J. Plasma Physics (1999), *vol.* 62, *part* 5, *pp.* 485–491 Printed in the United Kingdom

Millimetre-wave second-harmonic generation in an underdense magnetoplasma with a density ripple

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(Received 2 November 1998 and in revised form 12 June 1999)

Abstract. A high-power millimetre wave $(\omega_1, \mathbf{k}_1)$ propagating through a magnetized plasma at an angle to a density ripple $(0, \mathbf{k}_0)$ produces density perturbations $(\omega_1, \mathbf{k}_1, \mathbf{k}_0)$. The density perturbations couple with the oscillatory velocity at $(\omega_1, \mathbf{k}_1)$ to produce a nonlinear current at $(2\omega_1, 2\mathbf{k}_1 + \mathbf{k}_0)$ driving second-harmonic electromagnetic radiation. The efficiency of the process is sensitive to the angle between the density ripple and the millimetre wave.

Coherent radiation sources have been an active area of research for the past few decades. The free-electron laser (FEL) (Marshall 1985; Roberson and Sprangle 1989; Liu and Tripathi 1994), the gyrotron (Nasinovich 1992a, b; Singh et al. 1992), the Čerenkov free-electron laser (Tripathi and Liu 1989) and the backward-wave oscillator (Carmel et al. 1990) are a few of these devices that operate at millimetre wavelengths with high powers. The introduction of a plasma in the interaction region of these devices has shown some exciting results on efficiency enhancement (Carmel et al. 1990), radiation guiding (Tripathi and Liu 1990) and the use of plasma eigenmodes as wigglers in the FEL (Joshi et al. 1987; Tran et al. 1987). Chen and Dawson (1992a, 1993) have proposed an ion ripple laser to generate tunable coherent radiation from the microwave to the ultraviolet wavelength region. In their scheme, a relativistic electron beam propagating obliquely to an ion ripple generates electromagnetic radiation via the Raman scattering process. They have shown that on going to higher frequencies, the efficiency of the device decreases. Therefore it is worthwhile studying the generation of harmonics of the fundamental frequency of the device (Nusinovich 1992a, b).

In the second-harmonic generation process, two photons of energy $\hbar\omega_1$ and momentum $\hbar\mathbf{k}_1$ each combine to generate a photon of energy $\hbar\omega_2$ and momentum $\hbar\mathbf{k}_2$, where $(\omega_1, \mathbf{k}_1)$ and $(\omega_2, \mathbf{k}_2)$ satisfy the linear dispersion relation for electromagnetic waves (Bloembergen 1965; Grebogi et al. 1983). Energy and momentum conservation in a second-harmonic process demand that $\omega_2 = 2\omega_1$ and $\mathbf{k}_2 = 2\mathbf{k}_1$. However, since a plasma is a dispersive medium with refractive index increasing with wave frequency, the wave vector of the second harmonic is more than twice the wave vector of the fundamental millimetre wave, i.e. $k_2 > 2k_1$. This implies that the process is only a weak one. It can be important only

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https://doi.org/10.1017/S0022377899007965 Published online by Cambridge University Press

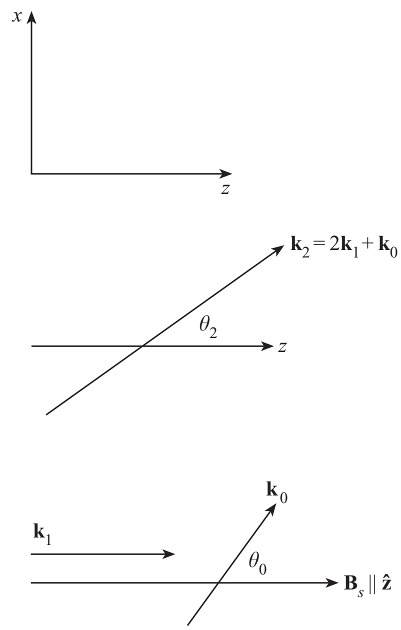

Figure 1. Schematic of the process.

when an additional momentum $\hbar\mathbf{k}_2 - 2\hbar\mathbf{k}_1$ is made available for each second-harmonic photon in the system. A density ripple or a wiggler magnetic field can be used to provide the additional momentum to make the process a resonant one (Parashar and Pandey 1992, 1993). In the present paper, we study second-harmonic generation in a magnetized plasma with a density ripple. The scheme could be useful in frequency upconversion of an ion ripple laser.

