Robotica (2000) volume 18, pp. 71–80. Printed in the United Kingdom © 2000 Cambridge University Press

Dynamics and control of a set of dual fingers with soft tips

Suguru Arimoto,* Pham Thuc Anh Nguyen,* Hyun-Yong Han* and Zoe Doulgeri†

SUMMARY

This paper attempts firstly to derive a mathematical model of the dynamics of a set of dual fingers with soft and deformable tips which grasps and manipulates a rigid object with some dexterity. To gain a physical insight into the problem, consideration is restricted to the case that the motion of the whole system is confined to a horizontal plane. Secondly on the basis of the derived model it is shown that the rotation of the object can be indirectly controlled by the change of positions of the center points of both contact areas on the object. Then, each of the center points of contact areas can be positioned by inclining the last link of each corresponding finger against the object. It is further shown that, when both forces of pressing the object becomes almost equal, the equation of motion of the object in terms of rotational angles assumes the form of a harmonic oscillator with a forcing term, which can be regulated coordinately by the relative angle between the two last links contacting with the object. It is also shown that dynamics of this system satisfy passivity. Finally, design problems of control for dynamic stable grasping and enhancing dexterity in manipulating things are discussed on the basis of passivity analysis.

KEYWORDS: Dual fingers; Multi-fingered hand; Soft fingers; Manipulation; Stable grasping.

1. INTRODUCTION

It was about two decades ago that robot engineers were optimistic about predicting that robots would quickly evolve into a higher form and be able to do any kind of tasks that a human being can do. However, even in the beginning of the new millennium 2000, it is said that robots are too clumsy to be used in ordinary tasks that humans must do in their everyday life. In fact, a variety of multi-fingered robot hands carefully designed and manufactured with very high precision has been brought into practice in manufacturing but mainly used in simpler repetitive tasks. Regardless of such fine hand mechanisms, it is quite a difficult problem to endow them with dexterity and versatility in execution of a variety of ordinary tasks. This paper is motivated by the observation that the prime cause of this difficulty may originate from the lack of our knowledge of physical characterizations of dynamics of such fine and sophisticated mechanisms physically interacting (grasping, handling, or manipulating) with things and environment. In particular, the paper attempts to explore dynamics of a set of dual fingers with soft and deformable finger-tips that are grasping and manipulating a rigid object, because most of the past literature treated only the case of multi-fingered hands contacting rigidly and point-wise with an object or environment [1-3] and proposed to use rolling of finger-tips on the surface of the object in order to enhance dexterity. Differently from the case of rigid finger-tips, it is possible to observe from dynamics of dual soft fingers that each center of contact areas can be changed by coordinately inclining the last links against the object. This means that the posture of the object can be controlled without making rigid and point-wise rolling of the finger-tips on the object surface, which may cause dry fiction and slipping. In this paper it is shown that the derived equation of motion of the system naturally satisfies passivity. Further, it is shown that the special equation of motion of the object in terms of rotation in a horizontal plane is of the form of motion equation of a pendulum with a forcing term composed of two moments, each of which is equal to the reproducing force of a deformed contact area times the length from each corresponding contact center to the horizontal center line of the object through the mass center. When two reproducing forces arose from deformed contact areas become almost equal, the rotational angle of the object can be controlled by changes of angles of two last links relative to the object, by which lengths of contact centers to the horizontal center line can be also regulated. Then a feedback control scheme for attaining stable grasping in a dynamic sense is proposed and it is shown that the closed-loop system approaches asymptotically an equilibrium manifold of still states of grasping. Another feedback scheme is also proposed, that can control the rotation angle of the object in addition to attaining a stable grasping.

In final two sections a variety of future research directions necessary to unveil secrets of dexterity and versatility of multi-fingered hands with soft finger-tips will be discussed. It is then claimed that, with the evolution of technology of tactile sensing as well as real-time robot vision, soft hands with multiple fingers will be used in more versatile everyday tasks that need automation.

2. DYNAMICS OF DUAL SOFT FINGERS MANIPULATING AN OBJECT

For the sake of simplifying the mathematical argument and gaining a physical insight into the problem why soft fingers can manipulate an object with some dexterity and versatility, we assume that motion of the set of dual fingers is confined to a horizontal plane (see Figure 1) and not affected by the gravity force. Further, we treat the case that the object is rigid with a rectangular shape and shape of

* Department of Robotics, Ritsumeikan University, H-1 Nojihgh-shi, Kusatsu, Shiga, 525-8577 (Japan)
† Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki 54006 (Greece)

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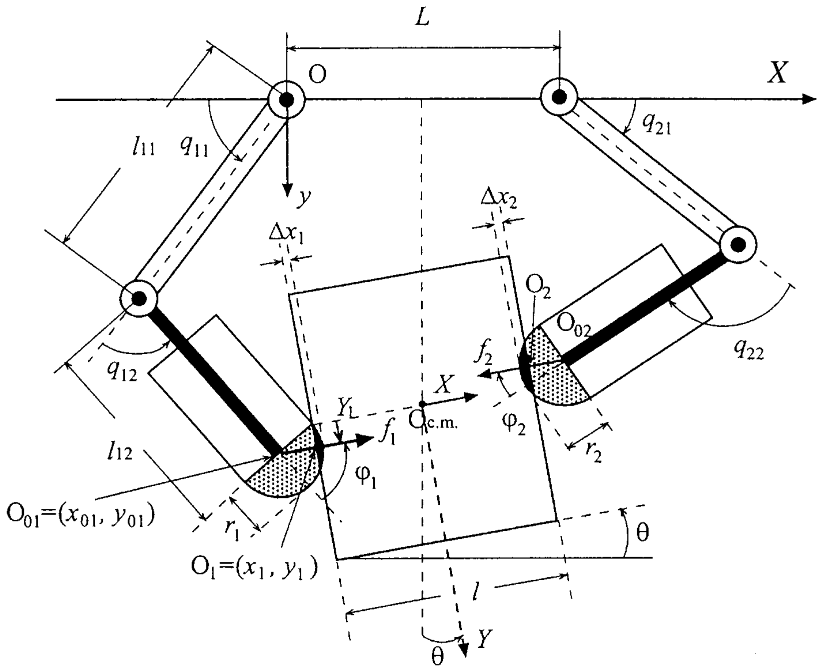

Fig. 1. A mechanical hand with dual fingers whose ends are covered with soft material.

each of soft and deformable finger-tips is spherical. Generalization of the argument will be discussed in later sections.

