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# A TRAGEDY OF ANNUITIZATION? LONGEVITY INSURANCE IN GENERAL EQUILIBRIUM

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We study the microeconomic and macroeconomic effects of longevity insurance. Using a tractable discrete-time overlapping-generations model of a closed economy we first study different types of government redistribution of accidental bequests in general equilibrium. Individuals face longevity risk, as there is a positive probability of passing away before the retirement period. We find nonpathological cases where it is better for long-run welfare to waste accidental bequests than to give them to the elderly. Next we study the introduction of a perfectly competitive life insurance market offering actuarially fair annuities. There exists a tragedy of annuitization: although full annuitization of assets is privately optimal, it is not socially beneficial, because of adverse general equilibrium repercussions.

**Keywords:** Longevity Risk, Risk Sharing, Overlapping Generations, Intergenerational Transfers, Annuity Markets

## 1. INTRODUCTION

Although death is one of the true certainties in life, the date at which it occurs is unknown to all but the most desperate. Faced with an uncertain length of life, rational nonaltruistic agents must balance the risk of leaving unconsumed wealth in the form of unintended (accidental) bequests against the risk of running out of resources in old age. As was shown in the classic analysis of Yaari (1965) and

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more recently by Davidoff et al. (2005), life annuities are very attractive insurance instruments in the presence of longevity risk. Intuitively, annuities allow risk sharing between lucky (long-lived) and unlucky (short-lived) individuals [Kotlikoff et al. (1986)]. These risk-sharing opportunities ensure that the introduction of life annuities is welfare-improving from a microeconomic perspective, i.e., in a partial equilibrium setting.

From a macroeconomic perspective, however, it is not immediately clear whether the availability of annuities is beneficial for everyone. Two key mechanisms are ignored in a partial equilibrium analysis. First, in the absence of private annuities, there will be accidental bequests, which, provided they are redistributed in one way or another to surviving agents, boost the consumption opportunities of these agents. See, among others, Sheshinski and Weiss (1981), Hubbard (1984), Abel (1985), Pecchenino and Pollard (1997), and Fehr and Habermann (2008) on this point. Second, the availability of annuities affects the rate of return on an individual's savings. As a result, aggregate capital accumulation will generally depend on whether annuity opportunities are available. Capital accumulation in turn determines wages and the interest rate if factor prices are endogenous.

The objective of this paper is to study the general equilibrium effects of life annuities. Our model has the following features. First, we postulate a simple general equilibrium model of a closed economy. On the production side we allow a capital accumulation externality of the form proposed by Romer (1989).

Second, we assume that the economy is populated by overlapping generations of two-period-lived agents facing longevity risk. Just as in the Diamond (1965) model, life consists of two phases, namely youth and old age, but in contrast to that model, there is a positive probability of death at the end of youth. At birth, agents are identical in the sense that they have the same preferences, labor productivity, and survival probability.

Third, in the absence of annuities we assume that the resulting accidental bequests flow to the government. We investigate the general equilibrium effects of three prototypical revenue-recycling schemes. In particular, the policy maker can (a) engage in wasteful expenditure (the WE scenario), (b) give lump-sum transfers to the old agents (the TO scenario), or (c) provide lump-sum transfers to the young (the TY scenario).

Fourth, we compare the different revenue-recycling schemes with the case in which annuities are available. In particular, we assume that private annuity markets are perfectly competitive. With perfect annuities (the PA scenario), the probability of death determines the wedge between the rate of return on physical capital and the annuity rate of return. Because the latter exceeds the former, rational nonaltruistic individuals fully annuitize their savings.

The main finding of the paper concerns the phenomenon that we call the *tragedy of annuitization*: although full annuitization of assets is privately optimal, it may not be socially beneficial, because of adverse general equilibrium repercussions. If all agents invest their financial wealth in the annuity market, then the resulting long-run equilibrium leaves everyone worse off than in the case where annuities are absent and accidental bequests are redistributed to the young (or even wasted by the government). We demonstrate the existence of two versions of the tragedy. In the *strong* version, opening up perfect annuity markets in an economy in which accidental bequests initially go to waste (switch from WE to PA) results in a decrease in the steady-state welfare of newborns. Interestingly, this rather surprising result holds for an empirically plausible (i.e., low) value of intertemporal substitution elasticity. In this case the beneficial effects of annuitization are more than offset by a substantial drop in the long-run capital intensity and in wages. Future newborns would have been better off if no annuity markets had been opened.[1]

There is also a *weak* version of the tragedy. If the economy is initially at equilibrium with accidental bequests flowing to the young, then opening up annuity markets will reduce steady-state welfare regardless of the magnitude of the intertemporal substitution elasticity. Intuitively, private annuities redistribute assets from deceased to surviving elderly in an actuarially fair way, whereas transferring unintended bequests to the young constitutes an intergenerational transfer. This intergenerational transfer induces beneficial savings effects, which, in the end, lead to higher welfare.

The structure of the paper is as follows. Section 2 presents the model and characterizes the steady-state social optimum. Section 3 studies the key analytical properties of the model under different redistribution schemes. Section 4 computes, both analytically and quantitatively, the allocation and welfare effects of changes in public redistribution scenarios. Section 5 is the core of the paper. It shows what happens to allocation and welfare if a perfectly competitive annuity market is opened up at some point in time. It also highlights the importance of initial conditions; i.e., it demonstrates that the results depend not only on the availability of annuities but also on the type of public redistribution scenario that is replaced by these insurance markets. Section 6 restates the main results. Technical details, proofs, and further results can be found in a long Mathematical Appendix published in Heijdra et al. (2012).

## 2. THE MODEL

### 2.1. Consumers

Each agent lives for a maximum of two periods and faces a positive probability of death between the first and the second period. Agents work full-time during the first period of their lives (termed "youth" ) and—if they survive—retire in the second period ("old age"). The expected lifetime utility of an individual born at time $t$ is given by

$$\mathbf{E}\Lambda_t^y \equiv U(C_t^y) + \frac{1-\pi}{1+\rho} U\left(C_{t+1}^o\right), \tag{1}$$

where $C_t^y$ and $C_{t+1}^o$ are consumption during youth and old age, respectively, $\rho > 0$ is the pure rate of time preference, and $\pi > 0$ is the probability of death.

Individuals have no bequest motive and therefore attach no utility to savings that remain after they die. We assume that the felicity function is of the CRRA type,

$$U(C) = \begin{cases} \dfrac{C^{1-1/\sigma} - 1}{1 - 1/\sigma} & \text{if } \sigma > 0,\ \sigma \neq 1, \\ \ln C & \text{if } \sigma = 1, \end{cases} \qquad \textbf{(2)}$$

where $\sigma$ is the elasticity of intertemporal substitution. The agent's budget identities for youth and old age are given by

$$C_t^y + S_t = w_t + Z_t^y, \qquad \textbf{(3a)}$$

$$C_{t+1}^o = Z_{t+1}^o + (1 + r_{t+1}) S_t, \qquad \textbf{(3b)}$$

where $w_t$ is the wage rate, $r_t$ is the interest rate, $S_t$ denotes the level of savings, and $Z_t^y$ and $Z_{t+1}^o$ are transfers received from the government during either youth or old age (see below). Combining the equations in (3) yields the consolidated lifetime budget constraint

$$C_t^y + \frac{C_{t+1}^o}{1 + r_{t+1}} = w_t + Z_t^y + \frac{Z_{t+1}^o}{1 + r_{t+1}}. \qquad \textbf{(4)}$$

Because of mortality risk, agents are not allowed to hold negative savings (i.e., loans). In the case of premature death their loans would be unaccounted for.

The agent chooses $C_t^y$, $C_{t+1}^o$, and $S_t$ to maximize expected lifetime utility (1) subject to the budget constraint (4) and a nonnegativity constraint on savings. Assuming an interior optimum ($S_t > 0$), the agent's optimal plans are fully characterized by

$$C_t^y = \Phi(r_{t+1}) \left[ w_t + Z_t^y + \frac{Z_{t+1}^o}{1 + r_{t+1}} \right], \qquad \textbf{(5)}$$

$$\frac{C_{t+1}^o}{1 + r_{t+1}} = [1 - \Phi(r_{t+1})] \left[ w_t + Z_t^y + \frac{Z_{t+1}^o}{1 + r_{t+1}} \right], \qquad \textbf{(6)}$$

$$S_t = [1 - \Phi(r_{t+1})] \left[ w_t + Z_t^y \right] - \Phi(r_{t+1}) \frac{Z_{t+1}^o}{1 + r_{t+1}}, \qquad \textbf{(7)}$$

where $\Phi(r_{t+1})$ is the marginal propensity to consume out of total wealth (wage income and transfers) in the first period:

$$\Phi(r_{t+1}) \equiv \left[ 1 + \left( \frac{1 - \pi}{1 + \rho} \right)^{\sigma} (1 + r_{t+1})^{\sigma - 1} \right]^{-1}, \qquad 0 < \Phi(\cdot) < 1. \qquad \textbf{(8)}$$

The impact of a change in the future interest rate on current savings is fully determined by the elasticity of intertemporal substitution $\sigma$; i.e., $\partial \Phi(x) / \partial x \gtreqless 0$ for $\sigma \lesseqgtr 1$.

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From an empirical perspective the most relevant case appears to be the one with $0 < \sigma < 1$. See, for example, Skinner (1985) and Attanasio and Weber (1995), who report estimates ranging between, respectively, 0.3 and 0.5, and 0.6 and 0.7. In our following discussion of the quantitative results, we will therefore consider the plausible case to be the one featuring $0 < \sigma < 1$. In this case the income effect is stronger than the substitution effect and savings decline as the interest rate rises.[2]

## 2.2. Demography

The population grows at an exogenous rate $n > 0$ so that every period a cohort of $L_t = (1+n)\,L_{t-1}$ young agents is born. In principle each generation lives for two periods, but not all of its members survive the transition from youth to old age. The total population at time $t$ is equal to $P_t \equiv (1-\pi)\,L_{t-1} + L_t$.

