# THE MASS AND LIGHT DISTRIBUTION OF THE GALAXY: A THREE-COMPONENT MODEL

J. P. Ostriker and J. A. R. Caldwell
Princeton University Observatory

ABSTRACT

The galaxy is represented schematically by a three-component model: a disc having the form of a modified exponential distribution, a spheroidal (bulge + nucleus) component and a dark halo component which, following the nomenclature of Einasto, we call the corona. The shapes of these components, chosen on the basis of observations of other galaxies, are consistent with imperfect knowledge of the Galaxy; values of the adjustable parameters are chosen by a least square minimization technique to best fit the most accurate kinematical and dynamical galactic observations. The local radius, circular velocity and escape velocity are found to be ($R_0$, $V_0$, $V_{esc}$) = (9.05 ± 0.33 kpc, 247 ± 13 km/s, 550 ± 24) quite close to the values determined from observations directly. The masses in the three components are ($M_D$, $M_{Sp}$, $M_C$) = (0.78 ± 0.13, 0.81 ± 0.09, 20.3) × $10^{11}$ $M_\odot$ for a model with coronal radius of 335 kpc. If the quite uncertain coronal radius is reduced to 100 kpc the model is essentially unchanged except that then $M_C$ = 6.65 × $10^{11}$ $M_\odot$. The disc and spheroidal components have in either case luminosities (in the visual band of ($L_D$, $L_{Sp}$) = (2.0 ,0.2 ) × $10^{10}$ $L_\odot$. The galaxy is a normal giant spiral of type Sb-Sc similar to NGC 4565.

## 1. MODEL COMPONENTS

### 1.1 Background

Many investigators have constructed models of the galaxy on the basis of observations made necessarily from the somewhat unfortunate vantage of the sun, and over the years both the modeling techniques and the accuracy of the input data have been steadily refined. But several aspects of the galaxy such as the ratio of the disc to spheroidal components in the inner parts or the mass distribution in the outer parts are essentially unobservable with present techniques. Recently, however, observations of other galaxies have improved so that we now know more about some of the dynamical and kinematic properties of M31 or



*W. B. Burton (ed.), The Large-Scale Characteristics of the Galaxy, 441–450.*
*Copyright © 1979 by the IAU.*

https://doi.org/10.1017/S0074180900014911 Published online by Cambridge University Press

NGC 4565 than of our own Galaxy. Thus, in this work we have chosen to assume that our galaxy is a normal giant spiral with two mass components following the light distribution observed in two-component fits to the surface brightness of other galaxies (e.g., de Vaucouleurs 1959) and to fix the adjustable parameters by fitting the best determined kinematical and dynamical properties of the Galaxy. While a number of components with varying degrees of flattening (as in the Schmidt, 1965, model) could have been used and may in fact be required, observational constraints on the inner parts of the galaxy do not allow one to subdivide the model into more than a flat and an approximately spherical part (the necessary flattening of the latter due to the gravitational field of the former being simply ignored). In addition, there is dynamical evidence (Ostriker et al. 1974; Turner 1976; Salpeter 1977) for an extended component of dark matter (cf. Spinrad et al. 1978) having a mass exceeding that in the inner parts of the galaxy and a rate of density decline of $\rho \propto r^{-2}$ approximately. Note that the modeling is based only on observations and not on any theoretical preconceptions, although there are reasons (Ostriker and Thuan 1975; Gunn 1977; Fall and Rees 1978) for believing that galaxy formation might occur in distinct stages corresponding to coronal, spheroidal and disc components. Let us now discuss in detail the parameterization of the three components.

## 1.2 The Disc

Freeman (1970) following de Vaucouleurs (1959) showed that the discs of flattened galaxies obey, in their outer parts, a simple exponential law with the surface brightness (in magnitudes/sec$^2$) falling linearly with projected radius. But in both its molecular and atomic components the gas content of our galaxy declines within 5 kpc of the center (cf. Scoville and Solomon 1975) after increasing in a more or less exponential fashion inwards towards this radius. Kormendy (1977a) finds that stellar discs also show a substantial decline in their inner parts (< 5 kpc) from that expected on the basis of the exponential law. We thus fit the discs with a surface mass distribution specified by three parameters,

$$\Sigma(\varpi) = \Sigma_D[\exp(-\varpi/\varpi_D) - \exp(-\varpi/\varpi_G)] , \qquad (1)$$

where $\varpi$ is the radial coordinate in the disc and ($\varpi_D$, $\varpi_G$) are radii characterizing the disc scale length and that of the central gap. A similar form has been used by Einasto (1970).