As it is shown in Figure 1 it is reasonably assumed that the deformed area of each finger-tip is crescent-shaped as black-painted in the figure and at the center point O_i $(=(x_i, y_i))$ there arises the concentrated pressure force f_i in the direction perpendicular to the surface of the object as shown by an arrow. It is also assumed in a reasonable way (as discussed in Appendix A) that the magnitude of the force f_i pressing the object is expressed as a function of Δx_i, the maximum displacement of deformation (see Figure 1), and must be a monotonically increasing function of Δx_i for $\Delta x_i \geq 0$ with $f_i(\Delta x_i) = 0$ for $\Delta x_i \leq 0$ (see Figure 2). Therefore, the endpoint of each bone of the second links expressed as $O_{0i}(=(x_{0i}, y_{0i}))$ receives the reproducing force in the opposite direction of f_i (the pressing force to the object) with the magnitude $f_i(\Delta x_i)$. We ignore the effect of mass transfer of the soft material used in covering the finger-tips in the opposite direction of f_i. Further, the adhesive force that may arise between the soft material and the surface of the object is so small that it does not affect motion of the set of fingers and the object. Then, it is possible to ignore the equation of motion of mass transfer in the direction of rotation φ_i. Finally, it is necessary to consider any possibility of forces that may arise from geometric constraints. However, the geometrical constraints

$$x=x_1+\tfrac{l}{2}\cos\theta-Y_1\sin\theta=x_2-\tfrac{l}{2}\cos\theta-Y_2\sin\theta \qquad (1)$$

$$y=y_1+\tfrac{l}{2}\sin\theta-Y_1\cos\theta=y_2-\tfrac{l}{2}\sin\theta-Y_2\cos\theta \qquad (2)$$

where

$$Y_1=c_1-r_1(\pi+\theta-q_{11}-q_{12})=c_1-r_1\varphi_1 \qquad (3)$$

$$Y_2=c_2-r_2(\pi+\theta-q_{21}-q_{22})=c_2-r_2\varphi_2 \qquad (4)$$

neither generate any force in the direction of Y_1 nor in that of Y_2 (see Figure 2) as discussed in Appendix B. Instead, the two geometric constraints may virtually produce two forces in directions of f_1 and f_2 respectively, but these forces should be merged into the reproducing forces with magnitudes of $f_1(\Delta x_1)$ and $f_2(\Delta x_2)$, respectively. On the other hand we must not ignore the effect of geometric constraints of the tightness of area-contacts as shown in equations (3) and (4) but we omit this purposely (a rough discussion is presented also in the last half of Appendix B).

Thus, it is possible to derive the following equation of motion for the setup of dual fingers manipulating an object (see Figure 1):

$$\left\{H_i(q_i)\frac{\mathrm{d}}{\mathrm{d}t}+\frac{1}{2}\dot{H}_i(q_i)\right\}\dot{q}_i+S_i(q_i,\dot{q}_i)\dot{q}+f_i(\Delta x_i)J_{oi}^T\begin{pmatrix}\partial\Delta x_i/\partial x_{o1}\\ \partial\Delta x_i/\partial y_{o1}\end{pmatrix}$$
$$=u_i,\ i=1,2 \qquad (5)$$

and

$$H\ddot{z}+\sum_{i=1,2} f_i(\Delta x_i)J_i^T(z)=0 \qquad (6)$$

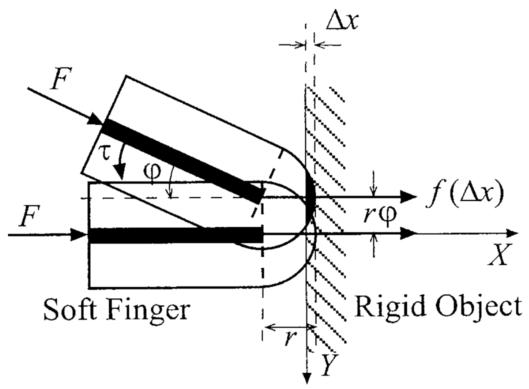

Fig. 2. The center of contact area moves on the object surface by inclining the last link against the object.

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where $q_i=(q_{i1}, q_{i2})^T$, $z=(x, y, \theta)^T$, $H_i(q_i)$ denotes the inertia matrix of finger i, $S(q_i, \dot{q}_i)$ is a skew-symmetric matrix including coriolis and centrifugal forces, $H=\text{diag}(M, M, I)$ denotes the diagonal matrix whose diagonal matrices stand for the mass of the object and the inertia moment at the mass center $O_{c.m.}(=(x, y)$ in the Cartesian coordinates fixed in the inertial frame). Further, J_{oi} denotes the Jacobian matix of the $x_{oi}, y_{oi})$ with respect to $(q_{i1}, q_{i2})$and $J_i(z)$ that of Δx_i at (x_i, y_i) with respect to $z=(x, y, \theta)^T$, and $f_i(\Delta x_i)$ the reproducing force of the deformed finger tip for finger i. Practical calculation of the two Jacobian matrices in (6) on the basis of geometrical relations is not a trivial task, but having a physical insight into the dynamics of the object exerted by the reproducing forces f_1 and f_2 leads us to the result that

$$\begin{pmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & I \end{pmatrix}\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{pmatrix} - f_1(\Delta x_1)\begin{pmatrix} \cos\theta \\ -\sin\theta \\ Y_1 \end{pmatrix} + f_2(\Delta x_2)\begin{pmatrix} \cos\theta \\ -\sin\theta \\ Y_2 \end{pmatrix} = 0 \qquad (7)$$

where Y_1 and Y_2 denote the Y-component of the center O_1 and O_2 of deformed areas expressed in terms of the coordinates (X, Y) fixed at the object (see Figure 1). Differently from the case of rigid fingers, the center point O_{01} where a force sensor may be implemented must move relative to the object when the last link of the left hand side finger inclines relative to the object (see Figure 2). This change of the position $(x_{01}\ y_{01})$ can be brought about by an inclination of the last link to the object without rigid rolling and is extremely important in acquiring dexterity of manipulation of the object, while in the case of rigid and point-wise contacts the change of the contact position must accompany rigid and point-wise rolling of the finger-tips on the surface of the object. More detailedly, the two Jacobian matrices in (7) can be formulated in the following way:

$$J_1^T(z)=\left(\frac{\partial \Delta x_1}{\partial x}, \frac{\partial \Delta x_1}{\partial y}, \frac{\partial \Delta x_1}{\partial \theta}\right)^T$$

$$=\begin{bmatrix} \frac{\partial x_1}{\partial x} & \frac{\partial x_1}{\partial y} & \frac{\partial x_1}{\partial \theta} \\ \frac{\partial y_1}{\partial x} & \frac{\partial y_1}{\partial y} & \frac{\partial y_1}{\partial \theta} \end{bmatrix}^T \begin{pmatrix} \frac{\partial \Delta x_1}{\partial x_1} \\ \frac{\partial \Delta x_1}{\partial y_1} \end{pmatrix}$$

$$=\begin{bmatrix} 1 & 0 & (l/2)\sin\theta + Y_1\cos\theta - r_1\sin\theta \\ 0 & 1 & (l/2)\cos\theta - Y_1\sin\theta - r_1\cos\theta \end{bmatrix}^T$$

$$\begin{pmatrix} -\cos\theta \\ \sin\theta \end{pmatrix} = [-\cos\theta, \sin\theta, -Y_1]^T, \qquad (8)$$

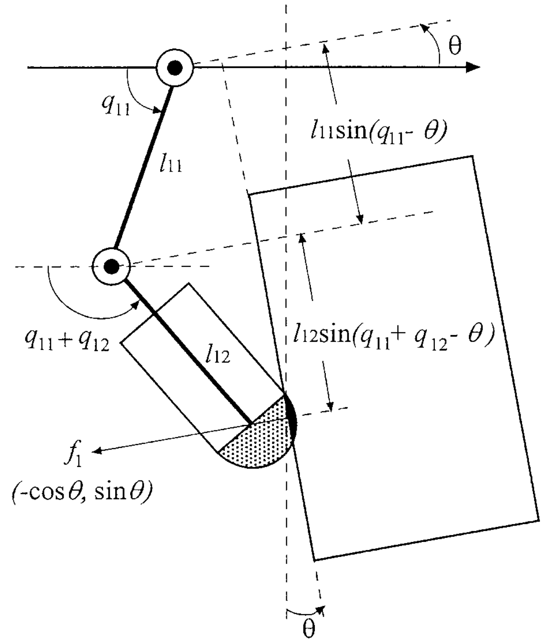

Fig. 3. Physical meanings of each component of equation (8).