Consider the propagation of an x-polarized millimetre wave

$$\mathbf{E}_1 = (\hat{\mathbf{x}} + \mathrm{i}\hat{\mathbf{y}})\, E_1 \exp[-i(\omega_1 t - k_1 z)], \tag{1a}$$

$$\mathbf{B}_1 = \frac{c\mathbf{k}_1 \times \mathbf{E}_1}{\omega_1}, \tag{1b}$$

$$k_1 = \frac{\omega_1}{c}\left[1 - \frac{\omega_p^2}{\omega_1(\omega_1 - \omega_c + i\nu)}\right]^{1/2} \tag{1c}$$

in a cold plasma in the presence of a static magnetic field $B_s\hat{\mathbf{z}}$, where

$$\omega_c = \frac{eB_s}{mc}, \qquad \omega_p = \left(\frac{4\pi n_0^0 e^2}{m}\right)^{1/2},$$

ν is the electron–ion collision frequency, and n_0^0, $-e$ and m are the plasma equilibrium density, charge and mass respectively. The plasma also contains a density ripple

$$n_0 = n_0 \exp(i\mathbf{k}_0 \cdot \mathbf{r}), \tag{2}$$

where the wave vector $\mathbf{k}_0$ is in the (x, z) plane, making an angle θ_0 with the z axis. The nonlinear coupling of the millimetre wave and the density ripple produces a second-harmonic wave

$$\mathbf{E}_2 = \mathbf{E}_2 \exp[-i(\omega_2 t - \mathbf{k}_2 \cdot \mathbf{r})],$$

where

$$\omega_2 = 2\omega_1, \qquad \mathbf{k}_2 = 2\mathbf{k}_1 + \mathbf{k}_0,$$

and $\mathbf{k}_2$ makes an angle θ_2 with the z axis (see Fig. 1). The value of $\mathbf{k}_0$ is chosen such that the second-harmonic electromagnetic wave of frequency ω_2 satisfies the dispersion relation

$$\left|\boldsymbol{\epsilon}_2 - \frac{k_2^2 c^2}{\omega_2^2}\mathbf{I} + \frac{c^2}{\omega_2^2}\mathbf{k}_2\mathbf{k}_2\right| = 0,$$

which can be written as

$$a\eta_2^4 + b\eta_2^2 + c = 0, \tag{3}$$

where $\eta_2 \equiv ck_2/\omega_2$, $\boldsymbol{\epsilon}_2$ is the plasma permittivity tensor at ω_2, and

$$a = \epsilon_{2xx}\sin^2\theta_2 + \epsilon_{2zz}\cos^2\theta_2,$$

$$b = -\epsilon_{2xx}\epsilon_{2zz}(1+\cos^2\theta_2) - \epsilon_{2+}\epsilon_{2-}\sin^2\theta_2,$$

$$c = \epsilon_{2zz}\epsilon_{2+}\epsilon_{2-},$$

$$\epsilon_{2\pm} = \epsilon_{2xx} \pm i\epsilon_{2xy},$$

$$\epsilon_{2xx} = 1 - \frac{\omega_p^2}{\omega_2^2 - \omega_c^2},$$

$$\epsilon_{2xy} = \frac{i\omega_c\omega_p^2}{\omega_2(\omega_2^2 - \omega_c^2)},$$

$$\epsilon_{2zz} = 1 - \frac{\omega_p^2}{\omega_2^2}.$$

For one value of θ_2, with given plasma parameters, the value of η_2 can be obtained. Using this value of η_2, one obtains $\mathbf{k}_0 = \mathbf{k}_2 - 2\mathbf{k}_1$:

$$k_0^2 = k_2^2 + 4k_1^2 - 4\mathbf{k}_1\cdot\mathbf{k}_2 = \frac{4\omega_1^2}{c^2}(\eta_2^2 + \eta_1^2 - 2\eta_1\eta_2\cos\theta_2),$$

$$\theta_0 = \tan^{-1}\left(\frac{\eta_2\sin\theta_2}{\eta_2\cos\theta_2 - 2\eta_1}\right),$$

where

$$\eta_1^2 = \frac{c^2k_1^2}{\omega_1^2} = 1 - \frac{\omega_p^2}{\omega_1(\omega_1 - \omega_c)}.$$

