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ARTICLE

Non-DC notional accounts: relaxing the trade-off between fairness and solvency of pay-as-you-go pension systems

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Abstract

Existing pay-as-you-go (PAYG) schemes based on notional accounts (NAs) have chosen the defined-contribution (DC) setting that forces the rate of interest credited to all individual accounts to change over time to ensure solvency. On the other hand, volatility of the rate of interest is the source of non-negligible disparities of individual internal rates of return (IRRs) both within and across generations. It is argued that these disparities represent a threat to the political appeal of PAYG DC systems, in particular in the present situation characterized by low growth rates of the contribution base. The paper uses a four-overlapping-generations model to prove that the DC setting is not a necessary ingredient of NAs and that their political appeal could be enhanced by extending their use to non-DC pension systems. In fact, redistributions can be avoided by crediting all individual accounts with a constant rate of interest while ensuring financial solvency by fine-tuning of the contribution rate to make the system's revenues grow at the same (constant) rate credited to all accounts. The proof requires constancy of the employment growth rate but not of average earnings. Changes in the employment growth rate produce small oscillations around perfect balance between contribution revenue and pension expenditure manageable with a small buffer fund.

Key words: Intergenerational fairness; NDC pension systems; notional accounts; solvency

JEL classification: H55; J26

Every time pay-as-you-go (PAYG) pension systems are forced to cope with population ageing and/or economic slowdown, they find themselves having to manage the inevitable intergenerational conflict. In fact, irrespective of the type of pension scheme, slower earnings growth and increase of the ratio of retirees to contributors requires harming the former with a reduction of their relative earnings and/or the latter with a heavier and/or longer contribution burden. This conflict, calling into question the very existence of public PAYG pension systems, is further exacerbated by frequent use of discretionary parametric reforms typical of traditional defined-benefit (DB) schemes that perform random intra- and inter-cohort redistributions. What to do with the dividends arising from long periods of expansion is also a crucial issue, albeit politically less demanding than the distribution of the sacrifices.

The Notional Defined Contribution (NDC) scheme was introduced in Italy, Latvia, Poland and Sweden in the 1990s (Chłoń-Domińczak *et al.*, 2012) and later in Norway (Christensen *et al.*, 2012) with the precise aim of eliminating the opaque redistributions inherent in PAYG-DB plans.[1] The NDC scheme attributes to each insured an interest-bearing personal account, where all individual contributions are recorded, though not accumulated and invested as in funded schemes. In fact,

[1]The most pervasive, and mainly regressive, opaque redistributions are generated by the so-called last-earning pension formulas. However, opaque and regressive redistributions persist also within DB systems adopting other pension formulas (Nisticò, 2019, pp. 67–71).

transparent redistributions are fully compatible with the working of the scheme, e.g., in the form of extra credits to the personal accounts of 'deserving' individuals. At retirement, the account balance is transformed into a life-long annuity, thus ensuring one-to-one correspondence between individual contributions and benefits, referred to in Sweden as the '1 Krona for 1 Krona' principle. The one-to-one correspondence holds only for the representative individual of each birth cohort, thus accepting the *ex post* redistributions from the shorter-lived to the longer-lived. The new system was also geared to ensure automatic financial solvency for any given contribution rate by choosing the appropriate rate of return to be credited to all notional accounts (NAs) in line with the growth rate of some measure of earnings growth and by computing the first pension through division of the account balance at retirement by a divisor reflecting life expectancy of retiring cohorts. On the other hand, since the essence of the NDC automatic adjustments lies precisely in fixing the contribution rate, as imposed by its defined-contribution (DC) nature, the flow of the contribution revenue is exogenously determined by the vagaries of average wage and employment growth, thus implying inherent volatility of the interest rate credited to all NAs. Accordingly, individual internal rates of return (IRRs) will differ within and across generations due to the differences of the vectors of annual rates of interest credited to individual accounts. For automatically solvent NDC schemes, this circumstance jeopardizes the 1-Krona-for-1-Krona principle, whose substance evaporates if different vectors of annual interest rates lie in-between the first 'inflow' and the last 'outflow' Krona for different individuals according to their career pattern and/or birth cohort. In other words, NA schemes, in their existing DC arrangement, exhibit a trade-off between solvency and fairness that can reduce their political appeal (Galasso, 2006) as a feasible reform option. Evidence of this political economy issue has just been revealed by the extremely low, or even negative, rates of return that both the Italian and the Swedish NDC schemes credited to all personal accounts during the recent recession.

The aim of this paper is to show that the spectrum of PAYG pension system can be extended to a new variety that resolves the intergenerational conflict inherent in financially solvent, unfunded pension schemes. In fact, insofar as intra- and inter-generational fairness is understood as uniformity of IRRs within and across generations, it is shown that it is indeed possible to conceive a pension scheme based on NAs that accommodates both automatic financial solvency and fairness. The paper is organized as follows. The first part summarizes the functioning of NAs in their DC version and reviews the literature on the relationship between the alternative mechanisms ensuring financial solvency in PAYG settings. The drawbacks of automatically solvent NDC schemes with respect to random redistributions, both within and across generations, are also discussed. The second part and the related Appendices describe an alternative NA scheme, relying on automatic, non-discretionary adjustments of the contribution rate bringing about a constant growth rate of the contribution revenue. It is proved that crediting such a constant rate as a return to all individual accounts ensures automatic financial solvency while precluding any intra- or intergenerational unfairness. Some *caveats* about the limits and actual implementation of the proposed scheme are also discussed together with some simulations showing the impact on the contribution rate of possibly implementing the proposed scheme in three NDC countries, Italy Latvia and Poland.

1. Ensuring sustainability with non-discretionary adjustments in NDC non-steady state systems

The NDC scheme computes the first pension (p) according to the following constraint:

$$\underbrace{p}_{n+1} + \frac{p(1+\sigma_{n+1})}{\underbrace{1+\pi_{n+1}}_{n+2}} + \frac{p(1+\sigma_{n+1})(1+\sigma_{n+2})}{\underbrace{(1+\pi_{n+1})(1+\pi_{n+2})}_{n+3}} + \cdots + \frac{p(1+\sigma_{n+1})\cdots(1+\sigma_{n+m-1})}{\underbrace{(1+\pi_{n+1})\cdots(1+\pi_{n+m-1})}_{n+m}}$$
$$= \underbrace{cw_1(1+\pi_1)\cdots(1+\pi_n)}_{1} + \underbrace{cw_2(1+\pi_2)\cdots(1+\pi_n)}_{2} + \cdots + \underbrace{cw_n(1+\pi_n)}_{n} \quad (1)$$

where n and m denote, respectively, the number of years of service and life expectancy at retirement, σ_i denotes the value taken on in year i ($i = n+1, n+2, \ldots, n+m$) by the indexation rate used to revalue pension benefits, c is the constant contribution rate, π_i denotes the rate of return credited to the NA balances of all members at the end of year i ($i = 1, 2, \ldots, n+m-1$) and w_i ($i = 1, 2, \ldots, n$) the individual's contribution base in year i. Constraint (1) formalizes the 1-Krona-for-1-Krona principle ensuring that the account balance at retirement (on the right-hand side), together with the interests credited to the account after retirement, 'finance' expected benefits (at the numerators on the left-hand side).[2]

Indicating the account balance at retirement with AB and solving for p we get:

$$p = AB/d \tag{2}$$

where

$$d = 1 + \frac{1+\sigma_{n+1}}{1+\pi_{n+1}} + \frac{(1+\sigma_{n+1})(1+\sigma_{n+2})}{(1+\pi_{n+1})(1+\pi_{n+2})} + \cdots + \frac{(1+\sigma_{n+1})\cdots(1+\sigma_{n+m-1})}{(1+\pi_{n+1})\cdots(1+\pi_{n+m-1})} \tag{3}$$

is the value of the balance divisor that satisfies constraint (1).

Condition (3) shows that if the rate of return to be credited in the m years after retirement should depend on future economic conditions, respecting constraint (1) cannot be guaranteed *ex ante* in year $n+1$; it can be guaranteed only by ensuring that, after choosing d, the values of the adjustment rate and the rate of return satisfy equation (3) year after year. In fact, assigning a value to d for each possible value of life expectancy m is equivalent to attributing a constant, arbitrary value to the recurrent ratio between the rate of return and adjustment rate on the right-hand side of (3) according to the following equality:

$$\frac{1+\sigma_i}{1+\pi_i} = \frac{1}{1+\delta} \forall i \tag{4}$$

such that condition (3) becomes

$$d = 1 + \frac{1}{1+\delta} + \frac{1}{(1+\delta)^2} + \cdots + \frac{1}{(1+\delta)^{m-1}} \tag{5}$$

where the arbitrary value attributed to δ enables calculation of d in the year of retirement according to life expectancy alone, regardless of the values to be taken on by π and σ in the m years to follow. The higher the value of δ, the higher will be, *ceteris paribus*, the first pension annuity. However, it is not as if choosing higher values of δ comes at no cost. In fact, given (4), computing d according to (5) implies the following adjustment rule:

$$\sigma_i = \frac{1+\pi_i}{1+\delta} - 1 \quad \forall i, \tag{6}$$

which ensures that constraint (1) is satisfied *ex post*. The adjustment rule expressed by (6) explains why the indexation rate, and hence the vector of the subsequent annuities, has an inverse relationship with the positive value of δ, whose higher values increase the first pension annuity. Therefore, δ can be referred to as the 'frontloading rate'.

[2]For a more rigorous version of constraint (1) in which the left-hand side shows the pensions to be paid for all admissible ages after retirement weighted by the respective compound probability of surviving at that age, see Gurtovaya and Nisticò (2019).