$$J_2^T(z)=\left(\frac{\partial \Delta x_2}{\partial x}, \frac{\partial \Delta x_2}{\partial y}, \frac{\partial \Delta x_2}{\partial \theta}\right)^T = [\cos\theta, -\sin\theta, Y_2]^T. \qquad (9)$$

It is also possible to calculate the Jacobian matrix in eq.(5), which results in (see Figure 3)

$$J_{01}^T\begin{pmatrix} \frac{\partial \Delta x_1}{\partial x_{01}} \\ \frac{\partial \Delta x_1}{\partial y_{01}} \end{pmatrix}$$

$$=\begin{pmatrix} l_{12}\sin(q_{11}+q_{12}) + l_{11}\sin(q_{11}) & l_{12}\cos(q_{11}+q_{12}) + l_{11}\cos(q_{11}) \\ l_{12}\sin(q_{11}+q_{12}) & l_{12}\cos(q_{11}+q_{12}) \end{pmatrix}\begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}$$

$$=\begin{pmatrix} l_{12}\sin(q_{11}+q_{12}-\theta) + l_{11}\sin(q_{11}-\theta) \\ l_{12}\sin(q_{11}+q_{12}-\theta) \end{pmatrix} \qquad (10)$$

$$J_{02}^T\begin{pmatrix} \frac{\partial \Delta x_2}{\partial x_{02}} \\ \frac{\partial \Delta x_2}{\partial y_{02}} \end{pmatrix}$$

$$=\begin{pmatrix} l_{22}\sin(q_{21}+q_{22}) - l_{21}\sin(q^{21}) & l_{22}\cos(q_{21}+q_{22}) + l_{21}\cos(q_{21}) \\ -l_{22}\sin(q_{21}+q_{12}) & l_{22}\cos(q_{21}+q_{22}) \end{pmatrix}\begin{pmatrix} -\cos\theta \\ \sin\theta \end{pmatrix}$$

$$=\begin{pmatrix} l_{22}\sin(q_{21}+q_{22}+\theta) + l_{21}\sin(q_{21}+\theta) \\ l_{22}\sin(q_{21}+q_{22}+\theta) \end{pmatrix} \qquad (11)$$

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3. PASSIVITY ANALYSIS

It is now possible to show that the input-output pair $(u_1, u_2),(\dot q_1, \dot q_2)$ for the system of equations (5) and (6) satisfies passivity. In fact, in the light of physical meanings of Jacobian matrices in (5) and (6) as described in (8) to (11), and inner product between (u_1, u_2) and $(\dot q_1, \dot q_2)$ is described as

$$\dot q_1^T u_1 + \dot q_2^T u_2 = \frac{d}{dt}\left\{\sum_{i=1,2}\frac{1}{2}\dot q_i^T H(q_i)\dot q_i\right\} + \{(\dot x_{01}, \dot y_{01}) f_1(\Delta x_1) - (\dot x_{02}, \dot y_{02})(f_2 \Delta x_2)\}\begin{pmatrix}\cos\theta \\ -\sin\theta\end{pmatrix} + \frac{d}{dt}\left\{\frac{1}{2}(M\dot x^2 + M\dot y^2 + I\dot\theta^2)\right\} - \{(\dot x_1, \dot y_1) f_1(\Delta x_1) - (\dot x_2, \dot y_2) f_2(\Delta x_2)\}\begin{pmatrix}\cos\theta \\ -\sin\theta\end{pmatrix}. \quad (12)$$

Since from a geometric relation as shown in Figure 1, it follows that

$$\begin{cases} x_1 - x_{01} = (r_1 - \Delta x_1)\cos\theta, \\ y_1 - y_{01} = -(r_1 - \Delta x_1)\sin\theta \end{cases} \quad (13)$$

from which we have

$$(x_1 - x_{01})\cos\theta - (y_1 - y_{01})\sin\theta = r_1 - \Delta x_1 \quad (14)$$

and

$$\Delta\dot x_1 = (\dot x_{01} - \dot x_1)\cos\theta - (\dot y_{01} - \dot y_1)\sin\theta \quad (15)$$

(Note that $\{(x_1 - x_{01})\sin\theta + (y_1 - y_{01})\cos\theta\}\dot\theta = 0$). Thus, by substituting (15) into (12) we obtain

$$\dot q_1^T u_1 + \dot q_2^T u_2 = \frac{d}{dt}K(\dot q_1, \dot q_2, \dot z, q_1, q_2) + \Delta\dot x_1 f_1(\Delta x_1) + \Delta\dot x_2 f_2(\Delta x_2) \quad (16)$$

where

$$K = \frac{1}{2}\sum_{i=1,2}\dot q_i^T H_i(q_i)\dot q_i + \frac{1}{2}(M\dot x^2 + M\dot y^2 + I\dot\theta^2). \quad (17)$$

Note that $f_i(\Delta x_i) \geq 0$, and f_i is strictly increasing with increasing Δx_i, and

$$\Delta\dot x_i f_i(\Delta x_i) = \frac{d}{dt}\int_0^{\Delta x_i} f_i(\xi)\, d\xi. \quad (18)$$

Since the integral of the right hand side is positive for $\Delta x_i > 0$, we have

$$\int_0^t (\dot q_1^T u_1 + \dot q_2^T u_2)\, d\tau = E(t) - E(0) \geq -E(0) \quad (19)$$

which shows the passivity, where

$$E = K + \sum_{i=1,2}\int_0^{\Delta x_i} f_i(\xi)\, d\xi. \quad (20)$$

Contrary to the above argument the passivity follows from the variational form defined by

$$\int_{t_1}^{t_2}\delta(K - P)\, dt = 0 \quad (21)$$

for any t_2, t_1 $(t_2 > t_1)$, where

$$P = \sum_{i=1,2}\int_0^{\Delta x_i} f_i(\xi)\, d\xi \quad (22)$$

and K is defined in (17).

Besides passivity, it is interesting to note that the last equation of (7) is governed by a kind of motion equation of a harmonic oscillator with a forcing term, that is,

$$I\ddot\theta + 2r f_d \theta = f_d[(c_1 - c_2) + r\{(q_{11} + q_{12}) - (q_{21} + q_{22})\}], \quad (*)$$

provided that $f_1 = f_2$ and $r_1 = r_2 = r$. In other words, the porture (rotational angle θ) of the object can be controlled indirectly by changing relative inclination angles of the two second finger links to the horizontal line, once the pressing forces are attained at the target value $f_1 = f_2 = f_d$ and this internal force can be maintained during inclining the second links to the horizontal line or the surface of the object.

