Astrometry and Fundamental Physics

Some of the Hipparcos satellite and Gaia mission leaders: (from left to right) Ulrich Bastian, Dafydd Wyn Evans, Lennart Lindegren, Jean Kovalevsky, François Mignard, Catherine Turon and Erik Hoeg.

François Mignard thanking Jean Kovalevsky after his tribute.

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The Tycho-Gaia Astrometric Solution

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Abstract. Gaia DR1 is based on the first 14 months of Gaia's observations. This is not long enough to reliably disentangle the parallax effect from proper motion. For most sources, therefore, only positions and magnitudes are given. Parallaxes and proper motions were nevertheless obtained for about two million of the brighter stars through the Tycho-Gaia astrometric solution (TGAS), combining the Gaia observations with the much earlier Hipparcos and Tycho-2 positions. In this review I focus on some important characteristics and limitations of TGAS, in particular the reference frame, astrometric uncertainties, correlations, and systematic errors.

Keywords. Catalogs, astrometry, reference systems, stars: distances, stars: general

1. Introduction

The *Tycho-Gaia astrometric solution* (TGAS) is the special astrometric processing applied to bright stars in Gaia Data Release 1 (DR1; Gaia Collaboration, Brown, *et al.* 2016) in order to provide full astrometric information (positions, parallaxes, and proper motions) for 2 057 050 sources. † For the remaining $\sim 1.14 \times 10^9$ sources in Gaia DR1, no parallaxes or proper motions are given. The theoretical background, contents, properties, and validation of TGAS are described in several papers (Michalik *et al.* 2015a; Lindegren *et al.* 2016; Arenou *et al.* 2017). Here I will focus on some properties of TGAS that were perhaps not sufficiently highlighted or explained in these papers.

2. Historical background and concept

TGAS dates back to an idea presented in 2009 by François Mignard to the ESA Gaia Science Team, called the *Hundred Thousand Proper Motions* project (HTPM; Mignard 2009). The basic idea is simple: to improve the proper motions of the ∼100 000 Hipparcos stars roughly by an order of magnitude by combining the early Gaia astrometry acquired for these stars with the Hipparcos positions at an epoch more than 20 years earlier. Simulations by Mignard showed that HTPM would be possible with just six months of Gaia data – the minimum needed to cover the whole celestial sphere – by using the Hipparcos parallaxes to transform the Gaia observations to barycentric directions, as needed for the accurate calculation of proper motions.

The combination of astrometric catalogues from different space missions (Hipparcos, Gaia, NanoJASMINE) was studied in a research project started in 2009 by D. Michalik at Lund Observatory, with support from ESA (Dr. U. Lammers). By rigorously formulating HTPM in a Bayesian framework, it was demonstrated how the solution could improve both the proper motions and parallaxes of the Hipparcos stars (Michalik *et al.* 2014). Since simulations showed that only the positions were needed from the earlier catalogue,

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† In Gaia terminology, a "source" is any approximately point-like object observed by Gaia. A TGAS source may be a single star, a component in a resolved double or multiple system, the photocentre of two or more stellar components, or a compact extragalactic object.

it was a very natural step to include also the Tycho-2 positions in the solution. This effectively led to TGAS (Michalik *et al.* 2015a), which then superseded HTPM.

The simplest kinematic model of a star assumes that it moves with uniform velocity relative to the Solar System Barycentre. Knowing the location and velocity of the observer, the apparent path of the source on the celestial sphere is then described by five astrometric parameters: the barycentric position (α, δ) at some chosen reference epoch (J2015.0 for TGAS), the parallax (ϖ), and the components of proper motion ($\mu_{\alpha *}$, μ_δ). (For a small number of nearby high-velocity stars, a spectroscopic radial velocity is also needed.) Successful estimation of all five parameters requires observations covering a time interval of at least a few years, with a suitable distribution in time to disentangle parallax from proper motion. With only 14 months of Gaia observations available for DR1, the astrometric solution would be highly degenerate for most sources, resulting in large uncertainties and strong correlations between the parallax and proper motion components. By incorporating positions from the Hipparcos and Tycho-2 catalogues at their much earlier epochs (around 1991.25), the proper motions are decorrelated from the other parameters, making a non-degenerate solution possible. In a Bayesian framework the early positions can be regarded as prior information for the estimation of the complete set of astrometric parameters.