## 1.3 Spheroid

The light profile in elliptical galaxies is fairly well fit by a Hubble law (cf. Oemler 1976; Kormendy 1977b) in which the surface brightness falls as $r^{-2}$ and the volume emissivity as $r^{-3}$. Since in the solar vicinity the population II tracers follow a similar distribution (Oort 1965), we thus adopt the Hubble law for the galactic surface mass density $\Sigma(\varpi) = \Sigma_{Sp}(1 + \varpi/\varpi_{Sp})^{-2}$. There is evidence (Kormendy (1977b) and

https://doi.org/10.1017/S0074180900014911 Published online by Cambridge University Press

Spinrad et al. (1978) that at large radii the surface brightness of other galaxies declines somewhat faster than given by the Hubble law, but a cutoff or correction to the Hubble law is unnecessary since at those radii the mass density in the computed models is determined by the corona, not the spheroidal component and thus the dynamical fitting procedure would be quite insensitive to any outer cutoff of the spheroid component. The three-dimensional ($r = |\mathbf{r}|$) density distribution $\rho$, determined by inverting Abell's equation is

$$\rho_{Hub}(r) = 3.75\,\rho_{Hub} \times \begin{cases} \left[\dfrac{3-z}{\sqrt{z}}\,\ell n\left(\dfrac{1+\sqrt{z}}{\sqrt{1-z}}\right) - 3\right]/z^2, & z < 1 \\ 1/3.75\,, & z = 1 \\ \left[\dfrac{3+z}{\sqrt{z}}\left(\tan^{-1}\left(\dfrac{1}{\sqrt{z}}\right) + \dfrac{\pi}{2}\right) - 3\right]/z^2, & z > 1 \end{cases} \quad (2a)$$

where

$$z \equiv |(r/r_{Sp})^2 - 1|\,. \quad (2b)$$

To this we add a small ($\sim 10^8\ M_\odot$) nuclear mass component $M_N$ which is seen in the infrared, and detected dynamically in M31 where a similar infrared nuclear profile is observed. Thus three parameters determine the spheroidal mass distribution:

$$\rho_{Sp}(r) = M_N \frac{\delta(r)}{4\pi r^2} + \rho_{Hub}(r)\,. \quad (2c)$$

## 1.4 The Dark Corona

We choose the analytically simple two-parameter form

$$\rho_C(r) = \frac{\rho_C}{1 + (r/r_C)^2}\,, \quad (3)$$

which at large radii $r \gg r_C$ approaches the density distribution in an isothermal sphere.

A cutoff radius $R_0$, applied to all components ($\rho = 0$ for $r > R_0$), is fixed in advance for each model rather than treated as an adjustable parameter to be determined by observations. In all we have nine parameters required to specify a given model.

https://doi.org/10.1017/S0074180900014911 Published online by Cambridge University Press

## 2. OBSERVATIONAL CONSTRAINTS

Given values of the model parameters, we can calculate the density, gravitational potential and any stellar orbit. Thus the galactic rotation curve, Oort constants,etc. that would be observed from any point in the Galaxy can be computed and compared with observations made locally. On minimizing the difference between observed and computed quantities we determine best values for the model parameters and for the point of observation.

The observational inputs used to constrain the model may conveniently be divided into three groups of four.

### 2.1 Local Constraints

The most important observed quantity for all the parameters is fortunately fairly well known. From a search of the literature we have arrived at the value of the sun's position $R_\Theta = 8.9 \pm 0.6$ kpc. Details of the determination will be given elsewhere but we have essentially combined 18 independent determinations (using R R Lyrae stars, globular clusters, etc.) weighting them in the final result inversely as the square of the individual fractional error.

For Oort's constant A an analysis of seven methods and 45 sources gave 15.2 ± 0.4 km/s/kpc.

