# About the most unstable modes encountered in beam plasma interaction physics

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### Abstract

A thorough analysis of the electromagnetic instabilities encountered in the beam plasma interaction physics shows that the most unstable modes are not the ones which are usually studies. We characterize these most unstable modes and determine the patterns they create.

Keywords: Filamentation instability; Two-stream instability; Weibel instability

# 1. INTRODUCTION

The fast ignition scenario for inertial confinement fusion (ICF) (Tabak *et al.*, 1994; Deutsch *et al.*, 1997) imparts a very important role to beam-plasma interaction physics, since a laser generated relativistic electron beam is supposed to ignite the pre-compressed target (Deutsch, 2004; Roth *et al.*, 2005). The basic physics underlying the phenomena has been investigated for decades. Nevertheless, new lights have been recently shed on the subject, mainly emphasizing the transverse instabilities undergone by the relativistic electron beam, as well as the way temperature may affect them.

When the beam enters the plasma, a return current neutralizes it and the resulting system is the well-known twostream configuration. The instabilities undergone by such a system can be classified in terms of their "polarization" (transverse or longitudinal), the orientation of their wave vector with respect to the beam and finally, their origin. By "origin," we mean that some instabilities depend on the beam density while others only depend on the plasma temperature anisotropy. Although the all wave vector space is unstable (Bret *et al.*, 2005*a*, 2005*b*), it is commonplace to single out three main instabilities, namely the two-stream, the filamentation, and the Weibel instabilities. Two-stream and filamentation, instabilities both depend on the beam, but the first one is longitudinal with a wave vector parallel to the beam while the second one is transverse with a wave vector normal to the beam. On the contrary, the Weibel instability is also transverse but simply relies on a plasma temperature anisotropy.

A full algebraic analysis (Bret *et al.*, 2004) of the system stability unravels the all unstable spectrum, including the unstable modes we just mentioned. At this juncture, some points need to be emphasized: (1) The two-stream and the filamentation instabilities are found on the same branch of the dispersion equation. The same root of the electromagnetic dispersion equation yields the two-stream instability when the wave vector is aligned with the beam, and the filamentation instability when the wave vector is normal to the beam. (2) It can be checked that two-stream and filamentation growth rates vanishes with the beam whereas Weibel growth rate is independent of the beam. (3) The largest growth rate all over the wave vector space is found on the two-stream/filamention branch. Interestingly, it is found for an intermediate orientation of the wave vector. We shall denote this intermediate mode as TSF mode, where "TSF" stands for two-stream/filamentation. (4) This TSF mode which yields the largest growth rate is quasi-longitudinal.

Considering an infinite and homogenous electron beam of density  $n_b$ , relativistic velocity  $V_b$  and relativistic factor  $\gamma_b = (1 - \beta^2)^{-1/2}$  ( $\beta = V_b/c$ ) passing through a non-magnetized infinite and homogenous plasma of electronic density  $n_p$ , growth rates for two-stream, filamentation and TSF instabilities, respectively, read in the zero temperature limit ( $\omega_p$  units,  $\alpha = n_b/n_p$ ),

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$$\delta^{TS} = \frac{\sqrt{3}}{2^{4/3}} \frac{\alpha^{1/3}}{\gamma_b}, \quad \delta^F = \beta \sqrt{\frac{\alpha}{\gamma_b}}, \quad \delta^{TSF} = \frac{\sqrt{3}}{2^{4/3}} \left(\frac{\alpha}{\gamma_b}\right)^{1/3}.$$

The growth rate for the TSF branch is plotted on Figure 1 in terms of the reduced wave vector  $\mathbf{Z} = \mathbf{k} V_b / \omega_p$ . The corresponding calculation have been made assuming a small normal temperature in the plasma (Normal thermal velocity =  $V_b/10$ ) and a simple "waterbag" distribution function. One clearly notices the occurrence of a maximum growth rate for an oblique wave vector as well as an angle in which direction modes are unstable for all k. This angle can be determined exactly and corresponds to the limit between quasi-longitudinal waves and the transition to purely transverse filamentation modes. It divides the k space in a two-stream like and a filamentation like regions. As we shall see, important effects upon the two-stream or the filamentation instabilities extend in the k space over the corresponding region. We shall now review the main temperature effects before discussing the respective role of each instabilities in the initial evolution of the beam.

# 2. TEMPERATURE EFFECTS

Temperature effects have been investigated so far using simple waterbag distributions in the non-relativistic temperatures limit (Bret *et al.*, 2005*c*). As far as fast ignition is concerned, the non-relativistic temperatures are not limitative for the plasma whereas they may be so for the beam. Concerning the simplified waterbag model for the distribution function, one can expect realistic results as long as temperatures are kept small, which, in the present setting, precisely means non-relativistic temperatures. These simplifications are useful to account for temperatures while implementing the full electromagnetic formalism required by the continuous transition from longitudinal modes (twostream) to transverse modes (filamentation) along the TSF branch.