## 2.3. Production

There are a constant and large number of identical and perfectly competitive firms. The technology available to each individual firm $i$ is given by

$$Y_{it} = \Omega_t K_{it}^{\alpha} L_{it}^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{9}$$

where $Y_{it}$ is output, $K_{it}$ and $L_{it}$ are, respectively, capital and labor used in production, $\alpha$ is the efficiency parameter of capital, and $\Omega_t$ is the aggregate level of technology in the economy, which is considered as given by each firm. Factor demands of the individual firm are given by the following marginal productivity conditions:

$$w_t = (1-\alpha)\,\Omega_t k_{it}^{\alpha}, \tag{10a}$$

$$r_t + \delta = \alpha \Omega_t k_{it}^{\alpha-1}, \tag{10b}$$

where $k_{it} \equiv K_{it}/L_{it}$ is the capital intensity of firm $i$ and $\delta > 0$ is the depreciation rate. Under the assumption of perfect competition in both factor markets, all firms face the same factor prices and, therefore, they all choose the same capital intensity $k_{it} = k_t$.

Generalizing the insights of Pecchenino and Pollard (1997, p. 28) to a growing population, we postulate that the interfirm investment externality takes the form

$$\Omega_t = \Omega_0 k_t^{\eta}, \qquad 0 < \eta < 1-\alpha, \tag{11}$$

where $\Omega_0$ is a constant, $k_t \equiv K_t/L_t$ is the economywide capital intensity, $K_t \equiv \sum_i K_{it}$ is the total stock of capital, and $L_t \equiv \sum_i L_{it}$ is the total labor force.

According to (11), total factor productivity increases in line with the aggregate capital intensity in the economy. That is, if an individual firm increases its capital stock, all firms benefit through a boost in the general productivity level $\Omega_t$. The strength of this interfirm investment externality is governed by the parameter $\eta$. Using the general productivity index (11), we can write output (9) and factor prices (10) in aggregate terms,

$$y_t = f(k_t) \equiv \Omega_0 k_t^{\alpha+\eta}, \tag{12}$$

$$w_t = (1-\alpha)\,\Omega_0 k_t^{\alpha+\eta}, \tag{13}$$

$$r_t = \alpha \Omega_0 k_t^{\alpha+\eta-1} - \delta, \tag{14}$$

where $y_t \equiv Y_t / L_t$ is the level of output per worker and $Y_t \equiv \sum_i Y_{it}$ is aggregate output. Because $\eta$ is strictly less than $1-\alpha$, there are diminishing returns to capital at the aggregate level and the long-run growth rate in per capita variables is exogenously determined and equal to zero.[3]

The economy-wide resource constraint is given by $Y_t + (1-\delta)\,K_t = L_t C_t^y + (1-\pi)\,L_{t-1} C_t^o + G_t + K_{t+1}$, where $G_t$ is unproductive government expenditure. Total available resources, consisting of output and the undepreciated part of the capital stock, are spent on consumption (by young and surviving old individuals and the government) and on the future stock of capital. In per capita terms, the resource constraint can thus be written

$$y_t + (1-\delta)\,k_t = C_t^y + \frac{1-\pi}{1+n} C_t^o + g_t + (1+n)\,k_{t+1}, \tag{15}$$

where $g_t \equiv G_t / L_t$.

## 2.4. First-Best Social Optimum

To prepare for the discussion to follow, we first characterize the properties of the steady-state first-best social optimum (FBSO). The social planner chooses nonnegative values for $C^y$, $C^o$, $k$, and $g$ to maximize steady-state welfare, $\mathbf{E}\Lambda^y \equiv U(C^y) + \frac{1-\pi}{1+\rho} U(C^o)$, subject to the steady-state resource constraint $f(k) - (\delta+n)\,k = C^y + \frac{1-\pi}{1+n} C^o + g$ and the production function $f(k) = \Omega_0 k^{\alpha+\eta}$. In addition to satisfying the constraints, the first-best social optimum has the following features:

$$\frac{U'(\tilde{C}^y)}{U'(\tilde{C}^o)} = \frac{1+n}{1+\rho}, \tag{S1}$$

$$f'(\tilde{k}) = n + \delta, \tag{S2}$$

$$\tilde{g} = 0. \tag{S3}$$

Using the terminology of Samuelson (1968), we refer to requirement (S1) of the FBSO as the biological-interest-rate golden rule (BGR), and to requirement (S2) as the production golden rule (PGR). Of course, requirement (S3) just states that the social planner does not waste valuable resources.

## 3. DECENTRALIZED EQUILIBRIA

Because agents face a risk of dying after the first period of life and do not have a voluntary bequest motive, it is necessary to make an assumption regarding the disposal of the assets of the unlucky individuals who perish before reaching old age. In the remainder of this paper we consider two canonical cases. In the first case, we assume that the government collects the accidental bequests and either consumes them itself or transfers the resources to agents who are still alive. In the second case, we postulate the existence of an actuarially fair private annuity market through which the resources left by those who pass away young boost the rate of return on the savings of surviving investors.

The remainder of this section contains details of the different public and private redistribution schemes. Next, we state and prove that the model is stable under every scenario.

### 3.1. Redistribution Schemes

*Government redistribution.* Consider first the case in which the government administers the collection and disposal of the accidental bequests. In doing so, it maintains a period-by-period balanced budget, without issuing debt or retaining funds. The government's budget constraint is then given by

$$\pi\,(1+r_t)\,K_t = (1-\pi)\,L_{t-1}Z_t^o + L_t Z_t^y + G_t. \tag{16}$$

Equation (16) shows that the assets left behind by the agents who die after the first period (left-hand side) are used to finance transfers to the survivors, $Z_t^o$, transfers to the newly arrived young, $Z_t^y$, and unproductive government expenditure, $G_t$.

We distinguish three pure government redistribution schemes:

WE: Wasteful expenditure by the government: $G_t > 0$, $Z_t^y = Z_t^o = 0$.
TO: Lump-sum transfers to the old: $Z_t^o > 0$, $G_t = Z_t^y = 0$.
TY: Lump-sum transfers to the young: $Z_t^y > 0$, $G_t = Z_t^o = 0$.

*Annuity market.* The fourth redistribution scheme that we consider is the one implied by the existence of a private annuity market. An annuity is a financial asset that pays a given return contingent upon survival of the annuitant to the second period of life. If the annuitant dies prematurely, then his assets accrue to the annuity firm. Let $r_{t+1}^{\mathrm{A}}$ denote the net rate of return on annuities. Assuming perfect competition among annuity firms,[4] the zero-profit condition is given by $(1+r_{t+1})L_t S_t = (1+r_{t+1}^{\mathrm{A}})(1-\pi)L_t S_t$, which implies

$$1 + r_{t+1}^{\mathrm{A}} = \frac{1+r_{t+1}}{1-\pi}. \tag{17}$$

It follows that $1+r_{t+1}^{\mathrm{A}} > 1+r_{t+1}$; i.e., the return on annuities exceeds the return on regular assets. Hence, in the absence of a bequest motive, it is optimal for the agent to fully annuitize his financial wealth [consistent with, inter alia, Yaari (1965) and Davidoff et al. (2005)]. Furthermore, there will be no accidental bequests. In terms of equations (5)–(8), $r_{t+1}$ is replaced by $r^{A}_{t+1}$ and transfers are zero.

In summary, in the presence of perfect markets for private annuities (PA), the fourth redistribution scheme takes the following form:

PA: Perfect private annuities and full annuitization of wealth: $G_t = Z^y_t = Z^o_t = 0$.

*Perturbation parameters.* We can combine the model equations for all four redistribution schemes by using perturbation parameters; see Table 1. We focus on pure recycling scenarios, which we distinguish with the aid of the zero-one perturbation parameters $z_1$, $z_2$, and $z_3$:[5]

WE: $z_1 = z_2 = z_3 = z_3^- = 0$.
TO: $z_1 = 1$, $z_2 = z_3 = z_3^- = 0$.
TY: $z_2 = 1$, $z_1 = z_3 = z_3^- = 0$.
PA: $z_1 = z_2 = 0$, $z_3 = z_3^- = 1$.

By including $z_3$ in the definition for $\Phi(\cdot)$ given in (T1.3), equations (T1.1)–(T1.2) and (T1.7) are sufficiently general to cover all redistribution schemes. Note furthermore that equation (T1.6) in Table 1 is obtained by writing (16) in per-worker format and using the perturbation parameters.

### 3.2. Equilibrium Existence and Stability

The fundamental difference equation (FDE hereafter) characterizing the model's existence and stability properties is obtained by substituting equations (T1.4)–(T1.6) into (T1.7):

$$\Psi(k_{t+1}, z_1, z_3) = \Gamma\left(k_t, z_2, z_3^-\right), \tag{18}$$

where $\Psi(k_{t+1}, z_1, z_3)$ and $\Gamma\left(k_t, z_2, z_3^-\right)$ are given by

$$\Psi(k_{t+1}, z_1, z_3) \equiv \frac{1 + z_1(1 - z_3)\frac{\pi}{1-\pi}\Phi(k_{t+1}, z_3)}{1 - \Phi(k_{t+1}, z_3)} k_{t+1}, \tag{19}$$

$$\Gamma\left(k_t, z_2, z_3^-\right) \equiv \frac{\left[1 - \alpha\left(1 - z_2\left(1 - z_3^-\right)\pi\right)\right]\Omega_0 k_t^{\alpha+\eta} + z_2\left(1 - z_3^-\right)\pi(1-\delta)k_t}{1+n}, \tag{20}$$

and $\Phi(k, z_3)$ is defined as follows:[6]

$$\Phi(k, z_3) \equiv \left[1 + (1 - z_3\pi)^{1-\sigma}\left(\frac{1-\pi}{1+\rho}\right)^{\sigma}\left(1 - \delta + \alpha\Omega_0 k^{\alpha+\eta-1}\right)^{\sigma-1}\right]^{-1}. \tag{21}$$

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**TABLE 1.** The model

(a) Individual choices

$$C_t^y = \Phi(r_{t+1}, z_3)\left[w_t + Z_t^y + \frac{Z_{t+1}^o}{1+r_{t+1}}\right] \quad \text{(T1.1)}$$

$$(1-z_3\pi)\frac{C_{t+1}^o}{1+r_{t+1}} = [1-\Phi(r_{t+1}, z_3)]\left[w_t + Z_t^y + \frac{Z_{t+1}^o}{1+r_{t+1}}\right] \quad \text{(T1.2)}$$

$$\Phi(r_{t+1}, z_3) \equiv \left[1 + (1-z_3\pi)^{1-\sigma}\left(\frac{1-\pi}{1+\rho}\right)^{\sigma}(1+r_{t+1})^{\sigma-1}\right]^{-1} \quad \text{(T1.3)}$$