As is well known, Oort's constant B is quite poorly known. We examined 18 determinations based either directly on proper motions or indirectly upon the ratio B/A obtained from the velocity ellipsoid axis ratios. Weighting together all determinations from the first method gave -10.4 ± 10 km/s/kpc and from the second gave -9.7 ± 2 km/s/kpc, but we noticed that the more recent studies gave systematically larger (in absolute value) estimates of B as well as larger estimates of the error, although the data had presumably improved! This makes a simple compounding of the weighted results nonsense and we decided to limit consideration to the most recent sources (Fatchikhin 1970; Vasilevskis and Klemola 1971; Fricke and Tsioumis 1975 for proper motions; Erikson (1975) for the velocity ellipsoid). These yielded B = -11.6 ± 2.6 and B = -11 ± 2 respectively. By averaging the results from both methods then assigning an estimated uncertainty of 20% to the mean we obtain our adopted value of B = -11.3 ± 2.3 km/s/kpc.

The adopted values in standard units of ($R_\Theta$, A, B) are (8.9 ± 0.6, 15.2 ± 0.4, -11.3 ± 2.3) which are consistent with the IAU (1964) system (10 ± 1, 15 ± 1.5, -10 ± 2) and Oort's suggested revision (8.7 ± 0.6, 16.9 ± 0.9, -9.0 ± 1.5) in Plaut and Oort (1975).

The remaining local constraint is $\mu_\Theta$ the local mass per unit area of the galactic plane in the flattened component. On the basis of VanderVoort's (1970) and Toomre's (1972) discussion of Oort's (1960) $K_z$ study we take $\mu_\Theta = 90 \pm 9\ M_\Theta/pc^2$.

https://doi.org/10.1017/S0074180900014911 Published online by Cambridge University Press

### 2.2 Rotation Curve Constraints

Following the method of Toomre (1972) we judged that the large scale galactic structure information inherent in the interior region could satisfactorily be incorporated into the model by having it fit the observed maximum recession velocities, $\Delta v$, at four longitudes corresponding to $\sin^{-1} \ell$ = (0.0671, 0.3448, 0.5747 and 0.8046) corresponding to R = .58, 3, 5, and 7 kpc, for $R_0$ scaled to 8.7. For the first of these we take $\Delta v = 250.5 \pm 8.0$ km/s from Rougoor and Oort (1960). For the remaining three velocities, which characterize the main hump of the rotation curve, we take $\Delta v$ = (131.2 ± 6.3, 98.1 ± 6.9, 54.8 ± 7.1) km/s from Tuve and Lundsager's (1973) smoothed rotation curve $B_0$. At the adopted separation in longitude, the points may be considered to be statistically independent; the uncertainties were set at 3% of the typical corresponding circular velocity following Burton's (1971) suggestion that streaming motions associated with the spiral arms of that order mask the underlying smooth curve.

### 2.3 Supplemental Constraints

These are additional observations which are necessary to define the model but which do not affect significantly the rotation curve in the vicinity of the sun.

The nuclear mass $M_N$ was taken to be $(1.6 \pm 0.4) \times 10^8 M_\odot$ from dynamical studies of M31. Oort (1977) and Tremaine (1976) obtain estimates of $(0.3, 0.6) \times 10^8$ respectively from analysis of the likely number of globular clusters sinking to the center due to dynamical friction so it is possible that our adopted value is somewhat too large. The halo core was fixed at $r_{sp} = 0.11 \pm 0.02$ kpc from comparison of infrared photometry of M31 and the Galaxy's nuclear regions.

The remaining two constraints are designed to determine the characteristics of the coronal component. From an analysis of Eggen's (1964) catalog of high velocity stars, the details of which are to be given elsewhere, we find $V_{esc} = 558 \pm 78$ km/s. This is to be compared with Schmidt's (1965) value of 380 km/s and Innanen's (1973) value of 374. Parenthetically we note that Hesser and Hartwick's (1976) observation of a globular cluster with radial velocity 273 km/s at a distance of R = 26 ± 5 kpc implies a minimum escape velocity of 430 km/s in the framework of this model. Finally, since galaxy rotation curves are typically flat or decreasing (but see the apparent counterexamples NGC 2590 and NGC 1620 found by Rubin <u>et al</u>. 1978) limits on the coronal radius $r_C$ can be set which, with the optimization scheme to be described shortly, determine that parameter.