3. Priors used in TGAS

Only the *positions* in the Hipparcos and Tycho-2 catalogues are used as priors in TGAS, leaving the parallaxes and proper motions in this solution statistically independent of their corresponding values in the Hipparcos catalogue.

For the Hipparcos sources in TGAS, positions at epoch J1991.25 were taken from the re-reduction by van Leeuwen (2007). For other TGAS sources, positions were taken from the Tycho-2 catalogue (Høg *et al.* 2000) using the "observed" coordinates at the original mean epochs roughly spanning 1990.9 to 1992.1. As shown in the left panel of Fig. 1 the two catalogues have very different positional uncertainties at these epochs, which makes it relevant to distinguish between the "Hip subset" (93 635 sources) and "Tyc subset" (1 963 415 sources) of TGAS. In the Gaia DR1 catalogue of the Gaia Archive† the flag `astrometric_priors_used` equals 3 for the Hip subset and 5 for the Tyc subset.

4. Reference frame of TGAS

The reference frame of TGAS is linked to the extragalactic radio frame ICRF2 (Fey *et al.* 2015) by means of the auxiliary quasar solution (Michalik & Lindegren 2016). The system of positions in TGAS is therefore accurately aligned (to < 0.1 mas) with ICRF2 at the reference epoch J2015.0. Since the Hipparcos catalogue was aligned with the radio frame to better than 0.6 mas at its epoch J1991.25, this makes the proper motion system of TGAS non-rotating with respect to the quasars to better than 0.03 mas yr^{-1}. TGAS thus supersedes the Hipparcos catalogue as the currently best optical realisation of the International Celestial Reference System (ICRS; Arias *et al.* 1995). When constructing TGAS it was found that the Hipparcos Reference Frame (HRF) rotates with respect to ICRS at a rate of 0.24 ± 0.03 mas yr^{-1}. This is still well within the stated uncertainty of the HRF, which is ± 0.25 mas yr^{-1} per axis (Kovalevsky *et al.* 1997).

If proper motions in the HRF (e.g. from the Hipparcos catalogue) are to be compared with TGAS (in the reference frame of Gaia DR1), they should first be corrected for the

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† `http://gea.esac.esa.int/archive/`

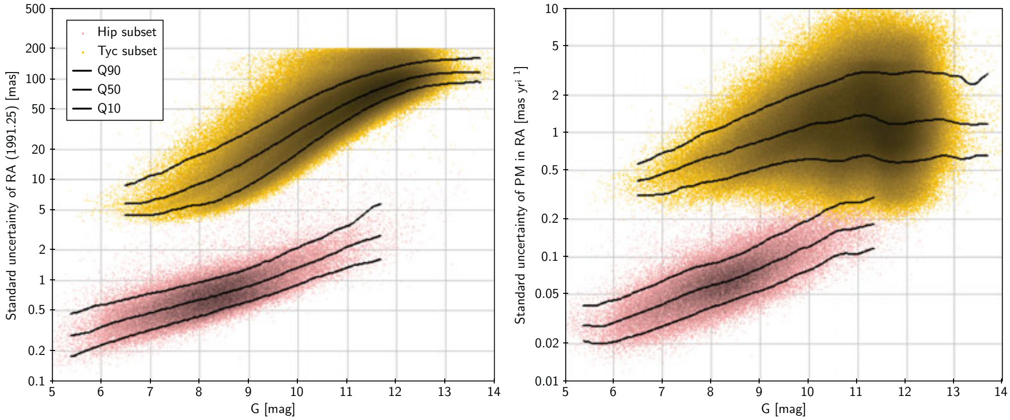

Figure 1. *Left:* Positional uncertainties at epochs around J1991.25 in the Hipparcos and Tycho-2 catalogues. These positions are used as priors in the TGAS solution. *Right*: Uncertainties of the proper motions in the TGAS catalogue. In both diagrams the lower cloud shows the ∼93 000 Hipparcos sources in TGAS, the upper cloud the ∼2 million Tycho-2 sources. The curves are quantiles at 10%, 50% (median), and 90%. Only the components in right ascension are shown.