(b) Factor prices and redistribution scheme

$$r_t = \alpha\Omega_0 k_t^{\alpha+\eta-1} - \delta \quad \text{(T1.4)}$$

$$w_t = (1-\alpha)\,\Omega_0 k_t^{\alpha+\eta} \quad \text{(T1.5)}$$

$$\begin{bmatrix} g_t \\ Z_t^o \\ Z_t^y \end{bmatrix} = \begin{bmatrix} (1-z_1-z_2)(1-z_3) \\ z_1\dfrac{1+n}{1-\pi}(1-z_3) \\ z_2(1-z_3^-) \end{bmatrix} \pi\,(1+r_t)\,k_t \quad \text{(T1.6)}$$

(c) Fundamental difference equation

$$(1+n)\,k_{t+1} = S_t = [1-\Phi(r_{t+1}, z_3)]\left[w_t + Z_t^y\right] - \Phi(r_{t+1}, z_3)\frac{Z_{t+1}^o}{1+r_{t+1}} \quad \text{(T1.7)}$$

*Note*: Definitions. Endogenous are $C_t^y$, $C_{t+1}^o$, $S_t$, $r_{t+1}$, $w_t$, $k_t$, and—provided government redistribution takes place—one of $Z_t^y$ or $Z_t^o$ or $g_t$. Parameters: mortality rate $\pi$, population growth rate $n$, rate of time preference $\rho$, capital coefficient in the technology $\alpha$, investment externality coefficient $\eta$, scale factor in the technology $\Omega_0$, and depreciation rate of capital $\delta$. Perturbation parameters: $z_1$, $z_2$, and $z_3$ (as well as its lagged value, $z_3^-$).

One of the crucial structural parameters is the intertemporal substitution elasticity, $\sigma$. Although the model can accommodate a wide range of values for $\sigma$, we nevertheless make the following assumption.

Assumption 1 [Admissible values for $\sigma$]. The intertemporal substitution elasticity satisfies

$$0 < \sigma \leq \bar{\sigma} \equiv \frac{2-\alpha-\eta}{1-\alpha-\eta}.$$

We defend this assumption on two grounds. First, the restriction is very mild. Indeed, empirical evidence suggests that $\sigma$ falls well short of unity, whereas—even in the absence of external effects ($\eta = 0$)—$\bar{\sigma}$ is much larger than unity. For example, for a capital share of $\alpha = 0.3$, we find that $\bar{\sigma} = 2.43$. In the presence of external effects ($\eta > 0$), $\bar{\sigma}$ is even larger. Second, by restricting the range of admissible values for $\sigma$, the existence and stability proofs are simplified substantially.

We can prove the following proposition.

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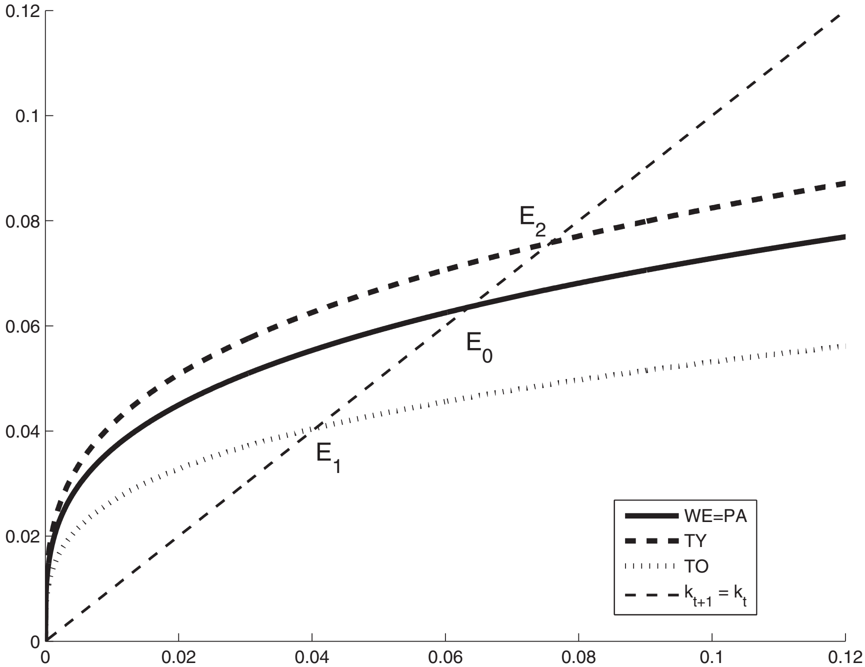


**FIGURE 1.** Phase diagram and steady-state equilibria (logarithmic felicity $\sigma = 1$).

PROPOSITION 1 [Existence and stability]. *Consider the model as given in (18) and adopt Assumption 1. The following properties can be established:*

(i) *For each scenario* $i \in \{\mathrm{WE}, \mathrm{TO}, \mathrm{TY}, \mathrm{PA}\}$*, the resulting model has two steady-state solutions; the trivial one features* $k_{t+1} = k_t = 0$*, and the economically relevant one satisfies* $k_{t+1} = k_t = \hat{k}^i$*, where* $\hat{k}^i$ *solves (18) with the relevant perturbation parameters substituted.*

(ii) *For each scenario i, the trivial steady-state solution is unstable, whereas the nontrivial solution is stable:*

$$0 < \frac{dk_{t+1}}{dk_t} < 1, \qquad \text{for } k_{t+1} = k_t = \hat{k}^i.$$

*For any positive initial value the capital intensity converges monotonically to* $\hat{k}^i$.

(iii) *The steady-state capital intensity satisfies the following inequalities:*

$$0 < \hat{k}^{\mathrm{TO}} < \hat{k}^{\mathrm{WE}} < \hat{k}^{\mathrm{TY}}, \qquad \hat{k}^{\mathrm{PA}} \lesseqqgtr \hat{k}^{\mathrm{WE}} \quad \Leftrightarrow \quad \sigma \lesseqqgtr 1.$$

Proof. See Heijdra et al. (2012). ■

In Figure 1 we visualize the phase diagrams for the different redistribution schemes. This figure is based on the following plausible parameter values, which are used throughout much of the paper. Each phase of life covers 40 years, the population grows by 1% per annum (so that $n = (1+0.01)^{40} - 1 = 0.49$), individuals face a probability of death between youth and old age of 30% ($\pi = 0.3$), the capital share of output is 30% ($\alpha = 0.3$), and the depreciation rate of capital is 6% per annum ($\delta = 0.92$). In the *benchmark* model we assume that the elasticity of intertemporal substitution is $\sigma = 1$ (i.e., log-utility) and that the investment externality is absent ($\eta = 0$). We set the production function constant and the time preference rate such that output per worker is equal to unity and the interest rate is 4% per annum ($\hat{r} = 3.80$) in the WE scenario. We obtain $\Omega_0 = 2.29$ and $\rho = 3.47\%$ or 3.82% annually. The resulting steady-state values of the key variables of the model are given in Table 2(a).[7] Note that Assumption 1 is easily satisfied for this calibration and that the equilibrium is dynamically efficient ($\hat{r} > n$).

In Figure 1 the solid line represents the FDE (18) for the WE and PA scenarios and the thin dashed line is the steady-state condition, $k_{t+1} = k_t$.[8] The economically relevant steady-state equilibrium is at point $E_0$, where the slope of (18) is strictly less than unity.[9] The thick dotted and dashed lines in Figure 1 represent the FDE for, respectively, the TO and TY scenarios. It is easy to prove that both the shapes and the relative locations of the FDEs for WE, TO, and TY are qualitatively the same for all values of $\sigma$. Furthermore, the FDE for PA lies below (above) the one for WE for $\sigma < 1$ ($\sigma > 1$). Proposition 1 confirms these results analytically.

### 3.3. Welfare Effects

Later, in Sections 4 and 5, we also study the welfare implications of the different scenarios. With bounded externalities ($0 \leq \eta < 1 - \alpha$), consumption by young and old agents ultimately converges to time-invariant steady-state values. As a result, we can compare the welfare effects of regime switches by evaluating the lifetime utility of newborns, both along the transition path and in the steady state. The welfare effect for the old at the time of the shock follows trivially from their budget identity (3b), which can be rewritten as

$$C_t^o = \left[1 + \frac{z_1 \pi}{1-\pi}\right] (1+r_t)(1+n)\, k_t, \tag{22}$$

where we have used the second expression in (T1.6) and recall that $S_{t-1} = (1+n)\, k_t$. For the shock-time old agents, all terms featured in (22) are predetermined except the transfers to the old, occurring exclusively in the TO scenario (for which $z_1 = 1$). Hence, $C_t^o$ will not be affected following a policy change, unless the switch involves the TO case.

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The (indirect) lifetime utility function of current and future newborns can be written as follows (for $\tau = 0, 1, \ldots$):

$$\mathbf{E}\Lambda^y_{t+\tau} \equiv \begin{cases} \dfrac{\Phi\left(r_{t+\tau+1}, z_3\right)^{-1/\sigma}\left(H^y_{t+\tau}\right)^{1-1/\sigma} - \dfrac{2+\rho-\pi}{1+\rho}}{1-1/\sigma} & \text{for } \sigma > 0,\ \sigma \neq 1, \\ \Xi_0 + \dfrac{2+\rho-\pi}{1+\rho} \ln H^y_{t+\tau} + \dfrac{1-\pi}{1+\rho} \ln\left(\dfrac{1+r_{t+\tau+1}}{1-z_3\pi}\right) & \text{for } \sigma = 1, \end{cases} \tag{23}$$

where $\Xi_0 \equiv \ln(\frac{1+\rho}{2+\rho-\pi}) + \frac{1-\pi}{1+\rho}\ln(\frac{1-\pi}{2+\rho-\pi})$ is a constant and human wealth at the birth of agents born $\tau$ periods after the policy change is given by

$$H^y_{t+\tau} \equiv w_{t+\tau} + Z^y_{t+\tau} + \frac{Z^o_{t+\tau+1}}{1+r_{t+\tau+1}}. \tag{24}$$

The expressions in (23)–(24) can be used to compute the transitions paths for $\mathbf{E}\Lambda^y_{t+\tau}$ under the different scenarios and the entries for $\widehat{\mathbf{E}\Lambda}^y$ in Table 2. For the analytical welfare effects at impact and in the long run, however, we employ the Envelope Theorem [see Heijdra et al. (2012)].

