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In Defense of Comparative Statics: Specifying Empirical Tests of Models of Strategic Interaction

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Beginning in 1999, Curtis Signorino challenged the use of traditional logits and probits analysis for testing discrete-choice, strategic models. Signorino argues that the complex parametric relationships generated by even the simplest strategic models can lead to wildly inaccurate inferences if one applies these traditional approaches. In their stead, Signorino proposes generating stochastic formal models, from which one can directly derive a maximum likelihood estimator. We propose a simpler, alternative methodology for theoretically and empirically accounting for strategic behavior. In particular, we propose carefully and correctly deriving one's comparative statics from one's formal model, whether it is stochastic or deterministic does not particularly matter, and using standard logit or probit estimation techniques to test the predictions. We demonstrate that this approach performs almost identically to Signorino's more complex suggestion.

1 Introduction

Political scientists are regularly concerned with the strategic nature of political decision making. This concern reflects the fact that political actors do not simply make choices independent of the anticipated actions and reactions of other political actors. Rather, they make their decisions specifically contingent on those anticipated choices. For example, when one country is considering whether to attack another country, the potential aggressor does so anticipating how the other country is likely to react. Will it respond militarily, or will it back down? Under many conditions, the potential aggressor's decision will be contingent on that expected response.

In a series of articles beginning in 1999, Curtis Signorino used this notion of strategic behavior to challenge how political scientists empirically study political behavior (see also Smith 1999; Lewis and Schultz 2003). In this challenge, Signorino argues that strategic behavior generates complex parametric relationships that confound traditional logit and probit

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analyses.[1] As a result, he claims, these traditional approaches can lead to wildly inaccurate inferences and should not be used. Signorino instead proposes an alterative approach of writing stochastic formal models from which one can directly derive maximum likelihood estimators (MLEs) to test strategic theories (Signorino 1999, 2003; Signorino and Yilmaz 2003).[2]

The importance of this challenge cannot be oversold. If Signorino is right, existing quantitative tests of strategic behavior in political science are potentially deeply flawed. Further, Signorino's identified solution is no quick fix. It requires scholars to rederive existing deterministic theories as stochastic formal models and then test the new stochastic model by deriving an MLE as well. Thus, there is no "off-the-shelf" solution on either the theoretical or empirical front.

This challenge has quickly gained wide notoriety and acceptance. Signorino's work has been published in the leading journals in the discipline, including the *American Political Science Review* (*APSR*), the *American Journal of Political Science*, and *Political Analysis*. That work is already widely cited, with the 1999 *APSR* article alone having over 60 citations, and National Science Foundation–funded Empirical Implications of Theoretical Models summer workshops make a point of teaching graduate students and young faculty his approach. Thus, it appears that Signorino has found a serious problem in the study of strategic behavior and that his solution is penetrating the discipline.

We very much agree with the emphasis Signorino places on careful theoretical and empirical modeling of strategic behavior. Strategic behavior will lead to complex parametric relationships, and, as a result, simply including a list of covariates in a linear-form logit is almost certainly a fatal misspecification of the theory. Any well-designed test of a strategic theory must entail accurate operationalization of precisely derived predictions.

However, we believe that Signorino's approach makes the process of accurately testing strategic theories appear unnecessarily complicated. The anticipatory behavior that generates complex parametric relationships can lead the incautious researcher to functional form misspecification; however, it does not cause the use of established logit techniques to fail. If the only complexity is anticipatory behavior, we propose an alternative, simpler

[1]As discussed in more detail below, it is unclear whether Signorino uses the term "traditional" to refer to the use of logits and probits in general or whether he is referring to just standard functional forms of these estimation techniques (specifically, the linear link). Thus, we leave the term intentionally undefined here as well.

[2]In a minimalist reading of this critique, Signorino simply is critiquing the existing applications of logit and probit. Scholars generally operationalize linear logits and probits, and these typical logits and probits fail because relationships between the independent variables, or parameters of the model, and the dependent variables, either observed behavior or outcomes, are often nonlinear. With this reading, Signorino is simply using the technique of writing a stochastic formal model and deriving MLEs directly from the theoretical model as a way of illustrating the problems with typical logit and probit specifications. In a maximalist reading of this critique, Signorino is critiquing the entire enterprise of using logits and probits to test strategic models period and is suggesting that the discipline needs new statistical techniques, that is, his approach, if one is going to accurately test models of strategic behavior.

Although nowhere in his published work does Signorino unambiguously state that he is making the maximalist critique, it is not an implausible conclusion. First, throughout his work, Signorino makes statements that read as critiques of using logits or probits in general. For example, in motivating the logit quantal response model in his 1999 article, Signorino (1990, 280) states, "[I]n sum, logit models of international conflict are unlikely to capture the real or theorized structure of strategic interaction." Similarly, he concludes his Monte Carlo analysis by stating: "The question posed at the beginning of this section was: How well does traditional logit model strategic interaction?" (Signorino 1999, 287). Thus, Signorino often does not qualify his statements by indicating that he is critiquing a particular empirical specification. Second, Signorino motivates using his LQRE approach as if it is necessary for incorporating strategic interdependence into a statistical model: "I analyze the effects of using a logit model when two states behave strategically. To do this, however, we need a method for incorporating the structure of strategic interdependence into statistical models of conflict" (Signorino 1999, 281). Third, and perhaps most importantly, at no point in any of his work does Signorino propose the obvious alternative to typical logit and probit specifications or to the LQRE approach, appropriately deriving standard comparative statics approaches to generate predictions and test them in a logit or probit model.

methodology. Simply put, we recommend that scholars directly derive comparative statics from their strategic models and use established estimation techniques to test the predictions generated through the comparative static analysis. Our proposed approach thus reinforces the importance of Signorino's insights while at the same time substantially simplifying their implementation.