1.1 The inverted Samuelson–Aaron theorem

About 60 years ago, Samuelson proved that individual well-being could be improved with the introduction of a social security system where each active worker participates in support of the retired population in exchange for her own after-retirement support from future generations of workers (Samuelson, 1958). According to Samuelson, 'every geometrically growing consumption-loan economy has an equilibrium market rate of interest exactly equal to its biological percentage growth rate' (*ibid*., p. 472). Some years later, Aaron (1966) further developed Samuelson's propositions, proving that a PAYG social security arrangement is still welfare improving in a steady economy where both wages and employment grow at constant rates, provided that the compound growth rate of wages and population is higher than the market interest rate. Since then, it has been taken as common knowledge that a PAYG-DB scheme, adjusting the contribution rate to guarantee financial solvency, implicitly rewards the contributions of its members with a rate of return equal to the growth rate of aggregate earnings – the Aaron–Samuelson theorem.

A few years after the approval of the first wave of NDC reforms, Valdés-Prieto (2000) proved that the Samuelson–Aaron rate is also the sustainable rate of return of NDC schemes. Later on, Gronchi and Nisticò (2006, 2008) presented a four-overlapping-generations model with two major extensions of the theorem, showing that: (i) where the contribution rate alone bears the burden of ensuring solvency of the PAYG scheme, the Samuelson–Aaron rate is to be understood as the implicit generational IRR, while marked disparities in intra-cohort rates of return according to the length and the shape of career patterns characterize traditional, earnings-related schemes; (ii) the 'inverted' Samuelson–Aaron theorem proves that *explicitly* crediting to all account balances the sustainable rate of return ensures long-term financial solvency of NDC schemes for whatever contribution rate and even when the growth rate of the average wage is not constant through time.

Gronchi and Nisticò (2008, pp. 139–141) also emphasized the following *caveats* for the 'inverted Samuelson–Aaron theorem':

- in the presence of employment cycles, the system will experience temporary deficits (surpluses) when the number of contributors falls short of (exceeds) its long-run path;
- given that retirement age flexibility is an essential feature of the NDC scheme, the possible volatility of the 'average' retirement age could also produce temporary deficits or surpluses;
- ever-decreasing mortality rates generate long-run imbalances due to the use of backward-looking annuity divisors based on period life tables;
- the use of forward-looking divisors based on projected, cohort-specific life tables generates long-run surpluses.

1.2 Automatic solvency under all possible circumstances

All existing NDC systems are exposed to the above risks of imbalances. Moreover, they are exposed to the political risk entailed by the policymakers' reluctance to acknowledge the importance of refraining from the discretionary adjustments typical of the old, DB systems. For many NDC countries, the temptation largely derives from the slow transition phase to the new system and from the consequent need to cope with the old system's liabilities.[3] The strong concern with the huge deficits of the old system can also explain why Italy, Latvia and Poland did not adopt any safety measures needed to counteract the risks of imbalances inherent in the working of the NDC system. This is not the case of Sweden that legislated a fast transition to the new system and was endowed with a large buffer fund to deal with the old system's liabilities. In fact, certainly aware of the *caveats* mentioned above and, in particular, of the fact that crediting the rate of return dictated by the inverted Samuelson–Aaron theorem to all NAs does not ensure solvency in non-steady-employment economies with

[3]This was clearly reflected e.g., in the continuous changes in the contribution rates in Poland as of 2010 (Buchholtz *et al.*, 2019, pp. 101–104) and in legal retirement ages in Italy (Gronchi *et al.*, 2019, pp. 84–85).

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increasing longevity, the Swedish reformers designed their NDC scheme to ensure intergenerational stability of the pension levels by using the average wage growth as standard rate of return while achieving financial independence from the fiscal revenue under all possible economic and demographic scenarios. The latter goal is pursued through the automatic balance mechanism (ABM), which was legislated in 2001, allowing to counter both the effects of non-constant growth rates of employment and non-constant mortality, as well as any other possible reason for imbalances arising in non-steady state economies. In essence, the ABM operates by curbing the rate of return to be credited to all NAs whenever the system's liabilities exceed its assets.

The main difficulty that the Swedish reformers had to face was measurement of the system's assets, given its PAYG financing architecture. Settergren and Mikula (2006) have shown that the 'contribution asset' (CA) is embedded in a factor referred to as the 'expected turnover duration' (TD), which can easily be measured as the difference between the pension-weighted average age of retirees and the earnings-weighted average age of contributors to the system. The importance of TD can readily be grasped on observing that it is a measure of the period of time that elapses before the PAYG system has to pay off pension obligations incurred in the year of TD assessment. The intuition of the Swedish reformers was that CA can be computed through multiplication of TD by the current contribution flow (Settergren, 2004). The solvency of the Swedish NDC scheme is then measured on the basis of the ratio of total assets (TA) – calculated by adding the buffer fund to CA – to overall pension liabilities. This balance ratio (*BR*) is, then, the crucial variable assessing the solvency of the scheme. In particular, in any year t, the system is solvent when $BR_t \geqslant 1$, meaning that the expected (under current conditions) flow of contributions and the buffer fund are sufficient to extinguish pension obligations 'as they go'. If, on the other hand, $BR_t < 1$, deficits are likely to occur and hence, after eventual exhaustion of the buffer fund, the system would become insolvent. To prevent the emergence of insolvency signalled by *BR* value, the Swedish pension system applies timely countermeasures to bring the growth of pension liabilities back in line with system's assets. Thus, if the *BR* declines below 1, the ABM is triggered, implying that current pensions and account balances are indexed according to a rate lower than the standard one, computed through multiplication of the average wage factor by *BR*, which reduces the dynamics of liabilities. When liabilities come back in line with TA (implying $BR_t \geqslant 1$), pensions and account balances will be indexed at a rate higher than average wage growth. Eventually, this 'compensation phase' ends and the ABM is deactivated when account balances reach the level they would have had if the ABM had not been triggered (Pensions Myndigheten, 2018). Having endowed the scheme with a 'brake' able to ensure solvency, the Swedish reformers have chosen the average wage growth rate as the 'ordinary' rate of return to be credited to all pension accounts, thus purging the Samuelson–Aaron rate of its 'demographic-employment' component. As mentioned above, the reason for this choice lies precisely in the relatively low volatility of the average wage growth rate, which favours intergenerational fairness (Settergren, 2003, p. 104), intended as a stable pensions-to-earnings ratio.

1.3 The shortcomings of the NDC adjustment mechanisms

If the NDC scheme credits to all account balances the Samuelson–Aaron rate of return every year, it gravitates towards financial solvency but is open to the imbalances described above. On the other hand, short-run and long-run changes of the rate of return credited to individual account balances (the growth rate of the contribution base) produces IRR disparities, both within and across generations. The Swedish ABM prevents the occurrence of even short-term insolvency. In fact, it constitutes a highly-sophisticated example of how the automatic adjustments of the NDC scheme can be applied to non-steady-growth economies while also managing changes in longevity. However, in light of the frequent triggering of the ABM recently occurred, the following non-trivial shortcomings should be taken into account:

- the 'compensation phase' produces unjustified premiums or penalties according to the size of the individual account balance when the rate of return actually used to index pension liabilities exceeds or falls short of the average wage growth;

- given the (regrettable) Swedish choice to frontload the disbursement profile by computing annuity divisors with a rather high frontloading rate (1.6%), negative nominal pension adjustments – as a result of equation (6) – are a likely outcome of ABM triggering;
- possible negative pension adjustments tend to produce a strong political resistance against NDC automatic adjustments.

In other words, NDC automatic adjustments, both in the Samuelson–Aaron and in the Swedish-ABM version, tend to generate the feeling that automatic financial solvency is given priority over social sustainability of pension levels, especially when it comes to low, or even negative, pension indexation.

2. The tricky issue of intergenerational fairness

In a paper published around 40 years ago in a volume bringing together the contributions to a symposium on *Alternate methods of social security financing* hosted by the U.S. Social Security Administration, Musgrave (1981) pointed out the risk that a PAYG scheme 'remains in a continuing state of disequilibrium, which tends to discredit its reliability' and rightly added that the 'long-run political viability of the scheme' crucially depends on whether it will 'be accepted as fair by both working population and retirees' (*ibid.*, pp. 96–97). With a touch of optimism, which I share, Musgrave added that 'the spectre of declining economic growth, does not require us to abandon social security. It merely calls for a reformulation of the underlying contract' (*ibid.*, p. 99). The problem, as Musgrave saw it, is that it is difficult to assess which of the traditional formulations of risk-sharing between workers and retirees, DB or DC, 'offers the most equitable solution' (*ibid.*, p. 102) unless one considers what happens to the relative position of workers and retirees under the impact of any possible shock affecting labour productivity and population (employment) growth.[4] Despite his admission that, of the traditional approaches, he preferred the DC solution 'which stipulates a constant effort obligation' to the DB one 'which stipulates constant entitlements', Musgrave's well known conclusion was that 'using the superior criterion of constant relative position, neither formulation is satisfactory' (*ibid.*, pp. 102–103). Musgrave added that this is true in particular under worsening demographic conditions, when DB arrangements 'impose a severe risk on working population' thus showing 'their very uneasy foundation for the social security contract', while DC schemes place 'the entire population risk on retirees, which is unacceptable (p. 103). In view of the above, Musgrave proposed revision of the intergenerational contract as calling for maintenance of a fixed ratio of per-capita benefits to retirees to per-capita earnings (net of social security tax) of workers (p. 104).

In practical terms, Musgrave's proposal implies that the contribution rate is adjusted on a regular basis to take into account both the change in the ratio of retirees to contributors and the need to stabilize the ratio of the average pension to the average wage net of contributions.[5] At the same time, pension benefits will 'somehow' have to be adjusted for pension expenditure to match the contribution revenues generated by the new 'fair' contribution rate. Musgrave's paper had the merit of revealing just how fundamental perceived fairness would become in safeguarding the intergenerational agreement supporting public PAYG systems given the gloomy prospects for the world economies after the glorious past of the 60s. On the other hand, the following limits of Musgrave's proposal are worth stressing:

[4]In his paper, Musgrave does not use the labels DB and DC. In Musgrave's terminology, the DB approach to risk sharing is represented by plans that promise a 'fixed replacement rate' (FRR) or a 'fixed replacement rate adjusted for productivity growth' (FRRA), as is the case when past earnings are indexed according to average wage growth. As for the DC approach to risk sharing, Musgrave refers to it with the label 'fixed contribution rate' (FCR).

