Earth and Environmental Science Transactions of the Royal Society of Edinburgh, **109**, 437–444, 2019 (for 2018)

Earth surface processes and environmental sustainability in China

Stochastic soil moisture dynamic modelling: a case study in the Loess Plateau, China

Cong WANG[1], Shuai WANG[2], Bojie FU[1*], Lu ZHANG[3], Nan LU[1] and Lei JIAO[1]

[1] State Key Laboratory of Urban and Regional Ecology, Research Center for Eco-Environmental Sciences, Chinese Academy of Sciences, PO Box 2871, Beijing 100085, PR China.
Email: bfu@rcees.ac.cn
[2] State Key Laboratory of Earth Surface Processes and Resource Ecology, Faculty of Geographical Science, Beijing Normal University, Beijing 100875, PR China.
[3] CSIRO Land and Water Flagship, Clunies Ross Street, Canberra ACT 2601, Australia.
*Corresponding author

ABSTRACT: Soil moisture is a key factor in the ecohydrological cycle in water-limited ecosystems, and it integrates the effects of climate, soil, and vegetation. The water balance and the hydrological cycle are significantly important for vegetation restoration in water-limited regions, and these dynamics are still poorly understood. In this study, the soil moisture and water balance were modelled with the stochastic soil water balance model in the Loess Plateau, China. This model was verified by monitoring soil moisture data of black locust plantations in the Yangjuangou catchment in the Loess Plateau. The influences of a rainfall regime change on soil moisture and water balance were also explored. Three meteorological stations were selected (Yulin, Yan'an, and Luochuan) along the precipitation gradient to detect the effects of rainfall spatial variability on the soil moisture and water balance. The results showed that soil moisture tended to be more frequent at low levels with decreasing precipitation, and the ratio of evapotranspiration under stress in response to rainfall also changed from 74.0% in Yulin to 52.3% in Luochuan. In addition, the effects of a temporal change in rainfall regime on soil moisture and water balance were explored at Yan'an. The soil moisture probability density function moved to high soil moisture in the wet period compared to the dry period of Yan'an, and the evapotranspiration under stress increased from 59.5% to 72% from the wet period to the dry period. The results of this study prove the applicability of the stochastic model in the Loess Plateau and reveal its potential for guiding the vegetation restoration in the next stage.

KEY WORDS: stochastic model, vegetation restoration, water balance, water stress.

Land use and climate change are major concerns for the sustainability of dryland ecosystems (Franz *et al*. 2010), which also affect the hydrological cycle in water-limited regions. The ecological and environmental degradations have been and will continue to be challenges in achieving global sustainability (Lü *et al*. 2015). In this context, ecological restoration and conservation have been implemented widely (Lü *et al*. 2015). Thus, it is necessary to assess the ecosystem responses to climate change with the restoration implemented in water-limited regions. Soil moisture is an integrative state variable that plays key roles in many ecosystem processes in water-limited regions (D'Odorico *et al*. 2007; Wang *et al*. 2017b). Characterising soil moisture variability will help us understand the ecosystem responses to land use and climate change in the water-limited regions, such as the Loess Plateau in China.

The Loess Plateau is located in the upper and middle reaches of the Yellow River in China (Fig. 1). It is dominated by a monsoon climate and an obvious rainfall gradient exists across this region (Wang *et al*. 2017a). Due to long-term cultivation, intensive rainfall, and the complex landform, the Loess Plateau was considered one of the most severely eroded areas in the world (Lu *et al*. 2015). Several large-scale restoration projects have been conducted to control soil erosion and restore ecological functions in the region (Liang *et al*. 2015). The Grain for Green (GfG) project was implemented in 1999, which converted sloping croplands to forest and pasture lands. With the implementation of the GfG project, the land use has changed dramatically and fast-growing trees and shrubs have been introduced extensively. However, recent studies have found that introduced plants degrade over time after initial normal growth due to severe water restriction and soil desiccation (Li 2001; Yu *et al*. 2015). Soil water availability is quite important for plant growth in terms of soil water balance in the water-limited regions. Understanding the relationships between soil moisture dynamics and vegetation is essential for vegetation restoration sustainability in the Loess Plateau (Zhang *et al*. 2016).

It has been shown that dryland ecosystems respond to shifts in rainfall climatology (Franz *et al*. 2010). The Loess Plateau has been found to be particularly sensitive to climate changes,