rotation of the HRF using the expressions (in mas yr^{-1}):

$$\left.\begin{aligned}\mu_{\alpha*}^{\mathrm{GRF1}} &= \mu_{\alpha*}^{\mathrm{HRF}} - 0.126\sin\delta\cos\alpha + 0.185\sin\delta\sin\alpha - 0.076\cos\delta\\ \mu_{\delta}^{\mathrm{GRF1}} &= \mu_{\delta}^{\mathrm{HRF}} + 0.126\sin\alpha + 0.185\cos\alpha\end{aligned}\right\} \tag{4.1}$$

The 1 billion non-TGAS sources in Gaia DR1 provide a dense optical reference frame all the way down to 20th magnitude, which is on the same positional system (ICRS) as TGAS. It is accurate to within a few mas at the reference epoch J2015.0 but quickly deteriorates when moving away from this epoch due to the unspecified proper motions.

5. Astrometric uncertainties

The standard uncertainties of the five astrometric parameters are the main indicators of the quality of the solution for a TGAS source. In the Gaia Archive they are given in the fields `ra_error`, `dec_error`, `parallax_error` (in mas) and `pmra_error`, `pmdec_error` (in mas yr^{-1}). `ra_error` is $\sigma_{\alpha*} = \sigma_\alpha \cos\delta$ and `pmra_error` is $\sigma_{\mu\alpha*} = \sigma_{\mu\alpha}\cos\delta$. They should be interpreted as estimated standard deviations of the actual total (random plus systematic) errors, remembering that actual error distributions may be non-Gaussian with an excess of errors beyond several standard deviations. Only the uncertainties in parallax and proper motion are discussed here.

5.1. *Proper motions*

As expected, the uncertainties of the TGAS proper motions strongly depend on the precision of the old (∼1991) positions used as priors, as shown in the right panel of Fig. 1. Superficially the diagram mirrors the prior uncertainties in the left panel, taking into account the epoch difference of ∼24 years, but the range of uncertainties is actually a lot smaller for the proper motions (a factor ∼250×) than it is for the prior positions (∼1000×). This is discussed in Sect. 6.

5.2. *Parallaxes*

By contrast, as shown in Fig. 2, the uncertainties in parallax are not drastically different between the Hip and Tyc subsets. The overall median uncertainty is 0.28 mas for Hip

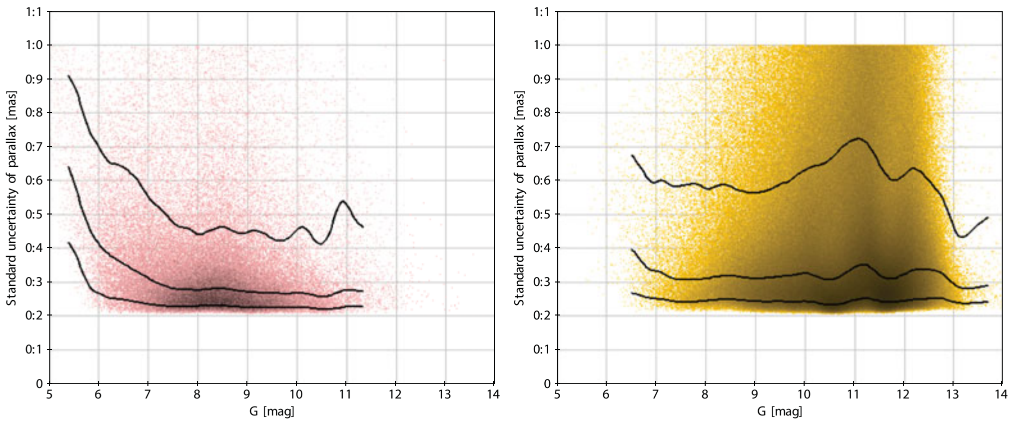

Figure 2. *Left:* Parallax uncertainties versus magnitude for the Hip subset of TGAS. *Right*: Same for the Tyc subset. The curves are quantiles at 10%, 50% (median), and 90%.