STABLE GRASPING

One merit of using soft fingers is possible design of a mechanical hand that realizes a smooth transition from the state of “non-contact” to “tight contact” via “instant of contact”, which does not incur discontinuous changes of the velocity vector of a finger-tip. Another merit is that it has capability of realizing stable grasping by a simple feedback control scheme with the aid of measurement of positions of center points of contact areas relative to the surface of the object together with measurement data on angles and angular velocities of finger links. More precisely, we assume measurements on vectors q_1, q_2, $\dot q_1$, $\dot q_2$, scalar values Y_1 and Y_2, and knowledge of kinematic parameters l_{11}, l_{12}, l_{21}, l_{22}, r_1, and r_2. Then, for a given desired internal force f_d we introduce control inputs defined by

$$u_1 = J_{01}^T\begin{pmatrix}\cos\theta \\ -\sin\theta\end{pmatrix} f_d - k_{v1}\dot q_1 - \frac{r_1 f_d}{r_1 + r_2}(Y_1 - Y_2)\begin{pmatrix}1 \\ 1\end{pmatrix}, \quad (23)$$

$$u_2 = J_{02}^T\begin{pmatrix}-\cos\theta \\ \sin\theta\end{pmatrix} f_d - k_{v2}\dot q_2 + \frac{r_2 f_d}{r_1 + r_2}(Y_1 - Y_2)\begin{pmatrix}1 \\ 1\end{pmatrix}. \quad (24)$$

It is then easy to see that substitution of (23) and (24) into (5) yields

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$$\left\{H_i(q_i)\frac{\mathrm{d}}{\mathrm{dt}}+\frac{1}{2}\dot{H}_i(q_i)+S_i(q_i,\dot{q}_i)\right\}\dot{q}_i+k_{vi}\dot{q}_i$$

$$=(-1)^i\left\{J_{0i}^T\begin{pmatrix}\cos\theta\\ -\sin\theta\end{pmatrix}\Delta f_i+\frac{r_if_d}{r_1+r_2}(Y_1-Y_2)\begin{pmatrix}1\\ 1\end{pmatrix}\right\},\qquad i=1,2 \quad (25)$$

where $\Delta f_i = f_i(\Delta x_i) - f_d$. On the other hand, equation (6) of motion of the object (or (7)) can be rewritten in the form

$$\begin{cases} M\ddot{x}-(\Delta f_1-\Delta f_2)\cos\theta=0,\\ M\ddot{y}-(\Delta f_1-\Delta f_2)\sin\theta=0,\\ I\ddot{\theta}-\Delta f_1Y_1+\Delta f_2Y_2=f_d(Y_1-Y_2).\end{cases} \quad (26)$$

By referring to the passivity analysis that could derive equation (19), taking an inner product between $\dot{q}_i$ and (25) for $i=1$, 2 and an inner product between (26) and $\dot{z}$ we obtain

$$\frac{\mathrm{d}}{\mathrm{dt}}\left\{(K+\Delta P)+\frac{f_d}{2(r_1+r_2)}(Y_1-Y_2)^2\right\}=-k_{v1}\|\dot{q}_1\|^2-k_{v2}\|\dot{q}_2\|^2 \quad (27)$$

where $\delta x_i=\Delta x_i-\Delta x_{di}$, $\Delta x_{di}=f_i^{-1}(f_d)$, and

$$\Delta P=\sum_{i=1,2}\int_0^{\delta x_i}\{f_i(\xi+\Delta x_{di})-f_d\}\,\mathrm{d}\xi. \quad (28)$$

Since $f_d>0$ and $f_i(\xi)$ is a strictly increasing with increasing ξ, each integral of the right hand side of (28) is positive definite in δx_i (see Figure 4). Hence, the content of bracket {} of the left hand side of (27) plays a role of Lyapunov's function. Further, it is possible to prove from (27) that $q_i(t)\rightarrow 0$ as $t\rightarrow\infty$ for $i=1$ and 2 (for example, see Appendix C in the book[4]). In fact, since all velocity vectors $\dot{q}_1$, $\dot{q}_2$ and $\dot{z}$ must be norm-bounded in the sense of $L^2(0,\infty)$-norm, all position vectors q_1, q_2, and z must be bounded. Then, from (25) it follows that $\ddot{q}_i$ becomes uniformly bounded, which means that $\dot{q}_i$ is uniformly continuous. Thus, by virtue of the fact that $\dot{q}_i$ belongs to

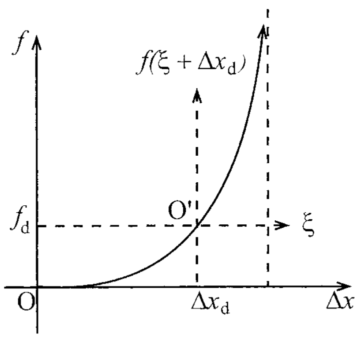

Fig. 4. $\Delta F(\delta x)=\int_0^{\delta x}\{f(\xi+\Delta x_d)-f_d\}\,\mathrm{d}\xi$ is a positive definite function of δx.

$L^2(0,\infty)$ and is uniformly continuous, it is concluded that $\dot{q}_i(t)\rightarrow 0$ as $t\rightarrow\infty$ for either $i=1$ or $i=2$. This means that the right hand side of (25) must converge to zero as $t\rightarrow\infty$. However, as is discussed later, the vector $J_{0i}^T(\cos\theta, -\sin\theta)^T$ (as is shown in Figure 3 and equations (10) and (11)) is independent to the vector $(1,1)^T$. Thus, both scalar values Δf_i and (Y_1-Y_2) in the right hand side of (25) must tend to zero as $t\rightarrow\infty$. That is, $f_i(\Delta x_i(t))\rightarrow f_d$ as $t\rightarrow\infty$ for $i=1$, 2, and $\theta(t)$ tends to a constant as $t\rightarrow\infty$ because $q_i(t)$ for $i=1$, 2 in (Y_1-Y_2) in (3) and (4) must tend to some constant vectors respectively as $t\rightarrow\infty$.

Moreover, it is easy to see from (26) and (27) that $\dot{x}$ and $\dot{y}$ tend to some constants, respectively as $t\rightarrow\infty$. Now, let us denote the sum of K, ΔP, and $(Y_1-Y_2)^2f_d/2(r_1+r_2)$ by $V(t)$ which is equivalent to the content of bracket {} in (27). Since V is a non-increasing function of t, the integral of the sum of $k_{vi}\|\dot{q}_1\|^2$ and $k_{v2}\|\dot{q}_2\|^2$ over $(0,\infty)$ can not become beyond the initial value of V at $t=0$. Thus, it is possible to consider a bounded open set D in R^{14} consisting of coordinates $(q_1, q_2, z, \dot{q}_1, \dot{q}_2, \dot{z})$ such that it contains at least one equilibrium state with $f_i=f_d$ for $i=1$, 2, $Y_1=Y_2$, $\dot{q}_1=\dot{q}_2=0$ and $\dot{z}=0$ and includes a smaller $D_0(\subset D)$ so that any solution to the set of equations (25) and (26) starting from any initial state in D_0 remains in D. We call such a domain D_0 a set of states of stable grasping. It is now possible to state the following theorem:

Theorem 1. Given a desired internal force $f_d>0$, there exists a domain D_0 of stable graspings in R^{14} such that any solution starting from an initial state in D_0 approaches asymptotically to a stable grasping state that satisfies $f_i=f_d$, $Y_1=Y_2$ $\dot{q}_1=\dot{q}_2=0$ and $\dot{z}=0$. More precisely, any solution to the equations of (25) and (26) with an initial state in D_0 approaches asymptotically a subset SG of $D(\subset R^{14})$ such that

$$SG=\{(q_1,q_2,z,\dot{q}_1,\dot{q}_2,\dot{z}):Y_1=Y_2, f_1=f_2=f_d, \dot{q}_1=\dot{q}_2=0, \dot{z}=0\}.$$

The above arguments in derivation of Theorem 1 are rather mathematical, but it is possible to give physical interpretations as in the following remarks:

Remark 1. The feedback control schemes defined by equations (23) and (24) need not use measurement data on reproducing forces of deformed finger-tips. However, instead of force sensors, measurements of both Y_1 and Y_2 are required, which can be realized from the data provided by vision or tactile sensing.