[5]For the chosen ratio (r) of the average pension to the average wage net of contributions to remain constant, the contribution rate must be adjusted as follows: $c = (r \cdot R)/(L + r \cdot R)$ where L and R denote the number of workers and retirees, respectively. See Musgrave (ibid., note 15).

- it requires that both the contribution rate and pension benefits be adjusted on a regular basis;
- it impacts on the IRR of both workers and retirees but with no guarantee that the change, be it positive or negative, will affect workers and retirees equally;
- it assumes that what matters for active generations is their wage net of contributions only, regardless of any possible concern about their pension wealth and, more importantly, the rate of return on their compulsory savings going to the PAYG system;
- it takes for granted that the existing ratio of the average benefit to the average wage net of contribution is a sort of agreed upon optimum to be preserved;
- it does not preclude the possible persistence of even severe intragenerational unfairness, as Musgrave himself admitted (*ibid*., p. 108);
- it does not specify how individual pension benefits will have to be transparently adjusted to respect the rule and the possible impact of such a redefinition of benefits on intragenerational fairness.

In the following section, it is demonstrated that designing a PAYG system based on the one-to-one correspondence between pensions and contributions and on the automatic financial solvency typical of NAs can actually be done outside the DC arrangement, thereby extending the policy options for policymakers sharing Musgrave's concern to promote a new intergenerational agreement for PAYG systems. In any case, this further option might help reduce much of the political scepticism towards NAs, so far limited to the DC variety that many governments perceive as unfair to retirees, considered the weaker party to the agreement. The proposed scheme ensures both intra- and inter-generational fairness by adjusting the contribution rate in order to ensure constancy of the rate of return credited to all NAs and, hence, uniformity of IRRs both within and across generations. Its fairness lies in the circumstance that, within the proposed fixed-return NA framework, no adjustment of the contribution rate will alter the workers' IRR, though possibly impacting on their pension wealth, as we will see below.

3. Automatic adjustments in a non-DC, NA pension scheme

One of the main advantages of the NDC scheme lies in its 'transparency'. Upon entering the labour market, individuals are provided with a personal account – supplemented with their contributions and adjusted according to a conventional rate of return – whose balance provides the individual with a clear-cut information on her pension wealth. Consequently, contributions can be perceived as compulsory savings and not as a tax needed to finance pensions to be disbursed in favour of individuals belonging to previous birth cohorts.[6] This is why NDC systems were supposed to foster the intergenerational agreement supporting PAYG financing. Moreover, they 'nudge', without forcing, higher retirement ages as a means to offset the effect of increasing life expectancy on pension levels. However, one should admit that crediting to individual account balances a vector of annual rates of return different from that credited to other individuals, e.g., to those belonging to birth cohorts contributing in periods characterized by much higher earnings growth, implies that contributions become quasi-taxes,[7] thus depriving NA schemes of much of their ability to count on the intergenerational agreement necessary to support their PAYG financing.

As a matter of fact, constancy of the contribution rate is by no means a necessary ingredient to preserve transparency and to reach the goals of NAs as summarized above. A scheme based on NAs could, for instance, bring in a one-off increase in the contribution rate either to extinguish a

[6]In what follows, I shall assume that the PAYG return is not lower than the rate of return of other forms of risk-free investment of individual savings for retirement. On whether old-age contributions to PAYG systems should be considered a tax, see Disney (2004).

[7]I am here referring to the meaning attributed to this expression by Jones (1914, p. 7), who argued that 'Mixed Taxes, or Quasi-Taxes, naturally arise when a governing body makes demands for payments, and gives something in return, but without any pretence of equivalence between individual payments and individual returns'.

share of unsustainable pension liabilities inherited from previous rules (Nisticò and Bevilacqua, 2018), or to foster both pension adequacy and the size of the buffer fund. Contrary to the outcome in earnings-related schemes, such a measure would not increase the tax component of the compulsory savings imposed by the public pension scheme since it would not reduce the IRRs of active cohorts required to pay a higher contribution rate; the higher contributions would still increase the individuals' pension wealth while the future, higher, contribution revenue would sustain the future, higher, pension expenditure entailed by higher account balances.

Adjustments of the contribution rate could also be automatic and continuous rather than discretionary and occasional. In fact, it is our contention that in NA schemes they could play a more significant role, namely ensuring that the growth rate of the contribution revenue does not deviate from a possibly predetermined, constant rate of return to be credited to all account balances, thus 'breaking' the trade-off between financial solvency and uniformity of IRRs both within and across generations. In particular, it can be proved that 'strong' uniformity of IRRs is compatible with financial solvency in a PAYG-NA scheme, provided that a constant rate of interest is credited to all NAs and the contribution rate varies through time in order to maintain the growth rate of the contribution revenue at the same (constant) rate credited to all accounts.

Proof of financial solvency of the proposed defined-IRR, PAYG scheme is provided in Appendix 1 with reference to a four-overlapping-generations model allowing for different career patterns and a variable growth rate of average earnings. The proof that the proposed NA scheme also ensures 'strong' uniformity of IRRs is provided in Appendix 2. Finally, Appendix 3 assesses the deviations from perfect solvency of the scheme under variable employment growth.

Note that:

- the proof is given in a context in which the employment growth rate is constant through time, and hence average wage growth is the sole cause of changes in the growth rate of the contribution base;
- allowing for variable employment growth rates produces only negligible deviations from the perfect short-term balance between contribution revenue and pension expenditure, thus significantly reducing the size of the buffer fund needed to cope with employment cycles (while strict IRR uniformity still holds).

4. Implications

After choosing the 'desired', or, so to speak, 'benchmark' contribution rate for the proposed public PAYG scheme, its actual level will have to go up when the growth rate of the contribution base falls short of the target rate of return, and vice versa. The following implications of this simple rule need to be discussed.

(1) The value of the constant target rate of return to be credited to the NAs should be set at a realistic level to prevent the contribution rate from being increased year after year, as required if the fixed rate of return systematically exceeded the growth rate of aggregate earnings (as happened with the very generous DB rules that forced the contribution rate to increase at very high levels in many countries as of the early 1980s). One interesting option is the expected long-run growth rate of labour productivity, i.e., the expected average wage growth in real terms, which for most developed economies is between 1% and 1.5%, but higher in most developing economies. Insofar as expectations are realized and employment oscillates around zero growth, the working of the proposed scheme will substantially replicate that of the canonical NDC scheme *à la Suédoise* that made a point of purging the benchmark rate of return of its employment/demographic component. In fact, the actual contribution rate would oscillate very close around its desired value, while individual pension wealth and pension levels would replicate those ensured by the NDC scheme (with the same contribution rate). On the other hand,

employment cycles would generate parallel cycles of the value of the buffer fund, which is a fundamental component of the proposed scheme.

(2) Since perfect foresight is a chimera, the interesting case to be discussed is the performance of our proposed scheme when the actual growth rate of the contribution base exceeds or falls short of the target rate of return for a sufficiently long period of time. Let us start with the latter case, i.e., with the case of lower-than-expected growth of the contribution base. For the sake of simplicity, we will assume that deviations from the target depend on deviations of employment growth only, while the average wage will be assumed to remain in line with its expected level.

2(a) Lower-than-target growth rates of the contribution base require the contribution rate to be raised to fill the gap. For instance, if the actual growth rate of the contribution base falls short of the target by 1%, the contribution rate will have to go up by 1% of its current value, say from 20% to 20.2%. Insofar as deviations from the target persist for a decade or so, the contribution rate will have to go up by some 10–15% of its benchmark value, say from 20% to 22% or 23%. Workers will be required to save more than expected for their old-age while the current average pension will be unaffected and its value relative to the average wage net of contributions will therefore go up. Note, however, that despite this apparent worsening of the workers' relative positions from the viewpoint of Musgrave's rule, the active cohorts' IRR will remain unaffected while their pension wealth will increase together with the contribution rate.[8] Note, moreover, that this is not the case of alternative PAYG schemes in the case of lower than expected growth rates of the contribution base. In particular, this is not the case of: (i) traditional DB schemes whose adjustment of the contribution rate leaves workers' pension wealth unaltered but at the expense of reducing their IRR, though not that of the retirees; (ii) point systems that reduce the IRR of active workers (but also of retirees, though in different proportions) by raising contribution rates while at the same time reducing their pension wealth; (iii) NDC schemes that, despite keeping the contribution rate unaltered, credit a lower than expected rate of return on workers' accounts thus reducing both their expected IRR and their pension wealth, the negative impact being stronger for workers approaching retirement, whose account balances are richer and so more sensitive to changes in the rate of interest.

2(b) Higher-than-target growth rates of the contribution base ask for the contribution rate to be reduced and, as explained in (2a) above, the reduction can be significant if the deviation from the target persists for a long enough period of time. Any such reduction of the contribution burden implies an improvement in the workers' positions according to Musgrave's criterion given that the average pension will not be affected while workers' earnings net of contributions will go up. The extent to which employers will also benefit from the reduction of the contribution rate depends on the incidence of old-age compulsory contributions, which is a long-debated issue.[9] However, the workers' pension wealth within the PAYG compulsory pillar will diminish,[10] given that the targeted, constant, interest rate will now be credited to lower-than-expected contribution balances. If, on the one hand, this circumstance could be considered a blessing by policymakers set on keeping compulsory savings in the public pillar as low as possible in order to free resources for privately managed pension plans, on the other hand it represents a serious threat when this is not the case, i.e., when policymakers are concerned that the public PAYG pillar be able to guarantee active workers a level of pension wealth appropriate to their positioning in the life cycle. The concern could be strong if the benchmark contribution rate was set at a

[8]Note that this is not the case for workers whose earnings decline during the employment downturn at a higher percentage than the percentage increase of the contribution rate, as is likely to be the case for the unemployed if the unemployment benefit on which old-age contributions are due falls significantly short of the previously earned wage.