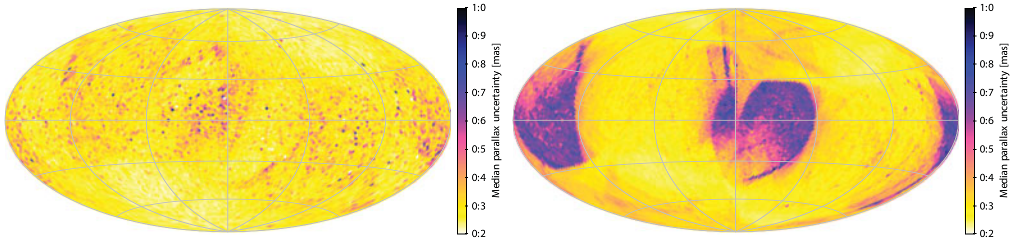

Figure 3. *Left:* Parallax uncertainty versus position for the Hip subset of TGAS. *Right*: Same for the Tyc subset. The maps use an equatorial projection with $\alpha = \delta = 0$ at the centre and α increasing from right to left. Median uncertainties are shown in cells of about 3.36 deg^2.

and 0.32 mas for Tyc, with relatively little variation with magnitude. One can conclude that the parallaxes in TGAS on the whole do not depend strongly on the prior information. Their uncertainties are dominated by errors in the Gaia data – mainly unmodelled calibration errors such as chromaticity (at all magnitudes) and saturation effects (for $G < 7$), both of which were not yet taken into account in the processing for Gaia DR1.

Figure 3 shows the median parallax uncertainty as a function of position for the Hip and Tyc subsets. The similarity in parallax uncertainty between the subsets does not extend to the whole celestial sphere. In the Tyc subset there are two big areas, each covering about 4000 deg^2, where the typical uncertainties are roughly twice as large as in comparable areas along the ecliptic. These areas do not generally have much fewer Gaia observations than other areas along the ecliptic; it is just that the particular temporal and directional distributions of the scans in these areas happen to be less favourable for disentangling the parallax from the proper motion. In the Hip subset this is not a problem because the proper motions are sufficiently well constrained by the priors.

6. Correlations

Correlations are an integral and essential part of the statistical description of the errors in an astrometric catalogue. Under the assumption of unbiased errors the covariance of any two parameters X and Y is $\text{Cov}(X, Y) = \text{E}(e_X e_Y) = \rho(X, Y)\sigma_X \sigma_Y$, where e_X, e_Y are the (unknown) errors in X and Y (deviations from the true values), σ_X, σ_Y their uncertainties, and $\rho(X, Y)$ their correlation, with $|\rho(X, Y)| < 1$.

Correlations matter whenever a calculation involves more than one astrometric parameter. To take a simple example, consider the calculation of the transverse velocity (in km s^{-1}) using the formula $v_{\rm T} = 4.7405\mu/\varpi$, where μ is the proper motion (in mas $\rm yr^{-1}$) along α or δ, and ϖ the parallax (in mas). In the limit of high signal-to-noise ratios ($\varpi/\sigma_\varpi \gg 1$, $|\mu/\sigma_\mu| \gg 1$), classical error propagation gives

$$\left(\frac{\sigma_{v{\rm T}}}{v_{\rm T}}\right)^2 = \left(\frac{\sigma_\mu}{\mu}\right)^2 + \left(\frac{\sigma_\varpi}{\varpi}\right)^2 - 2\rho(\varpi,\mu)\left(\frac{\sigma_\mu}{\mu}\right)\left(\frac{\sigma_\varpi}{\varpi}\right). \tag{6.1}$$