Remark 2. As far as stable grasping is concerned with, it is unnecessary to control both the position (x, y) and the posture θ of the object. Then, each mechanism of dual fingers need not have two controllable joints. If each one of dual fingers is designed as a single d.o.f. link mechanism whose end is covered by soft material, the position and posture of the object should be controlled externally by the movement of a wrist mechanism on which this dual-fingered hand is mounted.

Remark 3. Since the state vector $(q_1, q_2, z, \dot{q}_1, \dot{q}_2, \dot{z})$ of the system of differential equations (5) and (6) is of 14-dimension, a domain D_0 of stable grasping must be a 14-dimensional subset of R^{14}. From the practical point of view it is important to evaluate how large such a domain of stable graspings is.

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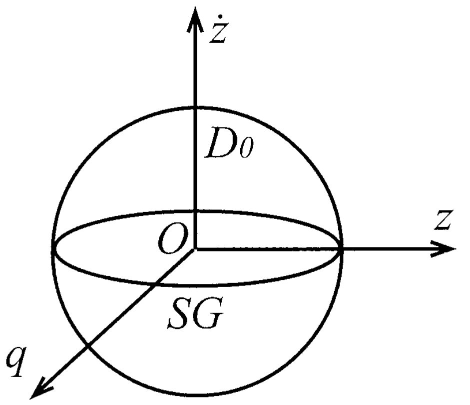

Fig. 5. A conceptual expression of an equilibrium manifold of stable grasping.

Remark 4. Recall again a subset D_0 of stable graspings and $D(D_0 \subset D)$. We define a manifold M as a set of states in D such that $Y_1 = Y_2, f_1 = f_2 = f_d$, $\dot{q}_1 = \dot{q}_2 = 0$ and $\dot{z} = 0$ and constraints (3) and (4). This subset M is a 2-dimensional manifold in D. We call it an equilibrium manifold of still states of grasping. By using these terminologies, Theorem 1 says that an equilibrium manifold $M_0 (= M \cap D_0)$ is asymptotically stable in the sense of Lyapunov.

Remark 5. The maximum set among all possible equilibrium manifolds of still states of grasping may contain all possible states of stable graspings introduced by Nguyen[5] from the static viewpoint. The maximum set among possibly many D_0's in R^{14} is defined from the dynamics viewpoint, which includes any transient states that may tend to an equilibrium still state of stable grasping inside M_0. (Figure 5)

5. FEEDBACK CONTROL OF ROTATION

In order to control the rotation angle θ of the object, it is necessary to introduce another feedback term in addition to each of control inputs (23) and (24). For a given target angle $\theta = \theta_d$, let us consider the control inputs

$$\bar{u}_1 = u_1 - \left\{ J_{01}^T \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix} - r_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\} (\alpha \Delta\dot{\theta} + \beta\Delta\theta), \quad (29)$$

$$\bar{u}_2 = u_2 + \left\{ J_{02}^T \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix} - r_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\} (\alpha \Delta\dot{\theta} + \beta\Delta\theta), \quad (30)$$

where $\Delta\theta = \theta - \theta_d$, $\Delta\dot{\theta} = \dot{\theta}$, and u_1 and u_2 are the same as those defined in (23) and (24) respectively. Then, the closed-loop system becomes of the form (by setting u_1 and u_2 in (5) as $\bar{u}_1$ and $\bar{u}_2$ defined in (29) and (30)):

$$\left\{ H_i(q_i)\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{2}\dot{H}_i(q_i) + S_i(q_i, \dot{q}_i) \right\} \dot{q}_i + k_{vi}\dot{q}_i$$

$$= (-1)^i \left[J_{0i}^T \left\{ \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} \Delta f_i + \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix} (\alpha\Delta\dot{\theta} + \beta\Delta\theta) \right\} \right.$$

$$\left. + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \left\{ \frac{r_i f_d}{r_1 + r_2}(Y_1 - Y_2) - r_i(\alpha\Delta\dot{\theta} + \beta\Delta\theta) \right\} \right]. \quad (31)$$

together with the motion equation of the object that is the same as in (26), where $\Delta\theta = \theta - \theta_d$ and $\Delta\dot{\theta} = \dot{\theta}$.

Now, by taking inner products of (31) with $\dot{q}_i$ for $i = 1, 2$ and (26) with $\dot{z}$ $(= (\dot{x}, \dot{y}, \dot{\theta}))$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}V(t) + k_{v1}\|\dot{q}_1\|^2 + k_{v2}\|\dot{q}_2\|^2$$

$$+ \Big\{ (\dot{x}_{01} - \dot{x}_{02})\sin\theta + (\dot{y}_{01} - \dot{y}_{02})\cos\theta - r_1(q_{11} + q_{12})$$

$$+ r_2(q_{21} + q_{22}) \Big\} (\alpha\Delta\dot{\theta} + \beta\Delta\theta) = 0 \quad (32)$$

where

$$V = K + \Delta P + \frac{f_d}{2(r_1 + r_2)}(Y_1 - Y_2)^2$$

and K and ΔP are defined in (17) and (28), repectively. According to Figure 1 or Figure 2, it is easy to see that x_{01} and y_{01} are related to x_1 and y_1 in the following formula:

$$\begin{cases} x_1 = x_{01} + (r_1 - \Delta x_1)\cos\theta, \\ y_1 = y_{01} - (r_1 - \Delta x_1)\sin\theta. \end{cases} \quad (33)$$

Similarly, it follows that

$$\begin{cases} x_2 = x_{02} - (r_2 - \Delta x_2)\cos\theta, \\ y_2 = y_{02} + (r_2 - \Delta x_2)\sin\theta. \end{cases} \quad (34)$$

From these equations it is easy to see that

$$(\dot{x}_{02} - \dot{x}_{01})\sin\theta + (\dot{y}_{01} - \dot{y}_{02})\cos\theta = (\dot{x}_1 - \dot{x}_2)\sin\theta$$
$$+ (\dot{y}_1 - \dot{y}_2)\cos\theta + \dot{\theta}\{(r_1 - \Delta x_1) + (r_2 - \Delta x_2)\} \quad (35)$$

On the other hand, it follows from the geometric constraints (see (B-1) and (B-2) in Appendix B and take differentiation on (B-3) in time t) that

$$(\dot{x}_1 - \dot{x}_2)\sin\theta + (\dot{y}_1 - \dot{y}_2)\cos\theta$$
$$= \dot{Y}_1 - \dot{Y}_2 - \{(x_1 - x_2)\cos\theta - (y_1 - y_2)\sin\theta\}\dot{\theta}. \quad (36)$$

Since the coefficient of $\dot{\theta}$ in the last term of the right hand side, that is, the content of bracket $\{\}$, is equivalent to $-l$, we have

$$(\dot{x}_1 - \dot{x}_2)\sin\theta + (\dot{y}_1 - \dot{y}_2)\cos\theta = \dot{Y}_1 - \dot{Y}_2 + l\dot{\theta}. \quad (37)$$

Substituting this into (35) and referring to equations (3) and (4), we finally obtain

$$(\dot{x}_{01} - \dot{x}_{02})\sin\theta + (\dot{y}_{01} - \dot{y}_{02})\cos\theta - r_1(\dot{q}_{11} + \dot{q}_{12})$$
$$+ r_2(\dot{q}_{21} + \dot{q}_{22}) = (l - \Delta x_1 - \Delta x_2)\dot{\theta}. \quad (38)$$

