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# ENDOGENOUS MARKET STRUCTURE, OCCUPATIONAL CHOICE, AND GROWTH CYCLES

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We model an industry that supplies intermediate goods in a growing economy. Agents can choose whether to provide labor or to become firm owners and compete in the industry. The idea that entry is determined through occupational choice has major implications for the economy's dynamics. Particularly, the results show that economic dynamics are governed by endogenous volatility in the determination of both the number of industry entrants and in the growth rate of output. Consequently, we argue that occupational choice and the structural characteristics of the endogenous market structure can act as both the impulse source and the propagation mechanism of economic fluctuations.

**Keywords:** Overlapping Generations, Endogenous Cycles, Market Entry, Industry Dynamics

## 1. INTRODUCTION

What are the fundamental causes behind economic fluctuations? The efforts to address this question have always been at the forefront of research in macroeconomics. Contrary to more conventional approaches that view exogenous (demand and/or supply) shocks as the initial impulse sources behind fluctuations in major economic variables, there is another strand of literature arguing that there is no reason to restrict attention to such exogenous processes as the generating causes of economic volatility.[1] Instead, its impulse source may be embedded in the deep structural characteristics that shape the economy's dynamics and may lead economic variables to display fluctuations, either through damped oscillations; or through periodic orbits of a more permanent nature; or through stochastic economic fluctuations generated by purely extrinsic uncertainty (i.e., sunspots) rather

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than shocks to such fundamentals as preferences or technologies. Analyses in this strand of the literature include the papers by Benhabib and Nishimura (1985), Grandmont (1985), Azariadis and Guesnerie (1986), Reichlin (1986), Azariadis and Smith (1996), Grandmont et al. (1998), Matsuyama (1999), Banerji et al. (2004), and Kaas and Zink (2007), among others. Our paper seeks to contribute to this strand of the literature by offering a theory that complements the existing ones in enriching our current understanding of the extent to which endogenous forces can be propagated and can manifest themselves in economic cycles.

We are motivated by an emerging literature of research papers that incorporate both endogenous entry and strategic interactions among firms as a means of enriching the potential propagation mechanisms in full-fledged dynamic general equilibrium models that include exogenous shocks.[2] The papers by Ghironi and Melitz (2005), Colciago and Etro (2010), Etro and Colciago (2010), and Bilbiie et al. (2012) analyze and discuss the ability of such frameworks to capture stylized facts of key economic variables over the cycle. We also incorporate an endogenous market structure, taking the form of an industry whose firms produce and supply intermediate goods in our dynamic model. Rather than analyzing how this structure can propagate the initial impact of an exogenous shock, however, we argue that the structural characteristics that determine the equilibrium dynamics of the industry act as both the impulse source and the propagation mechanism that generates fluctuations in output growth.[3] In this respect, our analysis is conceptually close to the work undertaken by Chatterjee et al. (1993) and Dos Santos Ferreira and Lloyd-Braga (2005), who find that the dynamic equilibrium can converge to endogenous cycles in models with imperfect competition and endogenous entry. In Chatterjee et al. (1993), a demand externality generates strategic complementarities in the decisions regarding market entry among different producers. As a result, the incentive of a potential entrant to create a firm and compete in the market is actually increasing in the existing number of firms, thus leading to multiple equilibria, as well as sunspots that are represented by a two-state Markov process. Dos Santos Ferreira and Lloyd-Braga (2005) build a model where households/workers and potential entrepreneurs are distinct in the sense that they are born with predetermined abilities that deter them from choosing a different occupation, apart from the one that nature dictates to the group that they belong to. The households' labor supply is elastic, whereas entrepreneurs face a fixed (exogenous) cost of entry to compete in the market of intermediate goods. The combination of these two factors affects the determination of the price markup, whereas the latter impinges on the dynamics of capital accumulation. These dynamics generate indeterminacy, stochastic sunspot equilibria, and deterministic cycles through Hopf bifurcations, even under circumstances that would rule out the existence such fluctuations in a perfectly competitive environment.

Similarly to these latter analyses, our model makes an explicit distinction between the different stages of an agent's lifetime, made possible by the OLG setting that we employ. The reason that the equilibrium number of competitors in the industry varies over time is radically different, however. In particular, the dynamics of the industry in our paper rests on the following structural characteristics. First and foremost, the number of agents who choose to become intermediate good producers and join the industry, rather than becoming workers in the final goods sector, is determined through an occupational choice process. In other words, agents are not born into distinct groups whose occupational possibilities are restricted. This is a significant departure whose empirical relevance is evident from arguments such as the one by Yu et al. (2009), who argue that "the decision to open a business reflects lifetime comparisons of anticipated earnings from self-employment with wage or salaried employment" [Yu et al. (2009, p. 3)]. In terms of our model, occupational choice replaces the more familiar zero-profit condition with a condition according to which agents compare the utility associated with particular choices of occupation. Consequently, the implied (utility) cost of market entry is not fixed; instead, it varies with both the preexisting and the anticipated future number of competitive firms—an outcome that has significant implications for the economy's dynamics, as will transpire in the main part of our paper. Second, contrary to labor, intermediate good production requires some specific training that delays the agent's entrance into the industry for the latter stage of her lifetime, thus allowing the number of market entrants to be effectively a state variable.

The combination of these characteristics in an OLG setting introduces rich dynamics with regards to the industry's structure. Particularly, the industry displays *endogenous* volatility; that is, fluctuations in market entry are not governed by the presence of exogenous shocks. Instead, they are manifested in either damped oscillations, sunspot equilibria, or limit cycles. Damped oscillations and sunspots occur when the steady state equilibrium is locally stable (a sink) but also locally indeterminate; limit cycles occur when the conditions for stability are not satisfied (the equilibrium is a saddlepoint) and the dynamics can display flip—or period-doubling—bifurcations. These cyclical trajectories rest on the strong nonmonotonicities that pervade the dynamics of the industry. Despite the fact that technological progress is exogenous and firms do not contribute to any productivity-enhancing R&D, these fluctuations generate endogenous cycles in the growth rate of output. These growth cycles are solely associated with the cyclical nature of entry and the corresponding variations in output that result from both the number of intermediate goods and the amount of labor. Again, growth cycles manifest themselves either through damped oscillations, sunspots, or periodic orbits, depending on the corresponding dynamics for the intermediate goods industry—dynamics to which we alluded earlier.

Our premise that occupational choice is a significant element in the emergence of endogenously driven cycles in market entry is not a trivial issue. Realistically, people are not born with predetermined career paths; instead, their choice of occupation is one of the most important economic decisions that they make over their lifetime—a decision that, as we argued previously, researchers view as fundamental in the process of business formation. Our argument is that the elements that affect this choice are important in generating fluctuations in entry and consequently economic activity. Indeed, if we remove occupational choice from our framework and assume instead that agents are born with predetermined employment possibilities, then endogenous fluctuations will be automatically ruled out. In terms of evidence, Koellinger and Thurik (2012) provide empirical support for this argument. They measure entrepreneurial activity as the share of self-employed individuals and owners/managers of businesses in the total labor force—suggestive of a choice between entrepreneurial activity and paid labor—and find evidence that fluctuations in entrepreneurship appear to cause fluctuations in GDP per capita. Further empirical support for the link between occupational choice and economic fluctuations is discussed in Carrasco (1999), who finds evidence that entry into self-employment is procyclical. This is another characteristic of our model, because the different age structures of the individuals engaged in alternative occupations allow a positive contemporaneous relation between output, the number of intermediate good firms (the "self-employed" of our model), and the amount of labor, despite the trade-off associated with occupational choice. Procyclical entry is also a characteristic in Chatterjee et al. (1993) and Dos Santos Ferreira and Lloyd-Braga (2005), but the mechanisms and implications of our paper are much different.