## 3. RESULTING MASS MODEL

Given a set of model parameters (including $R_0$) we calculate the potential by standard analytical and numerical methods and compare with

https://doi.org/10.1017/S0074180900014911 Published online by Cambridge University Press

the observational constraints, computing thereby

$$\chi^2 \equiv \sum_{i=1}^{12} (Q_{i,obs} - Q_{i,calc})^2/\sigma^2_{i,obs}$$

TABLE I. SOLUTION FOR MODEL PARAMETERS

| | | | |
|---|---|---|---|
| Observer's Position | $R_\odot = 9.05 \pm 0.33$ kpc | Spheroidal Component | $\rho_{Sp} = 145 \pm 63\ M_\odot/pc^3$ |
| | | | $r_{Sp} = 0.104 \pm 0.012$ kpc |
| Disc Component | $\Sigma_D = 7.23 \times 10^4\ M_\odot/pc^2$ | | $M_N = 1.6 \pm 0.4 \times 10^8\ M_\odot$ |
| | $\varpi_D = 2.31711$ kpc | | |
| | $\varpi_G = 2.27969$ kpc | Coronal Component | $\rho_C = 2.19^{+1.90}_{-1.69} \times 10^{-3}\ M_\odot/pc^3$ |
| | $M_D = 7.81 \pm 1.31 \times 10^{10}\ M_\odot$ | | $r_C = 15.4 \pm 5.3$ kpc |
| | $\langle r^2 \rangle^{1/2} = 7.96 \pm 0.41$ kpc | | |

TABLE II. THREE-COMPONENT GALACTIC MASS MODEL

| R kpc | $V_{cir}$ km/s | $V_{esc}$ | $-\Phi_D$ $10^3$(km/s)$^2$/kpc | $-\Phi_{Sp}$ | $-\Phi_C$ | $\Phi'_D$ $10^2$(km/s/kpc)$^2$ | $\Phi'_{Sp}$ | $\Phi'_C$ | $\Sigma_D$ $M_\odot/pc^2$ | $\Sigma_{Sp}$ | $\Sigma_C$ | $M_D$ $10^9 M_\odot$ | $M_{Sp}$ | $M_C$ | $\rho_{tid}$ $M_\odot/pc^3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| .125 | 251 | 913 | 73 | 257 | 86 | -19 | 5070 | 0.0 | 61 | 36300 | 106 | 0.0 | 1.8 | 0.0 | 238 |
| .25 | 274 | 858 | 74 | 208 | 86 | -27 | 3030 | 0.1 | 115 | 15200 | 106 | 0.0 | 4.4 | 0.0 | 83 |
| .5 | 270 | 795 | 74 | 155 | 86 | -31 | 1490 | 0.2 | 206 | 5210 | 106 | 0.1 | 8.7 | 0.0 | 23 |
| 1 | 245 | 734 | 76 | 107 | 86 | -23 | 623 | 0.4 | 331 | 1560 | 105 | 0.8 | 14 | 0.0 | 5.2 |
| 2 | 219 | 682 | 76 | 70 | 86 | 9.0 | 230 | 0.8 | 429 | 429 | 105 | 4.5 | 21 | 0.1 | .97 |
| 3 | 219 | 653 | 74 | 53 | 86 | 35 | 123 | 1.2 | 416 | 197 | 104 | 11 | 26 | 0.2 | .36 |
| 4 | 228 | 631 | 70 | 43 | 86 | 51 | 78 | 1.5 | 359 | 113 | 102 | 20 | 29 | 0.6 | .19 |
| 5 | 238 | 612 | 64 | 37 | 86 | 57 | 54 | 1.9 | 291 | 73 | 100 | 29 | 31 | 1.1 | .14 |
| 6 | 245 | 594 | 58 | 32 | 86 | 58 | 40 | 2.2 | 226 | 51 | 98 | 38 | 33 | 1.8 | .11 |
| 7 | 249 | 578 | 53 | 29 | 85 | 55 | 31 | 2.5 | 170 | 38 | 96 | 46 | 35 | 2.8 | 88 D-3 |
| 8 | 249 | 563 | 47 | 26 | 85 | 50 | 25 | 2.7 | 126 | 29 | 94 | 53 | 37 | 4.1 | 74 D-3 |
| 9 | 247 | 550 | 43 | 24 | 85 | 45 | 20 | 3.0 | 92 | 23 | 91 | 59 | 38 | 5.6 | 62 D-3 |
| 10 | 243 | 539 | 39 | 22 | 85 | 39 | 17 | 3.2 | 66 | 19 | 89 | 63 | 39 | 7.4 | 52 D-3 |
| 15 | 216 | 497 | 25 | 16 | 83 | 19 | 8.4 | 3.9 | 11 | 8.3 | 76 | 75 | 44 | 20 | 21 D-3 |
| 20 | 196 | 472 | 18 | 13 | 81 | 9.9 | 5.1 | 4.1 | 1.7 | 4.7 | 64 | 77 | 48 | 39 | 9.3 D-3 |
| 30 | 178 | 441 | 11 | 9.1 | 77 | 4.0 | 2.5 | 4.1 | 0.0 | 2.1 | 48 | 78 | 52 | 85 | 3.0 D-3 |
| 50 | 170 | 405 | 6.8 | 5.9 | 69 | 1.4 | 1.0 | 3.4 | 0.0 | 0.7 | 31 | 78 | 58 | 198 | 0.9 D-3 |
| 100 | 168 | 356 | 3.4 | 3.2 | 57 | 0.3 | 0.3 | 2.2 | 0.0 | 0.2 | 16 | 78 | 67 | 509 | 0.2 D-3 |