To demonstrate our claims, we proceed in three parts. First, we demonstrate that the complex parametric relationships derived by Signorino in his crisis bargaining models to illustrate the failure of traditional estimation techniques can be derived using standard comparative statics. Second, we demonstrate that these complex relationships do not rely upon the stochastic formal modeling assumption; that is, that one can get exactly those complex relationships using a simple deterministic formal model. Third, we demonstrate that if one directly derives the estimators in Signorino's examples, they are established variants of standard models for binary responses.

2 Nonlinearities, Stochastic Modeling, and Comparative Statics Analysis

This section proceeds in three parts. First, we summarize in more detail Signorino's critique of, and solution to, existing empirical work as argued in Signorino (1999) and Signorino and Yilmaz (2003). Second, we reanalyze the parametric relationships from these papers and demonstrate that correctly derived comparative statics capture the central nonlinearities used by Signorino to demonstrate the failure of traditional estimation techniques.[3] Finally, we also demonstrate that making the model stochastic is not central to deriving these nonlinear relationships.

2.1 *Signorino's Argument*

In "Strategic interaction and the statistical analysis of international conflict," Signorino (1999) first challenges the use of traditional logit analysis in the face of strategic interdependence. His challenge starts by characterizing "the problem with traditional methods of estimation" in empirical studies of international conflict (Signorino 1999, 280). The core of this critique is "the typical use of logit," by which he means including an array of plausible covariates in a logit predicting the occurrence of conflict at the nation-year, dyad-year, or monad-dispute level. Signorino states that there are a number of reasons to be wary of this form of analysis, including "if observed actions are the results of [perhaps complex] strategic interaction, then it is unlikely that a simple logit functional form will capture the structure of that strategic interdependence."

Focusing on this problem, Signorio argues that one needs "a method for incorporating the structure of strategic interdependence into statistical models of conflict" to demonstrate the failure of the traditional logit. To do so, he derives the logit quantal response equilibrium (LQRE). This is a statistical model in which one writes down a discrete-choice formal model, derives a quantal response equilibrium (QRE), and then directly derives the MLE from the statistical formal model.[4] By deriving such an estimator, one can be assured of capturing the behavioral interdependence that arises in strategic situations.

[3]This functional form critique is also central to part of Signorino's (2003) analysis. In the first part of that study, he demonstrates that the MLEs one derives from a strategic and nonstrategic model are fundamentally different. This is certainly true. However, this difference arises predictably from the fact that predicted behavior is going to differ across the two models, since incentives and choices differ. If one derived a series of comparative statics-based predictions from the two models, one would also arrive at different functional forms from one's logit or probit as well. Thus, in the end, this analysis is also a demonstration of the importance of correctly deriving one's predictions from one's theory.

[4]See McKelvey and Palfrey (1998) for derivation of the QRE solution concept. Also, note that QRE is another name for the agency error stochastic model mentioned in Signorino (2003).

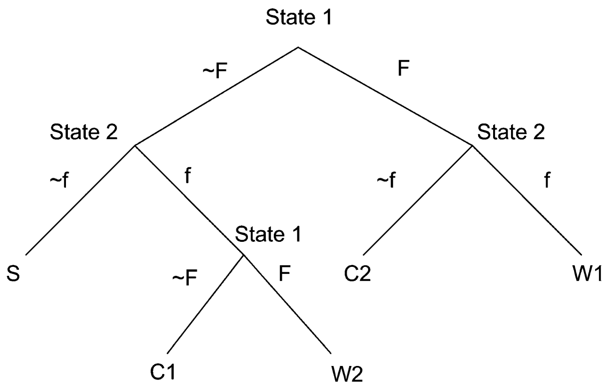

Fig. 1 A typical bilateral crisis game.

To demonstrate the superiority of the LQRE model over traditional logits, Signorino characterizes a "typical bilateral crisis game." This game is slightly more complex than the crisis bargaining game used in subsequent work (Signorino 2003; Signorino and Yilmaz 2003); Fig. 1 characterizes the game. First, player 1 chooses whether to fight (F) or not ($\sim F$). Once player 1 makes his/her decision, player 2 then chooses whether to fight (f) or not ($\sim f$). If player 1 choose not to fight and player 2 chooses to fight, player 1 then must choose whether to fight or not. If neither player chooses to fight, the outcome is the status quo (SQ); whereas if player i chooses to fight and player j chooses not to fight, the outcome is capitulation by player j (C_j), and if both players choose to fight, the outcome is war (W).

