

## Partial Observations

### 8.1 Introduction

In this chapter we discuss the quantized detector network (QDN) approach to *partial observations*, or the extraction, during an extended-in-time quantum process, of only some of the quantum information embedded in a detector amplitude. In order to deal with partial questions, we need first to discuss how QDN deals with *maximal questions*, which means looking at all the detectors at a given stage.

### 8.2 Observables

Our preoccupation with detectors rather than systems under observation (SUOs) is nothing new in quantum mechanics (QM). Indeed, the primary significance of what could be observed was a guiding principle when Heisenberg formulated his matrix mechanics approach to QM (Heisenberg, 1925). The conventional position in QM is that the only important operators in the theory are the *observables*. These correspond closely to the variables used in classical mechanics (CM) to describe measurable quantities, such as energy, momentum, electric charge, and so on.

In standard QM, observables are generally assumed to be self-adjoint operators (Hermitian operators in the case of finite-dimensional Hilbert spaces) so that they have real eigenvalues, and these are the QM analogues of CM variables. Apart from that proviso, operators representing observables have few restrictions.

This generality raises significant questions. A particularly critical question is whether a given “observable” makes empirical sense. There is no theorem that proves that every self-adjoint operator corresponds to something that can be actually observed in the laboratory. For example, in one-dimensional wave mechanics, the operator  $\widehat{pppxppp}$  is self-adjoint and its classical analogue,  $pppxppp$ , is a perfectly regular function over phase space. But we know of no experiment that can measure such a quantity directly, whereas experiments to measure  $p$  or  $x$  directly could be devised.

The problem as we see it in this book is that the *observable* concept in standard QM puts the cart before the horse: it is first implicitly assumed in QM that states of SUOs can be created, and only then is the question of what can be done to and on those states raised. QDN does things the other way around: the apparatus that creates the states and detects the signals has to come first. Indeed, that is all that is needed. “State preparation” defines the states contextually and the outcome detectors define what information can be obtained. In QDN, “observables” are nothing but the signals in final stage detectors plus the context that informs the observer as to the meaning of those signals.

### Transformations and Symmetries

In the twentieth century, great advances in quantum physics were made, driven by relativistic transformation theory, leading to the Lorentz covariant formulation of quantum fields known as relativistic quantum field theory (RQFT). That theory is generally regarded as the best theory to describe the phenomenology of elementary particles. However, there are some deep conceptual issues concerning the interface between the classical world of the observer’s knowledge base and the nonclassical behavior of labstates.

An important factor here is the relationship between different observers. In the standard QM approach to observables, the transformation properties of those observables are regarded as crucial. Indeed, Dirac’s approach to QM was developed in part as an analogue of CM “transformation theory” (Dirac, 1958; Goldstein, 1964; Leech, 1965).

In QDN the issue of transformation theory is avoided by the assertion that a given laboratory needs no transformation. And if it is required to discuss the relationship between two different laboratories (which we have to remind the reader consist of atoms and molecules that cannot just pass through each other without significant interaction, none of which is taken into account in standard QM), then we simply regard the two laboratories as a single larger laboratory defined by the context of the situation. For example, a Doppler shift experiment is conventionally described in terms of a source of light moving relative to a detector. In QDN terms, all of that can be described as one experiment in a single laboratory, albeit one that changes from stage to stage. Laboratories in QDN are not restricted to single inertial frames.

There is an interesting question that can be posed here, and its resolution has everything to do with the dominant principle of contextuality underpinning this book: *Given a distant source of light, such as a remote galaxy on the other side of the Universe, how can astronomers observing light from that source consider themselves to be part of a single laboratory that includes that galaxy (which might have long ago been destroyed)?*

The answer is *context*. A single detected photon, no matter how much “red shifted” it was,<sup>1</sup> could not carry any contextual information. But the actual

<sup>1</sup> Once again, we have to resort to realist language to convey our meaning, though it is an oversimplification to do so.

described scenario gives the game away: how could we even say that there was a galaxy acting as a source of light without having observed sufficient light from it to establish that fact? That degree of observation thereby provides the context that allows us to think of that remote galaxy and our detectors as part of the same experiment.