doi:10.1017/S1755691018000658

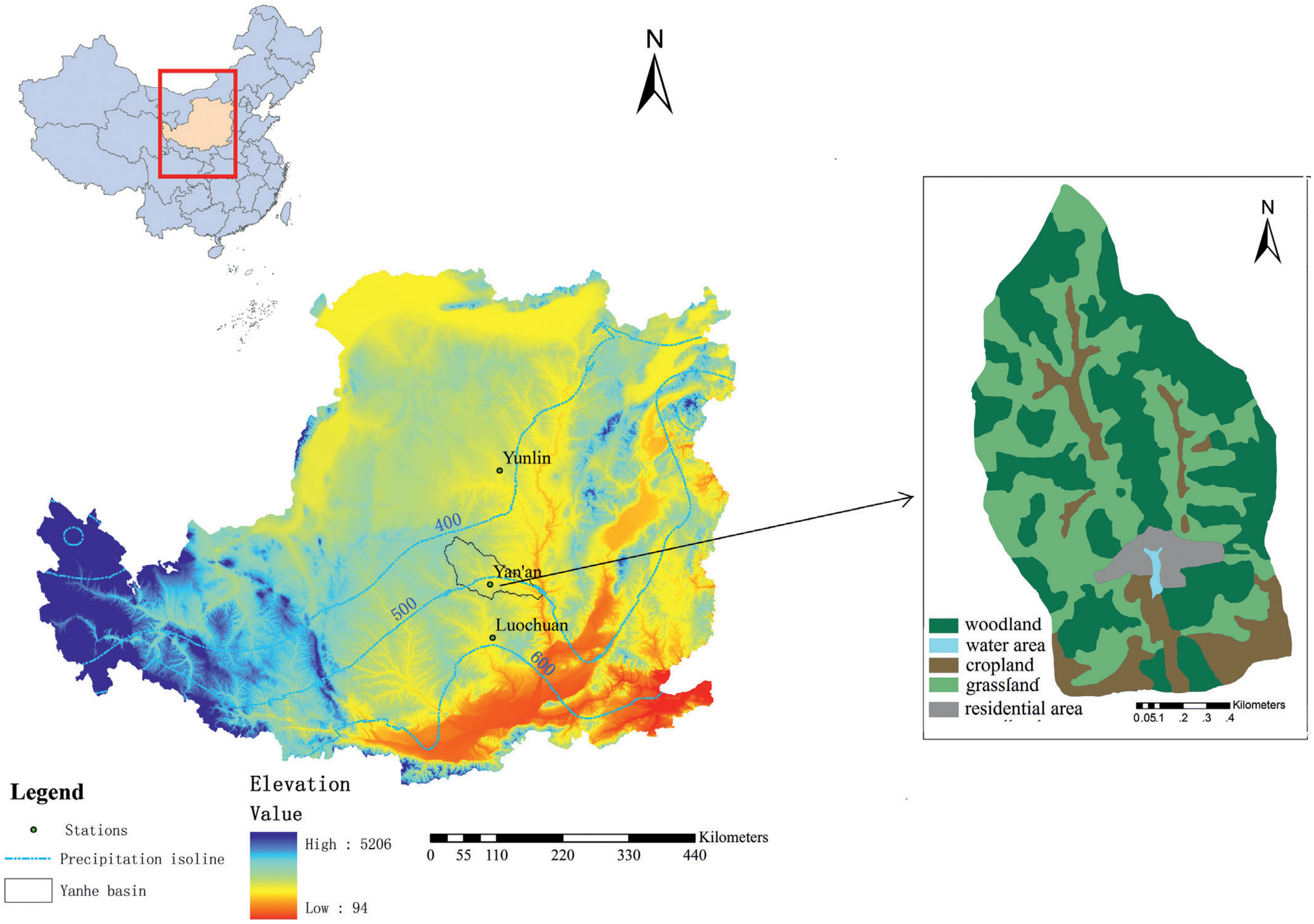

Figure 1 Location of Yangjuangou catchment and three meteorological stations in the Loess Plateau.

due to the fragile ecological environment and geographic features (Miao *et al.* 2016). Recent studies have shown that warming and drying trends exist in large areas of the Loess Plateau (Sun *et al.* 2015; Miao *et al.* 2016). The precipitation on wet days has decreased over large areas of the Loess Plateau during the past 50 years, which is either caused by a decrease in frequency despite a concomitant increase in rainfall intensity or caused by decreases in both intensity and frequency of rainfall (Sun *et al.* 2015). Rainfall in drylands is heterogeneous in space (D'Odorico & Porporato 2006; Franz *et al.* 2010), including the Loess Plateau (Liu 2007). In addition, rainfall is the only source of soil moisture in the Loess Plateau. Thus, it is necessary to characterise soil moisture variability under the influences of temporal change and spatial variability in rainfall regimes in the Loess Plateau.

Soil moisture modelling schemes consist of deterministic dynamics allowing for an analytical description of the feedbacks within the system, and the stochastic components, which are required to account for the unpredictability of rainfall, temperature, and variability inherent to storage–discharge relationships (Verma *et al.* 2011). Focusing on the soil moisture dynamics, recent studies have developed various descriptions of the components of the water balance (Cordova & Bras 1981; Daly & Porporato 2006; Vervoort & van der Zee 2008; Manfreda *et al.* 2010; Verma *et al.* 2011). Rawls *et al.* (1992) described the infiltration process and water movement in the soil. Milly (1993) provided an analytic solution of the stochastic storage problem by assuming that evapotranspiration occurred at a constant rate. Vervoort & van der Zee (2008) developed models that considered the capillary rise from groundwater to the unsaturated zone in a stochastic ecohydrological framework. Stochastic modelling is gaining more and more attention in soil moisture and water balance modelling in recent decades. As daily rainfall processes play a key role in dryland soil moisture variation, and they reveal obvious randomness, it is necessary to include daily rainfall as a stochastic forcing variable in soil moisture modelling schemes (Franz *et al.* 2010). In 1999, Rodriguez-Iturbe *et al.* (1999) developed a stochastic soil water balance model in which rainfall is represented by a marked Poisson process, and obtained an analytical solution for the soil moisture probability density function (pdf). Ridolfi *et al.* (2000) applied this model to explore the effect of climate variability on the water stress of the vegetation. Franz *et al.* (2012) developed a simple spatial-explicit daily stochastic ecohydrological model to explore the hillslope-scale vegetation patterns. Recently, a series of studies have developed and applied stochastic models similar to that of Rodriguez-Iturbe *et al.* (1999) in research focused on soil moisture dynamics.

In this study, the stochastic soil water balance model (Laio *et al.* 2001a, b; Porporato *et al.* 2001; Rodriguez-Iturbe *et al.* 2001) was applied to explore the interactions of the climate–soil–plant system in the Loess Plateau, China. Specifically, the objectives of the study are: (1) to test the applicability of this model to the Loess Plateau; and (2) to detect soil moisture responses to the rainfall regime change.

1. Material and methods

1.1. Study area

The Yangjuangou catchment [36°42′N, 109°31′E] was selected in this study to test the applicability of the stochastic soil water balance model. The catchment is located near Yan'an in Shaanxi province, China (Fig. 1), which is a typical hilly gully region of the Loess Plateau, China. Due to afforestation campaigns, black locust (*Robina pseudoacacia*) plantations were distributed widely in this catchment (Jiao *et al.* 2015). The growing season spans approximately from May to September for most deciduous plants (Jiao *et al.* 2015). The mean average precipitation (MAP) is 531 (±115) mm, and the average precipitation in the growing season is 422 (±103) mm, accounting for approximately 80 % of the annual precipitation (Jiao *et al.* 2015).

Two black locust plantation plots were monitored on the NW- and E-facing slopes in the catchment. The area of each plot was $10 \times 10\,\mathrm{m}^2$. The slope degrees of these plots were similar (approximately 25 °), and their slope positions were relatively low on the slopes. Field surveys were conducted at each plot, and the understory vegetation is mainly composed of liana (*Periploca sepoum*) and herb (*Artemisia sacrorum*) (Jiao *et al.* 2015, 2016). The volumetric soil water content of each plot was measured at 10, 20, 40, 60, 100, 120, 150, and 180 cm below the ground surface with the application of EC-5 sensors (Decagon Devices Inc., Pullman, WA, USA) in 2014. Then, the average volumetric water content at the rooting depth was calculated by the depth-averaged method based on the data recorded at the above depths. A HOBO logger (H21, Onset Computer Corp., Bourne, MA, USA) was used to record the soil moisture at 30 min intervals, and daily moisture was calculated from these data. The precipitation in 2014 was measured with a tipping bucket rain gauge (TE525), and the data were collected via a data logger (CR1000, Campbell Scientific Inc., Logan, UT, USA) at 30 min intervals. The observed soil moisture in the catchment was used to test the applicability of the stochastic soil water balance model.

1.2. Stochastic soil water balance model

1.2.1. Theoretical framework. The model applied in this study characterises the temporal dynamics of soil moisture at a given point without consideration of lateral moisture contribution (Rodriguez-Iturbe *et al.* 2001). The point water balance is shown as:

$$nZ_r \frac{ds(t)}{dt} = R(t) - I(t) - Q[s(t), t] - E[s(t)] - L[s(t)] \qquad (1)$$

where n is soil porosity, Z_r is the rooting depth, $s(t)$ is relative soil moisture ($0 \leq s(t) \leq 1$) in the rooting depth, which is calculated by dividing volumetric water content by soil porosity, $R(t)$ is rainfall rate, $I(t)$ is the rate of losses due to canopy interception, $Q(t)$ is the run-off rate, $E[s(t)]$ is the rate of evapotranspiration, and $L[s(t)]$ is the rate of leakage loss.