At this point, we should emphasize that other papers have also identified the importance of occupational choice for economic outcomes. For example, Banerjee and Newman (1993) analyze its significance for the dynamics of income inequality and economic development. Rampini (2004) builds a model where different occupational opportunities vary in terms of both risk and return and calibrates it in order to analyze the cyclical characteristics of entrepreneurial activities. Given the implications for indeterminacy, one of our results echoes the main implications of the analysis by Mino et al. (2005). They also use an overlapping-generations (OLG) setting to show that occupational choice can be responsible for dynamic indeterminacy. However, there are notable differences between their setting and ours. First, they do not endogenize the number of firms that operate in a particular sector; instead, they assume that both sectors in the economy (producing consumption and investment goods) are perfectly competitive. Second, the occupational choice entails a decision on whether to become a skilled worker or remain unskilled—with both types of labor being imperfect substitutes in production. Therefore, the aim and implications of our paper differ significantly.

All in all, our main message adds to the current understanding that stems from existing theories on endogenous market structures within dynamic general equilibrium setups, as well as existing theories on endogenous volatility. With respect to the former, we show that the endogenous determination of industry dynamics is not only a stronger propagation mechanism; it may also represent the actual impulse source of growth cycles. With respect to the latter, we show that the combination of occupational choice and endogenous market structure can represent yet another important explanatory factor in the emergence of recurrent cycles in economic activity.

Despite the fact that our endeavor is to present a theoretical framework that offers qualitative implications, rather than quantitative ones, it should be noted that our results are not alien to empirical facts. For example, the data seem to support the idea that business cycles are not just short-lived phenomena. On the contrary, existing work [e.g., Comin and Gertler (2006)] has offered evidence showing that cycles are relevant to lower frequencies as well—an outcome that corroborates our model's OLG structure. Furthermore, there is evidence to suggest the existence of medium- and long-term oscillations in industrial activity [e.g., Geroski (1995); Keklik (2003); Baker and Agapiou (2006)] in addition to the more commonly observed short-term movements related to the incidence of business cycles—again, a fact that is in accordance with the main mechanism of our equilibrium results.

Our analysis is organized as follows. In Section 2 we lay down the basic setup of our economy. Section 3 derives the temporary equilibrium, whereas Section 4 analyzes and discusses the dynamic equilibrium and its implications. In Section 5 we conclude.

## 2. THE ECONOMIC ENVIRONMENT

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. We consider an economy composed of a constant population of agents that belong to overlapping generations. Every period, a mass of $n > 1$ agents is born and each of them lives for two periods—youth and old age. During their youth, agents are endowed with a unit of time, which they can devote (inelastically) to one of the two available occupational opportunities. One choice is to be employed by perfectly competitive firms that produce the economy's final good. In this case, they receive the competitive salary $w_t$ for their labor services. Alternatively, they can devote their unit of time to some educational activity that will equip them with the ability to use managerial effort and produce units of a specific variety $j$ of an intermediate good when they are old. Intermediate goods are used by the firms that produce and supply the final good. We assume that, once made, occupational choices are irreversible.

The lifetime utility function of an agent born in period by $t$ is given by

$$u_j^t = \left(c_{t,j}^t\right)^{1-\beta}\left(c_{t+1,j}^t\right)^{\beta} - \psi V\left(e_{t+1,j}\right), \quad \textbf{(1)}$$

where $c_{t,j}^t$ denotes the consumption of final goods during youth, $c_{t+1,j}^t$ denotes the consumption of final goods during old age, and $\beta \in (0, 1)$ is the relative weight attached to the utility accrued from consumption during old age. Furthermore, $e_{t+1,j}$ denotes effort and $V(e_{t+1,j})$ is a continuous function that captures the disutility from effort and satisfies $V(0) = 0$ and $V' > 0$. The parameter $\psi$ is a binomial indicator that takes the value $\psi = 0$ if the agent is a worker and $\psi = 1$ if the agent is an intermediate good producer. As this notation is important for the clarity of the subsequent analysis, it is important to note that the time superscript indicates the period in which the agent is born, whereas the time subscript indicates the period in which an activity actually occurs. The subscript

$j$ will be applicable only to producers of intermediate inputs; thus, it will later be removed from variables that are relevant to workers.

We assume that the final good is the numéraire. The production of this good is undertaken by a large mass (normalized to one) of perfectly competitive firms. These firms combine labor from young agents, denoted $L_t$, and all the available varieties of intermediate goods, each of them denoted $x_{t,j}$, to produce $y_t$ units of output according to

$$y_t = A_t \left[ N_t^{-\frac{1}{\theta-1}} \left( \sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^a L_t^{1-a}, \qquad \textbf{(2)}$$

where $a \in (0, 1)$. The parameter $\theta > 1$ is the elasticity of substitution between different varieties of intermediate goods and $N_t$ gives the number of these different varieties [see Dixit and Stiglitz (1977)].[4] Therefore, the latter variable is the number of entrants operating in the oligopolistic industry at time $t$. The variable $A_t$ denotes total factor productivity, which we assume to grow at a constant rate $g > 0$ every period. Therefore,

$$A_t = (1+g)^t A_0, \qquad \textbf{(3)}$$

where the initial value $A_0 > 0$ is given. Note that, given the timing of events, the initial period's number of intermediate good firms is also exogenously given by $N_0 \in (1, n)$.

The production of intermediate goods takes place under Bertrand competition among producers. Each of them uses her managerial effort and produces units of an intermediate good according to

$$x_{t,j} = \gamma e_{t,j}, \quad \gamma > 0. \qquad \textbf{(4)}$$

With the price of each intermediate good denoted by $p_{t,j}$, the owner's revenue is given by $p_{t,j} x_{t,j}$. As we indicated earlier, the cost associated with the managerial activity is the effort/disutility cost characterized by the function $V(\cdot)$.

The process according to which agents choose their occupation involves comparison of the lifetime utility that corresponds to being either a worker or an intermediate good producer. This problem will be formally solved at a later stage in our analysis. Now we will identify the pattern of optimal consumption choices made by each agent, taking her occupational choice as given.

We shall assume that this is a small open economy in the sense that individuals can save or borrow funds at the fixed interest rate $r > 0$.[5] Furthermore, let us denote the present value of an agent's lifetime income by $i_t$. Given these, we can write her lifetime budget constraint as

$$c_{t,j}^t + \frac{c_{t+1,j}^t}{1+r} = i_t. \qquad \textbf{(5)}$$

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Substituting (5) into (1), we can calculate $\frac{\partial u_j^t}{\partial c_{t+1,j}^t} = 0$ in order to derive

$$c_{t+1,j}^t = \beta(1+r)i_i, \tag{6}$$

i.e., the demand for consumption goods during old age. Combining (6) and (5), we can derive the corresponding demand function for goods consumed during youth. This is given by

$$c_{t,j}^t = (1-\beta)i_t. \tag{7}$$

As expected, an individual's consumption expenditures during each period are proportional to her lifetime income (in present value terms). The relative utility weight $\beta$ is crucial in determining what proportion of lifetime income will be devoted to the satisfaction of consumption needs in either youth or old age.