Assuming that the world operates according to the crisis game and assuming the QRE solution concept, Signorino generates a Monte Carlo simulation and compares the performance of his LQRE estimation technique to that of the "typical" logits. In particular, he operationalizes a naive logit in which both player's military capabilities and assets are included linearly, a more sophisticated "balance of power" logit in which "military concentration" variables are created from the capabilities and assets variables, and finally another more sophisticated "joint-utility" logit in which the joint utility of war $u_1(W)u_2(W)$ is included as a regressor. Signorino demonstrates not only that only the LQRE model correctly retrieves the underlying parameters that generated the data set but also that one would actually draw incorrect inferences from the other three logits. To illustrate why these logit analyses fail, Signorino demonstrates that country 1's military power is nonlinearly related to the probability of war. Whereas the LQRE successfully recaptures this nonlinear relationship, the other logit models do not. From this analysis, Signorino concludes,

> The question posed at the beginning of this section was: How well does traditional logit model strategic interaction? At least for the simple crisis interaction model here, the answer appears to be: Not very well at all. Perhaps more troubling are the highly significant results in each case, which would be interpreted by the typical researcher as supporting one model or another. Hence, out of a single data set, support could be "found" for a number of different theories of international relations—all of which are wrong.

Signorino and Yilmaz (2003) extend this illustration of the failure of traditional logit by demonstrating that strategic misspecification is the equivalent of omitted variable bias. Once again, they rely upon a crisis bargaining model to illustrate the effects of strategic

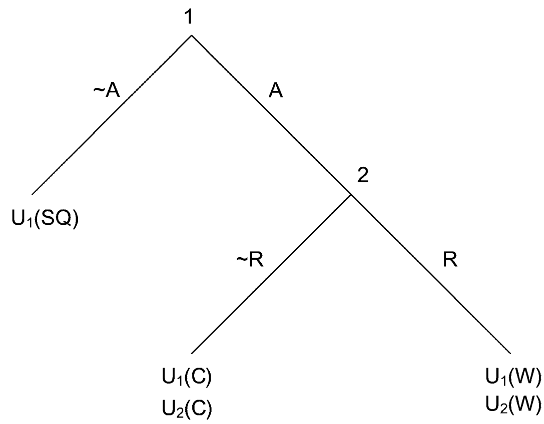

Fig. 2 A strategic deterrence model.

misspecification. Here, however, they use an even simpler model, as characterized in Fig. 2.[5] The first player chooses whether to attack (A) or not ($\sim A$). If he/she does not attack, the game ends and both players receive the payoff associated with the status quo. If he/she does attack, the second player gets to choose to retaliate (R) or not ($\sim R$). If player 2 retaliates, both players receive their payoffs associated with war. If player 2 does not retaliate, they both receive their payoffs associated with the second player capitulating.

To demonstrate the omitted variable bias, Signorino and Yilmaz show that the linear logit (i.e., a model with parameters included only linearly) omits higher order terms included in a Taylor expansion of the strategic model. Once again, this omission arises from the fact that strategic behavior generates nonlinear relationships among the parameters. In fact, Signorino and Yilmaz demonstrate that the linear logit is adequate only when relationships are unconditionally monotonic. Once again, they prove this point through simple illustration.

Signorino's proposed solution to the failure of the traditional logit is to directly derive one's MLE from a stochastic formal model. In his 1999 paper, this is the LQRE. However, the actual estimator will depend upon the precise error structure one assumes in the formal model (Signorino 2003). Although nowhere does Signorino explicitly rule out careful derivation of comparative statics, he also never proposes it as an alternative solution for dealing with complex strategic interdependence.

2.2 *Comparative Statics Analysis and Deterministic Modeling*

Signorino's observations are clearly important. However, they are a critique of poor hypothesis generation and testing, not of estimation failure. Careful comparative statics generation is more than adequate for resolving the complex relationships between parameters and outcomes that arise in the presence of strategic interdependence. To demonstrate this point, we reanalyze Signorino's two crisis bargaining models.

We first consider the simpler variant characterized in Signorino's later work (Signorino 2003; Signorino and Yilmaz 2003). For simplicity, we will discuss the solution in terms of a random-utility version of the model, though in this example the agency error version actually yields the same solution if we define $e_{1w} = e_{1c}$.

[5]Drawn from Signorino (2003) and Signorino and Yilmaz (2003).

The solution to this game is straightforward. Player 2 retaliates when $u_2(\text{War}) + e_{2w} > u_2(\text{Cap}) = 0 + e_{2c}$. Thus, if we define G to be the cumulative density function (CDF) of $e_{2c} - e_{2w}$, we can define the probability of player 2 retaliating as $Q \equiv G[u_2(\text{War})]$. Similarly, player 1 attacks when the expected utility of going to war is greater than the payoff for the status quo[6]; that is, when $E[u_1(A)] \equiv Q \times [u_1(\text{War}) + e_{1w}] + (1 - Q) \times [u_1(\text{Cap}) + e_{1c}] > 0 + e_{1\text{sq}}$. Thus, player 1 attacks when $e_{1\text{sq}} - Qe_{1w} + (1 - Q)e_{1c} < Q[u_1(\text{War})] + (1 - Q)[u_1(\text{Cap})]$. Defining F as the CDF of $e_{1\text{sq}} - Qe_{1w} + (1 - Q)e_{1c}$, the probability that player 1 attacks is $P = F\{u_1(\text{Cap}) - Q[u_2(\text{War})][u_1(\text{Cap}) - u_1(\text{War})]\}$.