[9]For a succinct survey of the literature, see Gronchi and Nisticò (2008, pp. 142–143).

[10]If, on the other hand, the extra growth rate of the contribution base depended on wage growth only, the workers' pension wealth would be unaffected.

particularly low level. To prevent this from coming about, our proposed scheme should be endowed with some sort of 'minimum-contribution-rate clause' according to which the actual contribution rate can, in no case, fall below a certain floor. A clause of this sort implies that in some years the growth rate of the contribution revenues could exceed the target rate of return, the excess contribution revenues being accumulated in a buffer fund to be used to temper the increase in the contribution rate needed when, in a downturn, the growth rate of the contribution base falls short of the target interest rate.[11] In this case, too, the other PAYG systems behave differently from our proposed scheme vis-à-vis active cohorts. In particular: (i) traditional DB schemes, being also required to reduce the contribution rate while leaving the workers' pension wealth unaltered, will increase the IRR of the active generations (but not of the retirees)[12]; (ii) point systems will cut the contribution rate while distributing the windfall to both workers and retirees, possibly in accordance with Musgrave's rule, thus increasing the workers' IRR and pension wealth at the same time though in different proportions from the retirees and (iii) NDC systems will keep the contribution rate unaltered while crediting the higher rate of return, as dictated by the contribution base growing faster than expected, thus increasing both the IRR and the pension wealth of active cohorts, not fairly but in proportion to the values their account balances.

Let me add that once the target rate has been set, according to adjustment rule (6), the frontloading rate should be set at a level that leaves room for non-negative real indexation, e.g., at 0.75% if the rate of return has been set at 1% in real terms, thus implying a constant indexation rate of 0.25% in real terms.

5. A retrospective and a prospective simulation: the cases of Italy, Latvia and Poland

Moving from concept to implementation is no easy and quick task. It took around 6 years for the Swedish experts to define the several details for the NDC idea to be transformed into a well-working pension system. The hastier implementation in other countries, Latvia, Poland and Italy, implied a significant deviation from the fundamental targets of the NDC scheme (Gronchi *et al.*, 2019). The new concept here proposed is no exception in this respect, although most of the implementation details, especially those related to the disbursement phase, could easily be 'imported' from the good practice of the Swedish NDC. Other issues are quite new, e.g., how ensuring that the contribution revenue grows yearly at the pre-set rate, given the temporal lag between the choice of the contribution rate and the information about the contribution base, but good solutions can certainly be devised in accordance with the administrative practices of each country. A no less important issue is the actual impact of the proposed scheme on the PAYG contribution rate, considering the dynamics of the contribution base and the choice of the target rate of return, given that, as mentioned above, significant deviations of the actual contribution rate from its desired level imply possible significant changes in the workers' pension wealth. The workers' income net of contributions, the firms' labour costs or some combination of the two will also be affected, according to the relative incidence of old-age contributions on wages and profits. To assess the impact, we must move on from theoretical to empirical grounds. To this end, we now simulate the functioning of our proposed scheme in Italy, Poland and Latvia in the likely economic scenario forecasted for those three countries in the next 50 years.[13]

[11]There is no reason why the gap in the workers' pension wealth should not be filled by diverting the excess contribution revenues in a second, funded personal-account pillar, if this is part of the public old-age insurance. In this case, the dynamics of the workers' pension wealth would compare with its expected level, i.e., that corresponding to the benchmark contribution rate, in accordance with the difference between the fund's return and the target rate of the PAYG pillar.

[12]Prefunding is, of course, also an option in traditional DB schemes.

[13]The data do not take into consideration the dramatic effects that the COVID-19 pandemic outbreak is having and will possibly continue to have on GDP growth in the coming years. Let us just add that the dramatic fall in GDP in almost all the

Table 1. A retrospective simulation of the working of the non-DC PA scheme in Italy, years 1996–2018 with a fixed rate of return of 1% in real terms

Year	Gross earnings (nominal) millions of Euros 1	Gross earnings (real – 2018 €) millions of Euros 2	Gross earnings growth rate (real) (%) 3	Real earnings index (real) 4	Present contribution rate 5	Contribution revenue index 6	Target contribution revenue index 7	Target contribution rate 8
1995	266,348.1	399,788.5		100	0.33	33.00	33.00	0.330
1996	280,910.8	405,916.1	1.53	101.53	0.33	33.51	33.33	0.328
1997	293,079.4	416,172.7	2.53	104.10	0.33	34.35	33.66	0.323
1998	303,184.8	422,942.8	1.63	105.79	0.33	34.91	34.00	0.321
1999	315,102.5	432,950.8	2.37	108.29	0.33	35.74	34.34	0.317
2000	330,624.8	442,706.6	2.25	110.74	0.33	36.54	34.68	0.313
2001	349,387.6	455,601.4	2.91	113.96	0.33	37.61	35.03	0.307
2002	365,514.2	465,299.6	2.13	116.39	0.33	38.41	35.38	0.304
2003	379,179.3	471,319.9	1.29	117.89	0.33	38.90	35.73	0.303
2004	393,472.6	479,643.1	1.77	119.97	0.33	39.59	36.09	0.301
2005	412,480.4	494,151.5	3.02	123.60	0.33	40.79	36.45	0.295
2006	434,412	510,434.1	3.30	127.68	0.33	42.13	36.82	0.288
2007	450,349.2	520,153.3	1.90	130.11	0.33	42.94	37.19	0.286
2008	466,476.7	521,987.4	0.35	130.57	0.33	43.09	37.56	0.288
2009	461,819.1	512,619.2	−1.79	128.22	0.33	42.31	37.93	0.296
2010	466,743.2	510,150.3	−0.48	127.61	0.33	42.11	38.31	0.300
2011	473,601.6	504,385.7	−1.13	126.16	0.33	41.63	38.70	0.307
2012	468,152.3	484,069.5	−4.03	121.08	0.33	39.96	39.08	0.323
2013	464,304.7	474,519.4	−1.97	118.69	0.33	39.17	39.47	0.333
2014	466,807.9	476,144.1	0.34	119.10	0.33	39.30	39.87	0.335
2015	477,637.5	487,667.9	2.42	121.98	0.33	40.25	40.27	0.330
2016	491,121.3	501,926.0	2.92	125.55	0.33	41.43	40.67	0.324
2017	502,449.5	507,976.4	1.21	127.06	0.33	41.93	41.08	0.323
2018	517,726	517,726.0	1.92	129.50	0.33	42.73	41.49	0.320

Table 1 simulates, retrospectively, the effects on the endogenous contribution rate of adopting our suggested scheme in Italy as of January 1996, when the NDC scheme came into effect. Given the Italian policy makers' concern with reducing the very high (33%) contribution rate, we assume that the target rate of return was set at 1% in real terms. Columns 1 and 2 show the actual evolution of gross earnings (the contribution base) in millions of Euros, nominal and real, respectively (ISTAT, 2019). The corresponding growth rates are shown in column 3. In order to facilitate reading of the table, the values of real earnings are normalized in column 4 (real earnings in 1995 = 100) while the evolution of the normalized real contribution revenue at the current contribution rate of 33% (column 5) is reported in column 6. Finally, columns 7 and 8 show the target contribution revenue, i.e., the contribution revenue in real terms that is each year 1% higher than that of the preceding year (column 7), and the corresponding contribution rate (column 8) necessary to reach the target. Close examination of column 8 shows that implementation of the suggested scheme would have produced a variance of the contribution rate within the range of a 28.6% low in 2007 and a 33.5% high in 2014 subsequent to the fall of gross earnings generated by the 2011 crisis. Note that the rise of the contribution rate in 2014 would have avoided the need for the Italian Parliament to legislate in 2015 a safety net that prevented the rate of return to be credited to all accounts to be negative as a consequence of the fall in gross revenues after 2011.

On the other hand, when choosing the specific value of the 'defined rate of return', experts and policy makers should look at numbers and figures prospectively rather than retrospectively.

countries around the world should be taken as an opportunity to understand the important role that buffer funds can play in safeguarding the intergenerational agreement supporting PAYG systems.

Take the case of another NDC country, Latvia, whose policy goal is to divert half of the 20% contribution rate to the NDC scheme to finance the funded DC scheme introduced as of 2001. At present, also because of the 2009–2012 crisis, only 6 percentage points accrue to the FDC pillar (Palmer and Stabina, 2019). According to the most recent forecasts (European Commission, 2018), Latvia will experience a growth rate of the contribution base of around 3.4% in real terms until 2030 with a subsequent slowdown to 1.8%, 1.3%, 1.0% and 1.5% in the following decades of the century. Table 2 simulates the working of the proposed PAYG scheme in Latvia in the next 50 years according to these EU forecasts used to fill the first four columns of the table. Given that the growth rate of the contribution base is projected to stay well above 1% in real terms for the next 30 years, it is no surprise that, with the adoption of our suggested scheme, the present contribution rate of 14% to the PAYG scheme would fall throughout the first three decades and then stabilize to around 10% in the following two decades; a result surprisingly in line with the policy goal to create two twin systems equally sharing the overall 20% contribution rate, one PAYG financed and one fully funded, both operating according to the personal account strategy.

A similar goal of creating two parallel DC systems, one PAYG financed and the other fully funded characterized the Polish 1999 reform. Despite the initial splitting of the overall contribution rate of 19.52% into a 12.22% financing the NDC pillar and 7.3% the FDC, at present only 2.92% goes to the funded scheme on a voluntary basis while 16.60% or the entire 19.52% for those who do not opt for the funded pillar goes to the NDC (Buchholtz *et al*., 2019). Table 3 simulates the working of the proposed scheme, again with a real rate of return of 1%, in the next 50 years in Poland based on the same EU forecasts. By looking at the future evolution of the contribution rate reported in column 9, it is easily seen that the contribution rate necessary to finance the PAYG component would fall to around 14.5% thus freeing more than two percentage points of contributions to be diverted to the FDC pillar.

