saving. The general equilibrium effect of lower GDP due to free riding, and hence the transmission from lower GDP to updated factor prices, would lead to yet another social cost of free riding. We will show that everyone gains from mandatory saving, even the free riders, without appealing to such general equilibrium effects. However, in Section 4 we redo our computational analysis for a full-blown production economy with uninsurable survival risk, lump-sum bequests, and endogenous factor prices.

8. An alternative assumption that we did not pursue is heterogeneity in earnings. This is because we want to avoid a model in which the welfare system serves the dual functions of transferring income from the rich to the poor and from savers to free riders. Our model with heterogeneity only in preferences is a clean way to abstract from the first channel while focusing all the attention on the second channel.

9. A separate issue that we do not focus on is how to *finance* social security benefits. For a discussion of this issue, see Huang et al. (1997) and Conesa and Garriga (2003).

10. The assumption that one must save nothing in order to qualify for welfare is unnecessarily strong. We could assume, for example, that individuals qualify as long as they save less than some given level, but this would only make it easier to generate free riding.

REFERENCES

Andersen, Torben M. and Joydeep Bhattacharya (2011) On myopia as rationale for social security. *Economic Theory* 47(1), 135–158.

Bucciol, Alessandro (2011) A note on social security welfare with self-control problems. *Macroeconomic Dynamics* 15(4), 579–594.

Caliendo, Frank N. (2011) Time-inconsistent preferences and social security: Revisited in continuous time. *Journal of Economic Dynamics and Control* 35(5), 668–675.

Conesa, Juan Carlos and Carlos Garriga (2003) Status quo problem in social security reforms. *Macroeconomic Dynamics* 7, 691–710.

Cremer, Helmuth, Philippe De Donder, Dario Maldonado, and Pierre Pestieau (2008) Designing a linear pension scheme with forced savings and wage heterogeneity. *International Tax and Public Finance* 15(5), 547–562.

Emre, Önsel (2007) Time Inconsistency and Social Security. Working paper, University of Chicago.

Feldstein, Martin (1985) The optimal level of social security benefits. *Quarterly Journal of Economics* 100(2), 303–320.

Guvenen, Fatih (2009) A parsimonious macroeconomic model for asset pricing. *Econometrica* 77(6), 1711–1750.

Hayek, Friedrich von (1960) *The Constitution of Liberty*. London: University of Chicago Press.

Homburg, Stefan (2000) Compulsory savings in the welfare state. *Journal of Public Economics* 77(2), 233–239.

Homburg, Stefan (2006) Coping with rational prodigals: A theory of social security and savings subsidies. *Economica* 73, 47–58.

Hosseini, Roozbeh (2010) Adverse Selection in the Annuity Market and the Role for Social Security. Working paper, Arizona State University.

Huang, He, Selahattin İmrohoroğlu, and Thomas J. Sargent (1997) Two computations to fund social security. *Macroeconomic Dynamics* 1(1), 7–44.

İmrohoroğlu, Ayşe, Selahattin İmrohoroğlu, and Douglas H. Joines (1995) A life cycle analysis of social security. *Economic Theory* 6(1), 83–114.

İmrohoroğlu, Ayşe, Selahattin İmrohoroğlu, and Douglas H. Joines (2003) Time-inconsistent preferences and social security. *Quarterly Journal of Economics* 118(2), 745–784.

Kotlikoff, Laurence J. (1987) Justifying public provision of social security. *Journal of Policy Analysis and Management* 6(4), 674–689.

Kotlikoff, Laurence J. (1989) On the contribution of economics to the evaluation and formation of social insurance policy. *American Economic Review* 79(2), 184–190.

Prescott, Edward C. (2004a) It's irrational to save. *Wall Street Journal*, pp. 1–3, December 29.

Prescott, Edward C. (2004b) Why does the government patronize us? *Wall Street Journal*, pp. 1–3, November 11.

Sheshinski, Eytan (2008) *The Economic Theory of Annuities*. Princeton, NJ: Princeton University Press.