The stochastic soil water balance model used in this study is based on the conservation of water within the rooting zone of each species, forced by daily precipitation (Franz *et al.* 2010). Eagleson (1978) indicated that probabilistic-trait methods could be applied to simplify the randomness of a rainfall event, in which the rainfall, $R(t)$, is represented as a marked Poisson process of storm arrivals in time with rate λ (day^{-1}), storm depth h (mm) treated as an exponentially distributed random variable with mean α (mm) (Franz *et al.* 2010). The infiltration depth is determined by the minimum storm depth and the remaining soil moisture capacity at the time of the storm (Xingyao *et al.* 2011). The run-off is considered to be dominated by the Dunne run-off processes (Xingyao *et al.* 2011). As for evapotranspiration, the model incorporated a description of vegetation to water stress: evapotranspiration declines linearly with soil moisture (Kiang 2002). The evapotranspiration, $E(s)$, was represented as a piecewise function for different soil moisture ranges (Laio *et al.* 2001b). The soil moisture cut-off points (in terms of values to saturation when $s = 1$) were set as field capacity, s_{fc}; onset of water stress, s_*; wilting point, s_w; and hygroscopic point, s_h (Laio *et al.* 2001b). Evapotranspiration arrives at its maximum, $E_{\max}$, then declines to some minimum level, E_w (soil evaporation), at the wilting point, s_w. The model described the function as below:

$$E(s) = \left\{ \begin{array}{cc} 0 & s_* < s \leq s_h \\ E_w \frac{s - s_h}{s_w - s_h} & s_h < s \leq s_w \\ E_W + (E_{\max} - E_w) \frac{s - s_w}{s_* - s_w} & s_w < s \leq s_* \\ E_{\max} & s_* < s \leq 1 \end{array} \right\} \qquad (2)$$

The deep percolation starts when the soil moisture is larger than s_{fc}, and the loss rate is assumed to reach its maximum when the soil moisture is saturated and then decays exponentially as the soil dries out, which follows the decrease of the hydraulic conductivity $K(s)$ (Laio *et al.* 2001b). Then, the general solution of the steady-state soil moisture pdf, $p(s)$, is shown in Eq. 3, where $m = Ks/nZr[\exp(\beta(1 - s_{fc})) - 1]$; $\eta = E_{\max}/nZ_r$; $\eta_w = E_w/nZ_r$; $\gamma = nZ_r/\alpha$; $\lambda' = \lambda e^{-\Delta/\alpha}$; α is the mean depth of rainfall depth; λ is the rainfall event frequency; Δ is a threshold for rainfall depth, below which no water could reach the ground (assumed to be 2 for trees); K_s is the saturated hydraulic conductivity; C is an integration constant which could be obtained by the identity $\int_0^1 p(s)ds = 1$ (Laio *et al.* 2001b); and β is the coefficient of the exponential relationship between soil moisture and water conductivity dependent on soil type, it equals $2b + 4$, where b is an experimentally determined parameter in soil water retention curves.

The soil water balance could be obtained based on the model presented in Eqs 1, 2, 3. It is noted that the different water balance components refer to the long-term averages of the respective components of the soil moisture dynamics.

$$p(s) = \begin{cases} \frac{C}{\eta_w}\left(\frac{s - s_h}{s_w - s_h}\right)^{((\lambda'(s_w - s_h))/(\eta_w)) - 1} e^{-\gamma s} & s_h < s \leq s_w \\ \frac{C}{\eta_w}\left[1 + \left(\frac{\eta}{\eta_w} - 1\right)\left(\frac{s - s_w}{s^* - s_w}\right)\right]^{((\lambda'(s^* - s_w))/(\eta - \eta_w)) - 1} e^{-\gamma s} & s_w < s \leq s^* \\ \frac{C}{\eta} e^{-\gamma s + (\lambda'/\eta)(s - s^*)}\left(\frac{\eta}{\eta_w}\right)^{\lambda'((s^* - s_w)/(\eta - \eta_w))} & s^* < s \leq s_{fc} \\ \frac{C}{\eta} e^{-(\beta + \gamma)s + \beta s_{fc}}\left(\frac{\eta e^{\beta s}}{(\eta - m)e^{\beta s_{fc}} + m2e^{\beta s}}\right)^{(\lambda'/(\beta(\eta - m))) + 1}\left(\frac{\eta}{\eta_w}\right)^{((\lambda'(s^* - s_w))/(\eta - \eta_w))} e^{(\lambda'/\eta)(s_{fc}(s_{fc} - s^*)} & s_{fc} < s \leq 1 \end{cases} \qquad (3)$$

Then, these components could be specified as (Rodríguez-Iturbe & Porporato 2004):

$$\left\{\begin{array}{c} \langle R\rangle = \alpha\cdot\lambda \\ \langle I\rangle = \alpha\lambda(1-e^{-(\Delta/\alpha)}) \\ \langle E_s\rangle \alpha\lambda^{'}P(s^*) - \alpha\eta p(s^*) \\ \langle E_{ns}\rangle = E_{\max}[1-P(s^*)] \\ \langle L\rangle = \alpha\left[\lambda^{'} - \lambda^{'}P(s_{fc}) - \left(\eta+\frac{K_s}{nZ_r}\right)p(1) + \eta p(s_{fc})\right] \\ -E_{\max}[1-P(s_{fc})] \\ \langle Q\rangle = \alpha\left(\eta+\frac{K_s}{nZ_r}\right)p(1) \end{array}\right. \tag{4}$$

where $\langle R\rangle$, $\langle I\rangle$, $\langle E_s\rangle$, $\langle E_{ns}\rangle$, $\langle L\rangle$, and $\langle Q\rangle$, are the mean rates of rainfall, interception, evapotranspiration-under-stress condition, unstressed evapotranspiration, leakage, and run-off, respectively, and $P(s)$ is the cumulative probability distortion of soil moisture.

1.2.2. Parameter estimation. Soil parameters such as field capacity (s_{fc}), onset of water stress (s_*), wilting point (s_w), and hygroscopic point (s_h) are determined by measurements through the soil water characteristic curve (the capillary suction method). Thus, reasonable values for the soil matrix potentials at these soil moisture cut-off points are important for the soil parameter estimation. In the Loess Plateau, typical values of soil matrix potentials at s_{fc} and s_w are -0.03 MPa and -2 MPa, respectively (Bai & Shao 2009). The soil matrix potential at s_h is set to be the reference value at -10 MPa (Rodríguez-Iturbe & Porporato 2004). The soil matrix potential at incipient stomatal closure (s_*) is determined through the black locust net photosynthetic rate observation with the soil matrix potential change (Wang *et al.* 2007) and the value is -0.06 MPa. The soil porosity is determined by field investigation. Using the typical values for sandy loam reported by Laio *et al.* (2001b), other soil parameters such as K_s and b are obtained because the soil texture in the study area is sandy loam according to the measurements of soil particle sizes. Since it is difficult to obtain soil parameters across the Loess Plateau, we only refer to variations due to different soil textures in this study. Thus, the soil parameters are constant in the study area due to the consistent soil texture.