Neglecting the last term, proportional to the correlation between ϖ and μ, could lead to $\sigma_{v{\rm T}}$ being either under- or over-estimated depending on the sign of $\rho(\varpi,\mu)$. More sophisticated tools to estimate astrophysically interesting quantities, e.g. using chi-square minimisation, maximum-likelihood estimation, or Bayesian methods, involve metrics like

$$\chi^2 = \left[\begin{array}{ccc} \Delta\varpi & \Delta\mu_{\alpha*} & \Delta\mu_\delta \end{array}\right] \mathbf{C}^{-1} \left[\begin{array}{c} \Delta\varpi \\ \Delta\mu_{\alpha*} \\ \Delta\mu_\delta \end{array}\right], \tag{6.2}$$

where the Δ-quantities are model residuals in the astrometric data, and $\mathbf{C}$ the covariance matrix of the astrometric parameters. Ignoring correlations, by using only the main diagonal in $\mathbf{C}$, could lead to biased estimates and under- or over-estimated uncertainties.

In an astrometric catalogue there are two kinds of correlations to worry about: between-source correlations and within-source correlations. *Between-source correlations* are important when combining astrometric data from several sources. Typical examples include the estimation of integral properties of a group of stars, such as the mean distance, radial extent, bulk velocity, or internal velocity dispersion of a stellar cluster. In the absence of between-source correlations, the mean parallax or mean proper motion of a cluster would improve as $\sigma/\sqrt{n}$, if σ is the uncertainty for individual member stars and n the number of such stars. The parallaxes or proper motions in a small area are however expected to have some degree of positive between-source correlation, due to shared errors from the attitude and instrument calibration. The uncertainty of the mean then decreases less rapidly with n, approaching a floor $\simeq \sigma\sqrt{\langle\rho\rangle}$ for large n, where $\langle\rho\rangle > 0$ is the average correlation between all pairs of cluster stars. In TGAS positive between-source correlations are expected on spatial scales up to a few tens of degrees. They are however very difficult to estimate, and are not quantified in the Gaia Archive for Gaia DR1. For all practical purposes they look just like the systematic errors discussed in Sect. 7.

Within-source correlations are the correlations between the different astrometric parameters of the same source, such as between μ and ϖ in Eq. (6.1). In TGAS there are always exactly five astrometric parameters per source, and consequently ten within-source correlations per source. In the Gaia Archive they are given in the fields `ra_dec_corr` for $\rho(\alpha,\delta)$, etc. Maps of their median values are shown in Fig. 7 of Lindegren *et al.* (2016). See also Sect. 8.1 of the Gaia DR1 validation paper (Arenou *et al.* 2017).

The complete 5×5 covariance matrix for the astrometric parameters of a TGAS source can be computed from the five uncertainties and ten correlations given in the Gaia Archive. This matrix is needed in some applications, for example to propagate the uncertainties to an epoch different from the TGAS reference epoch J2015.0, or to a different reference system such as galactic coordinates; see Sects. 4.3.2 and 3.1.7 of the Gaia DR1 on-line documentation.†

Users who care about the errors in ϖ, $\mu_{\alpha*}$, and μ_δ should also be concerned about the three correlations $\rho(\varpi,\mu_{\alpha*})$, $\rho(\varpi,\mu_\delta)$, and $\rho(\mu_{\alpha*},\mu_\delta)$. For TGAS as a whole, roughly

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† `https://gaia.esac.esa.int/documentation/GDR1/`