Having derived equilibrium behavior, we can now derive comparative statics. Equations (1)–(4) are the first derivatives of each player's probabilistic moves with respect to the parameters of interest.

$$\frac{\partial Q}{\partial u_2(\text{W})} = g[u_2(\text{W})] \geq 0, \tag{1}$$

$$\begin{aligned}\frac{\partial P}{\partial u_1(\text{W})} = {} & G[u_2(\text{W})] \\ & \times f\{u_1(\text{C}) - G[u_2(\text{W})][u_1(\text{C}) - u_1(\text{W})]\} \geq 0,\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial P}{\partial u_1(\text{C})} = {} & \{1 - G[u_2(\text{W})]\} \\ & \times f\{u_1(\text{C}) - G[u_2(\text{W})][u_1(\text{C}) - u_1(\text{W})]\} \geq 0,\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial P}{\partial u_2(\text{W})} = {} & -[u_1(\text{C}) - u_1(\text{W})] \times g[u_2(\text{W})] \\ & \times f\{u_1(\text{C}) - G[u_2(\text{W})][u_1(\text{C}) - u_1(\text{W})]\}.\end{aligned} \tag{4}$$

Equation (1) is obviously positive, whereas equation (2) is positive since it consists of a product of cumulative and density functions. Equation (3) is similarly positive since it consists of one minus a cumulative function and a density function. Equation (4) is unsigned because it can be either positive or negative depending on the relative size of the capitulation and war payoffs.

Substantively, these results are all intuitive. Most obviously, player 2 should be more likely to retaliate, the more player 2 values war. Also fairly clearly, player 1 should be more likely to attack, the more player 2 values both war and capitulation. These are the two possible outcomes if he/she attacks, and each has a strictly positive probability of occurring. Finally, and perhaps least obviously, whether player 1's probability of attacking is increasing or decreasing in player 2's value of war depends on whether player 1 values war or capitulation more. If player 1 values capitulation more, the expected value of attacking decreases as player 2's value of war increases because the war outcome is more likely, the more player 2 values war. The opposite relationship holds if player 1 values war more.

These relationships yield exactly the relationships that Signorino and Yilmaz derive in their MLE. In particular, note that we have derived, through standard comparative statics, both the nonlinear and the "conditionally monotonic" relationships illustrated in Fig. 5 of Signorino and Yilmaz (2003, 562) and reproduced below as Fig. 3 (with a relabeling of the

[6]The payoff is an expected value because player 1 does not know with certainty what player 2 is going to do if attacked.

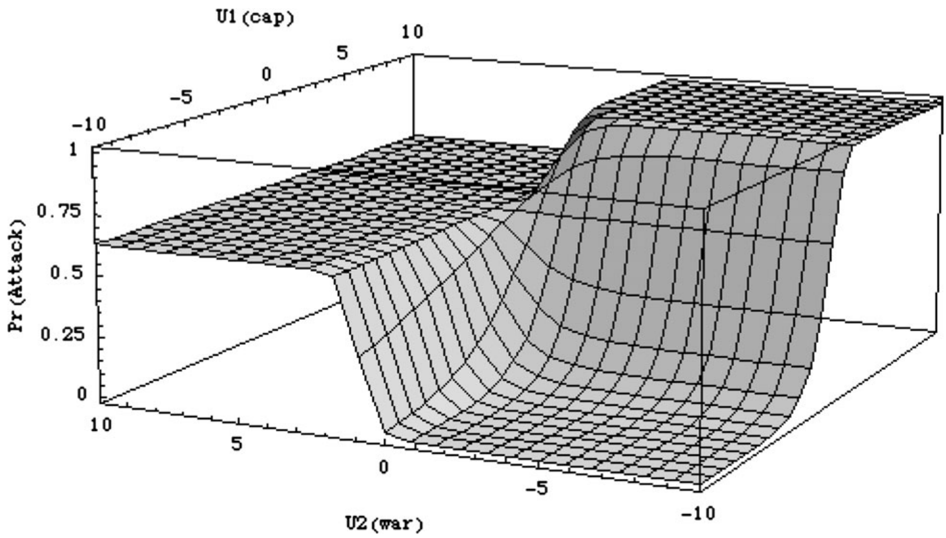

Fig. 3 Stochastic probabilities of attacking: strategic. Following Signorino (1999), this graph was generated using a type I extreme value distribution. The mean and variance, however, are different; here, the mean is 0 and the variance is 1, creating a graph that is shifted and stretched a little differently from Signorino's, but that still reflects the same dynamics for the game.

axes for ease of exposition). To see why, let us consider the two parameters of interest separately.