The maximum daily average evapotranspiration ($E_{\max}$) is difficult to estimate due to the lack of direct measurements (Rodríguez-Iturbe & Porporato 2004). Scholes & Walker (1993) indicated that typical $E_{\max}$ values could be obtained by the observation of an eight-hour transpiration period during the peak growing season after sizeable rainfall events. Then, the $E_{\max}$ (mm day^{-1}) for black locust in this study could be estimated using the soil moisture variation after several sizeable rainfalls on August 8, 2011, in the Yangjuangou catchment (Wang *et al.* 2012). The soil evaporation, E_w, is set to be 5% of $E_{\max}$, according to Franz *et al.* (2010). The effective soil depth (Zr) is set to be 180 cm according to the field investigation, which is also in accordance with Zhang & Xu (2011) in the Loess Plateau. The values of the parameters used in the model are summarised in Table 1. The rainfall parameters (λ and α) could be obtained through statistics on the observed daily precipitation data in the meteorological stations, and data could be obtained from the National Meteorological Information Center of the China Meteorological Administration. With the input of the climate, soil, and vegetation parameters, as previously mentioned, the soil moisture balance model could derive the pdf for soil moisture and water balance for given climates, soils, and vegetation.

1.3. Rainfall regimes

To illuminate the spatial variability of rainfall regimes across the Loess Plateau, three meteorological observation stations were selected along a precipitation gradient: Yulin (MAP = 420 mm), Yan'an (MAP = 500 mm), and Luochuan (MAP = 615 mm) (Fig. 1). The observed daily precipitation data in these three stations were obtained from the National Meteorological Information Center of the China Meteorological Administration. The rainfall parameters in the stochastic model for these three stations were calculated based on the rainfall data from 1999 to 2015, then the effects of rainfall pattern spatial variability on the soil moisture were explored. Meanwhile, the precipitation data from 1951 to 2015 in Yan'an station were applied to detect the effects of temporal changes in rainfall on the soil moisture variation. Figure 2 shows the annual rainfall (growing season, from May to September) fluctuations in Yan'an from 1951 to 2015, and obvious deviations existed from the long-term average rainfall over this region. The period from 1958 to 1988 was wet, considering the number of years with higher than average rainfall, and the period from 1989 to 2008 was dry, given the number of years with lower than average rainfall. For the wet period from 1958 to 1988, the rainfall parameters are $\lambda_{\text{wet}} = 0.37\,\text{day}^{-1}$ and $\alpha_{\text{wet}} = 8.2\,\text{mm}$, while for the dry period they are $\lambda_{\text{dry}} = 0.33\,\text{day}^{-1}$ and $\alpha_{\text{dry}} = 7.5\,\text{mm}$.

2. Results

2.1. Soil moisture prediction in black locust plantations

The predicted and observed soil moisture pdfs for the growing season of black locust in plots (plot 1 and plot 2) in the Yangjuangou catchment are shown in Figure 3. Figure 3 shows that there is good agreement between the predicted and observed pdfs in both plots, though some small deviations existed at the high soil moisture level. This verifies the applicability of the stochastic soil water balance model used in this study and shows the potential for it to be applied to the hilly gully regions of the Loess Plateau.

2.2. Soil moisture pdfs and water balance affected by spatial variability of rainfall regimes

The resulting soil moisture pdfs for the growing season of black locusts in the Yunlin, Yan'an, and Luochuan stations are shown in Figure 4. The effect of spatial variability in the rainfall regime is obvious. With decreasing MAP, it appears

Table 1 Parameters describing the soil and plant characteristics of the black locust in the Loess Plateau. Abbreviations: S_h = hygroscopic point; s_{fc} = field capacity; Ks = saturated hydraulic conductivity; s_w = wilting point; s_* = onset of water stress; Zr = rooting depth; $E_{\max}$ = maximum daily average evapotranspiration; E_w = soil evaporation; b = experimentally determined parameter.

Soil type	s_h	s_{fc}	**Ks (cm day^{-1})**	n	b
Sandy loam	0.095	0.292	80	0.53	4.9
	s_w	s_*	Zr (cm)	$E_{\max}$ (cm day^{-1})	E_w (cm day^{-1})
	0.107	0.234	180	0.32	5% of $E_{\max}$

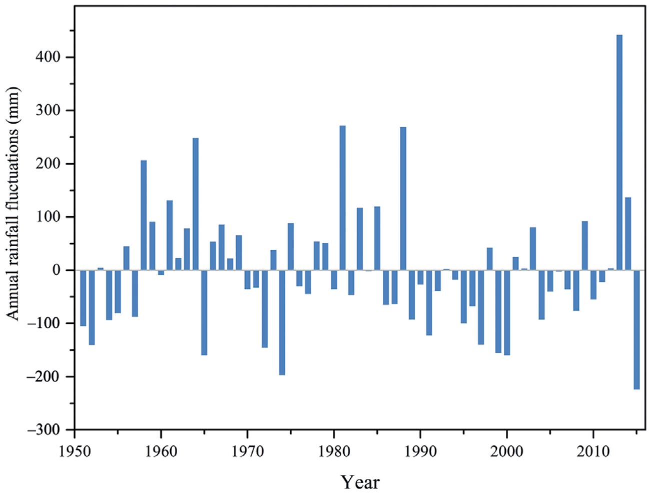

Figure 2 Fluctuations around the mean annual rainfall in Yan'an from 1951–2015.

that black locust tends to be more frequent at low soil moisture levels in Yulin. In Yulin, the soil moisture mainly ranges from s_w to s_*, while the proportion was lower in Yan'an and Luochuan. Regarding the differences among these stations, they have a fatter right tail in their pdf, which could be attributed to the more frequent soil moisture excursions at high soil moisture levels.

The comparison of water balances among the three stations is presented in Figure 5. It is evident that transpiration under stress makes up a majority of the water balance in these three regions, and this could be attributed to the frequent low-intensity rainfalls during the growing season. A large proportion of rainfall is transpired with the stomata partially closed. Obvious differences existed among these three stations; the proportion of evapotranspiration under stress decreased (from 74.0% in Yulin to 52.3% in Luochuan) with increasing MAP, while the proportion of unstressed evapotranspiration increased with increasing MAP.

2.3. Soil moisture pdfs and water balance affected by temporal changes in rainfall regime

Obvious fluctuations existed in Yan'an during the period from 1951 to 2015, and alternating dry (1958–1988) and wet (1989–2008) periods were detected throughout the whole period. Then, the effect of a temporal change in rainfall on soil moisture pdfs and water balance variations were explored. Comparisons of the soil moisture pdfs and water balance in the dry period and wet period are shown in Figure 6. The soil moisture pdfs in the dry and wet periods revealed a similar pattern, while the pdf in the wet period moved to the high soil moisture level. Regarding the water balance (Fig. 7), evapotranspiration under stress increased slightly (from 59.5 to 72%) in the dry period, while the unstressed evapotranspiration decreased greatly compared to the wet period.