Now, let us consider each group of agents separately, beginning with the workers for whom the lifetime income corresponds to that accruing from their labor services, i.e., $i_t = w_t$. Given the previous discussion and results, for those young agents whose choice is to provide labor we have $\psi = 0$, $c_t^{t,\text{worker}} = (1-\beta)w_t$ and $c_{t+1}^{t,\text{worker}} = \beta(1+r)w_t$, meaning that each worker saves an amount $s_t = \beta w_t$ during her youth. Therefore, we can use (1) to write the lifetime utility of a worker born in period $t$ as

$$u^{t,\text{worker}} = \zeta w_t, \tag{8}$$

where $\zeta$ is a composite parameter term given by

$$\zeta \equiv (1-\beta)^{(1-\beta)}\beta^{\beta}(1+r)^{\beta}. \tag{9}$$

For intermediate good producers, however, the equilibrium characteristics are different. Particularly, their lifetime income (in present value terms) equals $i_t = \frac{p_{t+1,j}x_{t+1,j}}{1+r}$. Therefore, those who decide to be intermediate good producers have $\psi = 1$, $c_{t,j}^{t,\text{producer}} = (1-\beta)\frac{p_{t+1,j}x_{t+1,j}}{1+r}$, and $c_{t+1,j}^{t,\text{producer}} = \beta p_{t+1,j}x_{t+1,j}$, meaning that each of them borrows the amount $b_{t,j} = (1-\beta)\frac{p_{t+1,j}x_{t+1,j}}{1+r}$ during her youth.[6] Using these results in (1), we get the lifetime utility of a producer born in period $t$ as

$$u_j^{t,\text{producer}} = zp_{t+1,j}x_{t+1,j} - V(e_{t+1,j}), \tag{10}$$

where $z$ is a composite parameter term given by

$$z \equiv (1-\beta)^{(1-\beta)}\beta^{\beta}(1+r)^{\beta-1}. \tag{11}$$

With this discussion we have completed the basic setup of our economy. In the sections that follow we derive the economy's temporary and dynamic equilibrium, with particular emphasis on the dynamics of the intermediate goods industry.

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## 3. TEMPORARY EQUILIBRIUM

For the producers of final goods, profit maximization implies that each input earns its marginal product. In terms of labor income, we have

$$w_t = (1-a)A_t\left[N_t^{-\frac{1}{\theta-1}}\left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right]^a L_t^{-a} = (1-a)\frac{y_t}{L_t}. \qquad \textbf{(12)}$$

For intermediate goods we have

$$p_{t,j} = A_t L_t^{1-a} a N_t^{-\frac{a}{\theta-1}}\left[\left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right]^{a-1}\left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}-1} x_{t,j}^{\frac{\theta-1}{\theta}-1}. \qquad \textbf{(13)}$$

Multiplying both sides of (13) by $x_{t,j}$ and summing over all $j$'s, we get

$$\sum_{j=1}^{N_t} p_{t,j}x_{t,j} = aA_t\left[N_t^{-\frac{1}{\theta-1}}\left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right]^a L_t^{1-a}. \qquad \textbf{(14)}$$

We can combine equations (13) and (14) and exercise some straightforward, but tedious, algebra to derive the demand function for an intermediate good. This is given by

$$x_{t,j} = \left(\frac{p_{t,j}}{P_t}\right)^{-\theta}\frac{X_t}{N_t}, \qquad \textbf{(15)}$$

where

$$X_t = N_t^{-\frac{1}{\theta-1}}\left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}. \qquad \textbf{(16)}$$

Furthermore, with $\sum_{j=1}^{N_t} p_{t,j}x_{t,j} = P_t X_t$, the price index is given by

$$P_t = \left(\frac{1}{N_t}\sum_{j=1}^{N_t} p_{t,j}^{1-\theta}\right)^{\frac{1}{1-\theta}}. \qquad \textbf{(17)}$$

The result in (15) is nothing else than the familiar inverse demand function in models with a constant elasticity of substitution between different varieties of goods [Dixit and Stiglitz (1977)]. In other words, the share of product $j$ in the overall demand for intermediate inputs is inversely related to its relative price. This effect is more pronounced with higher values of $\theta$, i.e., if different varieties are less heterogeneous, and thus more easily substitutable.

Now let us consider the equilibrium in the labor, the financial, and the final goods markets. With respect to the first, the demand for labor by firms ($L_t$) must be equal to the supply of labor by young agents. Recall that in period $t$, out of the total population mass of $n$, some agents will decide to set up firms and produce intermediate goods in period $t+1$. The number of these agents is $N_{t+1}$. Therefore, the labor market equilibrium is

$$L_t = n - N_{t+1}. \tag{18}$$

As for the financial market, let us define aggregate debt (denoted $d_t$) as the difference between borrowing and saving, i.e.,

$$d_t = \sum_{j=1}^{N_{t+1}} b_{t,j} - L_t s_t. \tag{19}$$

Naturally, debt evolves according to

$$d_t = (1+r)d_{t-1} + C_t - y_t, \tag{20}$$

where $C_t$ is the aggregate consumption expenditure in period $t$.[7] Furthermore, from the expressions in (2), (12), and (14), it becomes clear that the constant returns technology implies that

$$y_t = L_t w_t + \sum_{j=1}^{N_t} p_{t,j} x_{t,j}. \tag{21}$$

Combining (19)–(21), we can write equation (20) as

$$\begin{aligned} &\sum_{j=1}^{N_{t+1}} b_{t,j} - L_t s_t = (1+r)\left(\sum_{j=1}^{N_t} b_{t-1,j} - L_{t-1} s_{t-1}\right) + C_t - L_t w_t - \sum_{j=1}^{N_t} p_{t,j} x_{t,j} \\ &\Leftrightarrow \sum_{j=1}^{N_{t+1}} b_{t,j} + L_t(w_t - s_t) + \left[\sum_{j=1}^{N_t} p_{t,j} x_{t,j} - (1+r)\sum_{j=1}^{N_t} b_{t-1,j}\right] \\ &\quad + L_{t-1}(1+r)s_{t-1} = C_t. \end{aligned} \tag{22}$$

We can use previous results to rewrite equation (22) as

$$\sum_{j=1}^{N_{t+1}} c_{t,j}^{t,\text{producer}} + L_t c_t^{t,\text{worker}} + \sum_{j=1}^{N_t} c_{t,j}^{t-1,\text{producer}} + L_{t-1} c_t^{t-1,\text{worker}} = C_t, \tag{23}$$

an expression that corresponds to the equilibrium in the goods market.

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Now, we can use $\sum_{j=1}^{N_t} p_{t,j} x_{t,j} = P_t X_t$, (12), (17), and (21) in (15) to write the demand function for the intermediate good as

$$x_{t,j} = \frac{p_{t,j}^{-\theta}}{\sum\limits_{j=1}^{N_t} p_{t,j}^{1-\theta}} a y_t. \quad \textbf{(24)}$$

The result in (24) is more explicit on the interactions in the pricing decisions made by competing firms. It can be used to solve the utility maximization problem of an agent who produces intermediate goods. To this purpose, it will be useful to specify a functional form for the effort cost component $V(e_{t+1,j})$. For this reason, and to ensure analytical tractability, we specify

$$V(e_{t+1,j}) = m e_{t+1,j}, \quad m > 0. \quad \textbf{(25)}$$

Writing equation (24) in terms of period $t+1$ and substituting it together with (4) and (25) into (10) allows us to write the utility function of the producer $j$ as

$$u_j^{t,\text{producer}} = \left( z p_{t+1,j} - \frac{m}{\gamma} \right) \frac{p_{t+1,j}^{-\theta}}{\sum\limits_{j=1}^{N_{t+1}} p_{t+1,j}^{1-\theta}} a y_{t+1}. \quad \textbf{(26)}$$

Given that firm owners operate under Bertrand competition, their objective is to choose the price of their products in order to maximize their lifetime utility. In other words, their objective is

$$\max_{p_{t+1,j}} \left\{ \left( z p_{t+1,j} - \frac{m}{\gamma} \right) \frac{p_{t+1,j}^{-\theta}}{\sum\limits_{j=1}^{N_{t+1}} p_{t+1,j}^{1-\theta}} a y_{t+1} \right\}. \quad \textbf{(27)}$$

After some straightforward algebra, it can be shown that the solution to this problem leads to a symmetric equilibrium for which

$$p_{t+1,j} = p_{t+1} \quad \text{and} \quad x_{t+1,j} = x_{t+1} \forall j, \quad \textbf{(28)}$$

where the optimal price equals

$$p_{t+1} = \frac{m}{\gamma z} \frac{[\theta(N_{t+1} - 1) + 1]}{(\theta - 1)(N_{t+1} - 1)}. \quad \textbf{(29)}$$

In addition, given (24) and (28), the equilibrium quantity of the intermediate good produced by each entrepreneur is

$$x_{t+1} = \frac{a y_{t+1}}{N_{t+1}} \frac{\gamma z}{m} \frac{(\theta - 1)(N_{t+1} - 1)}{[\theta(N_{t+1} - 1) + 1]}. \quad \textbf{(30)}$$

The result in equation (29) resembles the familiar condition according to which the price is set as a markup over the marginal cost of production. In this case, each producer sets a markup over the marginal utility cost of producing the intermediate good, because one unit of production requires a utility cost of $m/\gamma$ units of effort. Naturally, the markup is decreasing in the number of producers, because an increase in the latter implies a more intensely competitive environment. Additionally, the markup is also decreasing in $\theta$ because higher values of this parameter increase the degree of substitutability between different varieties of intermediate goods—yet another structural characteristic that enhances the degree of competition. From equation (30), we can see that the inverse demand function implies that the components that reduce the relative price of the input increase its share of aggregate demand.