For $4\omega_1^2 \gg \omega_c^2$, one obtains from (3) $\eta_2^2 \approx 1 - \omega_p^2/\omega_2^2$. We have plotted in Fig. 2 the variation of the density ripple wave vector k_0, required for resonant second-harmonic generation, as a function of θ_2, the direction of propagation of the second-harmonic wave for the following parameters:

$$\frac{\omega_p^2}{\omega_1^2} = 0.3, \quad \frac{\omega_c^2}{\omega_1^2} = 0.2 \text{ and } 0.4.$$

The lowest value of k_0 is required at $\theta_2 \approx 45°$.

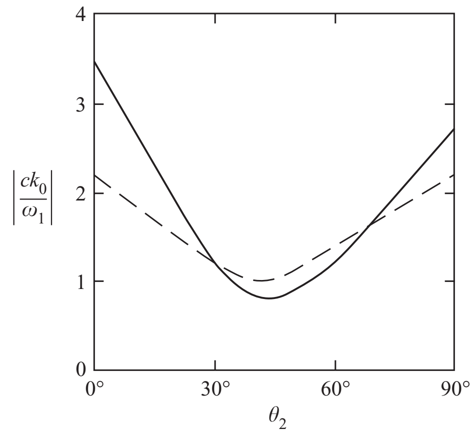

Figure 2. Density ripple wavenumber versus the angle θ_2 (the angle between $\mathbf{k}_1$ and $\mathbf{k}_2$) for $\omega_p^2/\omega_1^2 = 0.3$, and $\omega_c^2/\omega_1^2 = 0.2$ (– – –) and 0.4 (——).

The millimetre wave induces an oscillatory velocity $\mathbf{v}_1$ of the electrons. Solving the equation of motion

$$m\frac{d\mathbf{v}_1}{dt} = -e\left(\mathbf{E}_1 + \frac{1}{c}\mathbf{v}_1 \times \mathbf{B}_s\right) - m\nu\mathbf{v}_1, \tag{4}$$

we obtain

$$v_{1x} = -iv_{1y} = \frac{eE_1}{mi(\omega_1 - \omega_c + i\nu)}, \tag{5a}$$

$$v_{1z} = 0. \tag{5b}$$

$\mathbf{v}_1$ in conjunction with the density ripple produces a density perturbation

$$n_1' = n_1' \, e^{-i[\omega_1 t - (\mathbf{k}_1 - \mathbf{k}_0)\cdot\hat{\mathbf{z}}]},$$

which, on solving the equation of continuity, can be written as

$$n_1' = \frac{\mathbf{k}_0 \cdot \mathbf{v}_1}{2\omega_1} n_0. \tag{6}$$

$\mathbf{v}_1$ beats with n_1' to produce the nonlinear component of the second-harmonic current density at $(2\omega_1, 2\mathbf{k}_1 + \mathbf{k}_0)$:

$$\mathbf{J}_2^{NL} = -\tfrac{1}{2} n_1' \, e\mathbf{v}_1. \tag{7}$$

The self-consistent second-harmonic field $\mathbf{E}_2$ produces a second-harmonic current density $\mathbf{J}_2^L = \boldsymbol{\sigma}_2 \cdot \mathbf{E}_2$, where $\boldsymbol{\sigma}_2$ is the second-harmonic conductivity tensor. In the limit when $4\omega_1^2 \gg \omega_c^2$, $\boldsymbol{\sigma}_2$ reduces to a diagonal tensor,

$$\boldsymbol{\sigma}_2 \approx \frac{n_0^0 e^2}{im\omega_2} \mathbf{I}.$$

Using (7) and (8) in the continuity equation, one obtains the second-harmonic density perturbation