3. Discussion

3.1. Application and uncertainties of the stochastic soil moisture model in the Loess Plateau

Rainfall stochasticity is an important factor affecting the hydrological cycle and many hydrological processes; however, stochasticity issues have seldom been considered in soil moisture modelling in the Loess Plateau. The stochastic soil moisture model applied in this study would be beneficial to show how to cope with stochastic water availability in the Loess Plateau. The model applied in this study considers daily rainfall as an external random driving force (Rodríguez-Iturbe & Porporato 2004), in which the randomness of the rainfall event was simplified by the probabilistic-trait methods (Zhou *et al.* 2016). In addition, compared to other soil moisture models with physical interpretations, such as the Simultaneous Heat and Water model (SHAW) (Flerchinger *et al.* 1996) and the Soil Water Carrying Capacity for Vegetation model (Xia & Shao 2008), the parsimony in parameterisation of this model made it quite convenient for applications in data-sparse regions. More efforts are needed to better apply the stochastic soil moisture model to the Loess Plateau. First, although the black

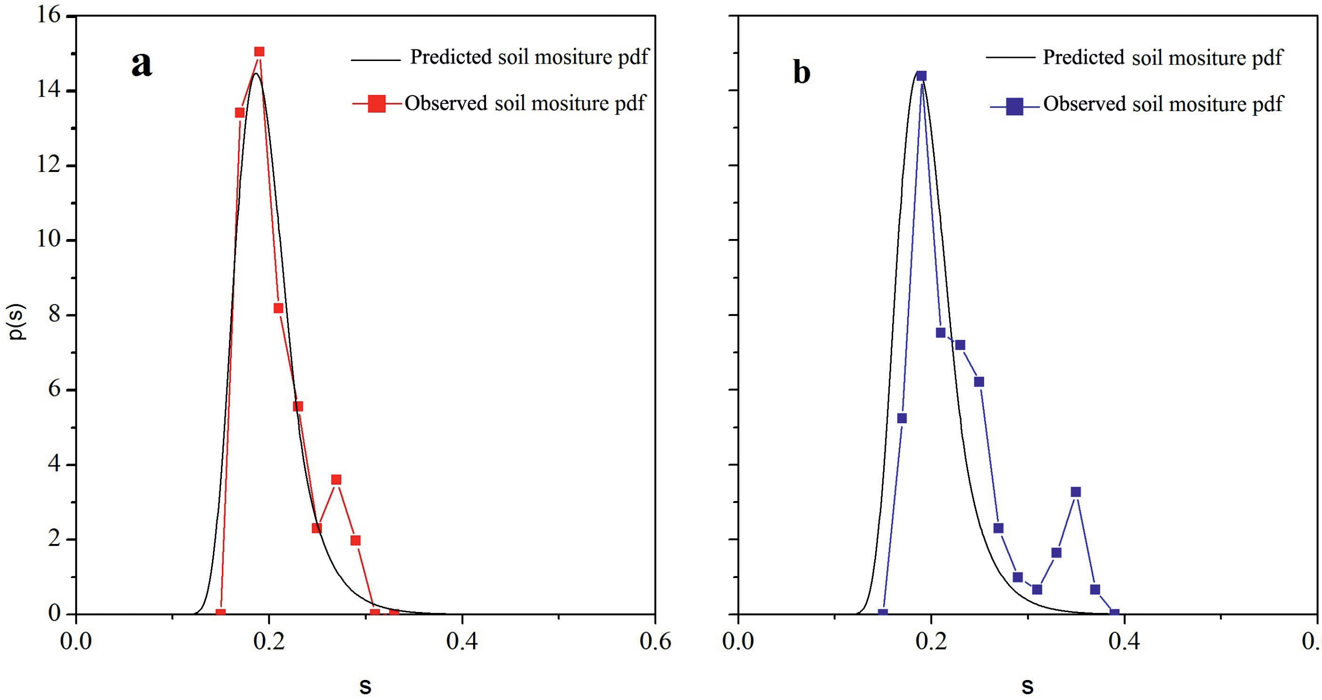

Figure 3 Soil moisture pdfs for black locust in the growing season in 2014. (a) Plot 1 in Yangjuangou catchment. (b) Plot 2 in Yangjuangou catchment.

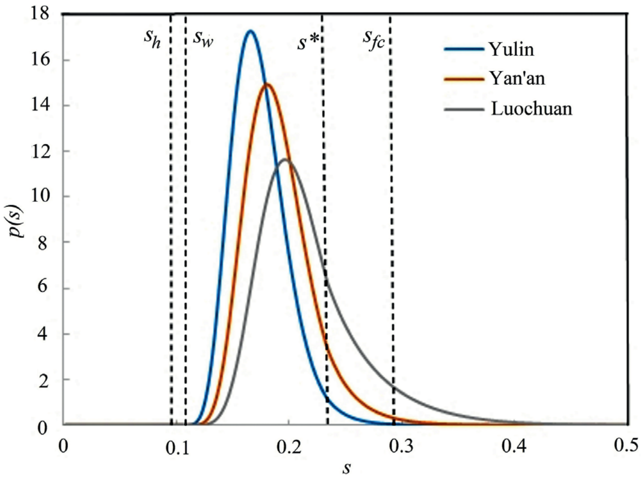

Figure 4 Predicted soil moisture pdfs in Yulin, Yan'an and Luochuan.

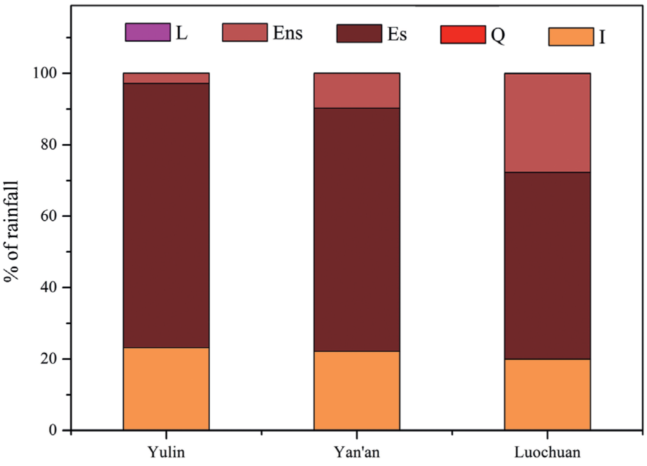

Figure 5 Components of the water balance normalised by the total rainfall in Yulin, Yan'an and Luochuan. The incoming rainfall is partitioned among leakage (L), unstressed evapotranspiration (Ens), evapotranspiration under stress (Es), runoff (Q) and canopy interception (I).

locust was selected to measure the impacts of soil moisture on ecosystem dynamics, other typical introduced vegetation in the Loess Plateau could also be used in soil moisture modelling due to their different transpiration characteristics. Second, more meteorological stations should be selected to reveal the spatial and temporal variability of the rainfall regimes with the spatial interpolation method so that the soil moisture pdf and water balance could be obtained at regional scales. Third, more long-term soil moisture datasets should be monitored and collected across the Loess Plateau, because these could further verify the stochastic soil moisture model.