Note that, given the gloomier prospects of other, more mature, European economies, the same goal of reducing the contribution rate could be reached only by setting the fixed rate of return at a lower level. This is simulated in Table 4, showing that according to the same EU forecasts, the growth rate of Italy's contribution base is deemed to stay well below 1% in the next two decades, thus suggesting that the target could be set at 0.75% in real terms, allowing the contribution rate to grow moderately up to 2040 and then declining to around 30% in 2070.

Were the policy makers more concerned with the robustness of the public PAYG pillar than with a reduction of the contribution rate or with nurturing the second, funded pillar, our scheme should be implemented with a higher target rate, say 1.5% or even 2% in real terms. Public pensions from the first pillar would be more generous than in the 1% case, though at the risk of rising contribution rates.

6. Conclusions

The NDC pension system responds to economic and/or demographic shocks with immediate adjustment of the rate of return and hence of the dynamics of the NA balances of both active workers and retirees. This is true of the systems crediting annually the Samuelson–Aaron growth rate to all account balances and indeed of the Swedish version of the NDC indexing its liabilities according either to the rate of growth of average wages or to the rate dictated by the ABM. Such an automatic adjustment, either positive or negative, modifies the dynamics of active workers' pension wealth. At the same time, it modifies the current income of the retirees, thus exposing the NDC scheme to the criticism of losing sight of the main social objective of public pension systems, i.e., ensuring an acceptable standard of living in old age. This is why policy makers could reject the use of automatic solvency opting for discretionary adjustments, such as those implied by the French and the German point systems, with the aim to distribute costs and benefits of demographic and economic shocks *à la* Musgrave. Although it can be proven (Gurtovaya and Nisticò, 2018, 2019) that the opaque adjustments of the point systems do not perform better than the transparent NDC adjustment mechanisms in terms of fairness, it should be admitted that automatic financial solvency may not be considered a

Table 2. A prospective simulation of the working of the non-DC PA scheme in Latvia, years 2020–2070 with a fixed rate of return of 1% in real terms

Year	GDP growth rate (real) (%) 3	Real earnings index (real) 4	Present contribution rate 5	Contribution revenue index at the 'old' contribution rate 6	Target contribution revenue index 7	Target contribution rate 8
2020	3.30	100	0.14	14.00	14.00	0.14000
2021	3.40	103.40	0.14	14.48	14.14	0.13675
2022	3.40	106.92	0.14	14.97	14.28	0.13358
2023	3.40	110.55	0.14	15.48	14.42	0.13048
2024	3.40	114.31	0.14	16.00	14.57	0.12745
2025	3.40	118.20	0.14	16.55	14.71	0.12449
2026	3.40	122.21	0.14	17.11	14.86	0.12160
2027	3.40	126.37	0.14	17.69	15.01	0.11878
2028	3.40	130.67	0.14	18.29	15.16	0.11602
2029	3.40	135.11	0.14	18.92	15.31	0.11333
2030	3.40	139.70	0.14	19.56	15.46	0.11070
2031	1.80	142.22	0.14	19.91	15.62	0.10983
2032	1.80	144.78	0.14	20.27	15.78	0.10896
2033	1.80	147.38	0.14	20.63	15.93	0.10811
2034	1.80	150.04	0.14	21.01	16.09	0.10726
2035	1.80	152.74	0.14	21.38	16.25	0.10642
2036	1.80	155.49	0.14	21.77	16.42	0.10558
2037	1.80	158.29	0.14	22.16	16.58	0.10475
2038	1.80	161.13	0.14	22.56	16.75	0.10393
2039	1.80	164.03	0.14	22.96	16.91	0.10311
2040	1.80	166.99	0.14	23.38	17.08	0.10230
2041	1.30	169.16	0.14	23.68	17.25	0.10200
2042	1.30	171.36	0.14	23.99	17.43	0.10169
2043	1.30	173.58	0.14	24.30	17.60	0.10139
2044	1.30	175.84	0.14	24.62	17.78	0.10109
2045	1.30	178.13	0.14	24.94	17.95	0.10079
2046	1.30	180.44	0.14	25.26	18.13	0.10049
2047	1.30	182.79	0.14	25.59	18.31	0.10020
2048	1.30	185.16	0.14	25.92	18.50	0.09990
2049	1.30	187.57	0.14	26.26	18.68	0.09960
2050	1.30	190.01	0.14	26.60	18.87	0.09931
2051	1.00	191.91	0.14	26.87	19.06	0.09931
2052	1.00	193.83	0.14	27.14	19.25	0.09931
2053	1.00	195.77	0.14	27.41	19.44	0.09931
2054	1.00	197.73	0.14	27.68	19.64	0.09931
2055	1.00	199.70	0.14	27.96	19.83	0.09931
2056	1.00	201.70	0.14	28.24	20.03	0.09931
2057	1.00	203.72	0.14	28.52	20.23	0.09931
2058	1.00	205.75	0.14	28.81	20.43	0.09931
2059	1.00	207.81	0.14	29.09	20.64	0.09931
2060	1.00	209.89	0.14	29.38	20.84	0.09931
2061	1.50	213.04	0.14	29.83	21.05	0.09882
2062	1.50	216.23	0.14	30.27	21.26	0.09833
2063	1.50	219.48	0.14	30.73	21.48	0.09785
2064	1.50	222.77	0.14	31.19	21.69	0.09737
2065	1.50	226.11	0.14	31.66	21.91	0.09689
2066	1.50	229.50	0.14	32.13	22.13	0.09641
2067	1.50	232.95	0.14	32.61	22.35	0.09594
2068	1.50	236.44	0.14	33.10	22.57	0.09546
2069	1.50	239.99	0.14	33.60	22.80	0.09499
2070	1.50	243.59	0.14	34.10	23.02	0.09452

priority by all actors (i.e., workers, retirees and policy makers), especially when it produces intergenerational disparities to the detriment in particular of retirees.

In his 1981 seminal paper on how to redesign a fair intergenerational agreement in support of PAYG systems that were going to be exposed to frequent, harmful, economic and demographic shocks,

Table 3. A prospective simulation of the working of the non-DC PA scheme in Poland, years 2020–2070 with a fixed rate of return of 1% in real terms

Year	GDP growth rate (real) (%) 3	Real earnings index (real) 4	Present contribution rate 5	Contribution revenue index at the 'old' contribution rate 6	Target contribution revenue index 7	Target contribution rate 8
2020	2.80	100	0.166	16.60	16.60	0.16600
2021	2.20	102.20	0.166	16.97	16.77	0.16405
2022	2.20	104.45	0.166	17.34	16.93	0.16212
2023	2.20	106.75	0.166	17.72	17.10	0.16022
2024	2.20	109.09	0.166	18.11	17.27	0.15834
2025	2.20	111.49	0.166	18.51	17.45	0.15648
2026	2.20	113.95	0.166	18.92	17.62	0.15464
2027	2.20	116.45	0.166	19.33	17.80	0.15283
2028	2.20	119.02	0.166	19.76	17.98	0.15103
2029	2.20	121.63	0.166	20.19	18.16	0.14926
2030	2.20	124.31	0.166	20.64	18.34	0.14751
2031	1.50	126.18	0.166	20.95	18.52	0.14678
2032	1.50	128.07	0.166	21.26	18.71	0.14606
2033	1.50	129.99	0.166	21.58	18.89	0.14534
2034	1.50	131.94	0.166	21.90	19.08	0.14462
2035	1.50	133.92	0.166	22.23	19.27	0.14391
2036	1.50	135.93	0.166	22.56	19.46	0.14320
2037	1.50	137.97	0.166	22.90	19.66	0.14250
2038	1.50	140.04	0.166	23.25	19.86	0.14179
2039	1.50	142.14	0.166	23.59	20.05	0.14109
2040	1.50	144.27	0.166	23.95	20.26	0.14040
2041	0.90	145.57	0.166	24.16	20.46	0.14054
2042	0.90	146.88	0.166	24.38	20.66	0.14068
2043	0.90	148.20	0.166	24.60	20.87	0.14082
2044	0.90	149.53	0.166	24.82	21.08	0.14096
2045	0.90	150.88	0.166	25.05	21.29	0.14110
2046	0.90	152.24	0.166	25.27	21.50	0.14124
2047	0.90	153.61	0.166	25.50	21.72	0.14138
2048	0.90	154.99	0.166	25.73	21.93	0.14152
2049	0.90	156.38	0.166	25.96	22.15	0.14166
2050	0.90	157.79	0.166	26.19	22.37	0.14180
2051	0.80	159.05	0.166	26.40	22.60	0.14208
2052	0.80	160.33	0.166	26.61	22.82	0.14236
2053	0.80	161.61	0.166	26.83	23.05	0.14264
2054	0.80	162.90	0.166	27.04	23.28	0.14293
2055	0.80	164.20	0.166	27.26	23.52	0.14321
2056	0.80	165.52	0.166	27.48	23.75	0.14349
2057	0.80	166.84	0.166	27.70	23.99	0.14378
2058	0.80	168.18	0.166	27.92	24.23	0.14406
2059	0.80	169.52	0.166	28.14	24.47	0.14435
2060	0.80	170.88	0.166	28.37	24.72	0.14464
2061	1.00	172.59	0.166	28.65	24.96	0.14464
2062	1.00	174.31	0.166	28.94	25.21	0.14464
2063	1.00	176.06	0.166	29.23	25.46	0.14464
2064	1.00	177.82	0.166	29.52	25.72	0.14464
2065	1.00	179.59	0.166	29.81	25.98	0.14464
2066	1.00	181.39	0.166	30.11	26.24	0.14464
2067	1.00	183.20	0.166	30.41	26.50	0.14464
2068	1.00	185.04	0.166	30.72	26.76	0.14464
2069	1.00	186.89	0.166	31.02	27.03	0.14464
2070	1.00	188.76	0.166	31.33	27.30	0.14464

Musgrave suggested a notion of intergenerational fairness grounded on the idea that the workers' satisfaction with the agreement depends solely on the contribution rate they are required to pay. The standpoint we have taken in this paper differs from Musgrave's in that we assumed that workers also, and above all, care about how the contributions they are required to pay affect their pension wealth, and