The preceding solutions allow us to rewrite the utility of an intermediate good producer, after substituting (28) and (29) into (26), as follows:

$$u^{t,\text{producer}} = \frac{zay_{t+1}}{\theta(N_{t+1}-1)+1}. \tag{31}$$

With this result at hand, we can now turn our attention to the occupational choice problem of an agent who is young in period $t$.

Our purpose is to determine how many agents will decide to become suppliers of intermediate inputs. Obviously, the equilibrium condition requires that an agent born in $t$ should be indifferent between the two different occupational opportunities. Formally, a condition that needs to hold in equilibrium is

$$u^{t,\text{producer}} = u^{t,\text{worker}}, \tag{32}$$

or, after (8), (12), and (31) are utilized,

$$\frac{zay_{t+1}}{\theta(N_{t+1}-1)+1} = \frac{\zeta(1-a)y_t}{L_t}. \tag{33}$$

We can manipulate the expression in (33) algebraically even further. First, we can use the symmetry condition of (28) in (2) to get

$$y_t = A_t(N_t x_t)^a L_t^{1-a}. \tag{34}$$

Next, we can use (30) to get

$$N_{t+1}x_{t+1} = ay_{t+1}\frac{\gamma z}{m}\frac{(\theta-1)(N_{t+1}-1)}{[\theta(N_{t+1}-1)+1]} \Leftrightarrow N_t x_t = ay_t\frac{\gamma z}{m}\frac{(\theta-1)(N_t-1)}{[\theta(N_t-1)+1]}. \tag{35}$$

Further substitution of (35) into (34) allows us to write

$$y_t = A_t^{\frac{1}{1-a}}\left\{\frac{a\gamma z}{m}\frac{(\theta-1)(N_t-1)}{[\theta(N_t-1)+1]}\right\}^{\frac{a}{1-a}} L_t. \tag{36}$$

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Finally, we can use (18) and (36) in (33), and rearrange to get

$$\frac{n - N_{t+2}^E}{\theta(N_{t+1} - 1) + 1} = \frac{(1-a)(1+r)}{a(1+g)^{1/(1-a)}} \left\{ \frac{[\theta(N_{t+1} - 1) + 1]}{(N_{t+1} - 1)} \frac{(N_t - 1)}{[\theta(N_t - 1) + 1]} \right\}^{a/(1-a)}, \qquad (\mathbf{37})$$

where $1 + g = A_{t+1}/A_t$ is derived by referring to (3). Note that the superscript in $N_{t+2}^E$ denotes the expectation formed on this variable.

The result in equation (37) is the most important in our setup. It implies that the determination of the equilibrium number of firms in the intermediate goods industry is not a static one. Instead, there will be some transitional dynamics as the number of producers converges to its long-run equilibrium. Particularly, we can see that the equilibrium number of firms in any given period depends on both the predetermined number of firms from the previous period and the expectation on the number of firms that will be active during the next period. Note that the endogenous occupational choice is critical for these dynamics. It is exactly because of this choice that the determination of $N_{t+1}$ is related to the previous period's demand conditions (and thus $N_t$) and the next period's labor market equilibrium (therefore $N_{t+2}^E$).

The intuition for these effects is as follows. If the existing number of intermediate good firms is large, then the overall amount of intermediate goods, and therefore the marginal product of labor, will be higher. This increases the equilibrium wage and thus the relative benefit from the utility of being a worker when young, rather than setting up a firm when old.[8] Now suppose that, while forming their occupational choice, the current young expect that the future number of firms in the intermediate goods industry will be high. For them, this implies that the amount of labor, and therefore total demand in the next period will be relatively low. Thus, the relative utility benefit of being a firm owner when old, rather than a worker when young, is reduced because the expectation of lower future demand for final goods will have corresponding repercussions in terms of reduced future demand for intermediate goods as well. Consequently, a reduced number of individuals out of the current young will opt for the choice of becoming intermediate good producers.

## 4. DYNAMIC EQUILIBRIUM

The remainder of our analysis will focus on the dynamics of the industry that produces intermediate goods. In what follows, we consider the economy's perfect foresight dynamics, i.e., equilibrium trajectories that satisfy $N_{t+2}^E = N_{t+2}$.

### 4.1. The Steady State

We can obtain the stationary equilibrium for the number of intermediate good firms after substituting $N_{t+2}^E = N_{t+2}$ into (37) and using the steady state condition $N_{t+2} = N_{t+1} = N_t = \hat{N}$. This procedure will eventually allow us to derive

PROPOSITION 1. *Suppose* $n > 1+\delta$, *where* $\delta = (1-a)(1+r)/a(1+g)^{1/(1-a)}$. *Then there exists a unique steady state equilibrium* $\hat{N} \in (1, n)$ *such that*

$$\hat{N} = \frac{n + (\theta - 1)(1 - a)(1 + r)/a(1 + g)^{1/(1-a)}}{1 + \theta(1 - a)(1 + r)/a(1 + g)^{1/(1-a)}}. \tag{38}$$

As long as the steady state solution is asymptotically stable, for any predetermined $N_0 \in (1, n)$ the equilibrium number of producers will eventually converge to $\hat{N}$ in the long run. Later, we are going to formally characterize the conditions for the (local) stability of this equilibrium. For now, it is instructive to undertake some comparative statics to identify the effects of the economy's structural parameters on the steady state number of firms competing in the intermediate goods industry. This is a task that can be easily undertaken through the use of equation (38). The results can be summarized in

PROPOSITION 2. *The long-run equilibrium number of firms in the intermediate goods industry is*

*i. increasing in the growth rate of total factor productivity* $(g)$*;*
*ii. decreasing in the relative share of labor income* $(1 - a)$*, the world interest rate* $(r)$*, and the degree of substitutability between different varieties of intermediate products* $(\theta)$.

The economic interpretation for these results is as follows. A permanent increase in the growth rate causes future demand to become even higher compared to current demand because of the increase in the economy's resources. This effect boosts the relative utility benefit of becoming a firm owner, with corresponding implications for the occupational choices made by young agents. An increase in the relative share of labor income will motivate more agents to work for final goods firms, as the income earned from activities in the intermediate goods industry becomes relatively low. The utility benefit of such activities is also impeded in an industry where goods are less heterogeneous. Finally, when individuals face a higher borrowing rate, then they have a lower incentive to opt for the occupation that renders borrowing necessary in order to finance the first period's consumption needs—in this case, firm ownership.

## 4.2. Transitional Dynamics

Let us use $N_{t+2}^E = N_{t+2}$ in (37) and solve the resulting expression for $N_{t+2}$. Eventually, we get

$$N_{t+2} = n - \frac{(1 - a)(1 + r)}{a(1 + g)^{1/(1-a)}} \frac{[\theta(N_{t+1} - 1) + 1]^{1/(1-a)}}{(N_{t+1} - 1)^{a/(1-a)}} \left[\frac{N_t - 1}{\theta(N_t - 1) + 1}\right]^{a/(1-a)} = F(N_{t+1}, N_t). \tag{39}$$

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As we can see, the dynamics of the intermediate goods industry is characterized by a nonlinear second-order difference equation in terms of the industry's size (i.e., the number of agents who compete in the industry).