In fact, such context is generally built up over relatively long periods of time and depends on technology. Thousands of years ago, astronomers could only see via their naked eyes a strange blob of diffuse light where M31, the Andromeda Galaxy, is in the night sky. Then as telescopes were developed, followed by long-exposure photography, sufficient observations were made to establish the context that we know today. Only that relatively recent context allows us to say whether a signal from Andromeda is red-shifted or blue-shifted light.<sup>2</sup>

### 8.3 Maximal Questions

In this section, dependence on the temporal index  $n$  is suppressed, as all questions are asked at some given stage  $\Sigma_n$ .

Given a rank- $r$  quantum register, an arbitrary pure labstate  $\Psi$  is of the general form

$$\Psi = \Psi^0 \mathbf{0} + \sum_{i=1}^r \Psi^i \widehat{\mathbb{A}}^i \mathbf{0} + \sum_{1 \leq i < j \leq r} \Psi^{ij} \widehat{\mathbb{A}}^i \widehat{\mathbb{A}}^j \mathbf{0} + \dots + \Psi^{12\dots r} \widehat{\mathbb{A}}^1 \widehat{\mathbb{A}}^2 \dots \widehat{\mathbb{A}}^r \mathbf{0}, \tag{8.1}$$

where we do not rule out superpositions of elements with different signality. Labstates are generally normalized to unity, so the coefficients in (8.1) satisfy the condition

$$\overline{\Psi} \Psi = |\Psi^0|^2 + \sum_{i=1}^r |\Psi^i|^2 + \sum_{1 \leq i < j \leq r} |\Psi^{ij}|^2 + \dots + |\Psi^{12\dots r}|^2 = 1. \tag{8.2}$$

**Example 8.1** An arbitrary normalized labstate in a rank-two quantum register is of the form

$$\Psi = \{\Psi^0 \mathbb{I}^{[2]} + \Psi^1 \widehat{\mathbb{A}}^1 + \Psi^2 \widehat{\mathbb{A}}^2 + \Psi^3 \widehat{\mathbb{A}}^1 \widehat{\mathbb{A}}^2\} \mathbf{0}, \tag{8.3}$$

with  $|\Psi^0|^2 + |\Psi^1|^2 + |\Psi^2|^2 + |\Psi^3|^2 = 1$ .

The interpretation of these coefficients is based on the Born rule in standard QM (Born, 1926): if the apparatus is in labstate (8.3) prior to the observer looking at both detectors “simultaneously” (which is possible in QDN by definition), then the probability of each detector being found in its ground state is  $|\Psi^0|^2$ , the probability of detector 1 being in its signal state *and* detector 2 being in its

<sup>2</sup> In fact, Andromeda is moving *toward* our galaxy, so light from ordinary sources in Andromeda should show a small blue shift.

ground state is  $|\Psi^1|^2$ , the probability of detector 1 being in its ground state *and* detector 2 being in its signal state is  $|\Psi^2|^2$ , and the probability of both detectors being in their signal states is  $|\Psi^3|^2$ . Note that  $|\Psi^1|^2$  is *not* the probability that there is a signal in detector 1. That probability is given by  $|\Psi^1|^2 + |\Psi^3|^2$ .

**Example 8.2** An observer prepares a pure labstate  $\Phi$  in a rank-four quantum register. Show that the probability  $Pr(\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4|\Phi)$  that the observer would find detectors 1 and 4 in their signal states and detectors 2 and 3 in their ground states is given by

$$Pr(\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4|\Phi) = \overline{\Phi}\widehat{P}^1P^2P^3\widehat{P}^4\Phi. \tag{8.4}$$