First consider the relationship between player 2's value of war and the probability of player 1 attacking. When player 1's value of capitulation is less than player 1's value for war, which is fixed at zero in the figure, the probability of attack is increasing in player 2's value of war, and when the opposite is true the probability of attack is decreasing in player 2's value of war. This conditional relationship is identical to the comparative static derived in equation (4).

Next consider the relationship between player 1's value of capitulation and player 1's probability of attacking. One might suppose that the comparative static yields an unconditionally monotonic relationship because its sign is unconditionally positive. However, in fact it does not. To see why, note that equation (3) consists of two parts, the density function $f(\cdot)$ and $1 - G[u_2(\text{War})]$. As player 2's value of war increases, the value of the first derivative $\partial P/[\partial u_1(\text{Cap})]$ decreases. Interpreting this as a slope, we see that the probability of player 1 attacking is much less sensitive to player 1's value of capitulation as player 2's value of war increases. Thus, we derive exactly the same relationships through comparative statics that we would if we wrote Signorino's version of a strategic empirical model. The only difference between traditional comparative statics and what we have just done is that we have taken advantage of all the information provided in the comparative static, not just its sign.

All this analysis has been performed assuming a stochastic formal model. However, none of these derived relationships rely upon the random-utility assumption. That is, we would derive exactly the same relationships between the parameters and the values of interest from a deterministic version of the same model. Using subgame perfection and solving backward, we know that player 2 will retaliate when $u_2(\text{Cap}) = 0 < u_2(\text{War})$. Further, we know that player 1's decision to attack will depend upon what player 1 anticipates player 2 will do. If $u_2(\text{War}) > 0$, then player 1 will attack if $u_1(\text{War}) > u_1(\text{SQ})$

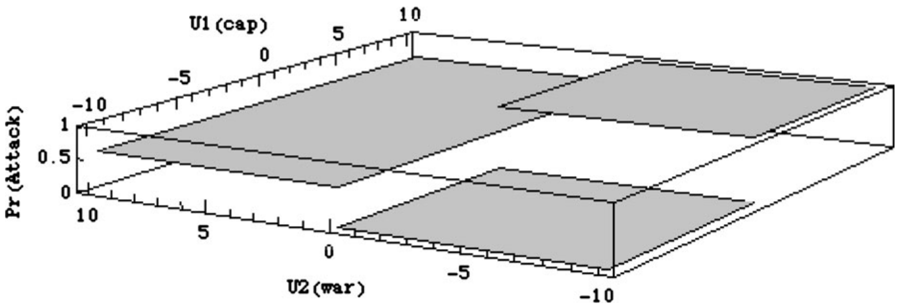

Fig. 4 Deterministic probabilities of attacking: strategic.

(i.e., if $u_1(\text{War}) > 0$), whereas if $u_2(\text{War}) < 0$, then player 1 will attack if $u_1(\text{Cap}) > u_1(\text{SQ})$.[7]

Now compare the probability of player 1 attacking across the two models. In the deterministic version, player 1's decision to attack depends conditionally on the three parameters. Player 1 attacks when u_1(War) is sufficiently large if u_2(War) is sufficiently large, and attacks when u_1(Cap) is sufficiently large if u_2(War) is not sufficiently large. Figure 4 compares this solution to the stochastic version's solution, focusing on player 1's utility for war and player 2's utility for war. As can be seen, the solutions are not as different as one might have initially suspected. When u_2(War) is small, the probability player 2 plays R is small, and both models predict that $p_1(A)$ is basically independent of u_1(War). When u_2(War) is large, the probability player 2 plays R is large, and both models predict that $p_1(A)$ is almost perfectly predicted by u_1(War). The only place that the two model's predictions substantially differ are when u_2(War) is moderate. In this case, the deterministic model either predicts no attack with certainty (when $u_2(\text{War}) < 0$) or attack with certainty (when $u_2(\text{War}) > 0$), whereas the stochastic model predicts that $p_1(A)$ probabilistically increases as u_1(War) increases. Thus, aside from some smoothing of the cut points, the predicted relationships across the two versions of the model are basically identical.[8]

Next consider the more complex crisis bargaining model from Signorino's (1999) article. For simplicity and brevity, we derive solely the subgame perfect, deterministic

[7]Note that this solution is identical to the solution for Signorino's (2003) regressor error version of a stochastic game theoretic model. In Signorino's regressor error model, as above, behavior is deterministic; only the regessor's ability to measure actor utility is stochastic.