The sources of uncertainties in this study could be mainly attributed to model parameterisation. In this study, the core of the model is its simplified characterisation of evapotranspiration variation with soil moisture, in which $E_{\max}$ is an important parameter for indicating the physiological processes of the plant. However, the physiological state of the plant would vary with the growth stage. Recent studies have shown that the transpiration rate in juvenile black locust plantations was higher than in the mature plantations (Zheng *et al.* 2011). The soil matrix potential at incipient stomatal closure, s^*, is obtained through a simulated drought experiment, which would deviate from field observations. Thus, more cautions should be taken to predict the soil moisture pdf and water balance variation in future studies.

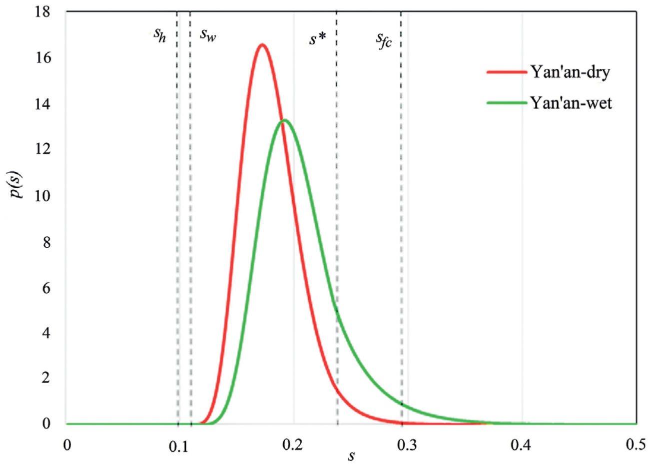

Figure 6 Predicted soil moisture pdfs in Yan'an in the dry and wet period.

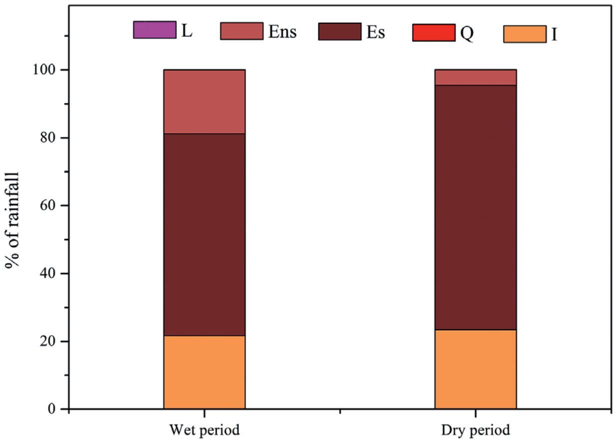

Figure 7 Components of the water balance normalised by the total rainfall in Yan'an in the dry and wet period. The incoming rainfall is partitioned among leakage (L), unstressed evapotranspiration (Ens), evapotranspiration under stress (Es), runoff (Q) and canopy interception (I).

3.2. Soil moisture and water balance responses to rainfall regimes

The results of this study have shown the obvious effects of a rainfall regime change on the soil moisture and water balance. The $s^* - s_w$ range for black locust in this study was relatively large, which indicated that the black locust could not quickly change from being wilted to unstressed (Rodríguez-Iturbe & Porporato 2004). Taking the Yangjuangou catchment as an example, the rainfall is intermittent and generally light at the beginning of the growing season. Heavy rains are relatively rare and concentrated in the peak of the growing season. Thus, the rainfall regime in the Yangjuangou catchment indicated that black locust would suffer greater water stress at the beginning of the growing season, as the winter charge is generally insufficient. The annual rainfall fluctuations result in Yan'an having alternating wet and dry periods from 1951 to 2015, which is also consistent with the warming and drying trends detected in large areas of the Loess Plateau. However, the temporal change in rainfall regime has been insufficiently accounted for in previous studies. Our study showed an example of how the soil moisture and water balance would vary from wet period to dry period, and the proportion of evapotranspiration under stress in response to the rainfall increased greatly from 59.5% to 72%. The results indicated that the warm and dry trends greatly affected the soil moisture dynamics and water balance. This study shows a simple way to assess how water stress would change from the wet period to the dry period, which could be quite practical for policy-making in the next stage of vegetation restoration in the Loess Plateau.

3.3. Implications for vegetation restoration in the Loess Plateau

Though the large-scale vegetation restoration projects in the Loess Plateau have achieved great outcomes in controlling soil and water loss since their implementation in recent years (Wang *et al.* 2015), dire challenges remain to be confronted regarding ecological and environmental degradations. Carefully considering plant species selection is imperative (Chen *et al.* 2010), and clarifying the suitable areas for the species is quite important in the implementation of vegetation restoration in the Loess Plateau. Several studies have been conducted to address this issue. McVicar *et al.* (2010) mapped the suitability of 38 predominately native species through an environmental variable spatial overlay approach and rule set defining the species' tolerances. Fu *et al.* (2012) applied the SHAW model to simulate soil moisture variations for a critical climatic year to determine the optimal plant coverage for two typical shrubs. Wang *et al.* (2017b) indicated that the soil moisture of woodland/shrubland was non-sustainable in the regions with MAP < 520 mm in the Loess Plateau. However, the interactions between soil moisture and the plant at the level of species were not fully considered in these previous studies. The results in this study could provide insights for guiding further vegetation restoration. In this study, the proportion of evapotranspiration under stress in response to the rainfall was calculated to quantify the water stress for the black locust. The high value in Yulin (74.0 %) was consistent with recent findings in the regions with MAP < 400 mm where the dwarfed trees have occurred throughout the reforestation areas (McVicar *et al.* 2010; Cao *et al.* 2011). The water stress for black locust in this study is calculated by dividing rainfall by the proportion of evapotranspiration under stress, which could be a quantitative and fundamental indicator assessing stress levels of the plant growth. However, the threshold for the normal growth of plants is not solely determined by the water stress; the ecohydrological optimality theory is an excellent way to explore this threshold. However, the complicated interactions among plants, soils, and climate make it difficult to obtain the ecohydrological optimisation mechanisms (Caylor *et al.* 2009). Instead of the maximisation of resource, resource scarcity accompanying limited resource consumption has drawn great considerations recently (Franz *et al.* 2010). Caylor *et al.* (2009) indicated that the ability of vegetation to maximise water use and minimise water stress determined their optimum spatial pattern. Though more species should be modelled, the relatively high water stress suffered by the black locust in the arid regions (e.g., Yulin) might also indicate that black locust is not the optimum vegetation type for these regions. Studies in the next stages should further focus on how to detect the water stress threshold of the black locust in the reforestation regions. Our results at least provided a quantitative index to assess the water stress on the plant at the level of species in the Loess Plateau. In addition, the mean transpiration rate is linked to ecosystem productivity from the vegetation's point of view (Rodríguez-Iturbe & Porporato 2004). Our results showed the potential to solve the issue of how to determine suitable regions for species. Based on the niche concept, the species suitability could be defined as the subset of environmental conditions affecting a particular organism (Franz *et al.* 2010). Suitability is a relative concept, which indicates that the black locust was more suitable in Luochuan than Yan'an, but we could not conclude that the black locust is the most suitable species in Luochuan. Thus, our results could be applied in suitability comparisons among the vegetation restoration types in the Loess Plateau, and used to evaluate the current restoration project.