Thus, the equations of motion (31) and (26), we obtain

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$$\frac{\mathrm{d}}{\mathrm{dt}}V=-k_{v1}\|\dot{q}_1\|^2-k_{v2}\|\dot{q}_1\|^2-(l-\Delta x_1-\Delta x_2)(\alpha\dot{\theta}^2+\beta\dot{\theta}\Delta\theta) \tag{39}$$

which is equivalent to

$$\frac{\mathrm{d}}{\mathrm{dt}}\{V+\frac{\beta}{2}(l-\Delta x_1-\Delta x_2)\Delta\theta^2\}$$
$$=-k_{v1}\|\dot{q}_1\|^2-k_{v2}\|\dot{q}_1\|^2-(l-\Delta x_1-\Delta x_2)\alpha\dot{\theta}^2$$
$$-\frac{\beta}{2}(\Delta\dot{x}_1+\Delta\dot{x}_2)\Delta\theta^2. \tag{40}$$

If $(\Delta\dot{x}_1+\Delta\dot{x}_2)$ were always positive then the right hand side of (40) would become always negative. Or, once the magnitudes of pressing forces f_1 and f_2 become almost convergent to f_d, then it may be possible to expect that the magnitude $\beta|\Delta\dot{x}_1+\Delta\dot{x}_2|\|\Delta\dot{\theta}\|^2$ becomes small relative to other dissipative terms $k_{v1}\|\dot{q}_1\|^2$ and $k_{v2}\|\dot{q}_2\|^2$. However, there arises a certain possibility that the right hand side of (40) becomes positive. In order to avoid this possibility, it is necessary to introduce another smaller terms in addition to $\bar{u}_1$ and $\bar{u}_2$ defined by (29) and (30) in the following way:

$$\Delta u_1=-J_{01}^T\left[\beta(1-\cosh(\Delta\theta))\begin{pmatrix}\cos\theta\\-\sin\theta\end{pmatrix}\right.$$
$$\left.+\beta(-\Delta\theta+\sinh(\Delta\theta))\begin{pmatrix}\sin\theta\\\cos\theta\end{pmatrix}\right] \tag{41}$$

$$\Delta u_2=J_{02}^T\left[\beta(1-\cosh(\Delta\theta))\begin{pmatrix}\cos\theta\\-\sin\theta\end{pmatrix}\right.$$
$$\left.+\beta(-\Delta\theta+\sinh(\Delta\theta))\begin{pmatrix}\sin\theta\\\cos\theta\end{pmatrix}\right]. \tag{42}$$

Note that the first terms in Δu_1 and Δu_2 are of order $\|\Delta\theta\|^2$ and the second terms are of order $\|\Delta\theta\|^3$. Then, we consider the control input

$$\bar{\bar{u}}_1=\bar{u}_1+\Delta u_1,\quad \bar{\bar{u}}_2=\bar{u}_2+\Delta u_2. \tag{43}$$

Then, it is possible in a similar way to the derivation of equations (32) to (39) that

$$\dot{q}_1^T\Delta u_1+\dot{q}_2^T\Delta u_2=$$
$$-\frac{\mathrm{d}}{\mathrm{dt}}\left[\beta(Y_1-Y_2)(\sinh(\Delta\theta)-\Delta\theta)\right.$$
$$+\beta(l+r_1+r_2-\Delta x_1-\Delta x_2)(\cosh(\Delta\theta)$$
$$\left.\frac{\Delta\theta^2}{-2}-1)\right]+\beta(\Delta\dot{x}_1+\Delta\dot{x}_2)\frac{\Delta\theta^2}{2} \tag{44}$$

(in detail, see Appendix C). Thus, as for the closed-loop equation when $\bar{\bar{u}}_1$ and $\bar{\bar{u}}_2$ are substituted into equation (5) by setting $u_1=\bar{\bar{u}}_1$ and $u_2=\bar{\bar{u}}_{12}$, it follows that

$$\frac{\mathrm{d}}{\mathrm{dt}}\left\{V+\frac{\beta}{2}(l-\Delta x_1-\Delta x_2)\Delta\theta^2\right.$$
$$+\beta(l+r_1+r_2-\Delta x_1-\Delta x_2)(\cosh(\Delta\theta)-\frac{\Delta\theta^2}{2}-1)$$
$$\left.+\beta(Y_1-Y_2)(\Delta\theta-\sinh(\Delta\theta))\right\}$$
$$=-k_{v1}\|\dot{q}_1\|^2-k_{v2}\|\dot{q}_2\|^2-(l-\Delta x_1-\Delta x_2)\alpha\dot{\theta}^2. \tag{45}$$

It should be noted that the last term in bracket {} of the left hand side is not positive definite but it is bounded from below in the following way:

$$\beta(Y_1\text{-}Y_2)(\Delta\theta-\sinh(\Delta\theta))\geq$$
$$-\frac{f_d}{4(r_1+r_2)}(Y_1-Y_2)^2-\frac{r_1+r_2}{f_d}\beta^2(\Delta\theta-\sinh(\Delta\theta))^2. \tag{46}$$

Since V includes the quadratic term of Y_1-Y_2 whose magnitude is the double of the maginitude of the first term of the right hand side of (46), it follows that the content of bracket {} of (45) becomes positive definite in $\Delta\theta$ and Y_1-Y_2 by choosing $\beta>0$ appropriately. Thus, it is possible to conclude the following theorem:

Theorem 2. Given a desired pressing force $f_d>0$ and a rotation angle θ_d for the object, there exists a domain D_0 of stable grasping in R^{14} such that any solution to the closed-loop system equation with inputs (41) to (43) starting from an initial state in D_0 approaches asymptotically a state of stable grasping with the conditions $\theta=\theta_d$, $f_1=f_2=f_d$, $Y_1=Y_2$, $\dot{q}_1=\dot{q}_2=0$, and $\dot{z}=0$.

Lyapunov's function shown in (45) implies that control of the posture of the object may tend to meet difficulty as the object becomes more slender in width relatively to the longitudinal length, that is, l becomes smaller, because $(l-\Delta x_1-\Delta x_2)$ may happen to be negative and thereby the condition of positive definiteness of the Lyapunov's function and non-negative definiteness of its time derivative is violated.

6. FUTURE RESEARCH SUBJECTS

Extensions of the arguments presented in the previous sections to more general problems in cases of three-dimensional motions of a mechanical hand with three or four soft fingers grasping and manipulating objects with arbitrary shape would be possible and must be the most interesting from a theoretical viewpoint. However, besides such a straightforward extension there still remains a variety of research problems to be attacked. In the following, we list up some of future research subjects to be explored.

(i) Even in the case of planer motion of a dual-fingered hand with soft tips, a lot of problems remains unsolved. For example, we are not satisfied with the proposed feedback control schemes, because it is still sophisticated and rather computational. To find a simpler but more efficient feedback scheme for stable

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grasping with a desired target rotation of the object is quite important and interesting.

(ii) In relation to this, is it possible to control the target position (x, y) of the object in addition to the given pressure force f_d and rotation angle θ_d? In usual cases of regular robot tasks using a mechanical hand the position of an object handled by the hand is controlled by means of position control of a wrist on which a frame of the hand is mounted.

(iii) The proposed feedback control schemes use the exact knowledge of Jacobian matrices. Since it is already shown that approximate Jacobian matrices work well in various cases[6,7] of feedback control of robot tasks, it may be possible to relax this condition. If some approximate Jacobian matrices can be used, then there arises a problem whether one or two of the four joints of dual fingers can be passive for only realizing stable grasping with a given rotation angle.