[8]The same findings hold with regards to the probability of war as well; we omit that discussion here for the sake of brevity. On a related note, Signorino (2003) makes a seemingly contradictory point of demonstrating that the type of error matters. That is, it matters whether one assumes regressor error, error on the part of the scholar in measuring parameters of the model, or agent or utility error, error built into the solution concept of the game itself. He demonstrates this point in two ways. First, he assumes that the random-utility model is the correct error specification and then demonstrates that the mean-squared error of the random-utility and agent error models are smaller than the regressor error model. Second, still assuming a data generation process based on the random-utility model, he demonstrates that the regressor error model will have higher average and maximal deviations from the true probabilities than the random-utility or agent error versions. He conjectures, but does not demonstrate, why this would be the case. We believe that all these differences are driven by the "smoothing" process of the random-utility and agent error models. Differences in point predictions along these curves should generate increased mean-squared errors, increased average differences in probabilities, and increased maximal differences in probabilities. Although plausible and interesting, this issue is not critical. If one assumes, as we stated above, that deterministic models are approximations of reality and that we generate implicitly probabilistic hypotheses from these deterministic models, then we get the same smoothed relationships whether we rely upon a stochastic or deterministic model. That is, the differences that Signorino identifies are a construct of the assumption that one would operationalize a deterministic model with hard cut points if one could.

conditions that lead to war. Referring back to Fig. 1, Signorino defines the payoffs for each outcome as follows:

$$u_i(SQ) = D_{ij},$$

$$u_i(C_i) = -A_i,$$

$$u_i(C_j) = A_j, \text{ and}$$

$$u_i(W) = p_iA_j + (1 - p_i)(-A_i - M_i),$$

where $p_i = M_i/(M_i + M_j)$. Substantively, M_i are state i's military assets, A_i are state i's other assets (e.g., land and natural resources), and p_i is the probability that state i wins a military conflict.

Assuming state 1 did not initiate conflict, state 1 fights back when $u_1(W_2) > u_1(C_1)$. Substituting and simplifying yields $M_2 < A_1 + A_2$; if this constraint holds, state 2 chooses to fight when $u_2(W_2) > u_2(\text{SQ})$. Similarly, substituting and simplifying yields $M_1 < (M_2A_1)/(A_2 + M_2)$; if this constraint does not hold, state 2 chooses to fight when $u_2(C_1) > u_2(\text{SQ})$, which in turn yields $A_1 > D_{ij}$. If, on the other hand, we assume state 1 initiated the conflict, state 2 fights back when $u_2(W_1) > u_2(C_2)$, which yields $M_1 < A_1 + A_2$. Finally, state 1 will initiate a fight when $u_1(\text{argmax}_2\{W_1, C_2\}) > u_1(\text{argmax}_2\{\text{SQ}, \text{argmax}_1\{C_1, W_2\}\})$, where $\text{argmax}_i\{\cdot\}$ identifies the outcome that maximizes the utility for player i. For brevity, we do not list out all the constraints generated by this condition here.

With this solution in hand, let us reconsider Signorino's conclusions. Using Monte Carlo analysis, Signorino demonstrates that his strategic model recaptures parameters correctly, whereas the naive, balance of power, and joint-utility models do not. He demonstrates that the failure arises from the inability of the other models to correctly capture the nonlinear relationships between parameters and outcomes that are generated by strategic interdependence.

Figure 5 reproduces Signorino's Fig. 3 and 4, in which he illustrates this point with regards to the relationship between country 1's military power and the probability of war. The relationship in the strategic model is curvilinear: as country 1's military capability increases, the probability of war increases quickly, drops close to zero, increases again, and then tails off. In contrast, the naive and balance of power logits predict the relationship to be monotonic and convex, respectively.

We certainly agree that Signorino has demonstrated the failure of these typical logits. However, none of the three logits estimated by Signorino rely upon correctly derived comparative statics. If one derives a comparative static over country 1's military capability from the deterministic solution characterized above, we observe exactly the curvilinear relationship derived in the QRE version of the game.[9] The probability of war is large when country 1's military capabilities are either moderately small or moderately large, whereas the probability of war is small when country 1's military capabilities are moderate, quite small, or quite large (see the Appendix for a proof); this solution is graphed in Fig. 6. As with the comparison between the deterministic and stochastic versions of the crisis bargaining game discussed in the previous section, the only difference is that the QRE version

[9]As in Signorino (1999), we fix $M_2 = 20$, $A_1 = 40$, $A_2 = 40$, and $D_{12} = 0$ to generate the graph.