4. Conclusions

The stochastic soil water balance model provides new insights into the soil moisture and water balance variation responses under rainfall regime changes. In this study, the stochastic soil water balance model developed by Rodríguez-Iturbe *et al.* (2004) was verified by monitoring soil moisture data in the Yangjuangou catchment in the Loess Plateau. The effects of rainfall regime spatial variability on the soil moisture and water balance of the black locust were detected in Yulin, Yan'an, and Luochuan. The soil moisture tends to be more frequent at low soil moisture levels with decreasing precipitation, and evapotranspiration under stress increased with decreasing precipitation. In addition, the effects of a temporal change in rainfall on the soil moisture and water balance was also detected in Yan'an. The soil moisture and water balance revealed obvious differences between the dry and wet periods. Though the variation in soil moisture and water balance with the rainfall regime change should be further modelled across the Loess Plateau, our results indicated that this model could assess the response of vegetation to water stress in the arid environment, which would be helpful in the next stage of vegetation restoration.

5. Acknowledgements

This work was funded by the State Key Project of Research and Development Plan (grant number 2016YFC0501603), the National Natural Science Foundation of China (grant numbers 41390464, 41722102), and the Chinese Academy of Sciences (grant number GJHZ 1502). Thanks are also expressed to the National Meteorological Information Center of the China Meteorological Administration for archiving the observed rainfall data (http://cdc.cma.gov.cn).

6. References

Bai, Y. & Shao, M. 2009. Soil water properties in a slope of different land use in the wind-water erosion crisscross region on the Loess Plateau. *Agricultural Research in the Arid Areas* **27**, 122–29. [In Chinese.]

Cao, S., Ge, S., Zhang, Z., Chen, L., Qi, F., Fu, B., Mcnulty, S., Shankman, D., Tang, J. & Wang, Y. 2011. Greening China naturally. *Ambio A Journal of the Human Environment* **40**, 828–31.

Caylor, K. K., Scanlon, T. M. & Rodriguez-Iturbe, I. 2009. Ecohydrological optimization of pattern and processes in water-limited ecosystems: a trade-off-based hypothesis. *Water Resources Research* **45**, W08407.

Chen, L., Wang, J., Wei, W., Fu, B. & Wu, D. 2010. Effects of landscape restoration on soil water storage and water use in the Loess Plateau region, China. *Forest Ecology & Management* **259**, 1291–98.

Cordova, J. R. & Bras, R. L. 1981. Physically based probabilistic models of infiltration, soil moisture, and actual evapotranspiration. *Water Resources Research* **17**, 93–106.

Daly, E. & Porporato, A. 2006. Impact of hydroclimatic fluctuations on the soil water balance. *Water Resources Research* **42**, 648–48.

D'Odorico, P., Caylor, K., Okin, G. S. & Scanlon, T. M. 2007. On soil moisture–vegetation feedbacks and their possible effects on the dynamics of dryland ecosystems. *Journal of Geophysical Research Atmospheres* **112**, 231–47.

D'Odorico, P. & Porporato, A., 2006. *Dryland ecohydrology*. Dordrecht, the Netherlands: Springer.

Eagleson, P. S. 1978. Climate, soil, and vegetation: 2. The distribution of annual precipitation derived from observed storm sequences. *Water Resources Research* **14**, 713–21.

Flerchinger, G. N., Baker, J. M. & Spaans, E. J. A. 1996. A test of the radiative energy balance of the SHAW model for snowcover. *Hydrological Processes* **10**, 1359–67.

Franz, T. E., Caylor, K. K., Nordbotten, J. M., Rodrigueziturbe, I. & Celia, M. A. 2010. An ecohydrological approach to predicting regional woody species distribution patterns in dryland ecosystems. *Advances in Water Resources* **33**, 215–30.

Franz, T. E., Caylor, K. K., King, E. G., Nordbotten, J. M. & Celia, M. A. 2012. An ecohydrological approach to predicting hillslope-scale vegetation patterns in dryland ecosystems. *Water Resources Research* **48**, 1515.

Fu, W., Huang, M., Gallichand, J. & Shao, M. 2012. Optimization of plant coverage in relation to water balance in the Loess Plateau of China. *Geoderma* **173–174**, 134–44.

Jiao, L., Lu, N., Sun, G., Ward, E. J. & Fu, B. 2015. Biophysical controls on canopy transpiration in a black locust (*Robinia pseudoacacia*) plantation on the semi-arid Loess Plateau, China. *Ecohydrology: Ecosystems, Land and Water Process Interactions, Ecohydrogeomorphology* **282**, 17–18.

Jiao, L., Lu, N., Fu, B., Gao, G., Wang, S., Jin, T., Zhang, L., Liu, J. & Zhang, D. 2016. Comparison of transpiration between different aged black locust (*Robinia pseudoacacia*) trees on the semi-arid Loess Plateau, China. *Journal of Arid Land* **8**, 1–14.

Kiang, N. Y.-l. 2002. Savannas and seasonal drought: the landscape-leaf connection through optimal stomatal control. Berkeley, CA: University of California. 322 pp.

Laio, F., Porporato, A., Fernandez-Illescas, C. P. & Rodriguez-Iturbe, I. 2001a. Plants in water-controlled ecosystems: active role in hydrologic processes and response to water stress: Iv. Discussion of real cases. *Advances in Water Resources* **24**, 745–62.

Laio, F., Porporato, A., Ridolfi, L. & Rodriguez-Iturbe, I. 2001b. Plants in water-controlled ecosystems: active role in hydrologic processes and response to water stress: Ii. Probabilistic soil moisture dynamics. *Advances in Water Resources* **24**, 707–23.

Li, Y. 2001. Effects of forest on water circle on the Loess Plateau. *Journal of Natural Resources* **16**, 427–32. [In Chinese with English abstract.]

Liang, W., Bai, D., Wang, F., Fu, B., Yan, J., Shuai, W., Yang, Y., Di, L. & Feng, M. 2015. Quantifying the impacts of climate change and ecological restoration on streamflow changes based on a Budyko hydrological model in China's Loess Plateau. *Water Resources Research* **51**, 6500–19.

Liu, Y. 2007. Analysis on the change trend of precipitation in North Shaanxi Province in the Loess Plateau. *Arid Zone Research* **24**, 49–55. [In Chinese.]