(iv) Sensing issues are extremely important. In this paper the knowledge of not only q_1 and q_2 (measured by internal sensors implemented in finger joints) but also Y_1, Y_2 *and* θ is implicitly assumed. The last three physical variables can be measured or evaluated with the aid of vision sensing. If original shapes of the finger-tips are known and initial positions of the centers of contacts at the instant when these finger-tips touch with the surface of the object are also known, it is possible to calculate Y_1 and Y_2 on the basis of measurements of q_1, q_2, and θ by using a geometrical relation like equations (3) and (4). Note that measurement of the rotation angle of the object is easier and more reliable than evaluations of Y_1 and Y_2 by means of image analysis.

(v) If tactile sensing is available for directly measuring Y_1 and Y_2 and contact-areas, then it must provide much information about the reproducing forces f_1 and f_2, and evaluations of maximum displacements Δx_1 and Δx_2. If both f_1 and f_2 can be evaluated in a real-time manner, it is possible to use integrals of $\Delta f_i(=f_i-f_d)$ with respect to time in the design of feedback signals. In fact, such an angular momentum feedback is considerably effective in smoothing the signals of reproducing forces[9,10].

(vi) The next interesting problem is to take into consideration a variety of more general geometric shapes of the object.

(vii) In parallel, an extension of the argument to the general case of non-spherical finger-tips is quite interesting. This problem will be attacked in our next paper.

(viii) It is also important from the practical viewpoint to extend the argument by taking into consideration the effect of gravity force. In that case, we need to develop some relations of stable graspings from the dynamic viewpoint with stability test problems of stable grasping from the static viewpoint[5]. Then, static frictions that may arise between the finger-tips and the surface of the object must play a crucial role.

(ix) As a matter of course, the most difficult and interesting problem is to develop a more systematic theory of mechanics that can attack the most general three-dimensional problem of grasping and manipulation of a three-dimensional rigid or deformable object with arbitrary shape by means of a multi-fingered hand with soft finger-tips.

(x) In the sequel it will be inevitable to consider a trajectory tracking problem while maintaining given internal forces when both the position and posture of the object are given as functions in time.

7. CONCLUSIONS

Although the importance of use of soft material in covering finger-tips was already pointed out in the literature[11] and actually some of manufactured hands are equipped with soft fingers, there is a dearth of papers that analyzed dynamics of multi-fingered hands with soft finger-tips. In this paper it is shown on the basis of passivity analysis that a mechanical hand with dual soft fingers with area-contacts with an object can grasp stably an object and manipulate it with some dexterity. Based on these theoretical findings, a variety of future research directions are discussed, which will eventually unveil the secret of human capability of execution of everyday tasks with certain dexterity and versatility.

References

1. R.S. Fearing, "Simplified grasping and manipulation with dexterous robot hand", *IEEE Trans. on Robotics and Automation* **2**(4), 188–195 (1986).
2. R.M. Murray, Z. Li, and S.S. Sastry, *A Mathematical Introduction to Robotic Manipulation* (CRC Press, London, UK, 1994).
3. M. Aribi, J. Chen and M.S. Mahmoud, "Coordination and control of multi-fingered robot hands with rolling on sliding contacts", *J. Intelligent and Robotic Systems* **24**(2), 125–149 (1999).
4. S. Arimoto, *Control Theory of Nonlinear Mechanical Systems: A Passivity-based and Circuit-theoretic Approach* (Oxford Univ. Press, Oxford, UK, 1996).
5. V. Nguyen, "Contructing stable grasps", *Int. J. Robotics Research* **8**(1), 26–37 (1989).
6. C.C. Cheah, S. Kawamura, and S. Arimoto, "Feedback control for robotic manipulator with an uncertain Jacobian matrix", *J. Robotic Systems* **16**(2), 119–134 (1999).
7. Z. Doulgeri and S. Arimoto, "A force commanded impedance control for a robot finger with uncertain kinematics", *Int. J. Robotics Research*, **18**(12), 1013–1029 (1999).
8. G. Ambrosi, A. Bicchi, D.D. Rossi and E.P. Scilingo, "The role of contact area spread rate in haptic discrimination of softness", *Proc. of the 1999 IEEE Int. Conf. on Robotics and Automation*, Detroit, USA (1999) pp.305–310.
9. L.L. Whitcomb, S. Arimoto, T. Naniwa and F. Ozaki, "Adaptive model-based hybrid control of geometrically constrained robot arms", *IEEE Trans. on Robotics and Automation* **13**(1), 105–116 (1997).
10. S. Arimoto, H.-Y. Han, C.C. Cheah and S.Kawamura, "Extension of impedance matching to nonlinear dynamics of robotic task", *Systems and Control Letters* **36**, 109–119 (1999).
11. M.R. Cutkosky, *Robotic Grasping and Fine Manipulation* (Kluwer Academic, Dordrecht, The Netherlands, 1985).

APPENDIX A

It is asummed that a narrow circular strip with width $d\theta$ and radius $r \sin \theta$ in the contact-area (see Figure 6) produces a

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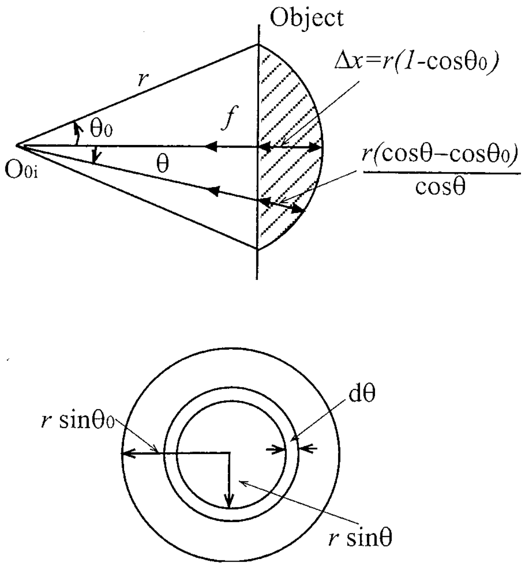

Fig. 6. Geometrical relations related to force generation from the area-contact.

reproducing force in the direction to the origin O_{0i} with the magnitude

$$k(2\pi r\sin\theta)d\theta\times r(\cos\theta-\cos\theta_0)\cos\theta \qquad \text{(A-1)}$$

where k denotes the stiffness parameter of the elastic material per unit area, $(2\pi r\sin\theta)\,d\theta$ the area of the narrow circular strip as shown in Figure 6, and

$$\frac{r(\cos\theta-\cos\theta_0)}{\cos\theta}=\left(r-\frac{r-\Delta x}{\cos\theta}\right)=r-\frac{r\cos\theta_0}{\cos\theta} \qquad \text{(A-2)}$$

denotes the length of deformation at angle θ. Since the total reproducing force with magnitude (A-1) generated from the narrow circular strip contributes to the direction of Δx (the arrow denoted by f in Figure 6) by $\cos\theta$, the total reproducing force can be expressed as

$$f=\int_0^{\theta_0}2\pi kr^2\sin\theta(\cos\theta-\cos\theta_0)\,d\theta$$
$$=\pi kr^2(1-\cos\theta_0)^2=\pi k\Delta x^2 \qquad \text{(A-3)}$$

This means that the reproducing force produced by the deformed area can be approximately expressed by an increasing function of Δx (the maximum length of displacement).