One way to analyze the transition equation in (39) is to define $Z_t = N_{t+1}$ and treat the dynamics as being generated by the following system of first-order difference equations:

$$Z_{t+1} = F(Z_t, N_t) = n - \delta \frac{[\theta(Z_t - 1) + 1]^{1/(1-a)}}{(Z_t - 1)^{a/(1-a)}} \left[ \frac{N_t - 1}{\theta(N_t - 1) + 1} \right]^{a/(1-a)}, \quad \textbf{(40)}$$

$$N_{t+1} = H(Z_t, N_t) = Z_t, \quad \textbf{(41)}$$

where $N_0, Z_0 \in (1, n)$ are taken as the initial conditions, the steady state satisfies $\hat{Z} = \hat{N}$, and $\delta$ is defined in Proposition 1. The Jacobian matrix associated with the planar system of equations (40) and (41) is

$$\begin{bmatrix} F_{Z_t}(\hat{Z}, \hat{N}) & F_{N_t}(\hat{Z}, \hat{N}) \\ H_{Z_t}(\hat{Z}, \hat{N}) & H_{N_t}(\hat{Z}, \hat{N}) \end{bmatrix},$$

where $\hat{N} = \hat{Z}$ is given in (38). Furthermore, the eigenvalues are the roots of the polynomial $\lambda^2 - T\lambda + D$, i.e.,

$$\lambda_1 = \frac{T - \sqrt{T^2 - 4D}}{2} \quad \text{and} \quad \lambda_2 = \frac{T + \sqrt{T^2 - 4D}}{2},$$

where

$$T = F_{Z_t}(\hat{Z}, \hat{N}) + H_{N_t}(\hat{Z}, \hat{N})$$

and

$$D = F_{Z_t}(\hat{Z}, \hat{N}) H_{N_t}(\hat{Z}, \hat{N}) - F_{N_t}(\hat{Z}, \hat{N}) H_{Z_t}(\hat{Z}, \hat{N})$$

are, respectively, the trace and the determinant of the matrix. As is well known [Azariadis (1993); Galor (2007)], the eigenvalues can be used to check the stability of the steady state solution and to trace the transitional dynamics toward it. Later, it will transpire that, under different conditions, $\hat{N}$ can be either (locally) stable or unstable. For now, we will focus our attention to a case in which the steady state equilibrium characterized by (38) is actually stable. The possible implications that arise in the scenario where $\hat{N}$ is unstable will be discussed subsequently.

Let us begin by defining $\Xi(\delta) \equiv \delta + \frac{\delta a(1+\delta\theta)}{1-a} \frac{2}{\delta\theta - 1}$ and $\tilde{\delta}$ such that $\Xi(\tilde{\delta}) = n - 1$. Furthermore, the analysis that follows will make use of the following assumptions:

Assumption 1. $n > 1 + 2(1 + \frac{3a}{1-a})$.

Assumption 2. $\delta\theta \leq 2$.

Both assumptions are employed to make the analysis of the transitional dynamics more precise, clear, and sharply focused. First, Assumption 2 is sufficient to guarantee that, at least for some range of parameter values, the steady state equilibrium will be stable. Second, we want to focus solely on the emergence of flip (period-doubling) bifurcations. Assumption 1 is sufficient to guarantee that the eigenvalues of the dynamical system in (40), (41) are real numbers. The proofs of the subsequent results are relegated to the Appendix.

The sufficient conditions for stability are formally described in

LEMMA 1. *If either (i) $\delta\theta \leq 1$ or (ii) $\delta\theta \in (1, 2]$ and $\delta < \tilde{\delta}$, then the steady state solution $\hat{N}$ is locally stable. In other words, the dynamics starting from an initial value $N_0 \in (1, n)$ will eventually converge to $\hat{N}$.*

With respect to output, once the industry converges to its steady state, the production of final goods will converge to a balanced growth path. Along this path, output will grow at a constant rate that is proportional to the growth rate of total factor productivity. It is straightforward to use equations (18) and (36) and Lemma 1 to establish that

$$\lim_{t\to\infty}\left(\frac{y_{t+1}}{y_t}-1\right)=(1+g)^{\frac{1}{1-a}}-1. \tag{42}$$

Nevertheless, during the transition to the balanced growth path, the dynamics of output will also be (partially) dictated by the transitional dynamics of the intermediate goods industry. A technical condition that can facilitate a better understanding of how the intermediate goods industry evolves over time is given by

LEMMA 2. *As long as the conditions for stability that are summarized in Lemma 1 hold, both eigenvalues are negative; i.e., $\lambda_1, \lambda_2 \in (-1, 0)$.*

Using Lemma 2, we can characterize the transitional behavior of the economy through

PROPOSITION 3. *Given Lemma 2, the number of firms in the intermediate goods industry converges to its long-term equilibrium through cycles. Consequently, output growth displays fluctuations as it converges to the balanced growth path.*

Recall that the number of producers in any given period is affected by both the predetermined number of producers from the previous period and the expectation on the number of producers that will be active in the future. The manner and direction of these effects, both discussed at an earlier point of our analysis, render the result of Proposition 3 a quite intuitive one. For example, consider a situation where the existing number of intermediate good producers is low relative to the steady state. For the current young agents, the incentive to opt for market entry when old is enhanced because the marginal product of labor (and therefore the wage) is currently low. As a result, an increased fraction of the current young will choose to become firm owners and compete in the intermediate goods industry when they become old. However, for this to happen, they also need to expect

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that, next period, a smaller fraction of the future generation's agents will decide to become producers, because this will increase labor, and therefore aggregate demand, during the period when producers will reap the benefits of their activity. The mechanism that we described previously does verify this expectation, granting the agents an even greater incentive to set up intermediate good firms. Furthermore, it explains why both the size of the intermediate goods industry and output growth converge to their long-run equilibrium through cycles.

For illustrative purposes, in what follows we will analyze the transition equation in (39) numerically, making sure to choose parameter values that render the solution in (38) stable, and hence a meaningful one. We should emphasize, however, that we undertake these numerical simulations solely as a means of illustrating the transitional behavior of the economy. The focus of our analysis is still purely qualitative; it is neither our intention nor do we claim any attempt to offer a quantitative match of key moments from stylized facts.

For the baseline parameter values, we choose $a = 0.5$, $g = 0.32$, $r = 0.3$, and $\theta = 1.3$, whereas the total population is set to $n = 100$.[9] The initial values are $N_0 = 15$ and $N_1 = 85$—recalling that $N_1$ corresponds to $Z_0$ in (41). In Figure 1 we see the transitional dynamics for the intermediate goods industry, based on this simulation. Given the numerical example, the steady state value for $\hat{N}$ is roughly 51 (50.8764 to be precise) and the industry converges to this equilibrium. Nevertheless, this convergence is clearly nonmonotonic. Instead, convergence takes place through damped oscillations, or cycles, during which the number of firms takes values above and below the stationary value as the industry approaches it. In Figure 2 we use the same baseline parameter values, in addition to $A_0 = 10$, to simulate the movements of the growth rate of output, $\frac{y_{t+1}}{y_t} - 1$. Again, we can see that, because of fluctuations that occupational choice generates in the determination of the number of producers each period, output converges to its balanced growth path (with a growth rate of 0.7424) through cycles. Note, however, that these fluctuations are not due to the fact that the intermediate goods industry is associated with some type of R&D that increases the rate of technological progress endogenously. Instead, they are purely associated with variations in output that result from the cyclical nature of $N_t$ and the corresponding variations in both the number of intermediate goods and the labor input.