Table 4. A prospective simulation of the working of the non-DC PA scheme in Italy, years 2020–2070 with a fixed rate of return of 0.75% in real terms

Year	GDP growth rate (real) (%) 3	Real earnings index (real) 4	Present contribution rate 5	Contribution revenue index at the 'old' contribution rate 6	Target contribution revenue index 7	Target contribution rate 8
2020	0.50	100	0.33	33.00	33.00	0.33000
2021	0.50	100.50	0.33	33.17	33.25	0.33082
2022	0.50	101.00	0.33	33.33	33.50	0.33164
2023	0.50	101.51	0.33	33.50	33.75	0.33247
2024	0.50	102.02	0.33	33.66	34.00	0.33330
2025	0.50	102.53	0.33	33.83	34.26	0.33412
2026	0.50	103.04	0.33	34.00	34.51	0.33496
2027	0.50	103.55	0.33	34.17	34.77	0.33579
2028	0.50	104.07	0.33	34.34	35.03	0.33662
2029	0.50	104.59	0.33	34.52	35.30	0.33746
2030	0.50	105.11	0.33	34.69	35.56	0.33830
2031	0.40	105.53	0.33	34.83	35.83	0.33948
2032	0.40	105.96	0.33	34.97	36.10	0.34066
2033	0.40	106.38	0.33	35.11	36.37	0.34185
2034	0.40	106.81	0.33	35.25	36.64	0.34304
2035	0.40	107.23	0.33	35.39	36.91	0.34424
2036	0.40	107.66	0.33	35.53	37.19	0.34544
2037	0.40	108.09	0.33	35.67	37.47	0.34664
2038	0.40	108.53	0.33	35.81	37.75	0.34785
2039	0.40	108.96	0.33	35.96	38.03	0.34906
2040	0.40	109.40	0.33	36.10	38.32	0.35028
2041	1.00	110.49	0.33	36.46	38.61	0.34941
2042	1.00	111.59	0.33	36.83	38.90	0.34855
2043	1.00	112.71	0.33	37.19	39.19	0.34769
2044	1.00	113.84	0.33	37.57	39.48	0.34683
2045	1.00	114.98	0.33	37.94	39.78	0.34597
2046	1.00	116.13	0.33	38.32	40.08	0.34511
2047	1.00	117.29	0.33	38.70	40.38	0.34426
2048	1.00	118.46	0.33	39.09	40.68	0.34341
2049	1.00	119.64	0.33	39.48	40.98	0.34256
2050	1.00	120.84	0.33	39.88	41.29	0.34171
2051	1.40	122.53	0.33	40.44	41.60	0.33952
2052	1.40	124.25	0.33	41.00	41.91	0.33734
2053	1.40	125.99	0.33	41.58	42.23	0.33518
2054	1.40	127.75	0.33	42.16	42.54	0.33303
2055	1.40	129.54	0.33	42.75	42.86	0.33089
2056	1.40	131.35	0.33	43.35	43.19	0.32877
2057	1.40	133.19	0.33	43.95	43.51	0.32667
2058	1.40	135.06	0.33	44.57	43.84	0.32457
2059	1.40	136.95	0.33	45.19	44.16	0.32249
2060	1.40	138.86	0.33	45.83	44.50	0.32042
2061	1.30	140.67	0.33	46.42	44.83	0.31868
2062	1.30	142.50	0.33	47.02	45.17	0.31695
2063	1.30	144.35	0.33	47.64	45.50	0.31523
2064	1.30	146.23	0.33	48.26	45.85	0.31352
2065	1.30	148.13	0.33	48.88	46.19	0.31182
2066	1.30	150.05	0.33	49.52	46.54	0.31013
2067	1.30	152.00	0.33	50.16	46.88	0.30844
2068	1.30	153.98	0.33	50.81	47.24	0.30677
2069	1.30	155.98	0.33	51.47	47.59	0.30510
2070	1.30	158.01	0.33	52.14	47.95	0.30345

so, about the IRR of the agreement they are asked to support. This is why we assumed that an intergenerational agreement should be considered fair insofar as it secures all generations with the same IRR.

In fact, only if the individual vectors of annual interest rates were all composed of the same, recurrent element, would individual IRRs prove uniform, regardless of the duration and pattern of individual careers, or indeed of the demographic and economic conditions prevailing during participation in the system. There emerges a trade-off between uniformity of IRRs and financial solvency of PAYG-DC schemes, given that the former is fully ensured by fixing the rate of return credited to all NAs at a constant level, while the latter requires appropriate adjustments of the rate of return according to wage and employment cycles. The trade-off could be avoided by abandoning the DC nature of NAs and adopting an automatic adjustment of the contribution rate ensuring that the growth rate of the system's revenue equals the targeted IRR. The ensuing defined-IRR scheme precludes both intra- and inter-generational redistributions by requiring workers to offset deviations of the contribution base growth rate from the target with changes of opposite sign in the contribution rate. There is no magic wand to avoid that someone be hurt by the necessary adjustment needed to counteract a fall in the growth rate of the contribution base, especially when it goes in the negative territory. However, the one-to-one correspondence between contributions and benefits embedded in the NA architecture ensures that when workers are asked to pay extra contributions, these will not be (and will not be perceived as) an unfair additional payroll tax required to finance the 'high' pensions of previous generations of workers, as it is in traditional DB schemes.

It took a 6-years debate for Swedish policy makers to accept the severe implications of the automatic mechanisms ensuring solvency for a given contribution rate. This is probably why the Swedish NDC has proved robust even when it was exposed to the criticism of being forced to apply a negative indexation rate to pensions in disbursement. On the other hand, policy makers of other countries could be reluctant to adopt NAs in a DC context that ensures solvency by adapting pension expenditure alone to the vagaries of economic conditions and life expectancy. I have tried to argue that the commitment not to change the contribution rate is a non-necessary ingredient of NAs and that this could be replaced by the commitment not to change the rate of return while legislating, once and a for all, an automatic adjustment rule for the contribution rate, as it is now for the rate of return in the NDC version of personal accounts. The simulations provided in the previous sections confirm the viability of the proposal. In fact, including the suggested non-DC version among the reform options could enhance the political appeal of NAs. Transparency, flexibility and automatic financial solvency would still be there, together with intra- and intergenerational uniformity of IRRs.

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Appendix 1: Automatic financial solvency of a non-DC, NA scheme

This section is devoted to proving that an NA system in which the contribution rate is adjusted yearly in order to generate a targeted growth of the contribution revenue, denoted as π, is financially solvent provided that the rate of return credited to all account balances is equal to the same targeted contribution-revenue growth rate π. Proof is provided for a four-overlapping-generation economy where individual life lasts 4 years deterministically under the following assumptions, replicating those used in Gronchi and Nisticò (2008):

- there are two earnings levels denoted as 'blue collar' and 'white collar' wages, the blue-collar wage, denoted as w for year t, being the entry wage for all individuals in their first year of life;
- individuals can choose to continue working and paying old-age contributions for 2 years, thus receiving an 'ordinary' pension for the other two, or to work just one year thus getting an 'early retirement' pension for 3 years;
- career frequencies are uniform for all cohorts, f_1 being the frequency with which workers choose early retirement, f_2 the frequency with which they work 2 years in blue-collar positions and f_3 the frequency with which they are promoted to white-collar status in the second year;
- each year, both blue-collar and white-collar wages grow at the same variable growth rate, denoted as α, which implies a constant percentage difference between the two, denoted as β;
- the size of each cohort of workers, denoted as N for year t, is $1 + \lambda$ times that of the preceding one so that employment grows at the same rate λ (> -1);
- the first pension instalment is obtained through division of the account balance at retirement by the annuity divisor d, as expressed in (5), with life expectancy being 3 for those choosing early retirement and 2 for the others;
- finally, pensions are indexed each year according to (6).

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Let us start by identifying the contribution revenue in year t as the following sum of the contributions paid by those born in the same year t and by those born in year $t-1$ who did not opt for early retirement:

$$c_t wN\left[1+\frac{f_2+f_3(1+\beta)}{(1+\lambda)}\right]. \tag{A1.1}$$

The following condition ensures that the contribution revenue in year t amounts to $(1+\pi)$ times the contribution revenue in year $t-1$:

$$c_t wN\left[1+\frac{f_2+f_3(1+\beta)}{(1+\lambda)}\right] = c_{t-1}wN\theta_t\left[1+\frac{f_2+f_3(1+\beta)}{(1+\lambda)}\right] \tag{A1.2}$$

where

$$\theta_t = \frac{(1+\pi)}{(1+\alpha_t)(1+\lambda)}\ . \tag{A1.3}$$

Let us also define the following:

$$\theta_{t-1} = \frac{(1+\pi)}{(1+\alpha_{t-1})(1+\lambda)} \tag{A1.4}$$

and

$$\theta_{t-2} = \frac{(1+\pi)}{(1+\alpha_{t-2})(1+\lambda)}\ . \tag{A1.5}$$

Simplifying and solving (A1.2) for c_t, we get:

$$c_t = \frac{c_{t-1}(1+\pi)}{(1+\alpha_t)(1+\lambda)} = c_{t-1}\theta_t \tag{A1.6}$$

Analogously:

$$c_{t-1} = \frac{c_{t-2}(1+\pi)}{(1+\alpha_{t-1})(1+\lambda)} = c_{t-2}\theta_{t-1} \tag{A1.7}$$

$$c_{t-2} = \frac{c_{t-3}(1+\pi)}{(1+\alpha_{t-2})(1+\lambda)} = c_{t-3}\theta_{t-2} \tag{A1.8}$$

Rearranging (A1.6) and substituting it into (A1.7) and (A1.8), we get:

$$c_{t-1} = \frac{c_t}{\theta_t} \tag{A1.9}$$

$$c_{t-2} = \frac{c_t}{\theta_t\theta_{t-1}} \tag{A1.10}$$

$$c_{t-3} = \frac{c_t}{\theta_t\theta_{t-1}\theta_{t-2}}\ . \tag{A1.11}$$

Pension expenditure in year $t(PE_t)$ is the following sum of the seven components corresponding to:

- the three cohorts of workers that have chosen early retirement;
- the two cohorts of workers who have retired in a blue-collar position;
- the two cohorts of workers who have retired in a white-collar position.

$$
\begin{aligned}
PE_t = & \Bigg\{ c_{t-1} \frac{w}{(1+\alpha_t)} \frac{N}{(1+\lambda)} f_1 (1+\pi) d(\delta, 3)^{-1} + c_{t-2} \frac{w}{(1+\alpha_t)(1+\alpha_{t-1})} \\
& \times \frac{N}{(1+\lambda)^2} f_1 (1+\pi) d(\delta, 3)^{-1} (1+\sigma) + c_{t-3} \frac{w}{(1+\alpha_t)(1+\alpha_{t-1})(1+\alpha_{t-2})} \\
& \times \frac{N}{(1+\lambda)^3} f_1 (1+\pi) d(\delta, 3)^{-1} (1+\sigma)^2 + \left[c_{t-2} \frac{w}{(1+\alpha_t)(1+\alpha_{t-1})} (1+\pi)^2 + c_{t-1} \frac{w}{(1+\alpha_t)} (1+\pi) \right] \\
& \times \frac{N}{(1+\lambda)^2} f_2 d(\delta, 2)^{-1} + \left[c_{t-3} \frac{w}{(1+\alpha_t)(1+\alpha_{t-1})(1+\alpha_{t-2})} (1+\pi)^2 + c_{t-2} \frac{w}{(1+\alpha_t)(1+\alpha_{t-1})} (1+\pi) \right] \\
& \times \frac{N}{(1+\lambda)^3} f_2 d(\delta, 2)^{-1} (1+\sigma) + \left[c_{t-2} \frac{w}{(1+\alpha_t)(1+\alpha_{t-1})} (1+\pi)^2 + c_{t-1} \frac{w(1+\beta)}{(1+\alpha_t)} (1+\pi) \right] \\
& \times \frac{N}{(1+\lambda)^2} f_3 d(\delta, 2)^{-1} + \left[c_{t-3} \frac{w}{(1+\alpha_t)(1+\alpha_{t-1})(1+\alpha_{t-2})} (1+\pi)^2 + c_{t-2} \frac{w(1+\beta)}{(1+\alpha_t)(1+\alpha_{t-1})} (1+\pi) \right] \\
& \times \frac{N}{(1+\lambda)^3} f_3 d(\delta, 2)^{-1} (1+\sigma) \Bigg\}
\end{aligned}
\tag{A1.12}
$$

Given that, according to (6)

$$1+\sigma = \frac{1+\pi}{1+\delta}$$

and taking into account (A1.3), (A1.4) and (A1.5), (A1.12) can be rewritten as:

$$
\begin{aligned}
PE_t = wN & \Bigg\{ c_{t-1} \theta_t f_1 d(\delta, 3)^{-1} + c_{t-2} \theta_t \theta_{t-1} \frac{f_1 d(\delta, 3)^{-1}}{(1+\delta)} \\
& + c_{t-3} \theta_t \theta_{t-1} \theta_{t-2} \frac{f_1 d(\delta, 3)^{-1}}{(1+\delta)^2} + \left[c_{t-2} \theta_t \theta_{t-1} + c_{t-1} \frac{\theta_t}{(1+\lambda)} \right] \\
& \times f_2 d(\delta, 2)^{-1} + \left[c_{t-3} \theta_t \theta_{t-1} \theta_{t-2} + c_{t-2} \frac{\theta_t \theta_{t-1}}{(1+\lambda)} \right] \frac{f_2 d(\delta, 2)^{-1}}{(1+\delta)} \\
& + \left[c_{t-2} \theta_t \theta_{t-1} + c_{t-1} (1+\beta) \frac{\theta_t}{(1+\lambda)} \right] f_3 d(\delta, 2)^{-1} \\
& + \left[c_{t-3} \theta_t \theta_{t-1} \theta_{t-2} + c_{t-2} (1+\beta) \frac{\theta_t \theta_{t-1}}{(1+\lambda)} \right] \frac{f_3 d(\delta, 2)^{-1}}{(1+\delta)} \Bigg\}
\end{aligned}
\tag{A1.13}
$$

Substituting (A1.9), (A1.10) and (A1.11) into (A1.13), we get:

$$
\begin{aligned}
PE_t = c_t wN & \left[f_1 \left(1 + \frac{1}{(1+\delta)} + \frac{1}{(1+\delta)^2} \right) d(\delta, 3)^{-1} + f_2 \left(1 + \frac{1}{(1+\lambda)} + \frac{1}{(1+\delta)} + \frac{1}{(1+\delta)(1+\lambda)} \right) \right. \\
& \left. \times d(\delta, 2)^{-1} + f_3 \left(1 + \frac{(1+\beta)}{(1+\lambda)} + \frac{1}{(1+\delta)} + \frac{(1+\beta)}{(1+\delta)(1+\lambda)} \right) d(\delta, 2)^{-1} \right],
\end{aligned}
$$

which, taking into account expression (5) of the annuity divisors and after rearranging becomes:

$$PE_t = c_t wN\left[f_1\left(1+\frac{1}{(1+\delta)}+\frac{1}{(1+\delta)^2}\right)\left(1+\frac{1}{(1+\delta)}+\frac{1}{(1+\delta)^2}\right)^{-1} + f_2\left(1+\frac{1}{(1+\lambda)}\right)\left(1+\frac{1}{(1+\delta)}\right)\left(1+\frac{1}{(1+\delta)}\right)^{-1} + f_3\left(1+\frac{(1+\beta)}{(1+\lambda)}\right)\left(1+\frac{1}{(1+\delta)}\right)\left(1+\frac{1}{(1+\delta)}\right)^{-1}\right],$$

and hence

$$PE_t = c_t wN\left[f_1 + f_2\left(1+\frac{1}{(1+\lambda)}\right) + f_3\left(1+\frac{(1+\beta)}{(1+\lambda)}\right)\right]. \tag{A1.14}$$

Given that the expression in square brackets within (A1.14) amounts to

$$\left[f_1 + f_2 + f_3 + \frac{f_2 + f_3(1+\beta)}{(1+\lambda)}\right]$$

and that the sum of the three frequencies necessarily amounts to 1, (A1.14) simplifies to (A1.1). Q.E.D.

Appendix 2: Strong uniformity of IRRs in a non-DC, NA scheme

This section shows that a non-DC scheme based on NAs whose balance is credited with a constant rate of return π ensures strong uniformity of IRRs, meaning that the cash flows paid in and withdrawn from the NA by all individuals imply the same rate of return, specifically π, regardless of career pattern and birth cohort. The proof is given by showing that π is precisely the rate of interest (r) ensuring that the present value of contributions equals the present value of expected benefits for the three career patterns specified in Appendix 1.

Early retirement

Let us start by setting the equality for an individual choosing early retirement in year t after having contributed in year $t-1$ and expecting to receive (three) pension benefits up to year $t+2$:

$$c_{t-1}\frac{w}{(1+\alpha_t)}(1+r) = \frac{c_{t-1}(w/(1+\alpha_t))(1+\pi)}{1+(1/(1+\delta))+(1/(1+\delta)^2)} + \frac{c_{t-1}(w/(1+\alpha_t))(1+\pi)}{(1+(1/(1+\delta))+(1/(1+\delta)^2))(1+r)}\frac{(1+\pi)}{(1+\delta)} + \frac{c_{t-1}(w/(1+\alpha_t))(1+\pi)}{(1+(1/(1+\delta))+(1/(1+\delta)^2))(1+r)^2}\frac{(1+\pi)^2}{(1+\delta)^2} \tag{A2.1}$$

By setting $r=\pi$, (A2.1) becomes:

$$c_{t-1}\frac{w}{(1+\alpha_t)}(1+\pi) = \frac{c_{t-1}(w/(1+\alpha_t))(1+\pi)}{1+(1/(1+\delta))+(1/(1+\delta)^2)} + \frac{c_{t-1}(w/(1+\alpha_t))(1+\pi)}{(1+(1/(1+\delta))+(1/(1+\delta)^2))(1+\pi)}\frac{(1+\pi)}{(1+\delta)} + \frac{c_{t-1}(w/(1+\alpha_t))(1+\pi)}{(1+(1/(1+\delta))+(1/(1+\delta)^2))(1+\pi)^2}\frac{(1+\pi)^2}{(1+\delta)^2} \tag{A2.2}$$

which, after simplification becomes:

$$1 = \frac{1}{1 + (1/(1+\delta)) + (1/(1+\delta)^2)}\left(1 + \frac{1}{(1+\delta)} + \frac{1}{(1+\delta)^2}\right) = 1,$$

showing that $(1+\pi)$ is one of the solutions of equation (A2.1) in $(1+r)$ while Descartes's rule ensures that no other economically meaningful solutions exists.[14] On looking into (A2.2), moreover, it is seen that the contribution rate and earnings growth cancel out on the two sides of the equation, which implies that π is the IRR regardless of the specific birth cohort of those who choose 'early retirement'.