Lu, N., Akujärvi, A., Wu, X., Liski, J., Wen, Z., Holmberg, M., Feng, X., Zeng, Y. & Fu, B. 2015. Changes in soil carbon stock predicted by a process-based soil carbon model (yasso07) in the Yanhe watershed of the Loess Plateau. *Landscape Ecology* **30**, 399–413.

Lü, Y., Zhang, L., Feng, X., Zeng, Y., Fu, B., Yao, X., Li, J. & Wu, B. 2015. Recent ecological transitions in China: greening, browning, and influential factors. *Scientific Reports* **5**, 8732.

Manfreda, S., Scanlon, T. M. & Caylor, K. K. 2010. On the importance of accurate depiction of infiltration processes on modelled soil moisture and vegetation water stress. *Ecohydrology: Ecosystems, Land and Water Process Interactions, Ecohydrogeomorphology* **3**, 155–65.

McVicar, T. R., Niel, T. G. V., Li, L. T., Wen, Z. M., Yang, Q. K., Li, R. & Jiao, F. 2010. Parsimoniously modelling perennial vegetation suitability and identifying priority areas to support China's re-vegetation program in the Loess Plateau: matching model complexity to data availability. *Forest Ecology & Management* **259**, 1277–90.

Miao, C., Sun, Q., Duan, Q. & Wang, Y. 2016. Joint analysis of changes in temperature and precipitation on the Loess Plateau during the period 1961–2011. *Climate Dynamics* **47**, 3221–34.

Milly, P. C. D. 1993. An analytic solution of the stochastic storage problem applicable to soil water. *Water Resources Research* **29**, 3755–58.

Porporato, A., Laio, F., Ridolfi, L. & Rodriguez-Iturbe, I. 2001. Plants in water-controlled ecosystems: active role in hydrologic processes and response to water stress: Iii. Vegetation water stress. *Advances in Water Resources* **24**, 725–44.

Rawls, W. J., Ahuja, L. R., Brakensiek, D. L., Shirmohammadi, A. & Maidment, D. R. 1992. Infiltration and soil water movement, pp. 5.1–5.51.

Ridolfi, L., D'Odorico, P., Porporato, A. & Rodriguez-Iturbe, I. 2000. Impact of climate variability on the vegetation water stress. *Journal of Geophysical Research Atmospheres* **105**, 18013–26.

Rodriguez-Iturbe, I., Porporato, A., Ridolfi, L., Isham, V. & Coxi, D. R. 1999. Probabilistic modelling of water balance at a point: the role of climate, soil and vegetation. *Proceedings Mathematical Physical & Engineering Sciences* **455**, 3789–805.

Rodriguez-Iturbe, I., Porporato, A., Laio, F. & Ridolfi, L. 2001. Plants in water-controlled ecosystems: active role in hydrologic processes and response to water stress : I. Scope and general outline. *Advances in Water Resources* **24**, 695–705.

Rodríguez-Iturbe, I. & Porporato, A. 2004. Ecohydrology of water-controlled ecosystems: soil moisture and plant dynamics. New York: Cambridge University Press, United States of America by Cambridge University Press.

Scholes, R. & Walker, B., 1993. An African savanna. Cambridge, UK: Cambridge University Press.

Sun, Q., Miao, C., Duan, Q. & Wang, Y. 2015. Temperature and precipitation changes over the Loess Plateau between 1961 and 2011, based on high-density gauge observations. *Global & Planetary Change* **132**, 1–10.

Verma, P., Yeates, J. & Daly, E. 2011. A stochastic model describing the impact of daily rainfall depth distribution on the soil water balance. *Advances in Water Resources* **34**, 1039–48.

Vervoort, R. W. & van der Zee, S. E. 2008. Simulating the effect of capillary flux on the soil water balance in a stochastic ecohydrological framework. *Water Resources Research* **44**, W08425.

Wang, C., Wang, S., Fu, B., Li, Z., Wu, X. & Tang, Q. 2017a. Precipitation gradient determines the tradeoff between soil moisture and soil organic carbon, total nitrogen, and species richness in the Loess Plateau, China. *Science of the Total Environment* **575**, 1538–45.

Wang, C., Wang, S., Fu, B., Yang, L. & Li, Z. 2017b. Soil moisture variations with land use along the precipitation gradient in the north-south transect of the Loess Plateau. *Land Degradation & Development* **28**, 926–35.

Wang, M. C., Wang, J. X., Shi, Q. H. & Zhang, J. S. 2007. Photosynthesis and water use efficiency of *Platycladus orientalis* and *Robinia pseudoacacia* saplings under steady soil water stress during different stages of their annual growth period. *Journal of Integrative Plant Biology* **49**, 1470–77.

Wang, S., Fu, B. J., Gao, G. Y., Yao, X. L. & Zhou, J. 2012. Soil moisture and evapotranspiration of different land cover types in the Loess Plateau, China. *Hydrology & Earth System Sciences* **16**, 2883–92.

Wang, S., Fu, B., Piao, S., Lü, Y., Ciais, P., Feng, X. & Wang, Y. 2015. Reduced sediment transport in the Yellow River due to anthropogenic changes. *Nature Geoscience* **9**, 38–41.

Xia, Y. Q. & Shao, M. A. 2008. Soil water carrying capacity for vegetation: a hydrologic and biogeochemical process model solution. *Ecological Modelling* **214**, 112–24.

Xingyao, P., Lu, Z., Nicholas, J. P., Jun, X. & Yongqiang, Z. 2011. Probabilistic modeling of soil moisture dynamics of irrigated cropland in the north China plain. *Hydrological Sciences Journal/journal Des Sciences Hydrologiques* **56**, 123–37.

Yu, Y., Wei, W., Chen, L. D., Yang, L., Jia, F. Y. & Zhang, H. D. 2015. Responses of vertical soil moisture to rainfall pulses and land uses in a typical loess hilly area, China. *Solid Earth* **6**, 595–608.

Zhang, L. & Xu, X. 2011. Distribution characters of *Robinia pseudoacacia* root in Yangou watershed in Yan'an. *Journal of Northwest Forestry University* **26**, 9–14. [In Chinese.]

Zhang, X., Zhao, W., Liu, Y., Fang, X. & Feng, Q. 2016. The relationships between grasslands and soil moisture on the Loess Plateau of China: a review. *Catena* **145**, 56–67.

Zheng, Y., Zhao, Z., Zhou, J., Zhou, H., Liang, Z. & Luo, Z. 2011. The importance of slope aspect and stand age on the photosynthetic carbon fixation capacity of forest: a case study with black locust (Robinia pseudoacacia) plantations on the Loess Plateau. *Acta Physiologiae Plantarum* **33**(2), 419–29.

Zhou, J., Fu, B., Gao, G., Lü, Y. & Wang, S. 2016. Effect of restoration vegetation on the stochasticity of soil erosion in a semi-arid environment. *Hydrology & Earth System Sciences Discussions* **2016**, 1–46.

MS received 17 December 2016. Accepted for publication 30 October 2017