We then adjust the values of the model parameters in an attempt to minimize $\chi^2$ using a variation of the standard Levenberg-Marquardt algorithm for nonlinear least square minimization. The model parameters determined by this method are given in Table I. The model itself is presented in Table II; column 1 is the distance from the center (in the galactic plane), 2 and 3 give the local circular and escape velocities, 3-5 the gravitational potential due to the three components, 6-8 the forces, 9-11 the projected mass densities, 12-14 the interior mass in each com-

https://doi.org/10.1017/S0074180900014911 Published online by Cambridge University Press

ponent and the last column gives $\rho_{tid}$. The row corresponding most closely to the solar position is shown in italics.

The model has a value of $\chi^2$ equal to 0.90 which for three degrees of freedom indicates a respectable probability of 82% that the model agrees with the observations. The $\chi^2$ test indicates a slightly worse result if the nuclear component is omitted and a much worse fit if the Plaut and Oort local constants are used (due to the considerably smaller value of B).

## 4. LIGHT DISTRIBUTION

Consistent with the approach we have taken up to this point we need only determine the local light per unit area in the disc and spheroidal components, compare with the model to find the local mass-to-light ratio, and applying that universally in the galaxy determine the light distribution. From Weistrop's (1972) counts of high latitude blue stars we find a local spheroidal visual luminosity of 0.5 $L_\odot/pc^2$ for a local mass-to-light ratio of approximately 40. The disc mass-to-light ratio is approximately 4.0 from Oort (1965) giving total visual luminosity in the two components of $2.0 \times 10^9$ and $2.0 \times 10^{10}$ $L_\odot$, with a total magnitude of -21.0 approximately the same as for the similar edge-on giant spiral NGC 4565.

## 5. CONCLUSION

We have constructed a mass and light distribution model which, while not greatly different in its details from existing models like those of Schmidt (1966) or Innanen (1973), has the virtues that a) it treats all observations on a comparable footing, b) it is based on our knowledge of external galaxies and c) it accommodates naturally the flat rotation curves found recently by Roberts (1974), Krumm and Salpeter (1977) and Rubin et al. (1978).

## ACKNOWLEDGEMENTS

We wish to thank J.H. Oort for helpful discussions and S. P. Bhavsar and R. L. Pariseau for their investigations of the properties of disc/halo galaxy models during an initial phase of this work. Support from National Science Foundation grant AST76-20255 is acknowledged.