**Solution** In this case, we need to ask the maximal question  $\overline{\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4}$  of the labstate  $\Phi$ , giving the amplitude  $\mathcal{A}(\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4|\Phi) \equiv \overline{\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4}\Phi$ . Then the Born rule gives

$$\begin{aligned} Pr(\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4|\Phi) &\equiv |\mathcal{A}(\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4|\Phi)|^2 \\ &= (\overline{\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4}\Phi)^*(\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4\Phi) \\ &= (\overline{\Phi}\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4)(\overline{\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4}\Phi) \\ &= \overline{\Phi}(\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4\overline{\mathbf{1}^1\mathbf{0}^2\mathbf{0}^3\mathbf{1}^4})\Phi \\ &= \underbrace{\overline{\Phi}(\mathbf{1}^1\overline{\mathbf{1}^1})}_{\widehat{P}^1} \underbrace{(\mathbf{0}^2\overline{\mathbf{0}^2})}_{P^2} \underbrace{(\mathbf{0}^3\overline{\mathbf{0}^3})}_{P^3} \underbrace{(\mathbf{1}^4\overline{\mathbf{1}^4})}_{\widehat{P}^4}\Phi \\ &= \overline{\Phi}\widehat{P}^1P^2P^3\widehat{P}^4\Phi. \end{aligned} \tag{8.5}$$

But it is straightforward to show that for a rank-four register,

$$\widehat{P}^1P^2P^3\widehat{P}^4 = \widehat{P}^1\widehat{P}^2\widehat{P}^3\widehat{P}^4, \tag{8.6}$$

where each of the operators on the right-hand side is a register operator. This then gives the required result (8.4).

For a rank- $r$  register, we shall call a register product of  $r$  distinct register projection operators a *maximal question*. The above example illustrates how any maximal question for a rank- $r$  quantum register can be related to the tensor product of  $r$  distinct bit projection operators, one for each detector in the register.

There are three points to note here. First, Eq. (8.6) holds only because the left-hand side is a register operator, being the tensor product of  $r$  individual bit operators, one for each detector in the register. Second, a maximal question can be identified with a specific element of the preferred basis. Since there are  $2^r$  elements in the latter, we deduce that there is a total of  $2^r$  distinct maximal questions. The third point is a technical one: the product concepts on each side of (8.6) are different. The left-hand side is the tensor product of bit operators, the right-hand side is the product of register operators.

### 8.4 Partial Questions

The above example shows how to ask a specific question of each and every detector in a quantum register at a given stage. For a rank- $r$  quantum register, any maximal question involves a product of  $r$  distinct register projection operators. For each detector  $i$ ,  $1 \leq i \leq r$ , there are two related register operators,  $\mathbb{P}^i$  and  $\widehat{\mathbb{P}}^i$ , which form a *conjugate pair*. Therefore there are exactly  $2^r$  distinct maximal questions, as stated above.

In the real world, however, observers could choose to ask *partial questions*, which involve looking at only some (or even none) of the detectors. The simplest example of a partial question involves the normalization condition

$$\overline{\Psi}\Psi = 1, \tag{8.7}$$

because we can always write  $\overline{\Psi}\Psi = \overline{\Psi}\mathbb{I}^{[r]}\Psi$ , where  $\mathbb{I}^{[r]}$  is the register identity operator. We may identify this operator with the question

**What is the probability that every detector is either in its ground state or signal state?**

and call this a *rank-zero partial question*, because it involves looking at no (i.e. zero) detectors.

Now suppose we wanted to ask a question involving just one detector, such as the  $a$ th, where  $1 \leq a \leq r$ . Given (8.7), we insert the register identity operator  $\mathbb{I}^{[r]}$  as before and use the property

$$\mathbb{P}^a + \widehat{\mathbb{P}}^a = \mathbb{I}^{[r]}, \quad a = 1, 2, \dots, r. \tag{8.8}$$

Using this property and linearity, we find

$$\overline{\Psi}\mathbb{P}^a\Psi + \overline{\Psi}\widehat{\mathbb{P}}^a\Psi = 1. \tag{8.9}$$

Each of the terms on the left-hand side is nonnegative. By inspection,  $\overline{\Psi}\mathbb{P}^a\Psi$  is the probability that detector  $a$  would be found in its ground state, while  $\overline{\Psi}\widehat{\mathbb{P}}^a\Psi$  is the probability that  $a$  would be found in its signal state, regardless of what was going on in any of the other  $r - 1$  detectors. We will refer to each of these partial questions as a *rank-one partial question*.