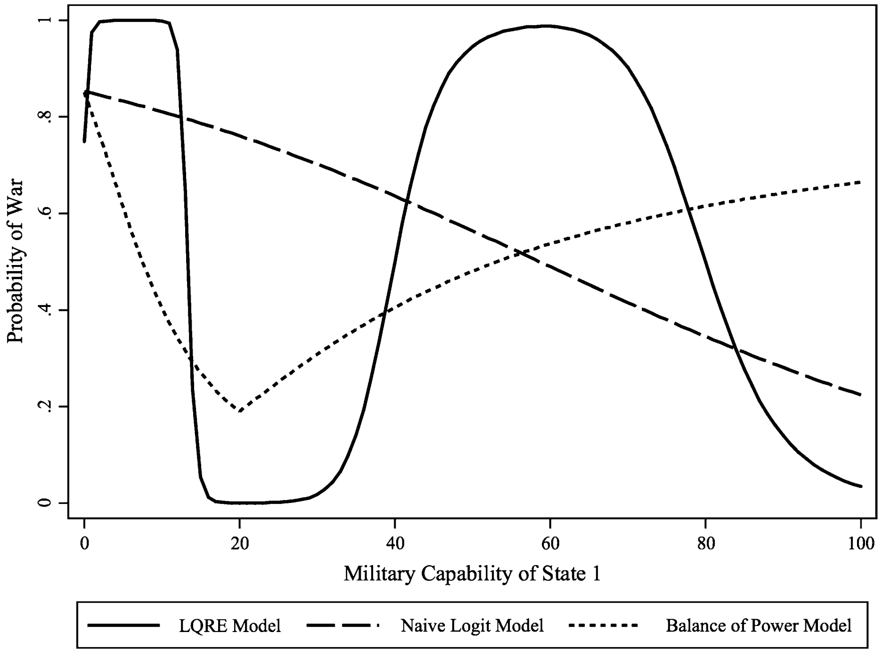

Fig. 5 Logit QRE–, naive logit–, and balance of power–predicted probabilities of war.

of the game "smooths" the cut points in the deterministic solution and explicitly makes all the derived relationships probabilitistic.

In sum, much of Signorino's critique is a functional form critique that has more to do with correctly deriving ones hypotheses from an underlying model than with the actual estimation technique used to test those hypotheses. Thus, much of the failure of traditional logit analysis can be resolved with more careful generation of comparative statics. Nothing in the functional form critique requires one to move to generating estimators from stochastic formal models. Next we consider appropriate estimation techniques for testing these models.

3 Estimation Techniques

Can we use existing logit and probit models to test strategic theories of politics? In fact, existing empirical models can be perfectly appropriate estimation techniques for testing such theories. To illustrate this point, we return to the simple crisis bargaining model used in the majority of Signorino's work.[10] For this discussion we assume errors are independently drawn, as Signorino does in all of his work except for that in 2002.[11] We derive three possible estimators and illustrate their validity through Monte Carlo simulations.

[10]Identical results hold for Signorino's LQRE version of the more complex crisis bargaining model. Because deriving through these results would be redundant with those for this simpler model, we omit the additional proofs.

[11]In his *International Interactions* paper, Signorino (2002) performs a simulation of a strategic model with correlated errors but concludes that such a model is extremely complicated and recommends using an estimator that correctly captures the strategic behavior at the sacrifice of modeling the correlated errors. Thus, because Signorino never promotes estimating a model with strategic behavior and correlated errors, in this study we focus on the model with uncorrelated errors.

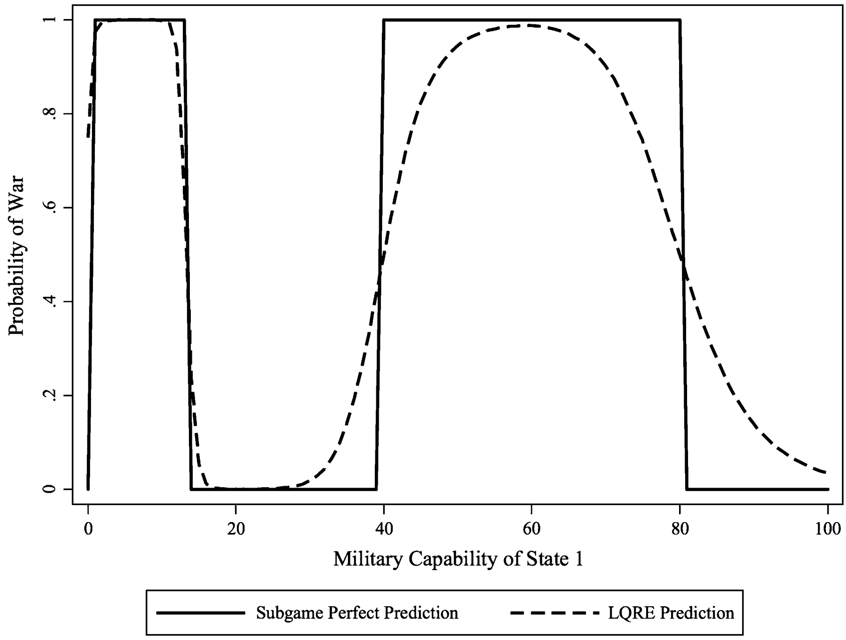

Fig. 6 LQRE- and subgame perfect–predicted probabilities of war.

3.1 *Some Estimators for Strategic Models*

Consider again the strategic deterrence model in Fig. 2. Note at the outset that we can reexpress the probabilities of the events of interest as

$$p_2(R) = \Pr[e_{2c} - e_{2w} \leq u_2(\text{War})] \tag{5}$$

and

$$p_1(A) = \Pr\{e_{1\text{sq}} - \Pr_2(R)e_{1w} + [1 - \Pr_2(R)]e_{1c} \leq u_1(\text{Cap}) - \Pr_2(R)[u_1(\text{Cap}) - u_1(\text{War})]\}. \tag{6}$$

Equations (5) and (6) are restatements of equilibrium behavior derived in the previous section; they characterize player 2's probability of retaliating and player 1's probability of attacking, respectively. In equation (5), the left-hand side is a difference of errors and the right-hand side is a parameterized cut point. Thus, equation (5) yields a simple logit or probit, depending upon whether we assume the errors have a type I extreme value or normal distribution.