## 4.3. Indeterminacy and Sunspot Equilibria

Before we proceed to the analysis of limit cycles, we will discuss the possibility of indeterminacy in the transitional dynamics of the economy. As we have seen from the second-order transition equation (39), or the equivalent dynamical system (40), (41), the transitional dynamics is traced after we consider two initial values $N_0$ and $Z_0$—the latter corresponding to $N_1$. Nevertheless, although $N_0$ is indeed predetermined, this is not the case for $N_1$. Instead, taking the value of $N_0$ as given, $N_1$ reflects an equilibrium formed on an expectation about $N_2$ and so on. In other words, the stability of the steady state equilibrium $\hat{N}$ implies that, for the same

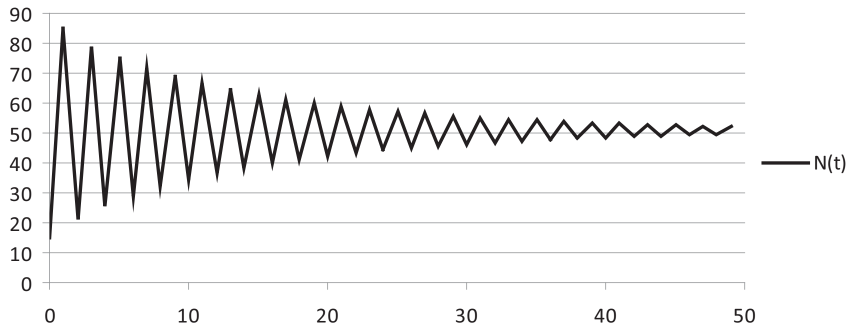


**FIGURE 1.** Damped oscillations in market entry.

$N_0 \in (1, n)$, there is certainly more than one trajectory that is consistent with the economy's convergence to the steady state. In other words, economies that are identical in terms of both structural parameters and predetermined conditions may display very different equilibrium characteristics for a large part of their transition toward the common steady state.

Nevertheless, the aforementioned arguments—indicative of local indeterminacy—have more implications for the evolution of output. Particularly, the self-fulfilling nature of one of the mechanisms that permeate occupational choice implies that, in addition to the stationary equilibrium derived in (38), the economy's dynamics may also be characterized by stochastic cycles due to sunspots. These cycles are not driven by shocks to any of the model's fundamental parameters of preferences or technologies. Instead, they manifest the element of extrinsic uncertainty inherent in the fact that individuals choose their occupation (labor or intermediate goods production) partially based on an expectation about future outcomes. Their choice, however, is itself conducive to the actual realization of

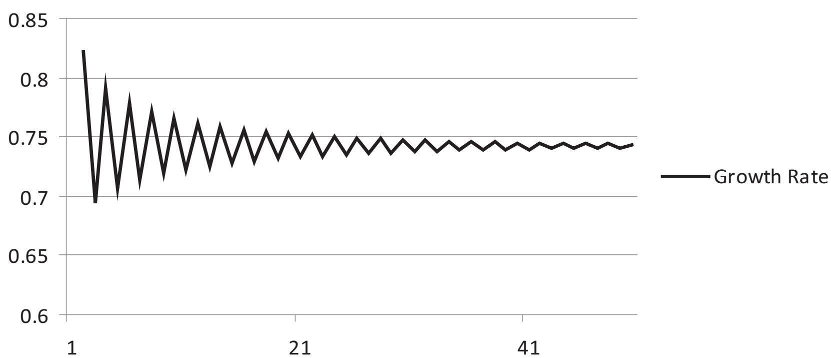


**FIGURE 2.** Damped oscillations in output growth.

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those outcomes. Indeed, previously we indicated that the incentive of agents to earn their lifetime income through intermediate goods production is increased when they expect that market entry will be reduced in the period afterward. Because current market entry is inversely related to the predetermined number of firms, these expectations can become self-fulfilling. If more agents join the industry now, then future market entry will actually be relatively low, thus verifying their initial expectation. We can summarize these implications through

PROPOSITION 4. *As long as the conditions for stability that are summarized in Lemma 1 hold, there exist sunspot equilibria in the neighborhood of the stationary solution* $\hat{N}$.

A formal proof for the previous result appears in Woodford (1986) and Grandmont et al. (1998)—the former for the case of a second-order scalar system with only one predetermined value and the latter for a planar system of two state variables in which only one is predetermined. The existence of sunspots represents one type of permanent economic fluctuations in our model. In the subsequent section, we show that there may also be another type of fluctuations—in that case, not stochastic but deterministic—that occur as a result of period-doubling bifurcations that result in oscillations, similar to the ones identified in Proposition 3, but at the same time rather different in the sense that these oscillations will be permanent.

## 4.4. Periodic Equilibrium

So far, we have seen scenarios in which oscillations in economic variables are not permanent—an outcome related to our restriction on conditions that guarantee the stability of the steady state. Nevertheless, it will also be interesting to examine the possibilities that arise when the steady state in (38) does not satisfy these stability conditions. This may happen in circumstances that are described in

LEMMA 3. *Suppose that* $\delta\theta \in (1, 2]$ *and* $\delta > \tilde{\delta}$. *In this case, the two eigenvalues* $\lambda_1$ *and* $\lambda_2$ *satisfy* $0 > \lambda_2 > -1 > \lambda_1$. *Therefore, the steady state solution* $\hat{N}$ *is a saddlepoint.*

The saddlepoint property of the steady state implies that, for a given $N_0$, there is only one corresponding $N_1 (= Z_0)$ such that the industry dynamics follow a path of convergence toward $\hat{N} (= \hat{Z})$. All other paths will diverge away from this point. Now, recall that the dynamics is traced after we consider two initial values $N_0$ and $N_1 = (Z_0)$, of which only $N_0$ is predetermined. This implies that we can rule out some divergent paths because they are clearly not optimal: as $N_t$ will at some point approach either 1 or $n$, output and consumption will become equal to zero. Nevertheless, there are paths that, although they do not converge toward $\hat{N}$, there is no reason to rule out. These paths entail the presence of a periodic equilibrium or limit cycles. We will use the previous numerical example to illustrate such cases, bearing in mind that parameter values must satisfy the conditions summarized in Lemma 3.

In the baseline numerical example, we set the TFP growth factor equal to $g = 0.26$. The simulation indicates that the steady state $\hat{N}$ falls to roughly 49 (the exact number is 48.55687), but this is an unstable solution. Instead, the dynamics of entry converges to a 2-period cycle equal to $\{N^1, N^2\} = \{83.02957452, 15.51242096\}$ that corresponds to a 2-period cycle for the growth rate $\{0.656219559, 0.521823448\}$. This periodic equilibrium is in fact robust to changes in the initial conditions—an outcome that, together with the fact that they surround a unique but unstable solution, indicates that the 2-period cycle is stable. Cycles with two periodic solutions appear as we reduce $g$ even further, until at some point we observe that 2-period cycles become unstable and are replaced by the emergence of a stable 4-period cycle. For example, setting $g = 0.205$ leads to an unstable steady state $\hat{N} = 46.33716$ and a 4-period cycle $\{N^1, N^2, N^3, N^4\} = \{89.6690622, 4.97178912, 93.46991857, 8.330862453\}$ that corresponds to a 4-period cycle for the growth rate $\{0.590587824, 0.226986784, 0.718944528, 0.325064367\}$.[10] Reducing $g$ even more leads to the emergence of cycles of period-8 (e.g., for $g = 0.199$), period-16 (e.g., for $g = 0.1983$), and so on, suggesting that the economy's dynamics undergoes a "period-doubling route to chaos" [Devaney (2003)]. Indeed, reducing the TFP growth factor to 0.185 generates cycles that are clearly aperiodic. Figure 3 provides the simulated graphs for all the examples that we offered previously.