For each scenario change, an important component of the long-run welfare effect consists of the induced weighted effect on factor prices. To prepare for the discussion to follow, we state a useful lemma that exploits an important property of the factor-price frontier.[10]

LEMMA 1 [Implications of the factor price frontier]. *Assume that the economy is initially in the steady state associated with the WE or TY scenario, and is dynamically efficient ($\hat{r} > n$). Let $dk_{t+\infty}/dz_i$ denote the long-run effect on the capital intensity of a unit perturbation in $z_i$ occurring at shock-time $\tau = 0$ and evaluated at $z_i = 0$. It follows that the long-run effect on weighted factor prices can be written as*

$$\frac{\hat{C}^o}{(1+\hat{r})^2}\frac{dr_{t+\infty}}{dz_i} + \frac{dw_{t+\infty}}{dz_i} = \Delta \frac{dk_{t+\infty}}{dz_i}, \tag{L1.1}$$

*where $\Delta$ is a positive constant:*

$$\Delta \equiv \left[\eta + \alpha\left(1-\alpha-\eta\right)\frac{\hat{r}-n}{1+\hat{r}}\right]\frac{\hat{r}+\delta}{\alpha} > 0. \tag{L1.2}$$

Proof. See Heijdra et al. (2012). ■

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**TABLE 2.** Steady-state equilibrium values

| | Panel A: $\eta = 0, \sigma = 1$ | | | | Panel B: $\eta = 0, \sigma = \frac{1}{2}$ | | | | Panel C: $\eta = 0, \sigma = \frac{3}{2}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (a) WE | (b) TO | (c) TY | (d) PA | (e) WE | (f) TO | (g) TY | (h) PA | (i) WE | (j) TO | (k) TY | (l) PA |
| $\hat{C}^y$ | 0.6053 | 0.5512 | 0.7218 | 0.6053 | 0.6053 | 0.5057 | 0.7393 | 0.5577 | 0.6053 | 0.5681 | 0.7145 | 0.6226 |
| $\hat{C}^o$ | 0.4546 | 0.5647 | 0.4804 | 0.6495 | 0.4546 | 0.5040 | 0.5002 | 0.5741 | 0.4546 | 0.5893 | 0.4725 | 0.6815 |
| $\hat{g}$ | 0.0916 | | | | 0.0916 | | | | 0.0916 | | | |
| $\hat{Z}^o$ | | 0.1694 | | | | 0.1512 | | | | 0.1768 | | |
| $\hat{Z}^y$ | | | 0.0968 | | | | 0.1008 | | | | 0.0952 | |
| $\hat{y}$ | 1.0000 | 0.8736 | 1.0542 | 1.0000 | 1.0000 | 0.7821 | 1.0957 | 0.8877 | 1.0000 | 0.9105 | 1.0377 | 1.0472 |
| $\hat{k}$ | 0.0636 | 0.0405 | 0.0758 | 0.0636 | 0.0636 | 0.0280 | 0.0862 | 0.0428 | 0.0636 | 0.0465 | 0.0720 | 0.0742 |
| $\hat{w}$ | 0.7000 | 0.6115 | 0.7380 | 0.7000 | 0.7000 | 0.5474 | 0.7670 | 0.6214 | 0.7000 | 0.6374 | 0.7264 | 0.7330 |
| $\hat{r}$ | 3.8010 | 5.5491 | 3.2541 | 3.8010 | 3.8010 | 7.4546 | 2.8954 | 5.3121 | 3.8010 | 4.9544 | 3.4106 | 3.3198 |
| $\hat{r}_a$ | 4.00 | 4.81 | 3.69 | 4.00 | 4.00 | 5.48 | 3.46 | 4.71 | 4.00 | 4.56 | 3.78 | 3.73 |
| $\hat{r}_a^{\mathrm{A}}$ | | | | 4.93 | | | | 5.65 | | | | 4.65 |
| $\widehat{\mathbf{E}\Lambda}^y$ | −0.6253 | −0.6851 | −0.4406 | −0.5695 | −0.7930 | −1.0930 | −0.4699 | −0.8801 | −0.5816 | −0.5988 | −0.4322 | −0.5003 |
| (m) $\mathbf{E}\Lambda_t^y$ | $-0.6253^{\ddagger}$ | −0.5435 | −0.4963 | −0.5695 | $-0.7930^{\ddagger}$ | −0.6532 | −0.6003 | −0.6782 | $-0.5816^{\ddagger}$ | −0.5145 | −0.4676 | −0.5426 |
| (n) $\mathbf{E}\Lambda_t^y$ | | | $-0.4406^{\ddagger}$ | −0.3848 | | | $-0.4699^{\ddagger}$ | −0.3664 | | | $-0.4322^{\ddagger}$ | −0.3928 |
| (o) $\mathbf{E}\Lambda_t^y$ | | $-0.6851^{\ddagger}$ | | −0.7112 | | $-1.0930^{\ddagger}$ | | −1.1215 | | $-0.5988^{\ddagger}$ | | −0.6267 |

*Notes*: Circumflexes denote steady-state values. To facilitate interpretation, $\hat{r}_a$ and $\hat{r}_a^{\mathrm{A}}$ are reported as annual percentage rates of return. In rows (m)–(o), $\mathbf{E}\Lambda_t^y$ is expected lifetime utility of the shock-time young, and the superscript ‡ denotes the initial steady-state equilibrium that is perturbed.

## 4. PUBLIC REDISTRIBUTION OF ACCIDENTAL BEQUESTS

Suppose that at some time $t$ the economy has converged to the steady state implied by the WE scenario, i.e., $k_t = \hat{k}^{\text{WE}}$. What would happen at impact, during transition, and in the long run if the government were to switch to a transfer scenario? We study two such policy changes in turn, from WE to TO and from WE to TY. The dynamic effects of a scenario switch on the capital intensity can be computed by perturbing (18).

### 4.1. Transfers to the Old

The effects of a policy change from the WE scenario to the TO scenario are obtained by perturbing the fundamental difference equation (18) from the initial steady state, $\Psi(\hat{k}^{\text{WE}}, 0, 0) = \Gamma(\hat{k}^{\text{WE}}, 0, 0)$, to $\Psi\,(k_{t+1}, 1, 0) = \Gamma\,(k_t, 0, 0)$. The policy switch thus consists of a unit increase in $z_1$ occurring at time $t$ in combination with the initial condition $k_t = \hat{k}^{\text{WE}}$. Proposition 1 proves (for the general case) that the capital intensity falls monotonically,

$$\left.\frac{dk_{t+\infty}}{dz_1}\right|_{k_t=\hat{k}^{\text{WE}}} < \left.\frac{dk_{t+1}}{dz_1}\right|_{k_t=\hat{k}^{\text{WE}}} < 0, \tag{25}$$

where $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{\text{TO}}$. In terms of Figure 1, the equilibrium shifts over time from $\text{E}_0$ to $\text{E}_1$. Intuitively, if transfers are given to old agents, the old at the time of the policy switch are able to increase their consumption, as they had not anticipated this windfall gain. The young at the time of the shock, however, react to the transfers they will receive in old age by reducing their saving below what it would have been in the WE scenario. This explains why the capital intensity starts to fall.

The quantitative long-run results are reported for various values of $\sigma$ in Table 2, columns (b), (f), and (j). Compared to the WE scenario, long-run output per worker falls substantially in the TO case. Quantitatively a relatively low (high) intertemporal substitution effect exacerbates (mitigates) the crowding-out effect on the capital intensity.

The welfare effects of the policy switch are as follows. Let $\hat{C}^o$, $\hat{C}^y$, $\hat{r}$, $\hat{w}$, and $\hat{k}$ denote steady-state values associated with the WE scenario. The welfare effect on the old at time $t$ is equal to

$$\frac{d\mathbf{E}\Lambda_{t-1}^{y}\,(z_1)}{dz_1} = \frac{1+n}{1+\rho}U'(\hat{C}^o)\pi\,(1+\hat{r})\,\hat{k} > 0. \tag{26}$$

The shock-time old are unambiguously better off because they receive a windfall transfer from the government. The welfare effect on the young at time $t$ is more complicated because they can still alter their consumption and savings decisions in the light of the policy shock. Although the wage rate faced by these agents is predetermined, their revised saving plans will induce a change in the future

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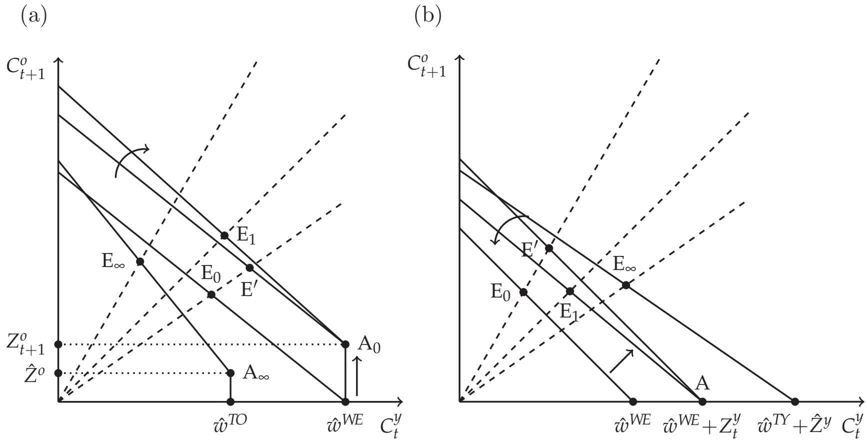


**FIGURE 2.** Effect of government transfers: (a) transfers to the old (TO); (b) transfers to the young (TY).

interest rate. After some manipulation we find

$$\frac{d\mathbf{E}\Lambda^y_t(z_1)}{dz_1} = U'(\hat{C}^y)(1+n)\hat{k}\left[\frac{\pi}{1-\pi} + \frac{1}{1+\hat{r}}\frac{dr_{t+1}}{dz_1}\right] > 0. \qquad \textbf{(27)}$$

The first term in square brackets represents the *direct effect* of the lump-sum transfer received in old age. Taken in isolation, this transfer expands the choice set and thus increases the expected lifetime utility of shock-time newborns. The direct effect can be explained with the aid of Figure 2a. The original budget line passes through $E_0$, which is the initial equilibrium. The shock-time young anticipate transfers in old age equal to $Z^o_{t+1}$. This shifts the budget line up in a parallel fashion.[11] Holding constant the initially expected future interest rate, the optimal point shifts from $E_0$ to $E'$. But this is not the end of the story, because it is only the partial equilibrium effect.