APPENDIX B (DERIVATION OF (44))

When the object is in contact with both finger-tips, the loop starting from the pivot O of the first finger through O_{01}, $O_{c.m.}$, O_2, O' (denotes the pivot of the second finger), and returning to the origin O is closed. This yields the geometrical relations

$$x=x_1+\frac{l}{2}\cos\theta-Y_1\sin\theta=x_2-\frac{l}{2}\cos\theta-Y_2\sin\theta, \qquad \text{(B-1)}$$

$$y=y_1-\frac{l}{2}\sin\theta-Y_1\cos\theta=y_2+\frac{l}{2}\sin\theta-Y_2\cos\theta. \qquad \text{(B-2)}$$

These two relations induce the two geometric constraints

$$(x_1-x_2)+l\cos\theta-(Y_1-Y_2)\sin\theta=0, \qquad \text{(B-3)}$$

$$(y_1-y_2)-l\sin\theta-(Y_1-Y_2)\cos\theta=0. \qquad \text{(B-4)}$$

By introducing the Lagrange multipliers λ_x and λ_y for (B-3) and (B-4) respectively, we define

$$Q=\lambda_x\{(x_1-x_2)+l\cos\theta-(Y_1-Y_2)\sin\theta\}$$
$$+\lambda_y\{(y_1-y_2)-l\sin\theta-(Y_1-Y_2)\cos\theta\} \qquad \text{(B-5)}$$

and consider the variational form

$$\int_{t_1}^{t_2}\{\delta(K-P+Q)\}\,dt=0 \qquad \text{(B-6)}$$

where K stands for the total kinetic energy defined by (17) and P for the potential energy defined by (22). Before taking the variation $\delta\theta$, it is necessary to note that

$$\begin{cases}\partial Y_1/\partial x_1=\sin\theta, & \partial Y_1/\partial y_1=\cos\theta,\\ \partial Y_2/\partial x_2=\sin\theta, & \partial Y_2/\partial y_2=\cos\theta\end{cases} \qquad \text{(B-7)}$$

which follows directly from constrains (B-3) and (B-4). Then, it is easy to see that

$$\begin{bmatrix}\partial Q/\partial x_1\\ \partial Q/\partial y_1\end{bmatrix}=(\lambda_x\cos-\lambda_y\sin\theta)\begin{bmatrix}\cos\theta\\ -\sin\theta\end{bmatrix}, \qquad \text{(B-8)}$$

$$\begin{bmatrix}\partial Q/\partial x_2\\ \partial Q/\partial y_2\end{bmatrix}=(\lambda_x\cos-\lambda_y\sin\theta)\begin{bmatrix}-\cos\theta\\ \sin\theta\end{bmatrix}, \qquad \text{(B-9)}$$

The vector defined by (B-8) has the same direction as that of the reproducing force in the task coordinates f_1 as shown in (10) (note that $\partial\Delta x_1/\partial x_{01}=\cos\theta$, $\partial\Delta x_1/\partial y_{01}=-\sin\theta$). This means that the force induced by this constraint should be already merged into the reproducing force f_1. As to the vector (B-9), it is possible to draw a similar conclusion. Thus, it is concluded that there does not arise any other alternative force that virtually originates from the geometircal constraints.

As to the constraint of equations (3) and (4), it is necessary to consider the dynamics of soft finger-tips in terms of $\varphi_i(i=1, 2)$. In this paper only a rough and intuitive treat of the problem is given in the following way. Since φ_i changes according to the change of centers of contact areas along the surfaces of finger-tips together with mass transfer, the quantity

$$S=\sum_{i=1,2}\left\{\frac{1}{2}\epsilon_i(\Delta x_i)\dot{\varphi}_i^2+\eta_i(Y_i-c_i+r_i\varphi_i)\right\} \qquad \text{(B-10)}$$

should be introduced and added to the Lagrangian given in (B-6), that is,

$$L=K-P+Q+S \qquad \text{(B-11)}$$

where η_i denotes a Lagrange multiplier and $\epsilon_i(\Delta x_i)$ an approximate increment of the mass concentration caused by

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each corresponding deformation of soft finger-tips. Then, the motion equation of mass-transfer can be described in such a way that

$$\frac{d}{dt}\{\epsilon_i(\Delta x_i)\dot{\varphi}_i\}+c_i(\dot{\varphi}_i)+r_i\eta_i=0,\ i=1,2 \tag{B-12}$$

and other terms

$$\begin{cases} \dfrac{\partial S}{\partial q_{ij}}=\dfrac{1}{2}\dfrac{\partial \epsilon_i(\Delta x_i)}{\partial q_{ij}}\dot{\varphi}_i^2+\eta_i\left(\dfrac{\partial Y_i}{\partial q_{ij}}+Y_i\dfrac{\partial \varphi_i}{\partial q_{ij}}\right),\ i=1,2 \\ \dfrac{\partial S}{\partial z}=\displaystyle\sum_{i=1,2}\eta_i\left(\dfrac{\partial Y_i}{\partial z}+Y_i\dfrac{\partial \varphi_i}{\partial z}\right) \end{cases} \tag{B-13}$$

are considered to be extra terms that must be added to equations (5) and (6) respectively. In (B-12), the last term of the left hand side comes from $\partial S/\partial\varphi_i$ and the second term must be a contribution due to the damping owing to some adhesive force between two surfaces of the object and finger-tips tightly contacted. Since $\epsilon_i(\Delta x_i)$ is small relative to the masses of finger links and the object, $\eta_i(i=1,2)$ must be very small. Therefore in the paper we omit the effect of the constraint described by (3) and (4). This approximation is almost equivalent to the assumption that the finger-tips are made of distributed massless springs. However, in case of computer simulation the terms (B-13) should be added to (5) and (6). Note again that additions of (B-12) and (B-13) to (5) and (6) do not violate all passivity relations discussed in sections 3 to 5.

APPENDIX C

In an analogous manner to the derivation of (37), it follows from the geometric constraints (B-1) and (B-2) that

$$(\dot{x}_{01}-\dot{x}_{02})\cos\theta-(\dot{y}_{01}-\dot{y}_{02})\sin\theta =(\Delta\dot{x}_1+\Delta\dot{x}_2)+(Y_1-Y_2)\dot{\theta}. \tag{C-1}$$

By referring to this equation together with (35) and (36), it is possible to calculate the inner product between ($\dot{q}_1$, $\dot{q}_2$) and (Δu_1, Δu_2) in the following way:

$$\begin{aligned} &-(\dot{q}_1^T\Delta u_1+\dot{q}_2^T\Delta u_2)\\ &=\beta(\cosh(\Delta\theta)-1)\{(\dot{x}_{01}-\dot{x}_{02})\cos\theta-(\dot{y}_{01}-\dot{y}_{02})\sin\theta\}\\ &\quad+\beta(-\Delta\theta+\sinh(\Delta\theta))\{(\dot{x}_{01}-\dot{x}_{02})\sin\theta+(\dot{y}_{01}-\dot{y}_{02})\cos\theta\}\\ &=\beta\Big[(\cosh(\Delta\theta)-1)\{(\Delta\dot{x}_1+\Delta\dot{x}_2)+(Y_1-Y_2)\dot{\theta}\}\\ &\quad+(\sinh(\Delta\theta)-\Delta\theta)\{(\dot{Y}_1-\dot{Y}_2)+(l+r_1+r_2-\Delta x_1-\Delta x_2)\dot{\theta}\}\Big]\\ &=\beta\frac{d}{dt}\Big[(l+r_1+r_2-\Delta x_1-\Delta x_2)(\cosh(\Delta\theta)\\ &\quad-\frac{\Delta\theta^2}{2}-1)+(Y_1-Y_2)(\sinh(\Delta\theta)-\Delta\theta)\Big]\\ &\quad-\beta(\Delta\dot{x}_1+\Delta\dot{x}_2)\frac{\Delta\theta^2}{2}. \end{aligned} \tag{C-2}$$

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