This process can be extended naturally to higher rank partial questions, until we reach rank  $r$ , which are the maximal questions we discussed in the previous section.

**Example 8.3** Given a labstate  $\Psi$  prepared in a rank-637 register, what is the probability that if the observer looked only at detectors 99, 323, and 438, they would find 99 and 323 each in its ground state and 438 in its signal state?

**Solution**

The required probability  $Pr$  is given by the expectation value

$$Pr = \overline{\Psi}\mathbb{P}^{99}\mathbb{P}^{323}\widehat{\mathbb{P}}^{438}\Psi. \tag{8.10}$$

It will be clear from the above that the set of all partial questions involves expectation values of all possible products of the register projection operators. This leads to the following theorem.

**Theorem 8.4** *For a rank- $r$  classical or quantum register  $\mathcal{Q}^{[r]}$ , the number of possible partial questions is  $3^r$ .*

*Proof* To prove the theorem, we determine the number of partial questions of each rank and then add up all those numbers.

There is only one rank zero partial question.

The observer could go to each of the  $r$  detectors one by one and ask one of two possible questions of it: the two questions that can be asked at detector  $i$  are given by the register projectors  $\mathbb{P}^i, \widehat{\mathbb{P}}^i$ . These are rank-one partial questions, so we conclude that there is a total of  $2^r$  rank-one partial questions.

Assuming  $r > 1$ , the observer could now ask rank-two partial questions, involving only two distinct detectors in the register. Given a rank- $r$  register, there is a total of  $r(r - 1) = \binom{r}{2}$  distinct pairs, and for each pair of choices,  $2^2$  alternative questions can be asked. For example, for the choice  $i < j$ , we can ask the four questions  $\mathbb{P}^i\mathbb{P}^j, \widehat{\mathbb{P}}^i\mathbb{P}^j, \mathbb{P}^i\widehat{\mathbb{P}}^j$ , and  $\widehat{\mathbb{P}}^i\widehat{\mathbb{P}}^j$ . Therefore, there is a total of  $2^2 \binom{r}{2}$  such partial questions.

We can continue this argument until we reach the maximal questions, which are rank- $r$  partial questions. There is only one way of choosing  $r$  objects from  $r$  objects, and  $2^r$  possible maximal questions to be asked of that choice. Hence we find the grand total  $T_Q$  of distinct partial question operators to be given by

$$T_Q = 1 + 2\binom{r}{1} + 2^2\binom{r}{2} + \dots + 2^r\binom{r}{r} = (1 + 2)^r = 3^r, \tag{8.11}$$

as asserted. □

### 8.5 Partial Question Eigenvalues

Every partial question has the property that each preferred basis element is an eigenstate of it, with an eigenvalue of either zero or one. Using the signal basis representation (SBR), the eigenvalue in each case can be readily read off. For example, in a rank-five register, the state  $\mathbf{0}^1\mathbf{1}^2\mathbf{1}^3\mathbf{0}^4\mathbf{1}^5$  is an eigenstate of the partial question operators  $\mathbb{P}^1$  and  $\mathbb{P}^1\widehat{\mathbb{P}}^4$ , with eigenvalues 1 and 0, respectively. On the other hand, using the computational basis representation (CBR) is not so convenient here. The given state  $\mathbf{0}^1\mathbf{1}^2\mathbf{1}^3\mathbf{0}^4\mathbf{1}^5$  has the CBR **22**, so we see

$$\mathbb{P}^1\mathbf{22} = \mathbf{22}, \quad \mathbb{P}^1\widehat{\mathbb{P}}^4\mathbf{22} = 0, \tag{8.12}$$

but the respective eigenvalues cannot now be directly read off. Since the CBR is generally useful, we need to quantify the action of the partial questions on the CBR. We do this as follows. The set  $\{\mathbb{I}^{[r]}, \mathbb{P}^1, \dots, \widehat{\mathbb{P}}^1\widehat{\mathbb{P}}^2 \dots \widehat{\mathbb{P}}^r\}$  of partial questions