The second term in square brackets on the right-hand side of (27) represents the *general equilibrium effect* of the policy change. It follows from (25) that the future capital stock is lower and the interest rate is higher as a result of the switch. In terms of Figure 2a the budget line pivots in a clockwise fashion around point $A_0$ and the optimal consumption bundle moves from $E'$ to $E_1$. At impact the general equilibrium effect thus brings about a further expansion of the choice set faced by the shock-time young. Not surprisingly, therefore, the change in welfare at impact is unambiguously positive for such agents. The quantitative welfare effects experienced by the shock-time young are reported in row (m) of Table 2.

The welfare effect experienced by future steady-state generations can be written

$$\frac{d\mathbf{E}\Lambda^y_{t+\infty}(z_1)}{dz_1} = U'(\hat{C}^y)\left[\frac{\pi(1+n)}{1-\pi}\hat{k} + \Delta\frac{dk_{t+\infty}}{dz_1}\right] \gtreqless 0, \qquad \textbf{(28)}$$

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where Lemma 1 implies that $\Delta > 0$ and we note that $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{\mathrm{TO}}$. The first term in brackets represents the steady-state direct effect, which is positive. The second term comprises the general equilibrium effect, which is negative because capital is crowded out in the long run [see (25) earlier]. On one hand, the reduction in the long-run capital intensity increases the interest rate, which affects welfare positively. But on the other hand, it also reduces the wage rate, which lowers welfare. In terms of Figure 2a, the budget line shifts to the left because of the fall in the long-run wage ($\hat{w}^{\mathrm{TO}} < \hat{w}^{\mathrm{WE}}$). In addition, long-run transfers are lower than anticipated transfers at impact ($\hat{Z}^{o} < Z^{o}_{t+1}$), so that the point $\mathrm{A}_\infty$ lies southwest from $\mathrm{A}_0$. The steady-state interest rate exceeds the future rate faced by shock-time newborns ($\hat{r}^{\mathrm{TO}} > r_{t+1}$); i.e., the budget line is steeper than at impact. The steady-state equilibrium is at the point $\mathrm{E}_\infty$.

Comparing columns (a) and (b) of Table 2 reveals that the long-run welfare effect of the policy switch is negative; i.e., the crowding out of capital induces a very strong reduction in wages, which dominates the joint effect of the transfers and the higher interest rate. Ignoring agents who are alive at the time of the shock, it is thus better to let the accidental bequests go to waste than to give them to the elderly. To better understand the intuition behind this paradoxical result, we next turn to the comparison of the steady states attained in the decentralized market outcome and in the FBSO, as covered in Section 2.4.

In the decentralized equilibrium for the WE scenario, the steady-state equilibrium satisfies the resource constraint, $f(\hat{k}) - (\delta + n)\,\hat{k} = \hat{C}^{y} + \frac{1-\pi}{1+n}\hat{C}^{o} + \hat{g}$, as well as the following conditions:

$$\frac{U'(\hat{C}^{y})}{U'(\hat{C}^{o})} = \frac{(1-\pi)\,(1+\hat{r})}{1+\rho}, \tag{W1}$$

$$\frac{\alpha}{\alpha+\eta} f'(\hat{k}) = \hat{r} + \delta, \tag{W2}$$

$$\hat{g} = \pi\,(1+\hat{r})\,\hat{k}. \tag{W3}$$

Comparing (W1)–(W3) with (S1)–(S3) in Section 2.4, we find that the WE equilibrium features four distortions. First, the government engages in wasteful expenditure ($\hat{g} > \tilde{g} = 0$). Second, the death probability affects the consumption Euler equation in the decentralized equilibrium; i.e., $\pi$ features in (W1) but not in (S1). There is a missing market, as agents cannot insure against longevity risk. Third, if $\eta$ is strictly positive, then the decentralized economy underinvests in physical capital because the capital externality is not internalized by individual agents. Fourth, by assumption, the steady-state interest rate exceeds the rate of population growth ($\hat{r} > n$).

We can rewrite the welfare effect on future steady-state generations—given in (28)—as follows:

$$\frac{d\mathbf{E}\Lambda^{y}_{t+\infty}(z_1)}{dz_1} = U'(\hat{C}^{y})\frac{\pi\,(1+n)\,\hat{k}}{1-\pi}\left[1-\Theta\right], \tag{29}$$

where $\Theta$ is defined as

$$\Theta \equiv \left[\frac{\eta}{\alpha\,(1-\alpha-\eta)} + \frac{\hat{r}-n}{1+\hat{r}}\right]\frac{1+\hat{r}}{1+n}\,\frac{\frac{\hat{r}+\delta}{1+\hat{r}}\Phi(\hat{k},0)}{1-(1-\sigma)\,\frac{\hat{r}+\delta}{1+\hat{r}}\Phi(\hat{k},0)} \geq 0. \tag{30}$$

In combination with the requirements of the FBSO discussed in Section 2.4, the expressions in (29) and (30) can be used to build intuition on the long-run welfare effect of the policy switch from WE to TO. In the adoption of the TO scenario, wasteful government expenditure is eliminated, which implies that one distortion is removed; i.e., (S3) holds for the TO case and $\hat{g}^{\mathrm{TO}} = \tilde{g} = 0$. If there were no capital externality ($\eta = 0$) *and* the steady-state interest rate equaled the rate of population growth ($\hat{r}^{\mathrm{TO}} = n$), then (S2) would also hold in the TO scenario; i.e., $\hat{k}^{\mathrm{TO}} = \tilde{k}$. The only distortion that would remain is the one resulting from the missing insurance market, i.e., $(1-\pi)\left(1+\hat{r}^{\mathrm{TO}}\right) < 1+n$. For $\hat{r} = n$ and $\eta = 0$, we find from (30) that $\Theta = 0$ and from (29) that the long-run welfare effect is strictly positive. The switch from WE to TO benefits all generations to the same extent in this hypothetical case because waste is eliminated, there is no transitional dynamics in the capital stock (and thus in factor prices), and the additional resources lead to an equiproportionate increase in youth and old-age consumption.

Matters are much more complicated in a dynamically efficient economy. For $\hat{r} > n$ and $0 \leq \eta < 1-\alpha$, it follows from (30) that $\Theta$ is strictly positive and, ceteris paribus $\hat{r}$ and $\hat{k}$, increasing in the externality parameter $\eta$. If $\eta = 0$ then WE and TO share two distortions, namely the missing insurance market and the violation of the BGR. It is a straightforward application of the theory of the second best that the welfare ranking between WE and TO is ambiguous in that case. In Table 3(a) we compute $\Theta$ for several values of the intertemporal substitution elasticity. Interestingly, $\Theta$ is strictly larger than unity for all but the most extreme values of $\sigma$. And for a relatively small capital externality [Table 3(b) with $\eta = \frac{1}{10}$], the same conclusion holds for *all* admissible values of $\sigma$!

In a plausibly parameterized dynamically efficient economy ($\hat{r} > n$), the switch from WE to TO is welfare-decreasing because it induces a decrease in the capital intensity and an increase in the interest rate in the long run. Hence, the policy change moves the economy further away from the FBSO.

## 4.2. Transfers to the Young

The effects of a policy switch from the WE case to the TY scenario are obtained by perturbing the fundamental difference equation (18) from the initial steady state,

**TABLE 3.** Value of $\Theta$ in a dynamically efficient economy ($\hat{r} > n$)

| | (a) $\eta = 0$ | (b) $\eta = \frac{1}{10}$ | (c) $\eta = \frac{1}{3}$ |
|---|---|---|---|
| $\sigma = \frac{1}{2}$ | 3.29 | 5.93 | 17.72 |
| $\sigma = 1$ | 1.89 | 3.41 | 10.19 |
| $\sigma = \frac{3}{2}$ | 1.33 | 2.39 | 7.15 |
| $\sigma = \bar{\sigma}$ | 0.85 | 1.41 | 3.07 |

$\Psi(\hat{k}^{\mathrm{WE}}, 0, 0) = \Gamma(\hat{k}^{\mathrm{WE}}, 0, 0)$, to $\Psi\,(k_{t+1}, 0, 0) = \Gamma\,(k_t, 1, 0)$. The policy change thus consists of a unit increase in $z_2$ occurring at time $t$ in combination with the initial condition $k_t = \hat{k}^{\mathrm{WE}}$. Proposition 1 shows that the capital intensity increases monotonically,

$$\left.\frac{dk_{t+\infty}}{dz_2}\right|_{k_t=\hat{k}^{\mathrm{WE}}} > \left.\frac{dk_{t+1}}{dz_2}\right|_{k_t=\hat{k}^{\mathrm{WE}}} > 0, \tag{31}$$

where $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{\mathrm{TY}}$. In terms of Figure 1, the equilibrium shifts over time from $\mathrm{E}_0$ to $\mathrm{E}_2$. Intuitively, by transfers being given to young agents only, the old at the time of the policy switch experience no effect at all. They just execute the plans conceived during their youth. In contrast, the shock-time young react to these transfers by increasing their savinsg above what they would have been under the WE scenario. This explains why the capital intensity starts to increase after the shock.

The quantitative long-run results are reported in Table 2, columns (c), (g), and (k). Compared to the WE scenario, long-run output per worker increases substantially under the TY case. Quantitatively a relatively low (high) intertemporal substitution effect exacerbates (mitigates) the crowding-in effect on the capital intensity.