It is possible to generalize the implications offered by these numerical examples. We can start with

LEMMA 4. *Suppose that* $\delta\theta \in (1, 2]$. *The dynamical system of (40) and (41) undergoes a flip (period-doubling) bifurcation at* $\delta = \tilde{\delta}$. *Hence, there exist stable limit cycles.*

Given this, we can characterize the dynamics in this case through

PROPOSITION 5. *Under the conditions in Lemmas 3 and 4, fluctuations in the number of intermediate good firms can become permanent. Therefore, output may not converge to its balanced growth path; instead it will fluctuate permanently around it.*

Recall that $\delta$ is a composite parameter term that is negatively related to $g$. Given Lemma 4, it is not difficult to understand why our previous simulations revealed that reductions of the TFP growth factor generate period-doubling bifurcations. In terms of intuition, we can allude to the forces of industry dynamics that we described previously. Now, however, the impact of nonmonotonicities is strong enough so that cycles do not dissipate over time. On the contrary, they become a permanent characteristic of the industry's dynamics and consequently the evolution of output. These fluctuations do not rest on any exogenous shocks. Instead, both the impulse source and the propagation mechanism lie with the structural characteristics of the economic environment. In particular, the occupational choice is the source of nonmonotonicities that generate fluctuations and propagate them into fluctuations of output growth.

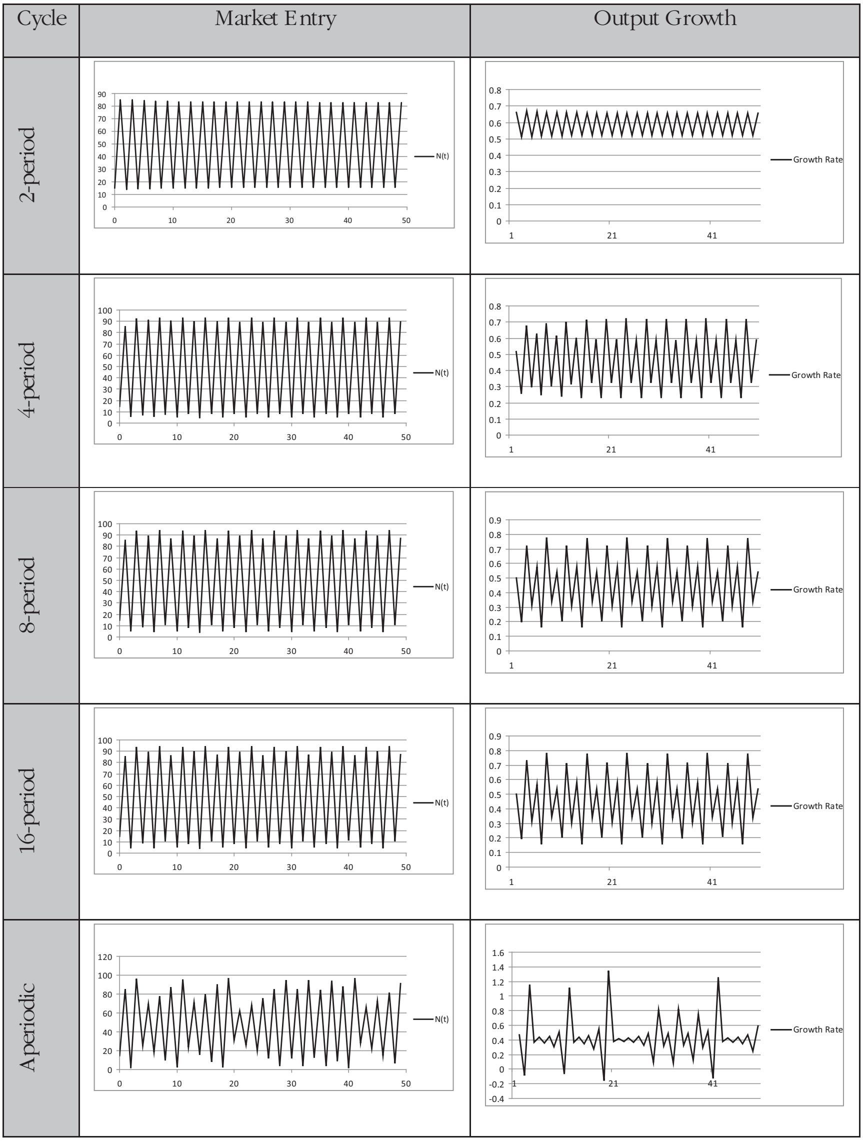


**FIGURE 3.** Permanent deterministic cycles.

## 5. CONCLUSION

In this paper, our endeavor was to contribute to the emerging body of literature that studies the dynamic behavior of endogenous market structures in dynamic general equilibrium models. We showed that an OLG setting, combined with the idea that entry decisions are made through an occupational choice process, can

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lead to interesting implications concerning these dynamic patterns. We showed that the intrinsic dynamics of the industry can lead to fluctuations, either though damped oscillations, stochastic sunspot equilibria, or limit cycles. These results represent yet another example on how endogenous forces can cause fluctuations in economic dynamics.

A note of caution merits discussion here, given that that our paper's dynamics are characterized by periodic orbits that may resemble the type of fluctuations we observe in the data. We believe that a better interpretation of our results should entail a correspondence to low-frequency waves in industry activity, such as those presented by Comin and Gertler (2006), rather than the high-frequency fluctuations that are more suitably attributed to the occurrence of short-term business cycles. For this reason, we need to clarify that our analysis is under no circumstances an attempt to invalidate other explanations for the cyclicality of economic dynamics, based on the idea of exogenous shocks—explanations that we actually view as being indubitably important. The main message form our work is that the (medium-term) cyclical behavior of economies, in addition to being a response to changing economic conditions, may also reflect characteristics that render them inherently volatile. As we indicated at the very beginning of this paper, other authors have asserted the same through their research work, thus giving this idea some momentum.

The model we presented is simple enough to guarantee a clear understanding of the mechanisms that are involved in the emergence of the basic results, without blurring either their transparency or their intuition. Of course, there is certainly a large scope for obtaining additional implications by modifying or enriching some of the model's founding characteristics. One obvious direction is to assume that the oligopolistic industry supplies firms with different varieties of capital goods while, at the same time, retaining the important characteristic of endogenous occupational choice. The ensuing process of capital accumulation could set in motion some very interesting implications concerning economic dynamics. Another potential direction is to endogenize the exit rate, perhaps by assuming that firm ownership is bequeathed from parents to children. Again, such a setup could initiate even richer dynamics; thus it offers a potentially fruitful avenue for future research work.

## *NOTES*

1. We refer to analyses that view economic fluctuations as only transitory or short-term phenomena, commonly known as "business cycles." The main idea is that various exogenous shocks represent the initial impulse sources whose effect is propagated and manifested in fluctuations of major economic variables. Different strands of the literature, such as the real business cycle and the new Keynesian approaches, have debated both the impulse sources and the propagation mechanisms that lead to economic fluctuations.

2. See Etro (2009) and the references therein for a more detailed discussion of this strand of the literature.

3. Other macroeconomic analyses that explicitly account for entry dynamics are, among others, those by Gil et al. (2013), Sanders (2013), and Zeng (2013).

4. The scale factor $N_t^{-1/(\theta-1)}$ implies that, in a symmetric equilibrium, $N_t^{-1/(\theta-1)}(\sum_{j=1}^{N_t} x_{t,j}^{(\theta-1)/\theta})^{\theta/(\theta-1)} = N_t x_t$.

5. The assumption of a small open economy implies that domestic borrowing and lending cannot affect the world interest rate $r$.

6. Note that the implied cost of the activity that equips an agent with the ability to become an intermediate good producer corresponds to the foregone labor income. We could have allowed an additional direct cost, taking the form of a proportion of (the present value of) her second period earnings. This would have introduced an additional scale factor into the model, without altering our results.

7. Note that aggregate debt can be negative, in which case equation (20) is the accumulation of assets in the economy.

8. According to Parker (2009), this "wage-effect" is an empirically important determinant when individuals choose whether to open a business or seek paid employment.

9. It can be easily established that these parameter values lie in the permissible range that guarantees stability according to Lemma 1.

10. In these examples, we have ignored the discrete nature of $N_t$. The reader may approximate the appropriate value by using the closest integer.