Table 8.1 Question eigenvalues for a rank-two register

	$\mathbf{0^1 0^2 = 0}$	$\mathbf{1^1 0^2 = 1}$	$\mathbf{0^1 1^2 = 2}$	$\mathbf{1^1 1^2 = 3}$
$\mathbb{Q}^1 \equiv \mathbb{I}^{[2]}$	$Y^{1,0} = 1$	$Y^{1,1} = 1$	$Y^{1,2} = 1$	$Y^{1,3} = 1$
$\mathbb{Q}^2 \equiv \mathbb{P}^1$	$Y^{2,0} = 1$	$Y^{2,1} = 0$	$Y^{2,2} = 1$	$Y^{2,3} = 0$
$\mathbb{Q}^3 \equiv \widehat{\mathbb{P}}^1$	$Y^{3,0} = 0$	$Y^{3,1} = 1$	$Y^{3,2} = 0$	$Y^{3,3} = 1$
$\mathbb{Q}^4 \equiv \mathbb{P}^2$	$Y^{4,0} = 1$	$Y^{4,1} = 1$	$Y^{4,2} = 0$	$Y^{4,3} = 0$
$\mathbb{Q}^5 \equiv \widehat{\mathbb{P}}^2$	$Y^{5,0} = 0$	$Y^{5,1} = 0$	$Y^{5,2} = 1$	$Y^{5,3} = 1$
$\mathbb{Q}^6 \equiv \mathbb{P}^1 \mathbb{P}^2$	$Y^{6,0} = 1$	$Y^{6,1} = 0$	$Y^{6,2} = 0$	$Y^{6,3} = 0$
$\mathbb{Q}^7 \equiv \widehat{\mathbb{P}}^1 \mathbb{P}^2$	$Y^{7,0} = 0$	$Y^{7,1} = 1$	$Y^{7,2} = 0$	$Y^{7,3} = 0$
$\mathbb{Q}^8 \equiv \mathbb{P}^1 \widehat{\mathbb{P}}^2$	$Y^{8,0} = 0$	$Y^{8,1} = 0$	$Y^{8,2} = 1$	$Y^{8,3} = 0$
$\mathbb{Q}^9 \equiv \widehat{\mathbb{P}}^1 \widehat{\mathbb{P}}^2$	$Y^{9,0} = 0$	$Y^{9,1} = 0$	$Y^{9,2} = 0$	$Y^{9,3} = 1$

contains  $3^r$  elements. We define  $\mathbb{Q}^1 \equiv \mathbb{I}^{[r]}, \mathbb{Q}^2 \equiv \mathbb{P}^1, \dots, \mathbb{Q}^{3^r} \equiv \widehat{\mathbb{P}}^1 \widehat{\mathbb{P}}^2 \dots \widehat{\mathbb{P}}^r$ . This choice of labeling is arbitrary, there being no obvious way to order a complete set of partial questions. Then for any partial question  $\mathbb{Q}^P, P = 1, 2, \dots, 3^r$ , and any CBR element  $\mathbf{i}, i = 0, 1, 2, \dots, 2^r - 1$ , we write

$$\mathbb{Q}^P \mathbf{i} = Y^{P,i} \mathbf{i}, \tag{8.13}$$

where the *question eigenvalue*  $Y^{P,i}$  is either zero or unity.

**Example 8.5** For a rank-two register, there are  $3^2 = 9$  distinct partial questions and  $2^2 = 4$  distinct CBR elements. Table 8.1 shows the question eigenvalues.

### 8.6 Identity Classes

A full partial question set can be divided into groups of operators that sum up, in that group, to the register identity. Each such group will be called an *identity class*. A rank- $r$  register has  $2^r$  identity classes.

**Example 8.6** A rank-one register has two identity classes, given by  $C^{1,1} \equiv \{\mathbb{I}^{[1]}\}$  and  $C^{2,1} \equiv \{\mathbb{P}^1, \widehat{\mathbb{P}}^1\}$ .