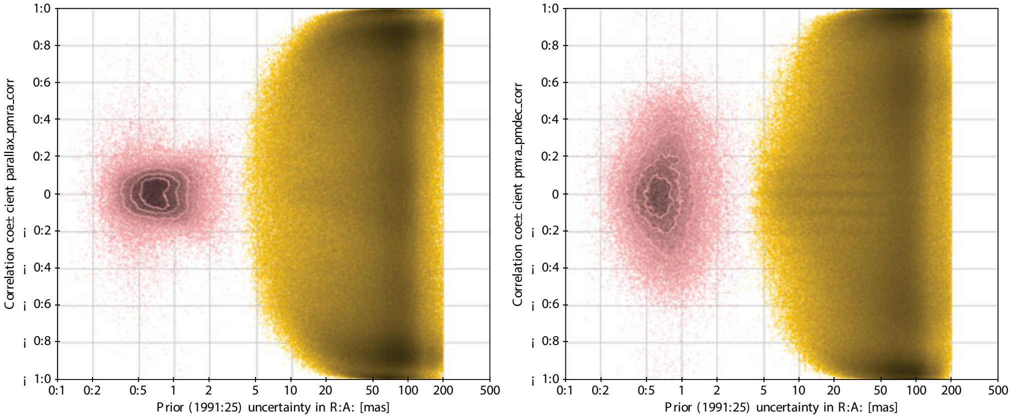

Figure 4. *Left:* Correlation coefficient $\rho(\varpi, \mu_{\alpha *})$ plotted against the positional prior in α at the Hipparcos/Tycho-2 epoch. *Right*: Same for $\rho(\mu_{\alpha *}, \mu_\delta)$. In both panels the (pink) cloud at prior $\lesssim 3$ mas is the Hip subset, the bigger (orange) cloud is the Tyc subset.

half of them exceed 0.5 in absolute value, and many are quite close to ±1. For the Hip subset, however, only a few per cent exceed 0.5 in absolute value. This difference between the Hip and Tyc subsets is clearly related to the strength of the prior. In Fig. 4 two of the correlation coefficients are plotted against the prior uncertainty at 1991.25 (a plot of $\rho(\varpi, \mu_\delta)$ looks similar). The correlations are moderate for sources with a strong prior (positional prior uncertainty < 2 mas), but start to bifurcate towards larger positive and negative correlations for Hip sources with a weaker prior. The progression towards larger (positive or negative) correlations with increasing prior uncertainty continues in the Tyc subset, but the paucity of sources with intermediate prior uncertainty, around 3 mas, still makes it relevant to think about Hip and Tyc as distinct subsets.

The interpretation of Fig. 4 is straightforward. Recalling (Sect. 2) that the purpose of the prior was to decorrelate parallax from proper motion, we must conclude that this was not fully achieved for most Tyc sources due to a too weak prior position in Tycho-2. Effectively, the TGAS results for many sources in the Tyc subset are almost entirely based on the 14 months of Gaia observations covered by DR1. This also explains the reduced range of proper motion uncertainties seen in the right panel of Fig. 1.

7. Systematic errors

A systematic error (bias) may be understood as the part of an observational error that cannot be eliminated by averaging many measurements. This definition is not very useful in the present context, where there is just one “measurement” of each quantity, namely the particular value given in the catalogue. It is more useful to regard the bias as an (essentially) unknown function of several “circumstantial variables”, such as the position, magnitude, and colour of the source. For example, if $e_i = \varpi_i^{\rm TGAS} - \varpi_i^{\rm true}$ is the (unknown) error of the parallax of source i, we imagine that it consists of two parts,

$$e_i = r_i + s(\alpha_i, \delta_i, G_i, C_i, p_i), \tag{7.1}$$

where r_i is the random error and s the systematic error as function of the circumstantial variables. The random errors are unbiased and uncorrelated between the sources.

This error model implies that several sources having similar circumstantial variables also have similar bias s. Averaging many such sources will not reduce the error below s. This behaviour is analogous to the effect of between-source correlations discussed in

Sect. 6. In practice there is no way to separate the between-source correlations from the systematic errors, and the discussion below applies to both.

It is useful to consider which circumstantial variables are believed to be most relevant for the systematics in TGAS, and why. The variables in Eq. (7.1) are as follows.

Position (α_i, δ_i) is important because the scanning geometry and temporal distribution of the scans across a source depend on its approximate position. Similar calibration errors affect the calculated astrometric parameters differently depending on these factors. The scanning law (Gaia Collaboration, Prusti, *et al.* 2016) imprints a complex pattern on the celestial sphere, where the number and distribution of observations may vary discontinuously on angular scales of a few arcmin. This spatial complexity can be seen in Fig. 3 and, on smaller scales, in high-resolution images of the full Gaia DR1 source density (e.g. Fig. 2 in Gaia Collaboration, Brown, *et al.* 2016). Even within apparently "smooth" areas of the sky the bias could vary significantly since the sources will have been observed in different parts of the field of view with different calibration errors.