## REFERENCES

Burton, W.:1971, Astron. Astrophys. 10, 76.
deVaucouleurs, G.:1959, Handbuch der Physik, 53 (Springer-Verlag:Berlin).
Eggen, O. J.:1964, Royal Obs. Bulletin No. 84.
Erikson, R.:1975, Astrophys. J. 195, 343.

https://doi.org/10.1017/S0074180900014911 Published online by Cambridge University Press

Einasto, J.:1970, Teated Tartu Obs.26,1.
Fall, M. and Rees, M.:1978, preprint.
Fatchikhin, N.:1970, Soviet Astron. J.,14,495.
Freeman, K.:1970, Astrophys. J. 160,811.
Fricke, W. and Tsioumis, A.:1975, Astron. Astrophys.,42,449.
Gunn, J. E.:1977, Astrophys. J.,218, 592.
Harris, W. and Hesser, J.:1976, Publ.Astron.Soc.Pacific,88,377.
Innanen, K.:1973, Astrophys.Space Sci.,22,393.
Kormendy, J.:1977a, Astrophys.J.,217,406.
Kormendy, J.:1977b, Astrophys. J.,218,333.
Krumm, N. and Salpeter, E. E.:1977,Astron. Astrophys.,56,465.
Oemler, G.:1976, Astrophys.J.,209,693.
Oort, J.:1960,
Oort, J.:1965,Stars & Stellar Systems V, p.483 (U of Chicago Press: Chicago).
Oort, J.:1977, Astrophys. J.,218,L97.
Ostriker, J.P., Peebles, P.E.J. and Yahil, A.:1974,Astrophys.J.Letters, 193,L1.
Ostriker, J.P. and Thuan, T.X.:1975, Astrophys. J.,202,353.
Plaut, L. and Oort, J.:1975, Astron. Astrophys., 41, 71.
Roberts, M.S.:1974,Rotation Curve of Galaxies, in Dynamics of Stellar Systems, ed. A. Hayli (Reidel Pub.: Dordrecht), 331
Rougoor, G. and Oort, J.:1960, Proc.Nat.Acad.Sci.,46,1.
Rubin, V.C., Ford,W.K. and Thonnard, N.:1978, preprint.
Salpeter, E.E.:1977, IAU Symp. No. 77.
Schmidt, J.:1965,Stars & Stellar Systems, V,513 (U of Chicago Press: Chicago).
Scoville, N.Z. and Solomon, P.M.:1975,Astrophys. J., 199,L105.
Spinrad,H., Ostriker, J.P., Stone, R.P.S., Chiu, G.L-T. and Bruzual, A.: 1978,Astrophys. J., in press.
Toomre, A.:1972, Quart.J.Roy.Astron.Soc.,13, 241.
Tremaine, S.D.:1976,Astrophys. J.,203,345.
Turner, E.L.:1976,Astrophys. J.,208,304.
Tuve, M. and Lundsager, S.:1973, Velocity Structures in Hydrogen Profiles, Carnegie Inst. of Washington Publ. 630.
Vandervoort, P.:1970,Astrophys. J.,162,453.
Vasilevskis,S. and Klemola,A.:1971,Astron.J.,76,508.
Weistrop,D.:1972,Astron.J.,77,366.

DISCUSSION

de Vaucouleurs: Is your escape velocity, which is estimated from rotation curves of other galaxies, sensitive to the assumed distance scale?

Sinha: How sensitively does the determination of the dark halo depend upon observations interior to the Sun? Based on the tangential point velocities of HI, I have derived a rotation curve (Astron. & Astrophys. in press) with three components: a nuclear disk (similar to one of Oort or Sanders and Lowinger) to match IR isophotes, a spherical $1/r^3$ halo to explain excess rotational velocities between 2 and 4 kpc, and a Toomre (n=5) disk. In this model I can explain the steep gradient at

https://doi.org/10.1017/S0074180900014911 Published online by Cambridge University Press

1 kpc without invoking a disk with a hole, and I get a rotation curve, less steep than the Schmidt curve outside the solar circle, which is very similar to your model (for a bulge mass 1/4 x disk mass). The flatness of the rotation curve in our Galaxy found by Moffat and his coworkers and reported by Dr. Jackson, can be fit by introducing an extra halo-like component.

Bok: I hope that Dr. Ostriker will go as far as he can in making specific suggestions for work by observers. Modern techniques make it by now a relatively simple matter to study density and velocity distributions for objects like F stars. Radial velocities can now be measured to 18th magnitude--good spectra for classification purposes to the same limits--and photometry in established color systems (including the near infrared!) can be carried out to 21st magnitude (and fainter if need be!).