The welfare effects of the policy switch are as follows. Let $\hat{C}^o$, $\hat{C}^y$, $\hat{r}$, $\hat{w}$, and $\hat{k}$ denote the steady-state values associated with WE. In the TY scenario the shock-time old do not receive any additional resources; i.e., $d\mathbf{E}\Lambda^y_{t-1}\,(z_2)\,/\,dz_2 = 0$. The welfare effect on the young at the time of the policy switch is given by

$$\frac{d\mathbf{E}\Lambda^y_t\,(z_2)}{dz_2} = U'(\hat{C}^y)\,(1+n)\,\hat{k}\left[\pi\frac{1+\hat{r}}{1+n} + \frac{1}{1+\hat{r}}\frac{dr_{t+1}}{dz_2}\right] > 0, \tag{32}$$

where the first term in square brackets is the direct effect and the second term is the general equilibrium effect. The direct effect is positive, but the general equilibrium effect is negative because the policy switch boosts capital accumulation, which leads to a reduction in the future interest rate. It is not difficult to show, however, that the direct effect is dominant so that welfare rises at impact. In terms of Figure 2b,

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the initial budget line passes through point $E_0$, the lump-sum transfer shifts the line in a parallel fashion to the right, and the decrease in the future interest rate rotates it in a counterclockwise fashion around point A. The direct effect consists of the move from $E_0$ to $E'$ and the general equilibrium effect is the move from $E'$ to $E_1$.

The change in welfare of the future steady-state generations can be written as

$$\frac{d\mathbf{E}\Lambda^{y}_{t+\infty}(z_2)}{dz_2} = U'(\hat{C}^y)\left[\pi\,(1+\hat{r})\,\hat{k} + \Delta\frac{dk_{t+\infty}}{dz_2}\right] > \frac{d\mathbf{E}\Lambda^{y}_{t}(z_2)}{dz_2} > 0, \quad \textbf{(33)}$$

where we have used Lemma 1 ($\Delta > 0$) and note that $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{\mathrm{TY}}$. Both terms in square brackets are positive, so that welfare unambiguously rises in the long run. Indeed, the general equilibrium effect ensures that future generations gain even more than the shock-time generation. The quantitative effects in columns (c), (g), and (k) of Table 2 confirm that, regardless of the magnitude of the intertemporal substitution elasticity, expected lifetime utility increases dramatically as a result of the policy switch. In terms of Figure 2b, the budget line shifts further to the right in the long run, both because the wage increases and because transfers are boosted. The decreased interest rate further rotates the budget line, but this effect is not large enough to lead to a reduction in the choice set for future generations.

To develop the economic intuition behind the strong steady-state welfare gain, we rewrite (33) as follows:

$$\frac{d\mathbf{E}\Lambda^{y}_{t+\infty}(z_2)}{dz_2} = U'(\hat{C}^y)\frac{\pi\,(1+n)\,\hat{k}}{1-\pi}\left[1 + \Theta\frac{1-\Phi(\hat{k},0)}{\Phi(\hat{k},0)}\right] > 0, \qquad \textbf{(34)}$$

where $\Theta$ is defined in (30). The switch from WE to TY is welfare-increasing because it induces an increase in the capital intensity and a decrease in the interest rate in the long run; i.e., the policy switch moves the economy closer to the FBSO.

## 5. TRAGEDY OF ANNUITIZATION

In this section we step away from the assumption that the government redistributes accidental bequests or wastes them completely. Instead we analyze the allocation and welfare effects of opening up a perfect annuity (PA) market at time $t$. We first study the case for which the initial scenario is WE; i.e., the switch is from WE to PA and the initial capital stock features $k_t = \hat{k}^{\mathrm{WE}}$. Next we study the case in which the switch is from the TY scenario to perfect annuities. In this case the initial capital stock satisfies $k_t = \hat{k}^{\mathrm{TY}}$. Surprisingly, in both cases it is quite possible that long-run welfare is *decreased* as a result of the introduction of a perfect annuity market, a phenomenon we label the tragedy of annuitization.

### 5.1. Strong Version

The effects of a switch from the WE scenario to the PA scenario are obtained by perturbing the fundamental difference equation (18) from the initial steady state, $\Psi(\hat{k}^{\text{WE}}, 0, 0) = \Gamma(\hat{k}^{\text{WE}}, 0, 0)$, to $\Psi\,(k_{t+1}, 0, 1) = \Gamma\,(k_t, 0, 1) = \Gamma\,(k_t, 0, 0)$. The policy switch thus consists of a unit increase in $z_3$ occurring at time $t$ in combination with the initial condition $k_t = \hat{k}^{\text{WE}}$. Proposition 1 establishes that the impact and long-run effects on the capital intensity depend on the magnitude of the intertemporal substitution parameter,

$$\sigma \lesseqqgtr 1 \quad \Rightarrow \quad \left.\frac{dk_{t+\infty}}{dz_3}\right|_{k_t=\hat{k}^{\text{WE}}} \lesseqqgtr \left.\frac{dk_{t+1}}{dz_3}\right|_{k_t=\hat{k}^{\text{WE}}} \lesseqqgtr 0, \tag{35}$$

where $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{\text{PA}}$.

In the benchmark case the intertemporal substitution elasticity is equal to unity, and it follows from (35) that the opening up of annuity markets has no effect on the capital intensity at all; i.e., the economy with perfect annuities features the same steady-state capital intensity as under the WE scenario ($k_t = \hat{k}^{\text{PA}} = \hat{k}^{\text{WE}}$ for all $t$). In terms of Figure 1, the phase diagrams for WE and PA coincide in that case. Youth consumption is unchanged and the additional resources resulting from annuitization are shifted entirely to old age.

Table 2(d) confirms that old-age consumption is significantly higher following the policy shock. Note also that the switch from WE to PA is quite different from the switch from WE to TO, even though both introduce risk sharing among old agents. In the latter case the anticipated transfers in old age lead to reduced saving during youth, which ultimately results in capital crowding-out. In contrast, in the former case the savings rate is unaffected by the policy change.

Because transfers are absent both before and after the opening up of annuity markets, the shock-time old are unaffected by this event; i.e., $d\mathbf{E}\Lambda^{y}_{t-1}\,(z_3)\,/dz_3 = 0$. The welfare effect on the young at the time of the policy switch is given by

$$\frac{d\mathbf{E}\Lambda^{y}_{t}\,(z_3)}{dz_3} = U'(\hat{C}^{y})\,(1+n)\,\hat{k}\left[\pi + \frac{1}{1+\hat{r}}\frac{dr_{t+1}}{dz_3}\right] > 0, \tag{36}$$

where the first term in square brackets is the direct effect and the second term is the general equilibrium effect. In the special case with $\sigma = 1$ and $k_t = \hat{k}^{\text{PA}}$, the latter effect is absent. It is easy to show that for all admissible values of $\sigma$, welfare unambiguously rises for the shock-time young—see also the entries in row (m) and columns (d), (h), and (l) of Table 2.

The long-run welfare effect is given by

$$\frac{d\mathbf{E}\Lambda^{y}_{t+\infty}\,(z_3)}{dz_3} = U'(\hat{C}^{y})\left[\pi\,(1+n)\,\hat{k} + \Delta\frac{dk_{t+\infty}}{dz_3}\right] \gtreqqless 0, \tag{37}$$

where we have used Lemma 1 ($\Delta > 0$) and note that $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{\text{PA}}$. The second term in square brackets represents the general equilibrium effect on factor prices. Of course, for $\sigma = 1$, these effects are absent and the impact and long-run effects coincide.

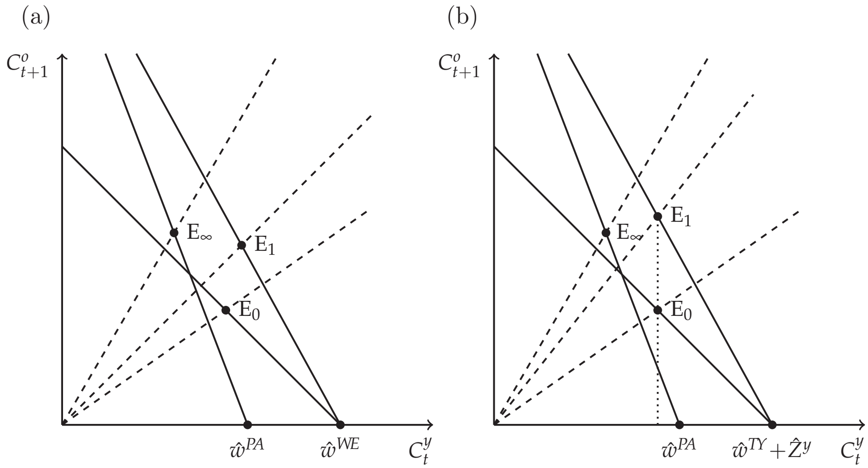


**FIGURE 3.** Opening up of perfect annuity markets: (a) initially in WE ($\sigma = \frac{1}{2}$); (b) initially in TY ($\sigma = 1$).

Empirical evidence, however, suggests that $\sigma$ falls well short of unity. It follows readily from (35) that for $\sigma < 1$ the impact and long-run effects on the capital intensity of the opening up of annuity markets are both negative.[12] Equation (36) shows that welfare of the shock-time young increases both because of the direct effect and because of the increase in the future interest rate. In the long run, however, capital crowding-out results in a reduction in wages, which shrinks the choice set and reduces welfare for future generations. Panel B of Table 2 provides quantitative evidence for the case with $\sigma = \frac{1}{2}$. As the comparison between columns (e) and (h) of Table 2 reveals, capital crowding-out is so strong that steady-state welfare is lower under perfect annuities than it is under the WE scenario! This is the first instance of a phenomenon that we call the *tragedy of annuitization*. Even though it is individually advantageous to make use of annuity products if they are available, their long-run general equilibrium effects lead to a reduction in the welfare of future generations.