Magnitude G_i: sources brighter than $G \simeq 12$ obtain shorter CCD integration times, to avoid detector saturation, through the use TDI gates (Gaia Collaboration, Prusti, *et al.* 2016). Each gate effectively means a different geometric instrument, requiring a dedicated geometric calibration with its own calibration errors. This results in different s as a function of G_i even for sources at practically the same position.

Colour C_i: the image centroiding in Gaia DR1 and the subsequent TGAS solution did not take into account that sources have different colours. The combination of wavelength-dependent diffraction and unavoidable optical aberrations makes the image centroid position sensitive to the colour. The colour-coefficient of the shift depends both on the time of the observation and on its location in the field of view. This so-called chromaticity can be calibrated, and thus eliminated, using stars of known colour index. In TGAS this was not yet possible, but it will be done for future data releases. The colour-dependent shifts of the individual CCD observations of a given source combine to create astrometric biases that depend not only on the colour of the source but also on the detailed scanning geometry and the times of the observations.

Prior p_i: in Eq. (7.1) this represents the strength of the prior position taken from the Hipparcos or Tycho-2 catalogue. As shown in Sect. 6 the TGAS astrometry for a source with a weak prior is almost unconstrained by the old position, and it is also expected to be more susceptible to systematic errors from the incomplete instrument calibration. This concerns mainly the Tyc subset and especially the faint part of it. The dependence on p_i will again be a complicated function of the detailed scanning geometry, etc.

On the other hand, the systematic errors are *not* expected to depend on the actual (true) values of the astrometric parameters at the sub-arcsec level. For example, while the systematic error in parallax may depend on the (approximate) position and magnitude of the source, it should not depend on the actual parallax, i.e. on the barycentric distance to the source, other factors being equal. The reason for this is the linearity of the astrometric model, where both the fitted astrometric parameters and their errors are linear combinations of the corresponding centroid shifts on the detectors.

From the discussion above it should be clear that it is very difficult to characterise the function s except in the broadest terms. For a limited number of sources some conclusions can be drawn from comparisons with external data. In their independent validation of the Gaia DR1 catalogue, Arenou *et al.* (2017) confirm the overall quality of the data but also find evidence for a global parallax zero point error of about -0.04 mas, as well as spatially varying offsets on the level of one or a few tenths of a mas.

As described in Appendix E of Lindegren *et al.* (2016), special validation solutions were computed by the Gaia astrometry team as part of their internal assessment of TGAS.

The differences between these solutions and the published catalogue give an idea about the likely sizes and spatial correlations of the systematic errors. In one experiment the input data were partitioned in two halves according to which part of the astrometric field of view was used. This is probably the closest one can get to having two independent Gaia instruments, as the resulting two solutions are expected to have very different $s(\alpha_i, \delta_i, G_i, C_i, p_i)$. A map of the median parallax differences at a resolution of a few degrees shows large-scale patterns with a standard deviation of about 0.1 mas and exceeding ±0.3 mas in less than 1% of the sky. Other validation solutions give similar results, and these numbers are our best guess at the overall statistics of s at an angular resolution of a few degrees and averaged over the other circumstantial variables.

The parallax uncertainties in TGAS refer to the total error e_i, i.e. the σ_ϖ given in the Gaia Archive is our best estimate of the standard deviation of e_i. This follows from the way the TGAS parallax uncertainties were calibrated (and inflated) through a comparison with the independent Hipparcos catalogue (Appendix B in Lindegren *et al.* 2016).

Gaia Collaboration, Brown, *et al.* (2016) recommend that uncertainties on the parallaxes are quoted as $\varpi \pm \sigma_\varpi$ (random) $\pm$ 0.3 mas (systematic). In view of the previous discussion, this does not mean that the total (random + systematic) uncertainty is $\sqrt{\sigma_\varpi^2 + 0.3^2}$ mas. Since σ_ϖ already represents the total uncertainty, that would clearly be an overestimation. Furthermore, the ±0.3 mas (systematic) should not be interpreted as an RMS value, but rather as an amplitude of what can reasonably be expected. The user should however always be aware of the possibility of even larger systematics in some areas, for the faint part of the Tyc subset, or sources with very blue or red colours.

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