As part of theoretical model calculations, people like Dr. Ostriker should provide observers the force law perpendicular to the galactic plane at the Sun, which can then be checked by combined analyses of radial velocity and density distributions perpendicular to the galactic plane at the sun.

There are probably too few globular clusters to permit extensive use of them for studies of the dynamical properties of the halo. However, the search for and study of RR Lyrae variables in high galactic latitudes holds great promise. I hope that theorists in the future will not hesitate to make specific recommendations for observational tests and that in all of their reports on new models they will attempt to give us specific information on the field of force perpendicular to the galactic plane at the Sun.

Ostriker: Here are some recommendations. For the ratio of disc to spheroidal components, better determination of the local force law is critical.

For the mass distribution exterior to the Sun these items come to mind:

a. More observations of velocities of halo objects far from the Sun (like Hartwick and Sargent's work on globular clusters).

b. More and better high-velocity-star searches are important for determination of the local escape velocity.

c. Better measurements of the velocities and masses of the Galaxy's satellites.

Oort: Bok suggested that a strong attempt be made to get information about the structure of the very large halo or "corona" by surveys of distant stars or clusters. A thorough search for distant globular clusters might be feasible; for RR Lyrae variables it might be too time consuming, because they are so rare at the distance to be considered.

Ostriker's mass model seems a very acceptable one. One feature about which I feel some doubt, however, is the gap he assumed in the central part of the disk. It is evident that in many spirals there is

https://doi.org/10.1017/S0074180900014911 Published online by Cambridge University Press

a central hole in the gas distribution. The existence of such holes can be a natural consequence of the process of star formation, but I find it difficult to understand that there would be similar holes in the mass distribution. With regard to this point, Ostriker answered that it is difficult to represent the dip in the rotation curve between $R \sim 0.5$ and $\sim 3$ kpc without a hole in the disk component. Burton drew attention to the fact that this dip may not be real. The gas motions in this region are highly noncircular, and may well be considerably lower than the circular velocities.

Ostriker remarked that our Galaxy would be exceptional in that it has no black hole in its center. However, the NeII radial velocities in the 1-pc infrared nuclear disk indicate the presence of a mass of about $5 \times 10^6$ solar masses within $R \sim 1/2$ pc. It seems doubtful whether this can consist of stars.

Yahil: First a comment to Dr. de Vaucouleurs: Because Dr. Ostriker calculates an escape velocity squared from external galaxies, and not a mass, this value is not inversely proportional to the Hubble constant as you suggest.

Then a question to Dr. Ostriker: I am concerned that your rotation curve falls faster at large galactocentric radii than is observed in other galaxies. Which of your data points would have to be different if the rotation curve were indeed flatter?

Ostriker: The calculated escape velocity is only very weakly dependent on the assumed Hubble constant, as you state.

The outer part of the rotation curve is largely dependent on the poorly-known local escape velocity. We chose $558 \pm 78$ km $s^{-1}$. A larger value of the escape velocity would be required to have a flatter rotation curve.

Berman: It seems that the inner dip in your final rotation curve is due more to your choice of a model with a cut out disk and a spheroidal component rather than on your use of many measurements of the rotation curve in your least-squares fitting scheme: Is this right? Secondly, even though the final rotation curve falls very rapidly with radius, it seems to satisfy the law that the enclosed mass $M(R) \propto R$, obtained for massive extended galaxies and rotation curves that are constant by Dr. Rubin. Is this law $M(R) \sim R$ more common for flattened galaxies? Or is it due to your model choice of a halo component?

Ostriker: Both Rubin's galaxies and our model's have $M \propto R$ in the outer parts. The reason why our $v_{cir}$ declines slightly (not rapidly) is due to the chosen value of $v_{esc}(R_o)$. Had we chosen a larger value we would have obtained a flatter curve. With respect to your first point, I should note that we also made models (which we shall publish separately in a more complete discussion) without the central hole in the disc. These do not fit the apparent dip in the rotation curve as well as the model we present here. The innermost point made by us (neglecting the "nuclear" point) is at 3 kpc and may be influenced by expansion or circulations.

https://doi.org/10.1017/S0074180900014911 Published online by Cambridge University Press