Figure 3a illustrates the choices made by the shock-time young and the young born in the new steady state. The initial WE equilibrium is at point $E_0$. The budget line rotates in a clockwise fashion both because of annuitization and because the interest rate rises ($dr_{t+1}/dz_3 > 0$) and the new optimum for the shock-time young is at $E_1$. In the long run the decline in the capital intensity shifts the budget line to the left and steepens it, so the optimal choice facing the future steady-state young is at $E_\infty$. The pre-annuity point $E_0$ is no longer attainable to such generations.

The intuition behind the tragedy is not hard to come by. In the PA case the decentralized steady-state equilibrium is characterized by the steady-state resource constraint, $f(\hat{k}) - (\delta + n)\hat{k} = \hat{C}^y + \frac{1-\pi}{1+n}\hat{C}^o + \hat{g}$, as well as the following conditions:

$$\frac{U'(\hat{C}^y)}{U'(\hat{C}^o)} = \frac{(1-\pi)\left(1+\hat{r}^{\mathrm{A}}\right)}{1+\rho} = \frac{1+\hat{r}}{1+\rho}, \tag{P1}$$

$$\frac{\alpha}{\alpha+\eta} f'(\hat{k}) = \hat{r} + \delta, \tag{P2}$$

$$\hat{g} = 0. \tag{P3}$$

The PA equilibrium removes two of the distortions plaguing the WE equilibrium. First, the availability of annuities eliminates the missing-market distortion; i.e., $\pi$ does not feature in (P1), whereas it does in (W1). Second, there are no wasteful government expenditures. Indeed, in the absence of the capital externality ($\eta = 0$) and if $\hat{r} = n$, then the PA equilibrium decentralizes the FBSO—compare (S1)–(S3) with (P1)–(P3). But starting from a dynamically efficient economy ($\hat{r} > n$) featuring a plausible value of the intertemporal substitution elasticity ($\sigma = \frac{1}{2}$), the switch from WE to PA is welfare-decreasing because it induces capital crowding-out and an increase in the interest rate in the long run. Hence, the policy switch moves the economy further away from the FBSO.

## 5.2. Weak Version

We return to the benchmark case (with $\sigma = 1$) and assume that annuity markets are opened up with the economy located in the steady-state equilibrium of the TY scenario; i.e., $k_t = \hat{k}^{\mathrm{TY}}$ initially. A policy switch from the TY case to the PA scenario now involves two distinct changes. On one hand, the availability of annuities boosts the rate at which the young can save. On the other hand, full annuitization implies that accidental bequests are absent, so that the transfers to the *future* young are eliminated; i.e., $Z^y_{t+\tau} = 0$ for $\tau = 1, 2, \ldots$. The combined effect of these shocks can be studied as follows. At time $t$ there is a permanent switch from $z_3 = 0$ to $z_3 = 1$ and the fundamental difference equation (18) changes from the initial steady state, $\Psi(\hat{k}^{\mathrm{TY}}, 0, 0) = \Gamma(\hat{k}^{\mathrm{TY}}, 1, 0)$, to $\Psi\,(k_{t+1}, 0, 1) = \Gamma\,(k_t, 1, 0)$. At time $t+1$, $z_3^- = 1$ and the value of $z_2$ becomes irrelevant. Hence $\Psi\,(k_{t+2}, 0, 1) = \Gamma\,(k_{t+1}, 1, 0)$ switches to $\Psi\,(k_{t+2}, 0, 1) = \Gamma\,(k_{t+1}, 0, 1) = \Gamma\,(k_{t+1}, 0, 0)$. The resulting difference equations are solved using $k_t = \hat{k}^{\mathrm{TY}}$ as the initial condition.

With $\sigma = 1$, the marginal propensity to save out of current resources is constant. The shock-time young still receive transfers. It follows that there is no effect on saving; i.e., $k_{t+1} = \hat{k}^{\mathrm{TY}}$. Of course, the young from period $t+1$ onward no longer receive transfers and these generations will reduce their savings. Over time, the economy monotonically converges to $\hat{k}^{\mathrm{PA}}$, which is strictly less than $\hat{k}^{\mathrm{TY}}$ (because, for $\sigma = 1$, $\hat{k}^{\mathrm{PA}} = \hat{k}^{\mathrm{WE}}$ and $\hat{k}^{\mathrm{TY}} > \hat{k}^{\mathrm{WE}}$ by Proposition 1(iii)).

The key effects can be explained with the aid of Figure 3b. The initial steady state is at $\mathrm{E}_0$ and income during youth is equal to $\hat{w}^{\mathrm{TY}} + \hat{Z}^y$. At impact the transfers are predetermined, but the interest rate at which the young save increases because of annuitization; i.e., the budget line rotates in a clockwise direction. The new equilibrium is at point $E_1$, which lies directly above point $E_0$ (because $\sigma = 1$). In the long run, transfers are eliminated, capital is crowded out, the interest rate rises, and the wage rate falls. The long-run budget constraint passes through $E_\infty$, which is the new steady-state equilibrium.

The quantitative effects are summarized in Table 2(d). There is a strong crowding-out effect on the capital intensity. Youth consumption falls as a result of the elimination of transfers, whereas old-age consumption of survivors increases because of the higher return on savings. Comparing columns (c) and (d) in Table 2, we find that long-run output per worker falls by more than 5% .

The welfare effects are as follows. Because the shock-time old do not get any transfers either before or after the opening up of an annuity market, and they no longer save, they are unaffected by this event; i.e., $d\mathbf{E}\Lambda^{y}_{t-1}(z_3)/dz_3 = 0$. The welfare effect on the young at the time of the policy switch is given by

$$\frac{d\mathbf{E}\Lambda^{y}_{t}(z_3)}{dz_3} = U'(\hat{C}^{y})(1+n)\hat{k}\left[\pi + \frac{1}{1+\hat{r}}\frac{dr_{t+1}}{dz_3}\right] > 0. \qquad \textbf{(38)}$$

The shock-time young benefit for all values of $\sigma$, i.e., regardless of whether the next period's capital intensity falls ($\sigma = \frac{1}{2}$) or rises ($\sigma = \frac{3}{2}$)—see row (n) in Table 2. To this generation the benefits of annuitization are clear and simple.

Matters are not so clear-cut for future generations. Indeed, the long-run welfare effect is equal to

$$\frac{d\mathbf{E}\Lambda^{y}_{t+\infty}(z_3)}{dz_3} = U'(\hat{C}^{y})\left[-\pi(\hat{r}-n)\hat{k} + \Delta\frac{dk_{t+\infty}}{dz_3}\right] \lesseqqgtr 0, \qquad \textbf{(39)}$$

where we have used Lemma 1 ($\Delta > 0$) and note that $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{\mathrm{PA}}$. The first term in square brackets is negative in a dynamically efficient economy, but the sign of the second term depends on the strength of the intertemporal substitution effect. For the empirically relevant case, however, we have $0 < \sigma < 1$, capital is crowded out in the long run, and steady-state welfare unambiguously falls.[13]

Table 2 shows (for $\sigma = 1$) that future steady-state generations are worse off as a result of the opening up of private annuity markets. In fact, simulations reveal that only the shock-time young benefit from annuitization [see Heijdra et al. (2012)]. Effectively, private annuities redistribute assets from deceased to surviving elderly in an actuarially fair way, whereas transferring unintended bequests to the young constitutes an intergenerational transfer. This intergenerational transfer induces beneficial savings effects, which, in the end, lead to higher welfare under TY than under PA. This is the second example of a *tragedy of annuitization*. Even though it is individually rational to annuitize fully, this is not optimal from a social point of view. If all agents invest their financial wealth in the annuity market, then the resulting long-run equilibrium leaves everyone worse off than in the case where annuities are absent and accidental bequests are redistributed to the young.

### 5.3. The Role of Scale Economies

Although the theoretical results have been derived for the case of bounded externalities ($0 \leq \eta < 1-\alpha$), the numerical simulations presented thus far only cover a special case for which the capital externality is absent altogether ($\eta = 0$). In this section we briefly discuss the quantitative implications of including positive capital externalities. In addition, we touch on the knife-edge case featuring endogenous growth ($\eta = 1-\alpha$).

In Heijdra et al. (2012), we present the counterparts to Table 2 for, respectively, $\eta = 0.3$ and $\eta = 0.6$. All the qualitative conclusions reached in this and the preceding section are robust to these values of $\eta$. In particular, TO and TY remain, respectively, the worst and best steady-state equilibrium from a welfare perspective, and both the weak and strong versions of the tragedy of annuitization are valid. Quantitatively the capital externality gives rise to a multiplier that increases the absolute value of the changes induced by switching to different scenarios.

Heijdra et al. (2012) also discuss the knife-edge case with $\eta = 1-\alpha$. The endogenous growth rate is defined by

$$(1+n)(1+\gamma) = [1-\Phi(\bar{r}, z_3)]\left[(1-\alpha)\,\Omega_0 + \frac{Z_t^y}{k_t}\right] - \frac{\Phi(\bar{r}, z_3)}{1+\bar{r}}\frac{Z_{t+1}^o}{k_t}, \quad \textbf{(40)}$$

where $\gamma \equiv k_{t+1}/k_t - 1$ is the (time-invariant) equilibrium growth rate and we have used the fact that the interest rate is constant in this scenario to have $r_t = \bar{r} \equiv \alpha\Omega_0 - \delta$ for all $t$. Straightforward inspection of the growth rates reveals that $\gamma^{\mathrm{TY}} > \gamma^{\mathrm{WE}} > \gamma^{\mathrm{TO}}$ for all admissible values of $\sigma$. Hence, in terms of growth, it is better to give the accidental bequests to the young than to use them for wasteful expenditures, yet it is better to let the accidental bequests go to waste than to give them to the elderly.

Comparison with the private annuities scenario is more subtle. The introduction of such annuities increases the rate of return on savings. The savings response of consumers, and thereby the growth rate in the perfect annuities scenario relative to the various public recycling schemes, depends on the value of the intertemporal elasticity of substitution $\sigma$. For the benchmark case with $\sigma = 1$, savings are independent of the interest rate and $\gamma^{\mathrm{TY}} > \gamma^{\mathrm{PA}} = \gamma^{\mathrm{WE}} > \gamma^{\mathrm{TO}}$. If $0 < \sigma < 1$, the higher interest rate will lead to less saving than in the benchmark scenario, so that we get $\gamma^{\mathrm{TY}} > \gamma^{\mathrm{WE}} > \gamma^{\mathrm{PA}} > \gamma^{\mathrm{TO}}$. Finally, if $\sigma > 1$, the higher interest rate will lead to more saving, which results in $\gamma^{\mathrm{PA}} > \gamma^{\mathrm{WE}} > \gamma^{\mathrm{TO}}$ and, depending on the exact magnitude of $\sigma$, $\gamma^{\mathrm{PA}} \gtreqless \gamma^{\mathrm{TY}}$.