$$n_2 = n_2 \, e^{-i[\omega_2 t - (2\mathbf{k}_1 + \mathbf{k}_0)\cdot\hat{\mathbf{z}}]} = -\frac{1}{2\omega_1 \, ie} \nabla \cdot \mathbf{J}_2, \tag{9}$$

where

$$\mathbf{J}_2 = \mathbf{J}_2^L + \mathbf{J}_2^{NL}.$$

The wave equation for the second-harmonic field is written as

$$\nabla^2 \mathbf{E}_2 - \nabla(\nabla \cdot \mathbf{E}_2) = -\frac{8\pi}{c^2} i\omega_1 \mathbf{J}_2 - \frac{4\omega_1^2}{c^2} \boldsymbol{\varepsilon}_2 \cdot \mathbf{E}_2. \tag{10}$$

where

$$\boldsymbol{\epsilon}_2 = \mathbf{I} + \frac{4\pi i \boldsymbol{\sigma}_2}{\omega_2}.$$

We solve (10) in the limit when $4\omega_1^2 \gg \omega_c^2$, i.e. we retain the effect of the magnetic field on the fundamental wave but ignore it on the second harmonic. Then

$$\boldsymbol{\epsilon}_2 = \left(\frac{1-\omega_p^2}{\omega_2^2}\right)\mathbf{I}.$$

On using (9), Poisson's equation gives

$$\nabla \cdot \mathbf{E}_2 = -4\pi e n_2 = \frac{\pi e}{i\omega_1}\frac{n_1' k_{0x} v_{1x}}{\epsilon_2}.$$

Substituting for $\nabla \cdot \mathbf{E}_2$ in (10) and introducing a distance variable

$$\xi = x \sin\theta_2 + z\cos\theta_2,$$

we can write (10) as

$$\frac{\partial^2 \mathbf{E}_2}{\partial \xi^2} + \frac{(4\omega_1^2 - \omega_p^2)\mathbf{E}_2}{c^2} = \mathbf{R}, \tag{11}$$

where

$$\mathbf{R} = \frac{8\pi i\omega_1}{c^2}\mathbf{J}_2^{NL} + \nabla\left(\frac{\pi e}{i\omega_1}\frac{n_1' k_{0x} v_{1x}}{\epsilon_2}\right).$$

$\mathbf{R}$ can be resolved into two components, $\mathbf{R} = \mathbf{R}_\parallel + \mathbf{R}_\perp$, where $\mathbf{R}_\parallel$ is parallel to ξ and $\mathbf{R}_\perp$ is perpendicular to ξ:

$$\mathbf{R}_\perp = \mathbf{R} - \mathbf{R}_\parallel = \mathbf{R} - \frac{\mathbf{R}\cdot\mathbf{k}_2}{k_2}\frac{\mathbf{k}_2}{k_2}.$$

In the WKB approximation, we substitute

$\mathbf{E}_2 = \mathbf{A}_2 e^{ik_2\xi}$ into (11), and ignore $\partial^2 A_2/\partial\xi^2$, to obtain

$$\mathbf{A}_2 = \frac{\mathbf{R}_\perp}{2ik_2} L, \tag{12}$$

where

$$\mathbf{R}_\perp = -\frac{4\pi\omega_1}{c^2} n_1' v_{1x} e(i\hat{\mathbf{y}} + \cos\theta_2 \hat{\boldsymbol{\eta}}),$$

$\hat{\boldsymbol{\eta}} \perp \mathbf{k}_2$ in the (x, z) plane, and L is the length of the interaction region. The power of the incident millimetre wave is given by

$$P_1 = \frac{c^2 k_1 E_1^2}{8\pi\omega_1}, \tag{13}$$

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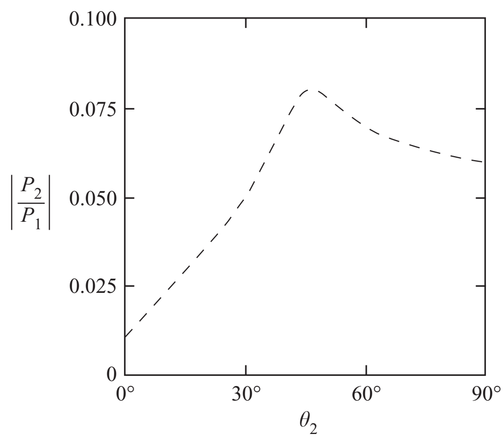