Old-age retirement as blue collar

The root of the following polynomial in $(1+r)$ identifies the IRR of those who retire in year t, at the ordinary retirement age, as blue-collar workers:

$$\begin{aligned} & c_{t-2}\frac{w}{(1+\alpha_{t-1})(1+\alpha_t)}(1+r)^2 + c_{t-1}\frac{w}{(1+\alpha_t)}(1+r) \\ &= \frac{c_{t-2}(w/((1+\alpha_{t-1})(1+\alpha_t)))(1+\pi)^2 + c_{t-1}(w/(1+\alpha_t))(1+\pi)}{1+(1/(1+\delta))} \\ &\quad + \frac{c_{t-2}(w/((1+\alpha_{t-1})(1+\alpha_t)))(1+\pi)^2 + c_{t-1}(w/(1+\alpha_t))(1+\pi)}{(1+r)(1+(1/(1+\delta)))}\frac{(1+\pi)}{(1+\delta)}. \end{aligned} \tag{A2.3}$$

After setting $r=\pi$, (A2.3) takes the following form

$$\begin{aligned} & c_{t-2}\frac{w}{(1+\alpha_{t-1})(1+\alpha_t)}(1+\pi)^2 + c_{t-1}\frac{w}{(1+\alpha_t)}(1+\pi) \\ &= \left(c_{t-2}\frac{w}{(1+\alpha_{t-1})(1+\alpha_t)}(1+\pi)^2 + c_{t-1}\frac{w}{(1+\alpha_t)}(1+\pi)\right) \\ &\quad \times\left(\frac{1}{1+(1/(1+\delta))} + \frac{(1+\pi)}{(1+\pi)(1+\delta)(1+(1/(1+\delta)))}\right) \end{aligned}$$

which, after simplification becomes:

$$1 = \left(1 + \frac{1}{(1+\delta)}\right)^{-1}\left(1 + \frac{1}{(1+\delta)}\right) = 1$$

showing that $(1+\pi)$ is one of the roots of polynomial (A2.3) in $(1+r)$ while Descartes's rule ensures that it is the only economically meaningful solution. Moreover, it will readily be seen that π is the IRR of any birth cohort of blue-collar workers, since $(1+\pi)$ would be the root of the polynomial also for a pair of contribution rates other than c_{t-2} and c_{t-1} and also for a pair of annual earnings other than $w/((1+\alpha_{t-1})(1+\alpha_t))$ and $w/(1+\alpha_t)$.

Old-age retirement as white collar

This section proves that the same conclusions can be reached with reference to the following polynomial representing the cash flows of a white-collar worker retiring in the same year t, differing from polynomial (A2.3) in the extra earnings of the second year of work:

$$\begin{aligned} & c_{t-2}\frac{w}{(1+\alpha_{t-1})(1+\alpha_t)}(1+r)^2 + c_{t-1}\frac{w(1+\beta)}{(1+\alpha_t)}(1+r) \\ &= \frac{c_{t-2}(w/((1+\alpha_{t-1})(1+\alpha_t)))(1+\pi)^2 + c_{t-1}((w(1+\beta))/(1+\alpha_t))(1+\pi)}{1+(1/(1+\delta))} \\ &\quad + \frac{c_{t-2}(w/((1+\alpha_{t-1})(1+\alpha_t)))(1+\pi)^2 + c_{t-1}((w(1+\beta))/(1+\alpha_t))(1+\pi)}{(1+r)(1+(1/(1+\delta)))}\frac{(1+\pi)}{(1+\delta)} \end{aligned} \tag{A2.4}$$

[14]In fact, polynomials representing pension plans with a contribution phase followed by a disbursement phase exhibit only one reversal of sign.

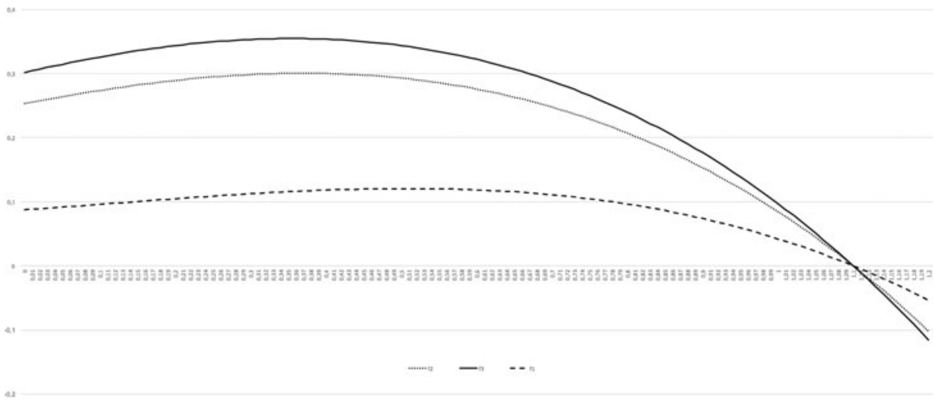

Figure 1. Graphs of three polynomials representing the careers of the model ($\pi = 10\%$).

After setting $r = \pi$, (A2.4) becomes:

$$c_{t-2}\frac{w}{(1+\alpha_{t-1})(1+\alpha_t)}(1+\pi)^2 + c_{t-1}\frac{w(1+\beta)}{(1+\alpha_t)}(1+\pi)$$
$$= \left(c_{t-2}\frac{w}{(1+\alpha_{t-1})(1+\alpha_t)}(1+\pi)^2 + c_{t-1}\frac{w(1+\beta)}{(1+\alpha_t)}(1+\pi)\right)$$
$$\times\left(\frac{1}{1+(1/(1+\delta))} + \frac{(1+\pi)}{(1+\pi)(1+\delta)(1+(1/(1+\delta)))}\right),$$

which also simplifies to

$$1 = \left(1+\frac{1}{(1+\delta)}\right)^{-1}\left(1+\frac{1}{(1+\delta)}\right) = 1.$$

The same considerations regarding Descartes's rule and invariance of the IRR for different cohorts of workers apply also to white-collar careers, thus showing that crediting a constant rate of return on all NAs ensures strong uniformity of IRRs. The graphs in Figure 1 of the polynomials in $(1 + r)$ whose coefficients correspond to the cash-flows of the three careers of the model clearly show the invariance of the IRR with respect to career patterns. The graphs refer to a hypothetical case in which a constant rate of return of 10% is credited to all the workers' NAs.

Appendix 3: The effects of changes in employment growth rate

Table 5 shows a dynamic simulation of the effects produced on the non-DC NA scheme by possible oscillations of the parameter λ (the new workers' growth rate). The simulation, covering 12 years from $t - 5$ to $t + 6$, is run in the same framework of four-overlapping-generations used in Appendix 1. The expression WB denotes the wage bill. Note that it is only from $t - 2$ that pension expenditure includes all seven components included in (A1.12). The simulation shows that the proposed non-DC NA scheme ensures substantial solvency also under employment cycles of significant amplitude. The contribution rate is arbitrarily fixed at 20% in the first 2 years and then adjusted yearly according to the discrepancy between the targeted growth rate of contribution revenue (set at 3%) and the growth rate of the wage bill (WB), as in formulas (A1.9)–(A1.11). The table also shows that the oscillations of the contribution rate needed to ensure a constant growth rate of the contribution revenue are fairly 'small'.

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Table 5. Simulations showing the negligible impact on short-run revenue/expenditure ratio of significant changes in employment growth

1. Parameters utilized												
π	3%					f_1	0.3000					
α $t+6$	2.0%	λ $t+6$	1.0%	θ $t+6$	1.00	f_2	0.5000	c $t+6$	20.5855%	WB $t+6$	476.79	
α $t+5$	2.0%	λ $t+5$	1.0%	θ $t+5$	1.00	f_3	0.2000	c $t+5$	20.5895%	WB $t+5$	462.81	
α $t+4$	3.0%	λ $t+4$	1.5%	θ $t+4$	0.99	δ	0.0000	c $t+4$	20.5935%	WB $t+4$	449.24	
α $t+3$	4.0%	λ $t+3$	2.5%	θ $t+3$	0.97	β	0.4000	c $t+3$	20.9024%	WB $t+3$	429.71	
α $t+2$	3.0%	λ $t+2$	1.5%	θ $t+2$	0.99	z	0.7800	c $t+2$	21.6330%	WB $t+2$	403.11	
α $t+1$	2.0%	λ $t+1$	1.0%	θ $t+1$	1.00	h (δ,3) (*)	0.333333	c $t+1$	21.9575%	WB $t+1$	385.58	
α t	−0.5%	λ t	−0.5%	θ t	1.04	h (δ,2) (*)	0.5	c t	21.9617%	WB $t-0$	374.28	
α $t-1$	0.0%	λ t–1	−0.5%	θ t–1	1.04	w $t-5$	10	c $t-1$	21.1094%	WB $t-1$	378.05	
α $t-2$	1.0%	λ t–2	0.0%	θ t–2	1.02	N $t-5$	20	c $t-2$	20.3921%	WB $t-2$	379.95	
α $t-3$	2.0%	λ t–3	1.0%	θ t–3	1.00			c $t-3$	19.9961%	WB $t-3$	376.19	
α $t-4$	2.0%	λ t–4	1.0%	θ t–4	1.00			c $t-4$	20.0000%	WB $t-4$	365.16	
								c $t-5$	20.0000%	WB $t-5$	200.00	
2. Results												
2.1. Time	−5	−4	−3	−2	−1	0	1	2	3	4	5	6
2.2. Revenue	40.0000	73.0320	75.2230	77.4796	79.8040	82.1982	84.6641	87.2040	89.8201	92.5148	95.2902	98.1489
			+3.0%	+3.0%	+3.0%	+3.0%	+3.0%	+3.0%	+3.0%	+3.0%	+3.0%	+3.0%
2.3. Expenditure		4.1200	39.7300	77.1606	79.6533	82.6472	85.2203	87.4905	89.7237	92.1205	94.8841	98.0436
					+3.2%	+3.8%	+3.1%	+2.7%	+2.6%	+2.7%	+3.0%	+3.3%
2.4. Revenue/expenditure				1.004134	1.001893	0.994566	0.993474	0.996726	1.001074	1.004280	1.004280	1.001074
2.5. Expenditure for new instalments		4.1200	35.4864	36.2388	37.8244	39.0501	40.2216	41.1418	42.2796	43.3523	44.8545	46.3054
				+2.1%	+4.4%	+3.2%	+3.0%	+2.3%	+2.8%	+2.5%	+3.5%	+3.2%
2.6. Expenditure for continuing pensions			4.2436	40.9219	41.8288	43.5972	44.9987	46.3487	47.4441	48.7681	50.0296	51.7381
					+2.2%	+4.2%	+3.2%	+3.0%	+2.4%	+2.8%	+2.6%	+3.4%

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