A detailed investigation where parallel and normal temperatures are singled out for both the beam and the plasma

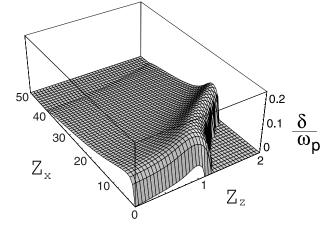


Fig. 1. Growth rate in terms of the reduced wave vector  $\mathbf{Z} = \mathbf{k} V_b / \omega_p$ .

shows that two temperatures are playing a very significative role as far as instabilities are concerned: *transverse beam* temperature and *parallel plasma* temperature. Starting with transverse beam temperature effects, it is already known that it may reduce, and even suppress, the filamentation instability (Silva *et al.*, 2002). Since it leaves the two-stream instability almost unaffected, the question comes to find out where, in the **k** space, starts the influence. As previously mentioned, the answer lies in the critical angle evidenced in Figure 1. Modes located beyond the critical angle are damped like filamentation instability, whereas modes located below are almost unaffected. The transverse thermal velocity  $V_{tbL}$  required in the beam is given in the weak beam density limit by Silva *et al.* (2002)

$$V_{tb\perp} = V_b \sqrt{\alpha \gamma_b}$$

As a consequence, even a small transverse beam temperature may cancel the filamentation instability, while leaving the *TSF* one almost unchanged (Bret *et al.*, 2005*c*).

The investigation of parallel plasma temperature reveals a very interesting connection with the Weibel instability. As is know, the Weibel instability comes from an anisotropy of the plasma rather than from the passing of a beam. Assuming a plasma hotter in one direction, Weibel transverse modes are found for wave vectors normal to the "hot" axis. This means that if the plasma is too hot in the beam direction, Weibel modes shall be found for wave vectors normal to the beam, which is also the direction of the filamentation instability wave vectors. What we find here is that a plasma with a temperature higher in the beam direction than in the others shall prompt transverse unstable Weibel modes with wave vector normal to the beam. On the other hand, filamentation modes are prompted by the beam and are also transverse with wave vectors also normal to the beam. Filamentation and Weibel modes therefore share the same wave vectors and the same "polarization." It turns out that they eventually mix so that filamentation modes continuously switch to Weibel modes as the plasma gets more and more anisotropic. Consequently, filamentation instability decoupled from the beam in the process (since Weibel modes are not beam based) so that damping mechanisms such as transverse beam temperature are progressively loosing efficiency. Quantitatively speaking, the transverse beam thermal velocity required to cancel the filamentation instability is given by

$$V_{tb\perp} = V_b \frac{\sqrt{\alpha \gamma_b}}{\sqrt{1 - \frac{1}{3} (V_{tp\parallel} / V_{tp\perp})^2}}$$

For an isotropic plasma, filamentation instability vanishes for  $V_{tb\perp} > V_b \sqrt{3\alpha\gamma_b/2}$ . But the required temperature diverges for  $V_{tp\parallel} = \sqrt{3}V_{tp\perp}$ , which means the instability can no longer be damped beyond this limit. Meanwhile, the

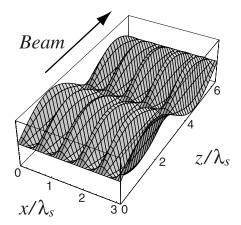


Fig. 2. Typical electronic density perturbations arising from the growth of the most unstable mode. Density units are arbitrary. The length unit is the plasma skin depth  $\lambda_s$ .

filamentation growth rate switches from the expression  $\delta^F$  given by Eq. (1) to the Weibel growth rate in the waterbag model  $\delta^W = V_{tpll} / (c\sqrt{3})$ . As expected, the expression no longer depends on the beam density because filamentation instability has switched to a Weibel regime mainly governed by plasma temperature anisotropy.

The reader may have noticed that the temperature effects we just discussed have to do with the filamentation instability, and by extension, with the filamentation region of the wave vector space. Indeed, virtually no non-relativistic temperature have a significant effect upon the two-stream instability and the two-stream region. As a consequence, the most unstable mode, located "below" the critical angle, is almost unaffected by any temperature of any kind.

## 3. DISCUSSION AND CONCLUSION

The previous analysis of the instability map all over the Fourier space allows now for a rather detailed characterization of the early linear phase of the evolution of the beam. Even if the outcome of this linear phase does not only rely on the fastest growth rates, but also depends on the saturation levels of each mode, we have found that even some small transverse beam temperature can completely stabilize the filamentation instability without stabilizing the oblique one. Situations are therefore evidenced where the system seem stable when looking at wave vectors normal to the beam, while many unstable modes are left in oblique directions. Figure 2 displays a two-dimensional projection of the pattern generated by the most unstable mode, and one recovers two-stream like structures associated with smaller scale transverse filaments. More precise calculations using Maxwellian functions and extending to relativistic beam temperature are now required. On one hand, it is important to quantify more accurately the interplay between filamentation and Weibel instabilities. On the other hand, one can expect a relativistic parallel beam temperature to significantly reduce the TSF instability and thus, really stabilize the system.

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