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# APPENDIX: PROOFS OF LEMMAS 1–4

The Jacobian matrix associated with the planar system in (40), (41) is

$$\begin{bmatrix} F_{Z_t}(\hat{Z}, \hat{N}) & F_{N_t}(\hat{Z}, \hat{N}) \\ H_{Z_t}(\hat{Z}, \hat{N}) & H_{N_t}(\hat{Z}, \hat{N}) \end{bmatrix},$$

where $\hat{N} = \hat{Z}$ is given in (38). Some straightforward algebra with equations (38), (40), and (41) reveals that the trace ($T$) and the determinant ($D$) are equal to

$$T = F_{Z_t}(\hat{Z}, \hat{N}) + H_{N_t}(\hat{Z}, \hat{N}) = -\delta \left\{ \theta - \frac{a(1+\delta\theta)}{(1-a)[n-(1+\delta)]} \right\} \tag{A.1}$$

and

$$D = F_{Z_t}(\hat{Z}, \hat{N}) H_{N_t}(\hat{Z}, \hat{N}) - F_{N_t}(\hat{Z}, \hat{N}) H_{Z_t}(\hat{Z}, \hat{N}) = \frac{\delta a(1+\delta\theta)}{(1-a)[n-(1+\delta)]}, \tag{A.2}$$

respectively. Furthermore, the eigenvalues are the roots of the polynomial $\lambda^2 - T\lambda + D$, i.e.,

$$\lambda_1 = \frac{T - \sqrt{T^2 - 4D}}{2} \quad \text{and} \quad \lambda_2 = \frac{T + \sqrt{T^2 - 4D}}{2}. \tag{A.3}$$

To ensure the stability of the steady state, we want the eigenvalues to satisfy $|\lambda_1| < 1$ and $|\lambda_2| < 1$. Given that $\lambda_1 + \lambda_2 = T$ and $\lambda_1 \lambda_2 = D$, two necessary but not sufficient conditions for stability are $-1 < D < 1$ and $-2 < T < 2$. Evidently, the determinant is positive by virtue of (A.2); therefore we can use (A.2) to find that $D < 1$ corresponds to the restriction

$$n - 1 > \delta + \frac{\delta a(1 + \delta\theta)}{1 - a}. \tag{A.4}$$

Furthermore, note that we can use (A.1) and (A.2) to get

$$T = D - \delta\theta. \tag{A.5}$$

As we constrain ourselves to $D < 1$, equation (A.5) reveals that $T < 2$. Therefore, we want to obtain a restriction for which $T > -2$. Using (A.5), it can be very easily established that a sufficient (but not necessary) condition for this is given by

$$\delta\theta < 2. \tag{A.6}$$

In addition to the preceding, we will rule out complex eigenvalues by imposing a parameter restriction that ensures that $T^2 - 4D \geq 0$. Specifically,

$$\begin{gathered} T^2 \geq 4D \Leftrightarrow \\ (D - \delta\theta)^2 \geq 4D \Leftrightarrow \\ D^2 - 2(2 + \delta\theta)D + (\delta\theta)^2 \geq 0 \Leftrightarrow \\ \Phi(D) \geq 0. \end{gathered} \tag{A.7}$$

Given (A.7), we have $\Phi(0) = (\delta\theta)^2 > 0$ and $\Phi(1) = 1 - 2(2 + \delta\theta) + (\delta\theta)^2 < 0$ by virtue of (A.6). Furthermore, $\Phi' = 2D - 2(2 + \delta\theta) < 0$ for $D \in (0, 1)$. Hence, for $T^2 - 4D \geq 0$ to hold we need the restriction $D < \tilde{D}_{\min}$, where $\tilde{D}_{\min}$ is the lowest-valued root of $\Phi(\tilde{D}) = 0$. We can then use (A.2) to establish that

$$\begin{gathered} D < \tilde{D}_{\min} \Leftrightarrow \\ D < 2 + \delta\theta - 2\sqrt{1 + \delta\theta} \Leftrightarrow \\ n - 1 \geq \delta + \frac{\delta a(1 + \delta\theta)}{(1 - a)(2 + \delta\theta - 2\sqrt{1 + \delta\theta})} \equiv \Omega(\delta). \end{gathered} \tag{A.8}$$

However, notice that $2 + \delta\theta - 2\sqrt{1 + \delta\theta} \in (0, 1)$ by virtue of (A.6). This implies that the restriction in (A.8) ensures that the condition in (A.4) is also satisfied.

Now, check that $\delta\theta > 2 + \delta\theta - 2\sqrt{1 + \delta\theta}$. By virtue of (A.8), this means that

$$n \geq 1 + \delta + \frac{\delta a(1 + \delta\theta)}{(1 - a)\delta\theta}. \tag{A.9}$$

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Consequently, combining (A.9) and (A.5), we can establish that the trace $T$ is negative, i.e., $T \in (-2, 0)$ which, combined with (A.3), reveals that both eigenvalues $\lambda_1$ and $\lambda_2$ are negative. It can be easily established that $\lambda_2 > -1$, whereas $\lambda_1 > -1$ holds as long as

$$\frac{T - \sqrt{T^2 - 4D}}{2} > -1 \Leftrightarrow$$

$$2 + T > \sqrt{T^2 - 4D}.$$

Given $T > -2$, we can use this expression to get

$$(2 + T)^2 > (\sqrt{T^2 - 4D})^2 \Leftrightarrow \qquad \textbf{(A.10)}$$

$$4 + 4T > -4D \Leftrightarrow$$

$$D + T + 1 > 0 \Leftrightarrow$$

$$D > \frac{\delta\theta - 1}{2},$$

which holds unambiguously when $\delta\theta \leq 1$. Hence, in this case $0 > \lambda_2 > \lambda_1 > -1$ holds—a result ensuring that there is convergence to the long-run equilibrium and that it is oscillatory (or cyclical).

Now consider the case where $\delta\theta \in (1, 2]$. The condition in (A.10) can be written as

$$n - 1 < \Xi(\delta), \qquad \textbf{(A.11)}$$

where

$$\Xi(\delta) \equiv \delta + \frac{\delta a(1 + \delta\theta)}{1 - a} \frac{2}{\delta\theta - 1}. \qquad \textbf{(A.12)}$$

Recalling that we are considering values for which $\delta\theta \in (1, 2]$, we can determine that $\lim_{\delta \to (1/\theta)+} \Xi(\delta) = +\infty$ and $\Xi(\frac{2}{\theta}) = 2\left(1 + \frac{3a}{1-a}\right) < n - 1$ by assumption. As long as $\Xi(\delta)$ cuts the $n - 1$ line only once, there is $\tilde{\delta}$ such that $\Xi(\tilde{\delta}) = n - 1$. Furthermore, note that for $\delta\theta \in (1, 2]$ we have $\frac{1}{2+\delta\theta-2\sqrt{1+\delta\theta}} < \frac{2}{\delta\theta-1}$. Given (A.8), (A.12), and the assumption that $n > 1 + 2(1 + \frac{3a}{1-a})$, this implies that $\Xi(\delta) > \Omega(\delta)\ \forall \delta \leq 2/\theta$, i.e., the condition in (A.8) always holds given our assumptions.

The previous analysis implies that (A.11) holds when $\delta < \tilde{\delta}$ and therefore $0 > \lambda_2 > \lambda_1 > -1$. The steady state $\hat{N}$ is locally stable. However, when $\delta > \tilde{\delta}$, we have $0 > \lambda_2 > -1 > \lambda_1$ and the steady state $\hat{N}$ is a saddlepoint. Evidently, at $\delta = \tilde{\delta}$, we have $\lambda_1 = -1$. Combined with $\lambda_2 \in (-1, 0)$, we can use Theorem 8.4 in Azariadis (1993) to deduce that the dynamical system undergoes a flip (or period doubling) bifurcation so that there exists a 2-period cycle.