**Example 8.7** A rank-two register has four identity classes, given by

$$\begin{aligned} C^{1,2} &\equiv \{\mathbb{I}^{[2]}\}, \\ C^{2,2} &\equiv \{\mathbb{P}^1, \widehat{\mathbb{P}}^1\}, \\ C^{3,2} &\equiv \{\mathbb{P}^2, \widehat{\mathbb{P}}^2\}, \\ C^{4,2} &\equiv \{\mathbb{P}^1 \mathbb{P}^2, \widehat{\mathbb{P}}^1 \mathbb{P}^2, \mathbb{P}^1 \widehat{\mathbb{P}}^2, \widehat{\mathbb{P}}^1 \widehat{\mathbb{P}}^2\}. \end{aligned} \tag{8.14}$$

In Table 8.1, the four identity classes are separated by horizontal lines. A given identity class consists of partial questions of the same rank, so we shall refer to each class by its rank. In the above example,  $C^{1,1}$  and  $C^{1,2}$  are rank-zero identity classes;  $C^{2,1}$ ,  $C^{2,2}$ , and  $C^{3,2}$  are rank-one identity classes; and  $C^{4,2}$  is a rank-two identity class.

Identity classes are related to probability conservation. We shall find that if we want to conserve probability in any calculation, we need to restrict partial questions to the same identity class. It may then be necessary to relabel the partial questions and their question eigenvalues with an extra label identifying individual identity classes.

### 8.7 Needles in Haystacks

The classification of question rank and identity class sheds some light on how the unimaginable complexity of the real world can be comprehended by intelligent observers. Suppose an observer wanted to model the Universe by an enormously large number  $N$  of qubits, giving a quantum register  $\mathcal{Q}^{[N]}$  of rank  $2^N$ . Suppose now that that observer was investigating their environment by asking a limited number of partial questions. This is a typical scenario in empirical science: resources are not infinite and experimentalists can only do so much. By inspection of Table 8.1, we see an interesting pattern, one that would be repeated in the general case. If we ask no questions, then that is represented by the rank-zero identity class. We see from the question eigenvalues for such a question, denoted  $\mathbb{Q}^1$  in Table 8.1, that the answer for each possible labstate is one, meaning that we can extract no new information about the system under observation (SUO) from such a question. But that question has cost us nothing.

Moving on, we may now decide to ask rank-one questions, represented by  $\mathbb{Q}^2$  and  $\mathbb{Q}^3$  in Table 8.1. Now we start to get some real information about the state of the SUO, but it is also starting to become expensive.

This process could continue, with greater rank questions being posed, with more information coming out but at ever increasing cost.

There is an interesting trade-off. Looking at Table 8.1, we note that for a given labstate, the maximal rank identity class partial questions all have answer zero except one of them. This enormously simplifies the problem of finding a signal, in that should we find an answer of one halfway through this process, we can immediately stop, because we can be sure all the remaining answers are zero.

We can now appreciate what it means to do experiments and why they are done. Experiments are careful arrangements of partial questions of the same identity class, designed to eliminate as many zero-value answers and home in on unity answers (those for which  $Y^{\theta,i} = 1$ ), guided by theory. Such searches can require planning to choose the right questions and the expenditure of enormous resources as in the case of the search for the Higgs particle at the Large Hadron Collider. That task was far more of a technical problem than searching for a



needle in a haystack.<sup>3</sup> At the end of the day, we note that the result of the search was a single number, that is, one, which means *yes, the Higgs particle exists* (according to our contextual interpretation of the data).

This line of thinking also encourages experimentalists not to give up but to continue searches that appear to be unsuccessful. For instance, there is at this time (2017) no direct empirical evidence for supersymmetric partners of various particles such as electrons and photons. Because the number of zero-value question eigenvalues is potentially vast, a lack of confirmation so far does not mean that a *yes* answer does not exist.

<sup>3</sup> Which can be done quickly with the right apparatus, such as a metal detector.