To quantify the growth and welfare effects, we adopt the following approach. For $n$, $\pi$, $\alpha$, $\delta$, and $r$ we use the same values as for the exogenous growth model (see the text below Proposition 1). We calibrate an annual growth rate of 1% in the WE scenario ($\gamma^{\mathrm{WE}} = 0.49$) and obtain $\Omega_0 = 15.72$ and $\rho = 1.78$ (or 2.58% annually). The equilibrium growth rate under the various policy schemes is reported in Table 4 for different values of $\sigma$.

**TABLE 4.** Annual steady-state growth rates with endogenous growth $\eta = 1 - \alpha$

| | (a) $\sigma = \frac{1}{2}$ | (b) $\sigma = 1$ | (c) $\sigma = \frac{3}{2}$ |
|---|---|---|---|
| WE | 1.00 | 1.00 | 1.00 |
| TO | 0.26 | 0.26 | 0.26 |
| TY | 1.31 | 1.31 | 1.31 |
| PA | 0.64 | 1.00 | 1.35 |

In line with the exogenous growth model, we find that if the economy exhibits endogenous growth and the intertemporal substitution elasticity is in the realistic range ($0 < \sigma \leq 1$), then it is better to transfer the proceeds of accidental bequests to the young than to open up a private annuity market—see Table 4. In addition, we find that for low values of $\sigma$ it may even be better to waste the accidental bequests than to have a system of private annuities. Hence, both the weak and the strong version of the tragedy of annuitization show up in terms of economic growth rates.[14]

## 5.4. Discussion

In the preceding subsections we have seen two instances of the tragedy of annuitization, namely the *strong* version (from WE to PA) and the *weak* version (from TY to PA). The remaining question that must be answered is whether the tragedy is inescapable for realistic values of the intertemporal substitution elasticity. Does the introduction of a perfect annuity market in such a case always make future generations worse off? There are two theoretical cases in which the tragedy does not occur.

First, the tragedy of annuitization does not materialize if the initial equilibrium is a very bad one in welfare terms. Note that in Table 2, steady-state welfare is lowest for all scenarios considered in the case where accidental bequests are transferred to the old (the TO scenario). If the switch from TO to PA would still give rise to the tragedy, then this would be an even stronger version than the one resulting from the change from WE to PA. It turns out, however, that the tragedy does not arise when annuity markets are opened under the TO scenario.

We summarize the quantitative results in Table 2(d). Comparing columns (b) and (d), we find that there is a strong expansionary effect on the capital intensity. Consumption during youth and old age increase substantially as a result of the expansion in the choice set made possible by increased capital accumulation. Long-run output per worker increases by almost 15% . Table 2 also shows the welfare effect on shock-time and future newborns. Interestingly, the shock-time young are worse off as a result of the introduction of annuity products and the loss of old-age transfers—see row (o) in Table 2. For these agents, the increase in old-age consumption is insufficiently large to offset the strong decrease in youth consumption. All future newborns, however, are strictly better off because of the annuitization opportunities. Table 2 shows that the same conclusion holds for all values of $\sigma$ considered.[15]

The second case in which the tragedy of annuitization does not occur is if the policy maker provides an antidote to it in the form of a reverse pension scheme. Consider the weak version of the tragedy. The antidote works as follows. The shock-time young keep their transfers ($\hat{Z}^y$) but will be taxed during old age if they survive ($T^o_{t+1} > 0$). The tax could, for example, be set at such a level that the lifetime utility of the shock-time young is unaffected ($d\mathbf{E}\Lambda^y_t = 0$), and be held constant thereafter. From period $t+1$ onward, the tax on old agents is used to provide transfers to the young on a balanced-budget pay-as-you-go basis; i.e., $(1+n)\, Z^y_{t+\tau} = (1-\pi)\, T^o_{t+\tau}$ and $T_{t+\tau} = T_{t+1}$ for $\tau = 1, 2, \ldots$. It is easy to show that welfare of all future newborn generations will rise as a result of this scheme.[16]

We do not believe that these two theoretical "refutations" of the tragedy of annuitization are very compelling. First, accidental bequests typically flow to the young (from parent to offspring) and not to the old, thus making the first refutation highly unlikely to materialize. Second, assuming that the policy maker accompanies the opening up of annuity markets with a system of reverse pensions is tantamount to stating that market failures cannot exist because the government will always be able to find a sufficiently rich set of instruments to correct them. Taken in *isolation*, as we have demonstrated in this paper, annuities can have adverse long-run welfare effects.

## 6. CONCLUSION

We construct a tractable discrete-time overlapping-generations model of a closed economy featuring endogenous capital accumulation. We use this model to study government redistribution and private annuities in general equilibrium. Individuals face longevity risk, as there is a positive probability of passing away before the retirement period. With an uncertain life expectancy, nonaltruistic agents engage in precautionary saving to avoid running out of assets in old age. Although they refrain from leaving intentional bequests to their offspring, they will generally make *unintended* bequests, which we assume to flow to the government. Starting from a case in which the government initially wastes these resources, we investigate the effects of various revenue recycling schemes on allocation and welfare. Interestingly, we find nonpathological cases where it is better for long-run welfare to waste accidental bequests than to give them to the elderly. This is because transfers received in old age cause the individual to reduce saving, which at the macroeconomic level results in a dramatic fall in capital intensity and in wages.

Next we study the introduction of a perfectly competitive annuity market offering actuarially fair annuitization products. We demonstrate that there exists a *tragedy of annuitization*: although full annuitization of assets is privately optimal, it may not be socially beneficial because of adverse general equilibrium repercussions. For example, if the economy is initially at equilibrium with accidental bequests flowing to the young, then opening up annuity markets will reduce steady-state welfare regardless of the magnitude of the intertemporal substitution elasticity. Intuitively, private annuities redistribute assets from deceased (unlucky) individuals to surviving (lucky) elderly in an actuarially fair way, whereas transferring unintended bequests to the young constitutes an intergenerational transfer. This intergenerational transfer induces beneficial savings effects, which, in the end, lead to higher welfare.

The existence of the tragedy is the rule rather than the exception. We find an even stronger version, which states that revenue wasting dominates perfect annuitization.

## NOTES

1. As has been pointed out to us, there exists an older literature—hinting at the tragedy of annuitization—that has been largely forgotten by the profession. See Hubbard (1984), Abel (1985, pp. 787–788), and Kotlikoff et al. (1986).

2. If the government provides transfers to the old ($Z^o_{t+1} > 0$), there is also a positive human wealth effect on saving. In this paper, however, such transfers are proportional to the interest factor, $1 + r_{t+1}$, so that this human wealth effect is not operative. If the agent worked during old age, then the human wealth effect would result in an increase in the savings elasticity.

3. In the knife-edge case with $\eta = 1 - \alpha$, the investment externality exactly offsets the decrease in marginal productivity following an addition to the capital stock. The aggregate production sector then exhibits single-sector endogenous growth of the type described in Romer (1989). The knife-edge model is briefly studied in Section 5.3.

4. Heijdra and Mierau (2010) study the effects on economic growth of consumption and labor income taxes, employing a continuous-time overlapping-generations model featuring imperfect annuities and realistic demography. Bruce and Turnovsky (in press) study the relationships between the different types of overlapping-generations models that exist in the literature.

5. Any convex combination of these options is also feasible. We focus on pure scenarios for ease of illustration and interpretation.

6. Equation (21) is obtained by substituting (T1.4) into (T1.3).

7. For different values of $\sigma$, we recalibrate the WE model (by choice of $\rho$ and $\Omega_0$) so that the same steady state is attained (compare columns (a), (e), and (i) in Table 2).

8. For $\sigma = 1$ the savings function is independent of the interest rate, so that the FDEs for WE and PA coincide. As is explained in Section 5 below, this is no longer the case for $\sigma \neq 1$.

9. Li and Lin (2012) study existence and uniqueness in the standard Diamond model without lifetime uncertainty.

10. In the remainder of this paper we assume that the steady-state interest rate exceeds the rate of population growth. Empirical support for this assumption is provided by Abel et al. (1989).

11. Remember that agents are not allowed to borrow and that, therefore, consumption bundles with $C^y_t > w_t$ remain unattainable.

12. In terms of Figure 1, for $\sigma < 1$ ($> 0$), the phase diagram for PA lies below (above) the one for WE.

13. Indeed, the results in Table 2 confirm that the same conclusion holds for $\sigma = \frac{3}{2}$—compare columns (j) and (l). Of course, in that case, the capital intensity rises somewhat, so that the welfare loss from the switch from TY to PA is smaller.

14. As we explain in detail in Heijdra et al. (2012), the policy shocks feature both level and growth effects. In the long run, however, the latter will always dominate the former in the welfare comparison. At a result, it suffices to restrict attention to the growth rate effects.

15. These findings bear a strong resemblance to the literature on the reform of PAYG pensions. See, for example, Andersen and Bhattacharya (in press). In a dynamically efficient economy, a PAYG system is Pareto-efficient. A pension reform in the direction of a fully funded system increases welfare of steady-state generations but harms the shock-time old and possibly the young generations born close to the time of the reform. The scenario considered here differs from the pension reform case because the shock is not policy-induced but results from the emergence of a new longevity insurance market.

16. The steady-state welfare levels under this reverse pension scheme are $-0.2620$ (for $\sigma = 1$), $-0.2645$ (for $\sigma = \frac{1}{2}$), and $-0.2639$ (for $\sigma = \frac{3}{2}$). For the transition from WE to PA a similar reverse pension scheme can be designed. This leads to steady-state welfare levels of $-0.4240$ (for $\sigma = 1$), $-0.4553$ (for $\sigma = \frac{1}{2}$), and $-0.4140$ (for $\sigma = \frac{3}{2}$).

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