Figure 3. Power conversion efficiency $|P_2/P_1|$ versus θ_2.

and that of the generated wave is given by

$$P_2 = \frac{c^2\, k_2\, E_2^2}{8\pi\omega_2} \approx \frac{c}{8\pi}\frac{\mathbf{R}_\perp \cdot \mathbf{R}_\perp^*}{4k_2^2} L^2. \tag{14}$$

The ratio of the generated power P_2 to the incident power P_1 is given by

$$\left|\frac{P_2}{P_1}\right| \approx \frac{\pi^2 \omega_1\, k_{0x}^2\, |n_0|^2\, |v_{1x}|^4\, e^2 L^2 (1+\cos^2\theta_2)}{c^5\, k_2^2\, k_1\, E_1^2}. \tag{15}$$

The variation of $|P_2/P_1|$ with θ_2 is shown in Fig. 3 for the following typical parameter: a millimetre-wave power density of 3 MW cm^{-2} at 1 mm wavelength (Kartunen et al. 1991), a plasma density fluctuation level $n_0/n_0^0 \approx 10\%$, $\omega_p/\omega_1 = 0.56$, $\omega_c/\omega_1 = 0.66$ and $L \approx 4$ cm. We have used Fig. 2 to obtain the corresponding value of k_0 for phase matching for each θ_2. We observe that the second-harmonic power conversion efficiency peaks at around 8% at $\theta_2 = 45$ (see Fig. 3). The power conversion efficiency is quite sensitive to ω_c/ω_1, which for Fig. 3 has been chosen as 0.66. As one moves ω_c/ω_1 closer to 1, one must keep the electron density below the right-hand cut-off, $\omega_p^2/\omega_1^2 < 1-\omega_c/\omega_1$, so that the fundamental electromagnetic wave can propagate through the plasma. For a fixed ratio $\omega_p^2/[\omega_1(\omega_1-\omega_c)]$, P_2/P_1 scales as $(1-\omega_c/\omega_1)^{-3/2}$. However, as $\omega_c/\omega_1 \to 1$, the requisite value of k_0 goes up to unrealistic values. A large-amplitude electron density ripple can be produced by:

(i) shining two crossed electromagnetic beams to form a static interference pattern, exert a ponderomotive force on the electrons and cause ambipolar diffusion of the plasma to produce a static density ripple;
(ii) generating ion acoustic waves using two parallel grids and applying a periodic potential difference between them (Chen and Dawson 1992b; Liu and Tripathi 1994);
(iii) modulating the density of a neutral gas by a sound wave and then ionizing it with a laser pulse (Marshall 1985; Chen and Dawson 1992b).

In a plasma-filled cavity, one may create a standing wave that pushes plasma from the antinodes to nodal regions owing to the ponderomotive force.

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Experiments on backward-wave oscillators have shown strong modulations of density (Botton and Ron 1991). Linear scattering of the pump electromagnetic wave would be important when $(\omega_1, \mathbf{k}_1 + \mathbf{k}_0)$ satisfy the dispersion relation for an obliquely propagating electromagnetic wave

$$\left| \boldsymbol{\epsilon}_1 - \frac{c^2}{\omega_1^2} (\mathbf{k}_1 + \mathbf{k}_0)^2 \mathbf{I} + \frac{c^2}{\omega_1^2} (\mathbf{k}_1 + \mathbf{k}_0)(\mathbf{k}_1 + \mathbf{k}_0) \right| = 0, \qquad (16)$$

where $\boldsymbol{\epsilon}$ is the dielectric tensor at the frequency ω_1 and can be recovered from $\boldsymbol{\epsilon}_2$ by replacing ω_2 by ω_1. One must avoid values of $\mathbf{k}_0$ that satisfy (16).

Acknowledgements

J.P. wishes to thank the Department of Physics, University of Durban–-Westville, South Africa for the hospitality accorded. He is also grateful to Professor H. N. Silakari, Director, SATI, Vidisha (M.P.), India for granting study leave to visit UDW, South Africa.

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