Equation (6) is more complex because player 1 is making an expected utility calculation. Player 1 is comparing settling for the status quo to choosing the lottery of attacking, where the value of the lottery depends upon player 1's assessment of the likelihood that player 2 will retaliate, $\Pr_2(R)$. As a result, both the error distribution and the cut point are functions of $\Pr_2(R)$. Although this construction makes the error term appear more complex, it simply yields a weighted combination of errors. And, since the individual error terms are mean zero, we know the weighted combination of errors must also be mean zero (i.e., $E\{e_{1\text{sq}} - \Pr_2(R)e_{1w} + [1 - \Pr_2(R)]e_{1c}\} = 0$). Thus, the error distribution is unproblematic. The only substantively important added complexity arises on the right-hand side of the equation, where the cut point is a function of player 1's payoffs and the probability that player 2 retaliates.

Equations (5) and (6) provide the basis for three possible estimation techniques. First, one can use probits (or logits) "all the way up." That is, one can estimate the second stage [equation (5)] using a probit (or logit), recover the predicted values, and use those predicted values in equation (6) to estimate the first-stage equation.[12] Second, one could derive an MLE that allows one to estimate just the first-stage equation; such a model is similar in form to Achen's (2006) "double probit" model. Although this approach has the disadvantage of losing information (because the analyst is not taking advantage of having information on player 2's decision to retaliate or not), it nonetheless identifies a model that will recover unbiased and consistent estimates of all the coefficients, including the impact of the covariates on the decision of player 2 to retaliate. One might choose to use this approach if, for example, the second-stage dependent variable is of particularly poor quality. Finally, one can derive an MLE that allows one to estimate both equations simultaneously; this last option is equivalent to Signorino's strategic model.[13]

Finally, suppose one instead derives a deterministic model and wishes to test its predictions. Recall that the only difference between the deterministic and the stochastic versions of Signorino's models is that the deterministic version does not smooth the cut points. As such, the deterministic model's prediction is that the impact of player 1's utility of war and capitulation depends on whether player 2 is likely to choose to retaliate or not. If the researcher recognizes player 1's decision is implicitly an expected utility calculation, again assuming we do not believe that the world is actually deterministic, then the researcher could use any one of the three estimators characterized above. Alternatively, if the researcher did not recognize player 1's decision as an expected utility calculation, then a researcher might simply include interaction terms in the equation for player 1's decision $\{u_1(\text{War}) \times p_2(R), u_1(\text{Cap}) \times [1 - p_2(R)]\}$. This "interactive probit" fails to achieve precisely the right functional form because both $u_1(\text{War})$ and $u_1(\text{Cap})$ would be interacted linearly with the covariates for $p_2(R)$, $\mathbf{X\beta}$, rather than with a nonlinear transformation of those covariates, $\mathbf{\Phi(X\beta)}$. However, even this model would be right to a first-order approximation, particularly in instances where the probabilities in question were in the neighborhood of 0.5.

3.2 *Monte Carlo Analysis*

To assess the relative viability of these models, we conduct a series of Monte Carlo simulations. We estimate each set of simulations on data sets of size $N = 250$, $N = 1000$, and $N = 10{,}000$. We denote covariates that impact player 1's probabilities by Z:

$$u_1(\text{Cap})_i = Z_{C_i}, \tag{7}$$

$$u_1(\text{War})_i = Z_{W_i}, \tag{8}$$

$$u_1(\text{SQ})_i = Z_{\text{SQ}_i}, \tag{9}$$

[12]In a recent paper, Bas, Signorino, and Walker (2006) independently derive this technique and demonstrate its properties; see also Caldeira, Wright, and Zorn (1999), Brueckner (2003), Erdem et al. (2005), Bajari et al. (2006), and Haile, Hortaçsu, and Kosenok (2006).

[13]See Quinn, Martin, and Whitford (1999) for a treatment of higher order Markov Chain Monte Carlo models that could be another approach to estimating this sort of model.

whereas those influencing player 2's utility are denoted with X:

$$u_2(War)_i = X_i, \tag{10}$$

and where we assume $u_2(\text{Cap})_i = 0$ without loss of generality. We draw $Z_{C_i}, Z_{W_i}, Z_{\text{SQ}_i}$, and X_i from independent $N(0,\ 1)$ distributions, and generate

$$p_2(R)_i = \Phi(5X_i + e_{2i}), \tag{11}$$

where $e_{2i} \sim N(0,\ 1)$. As we note above, player 1's propensity to attack depends in part on its best guess as to player 2's probability of defending, denoted $\Pr_2(R)_i$. We therefore generate a continuous (and, under normal circumstances) latent propensity for player 1 to attack—corresponding to that in equation (6)—as:

$$\text{Attack}_i = \Pr_2(R)_i(5Z_{W_i}) + \{[1 - \Pr_2(R)_i] \times (5Z_{C_i})\} - (5Z_{\text{SQ}_i}) + e_{1i}, \tag{12}$$

where we let $\Pr_2(R)_i = \Phi(5X_i)$ and once again $e_{1i} \sim N(0, 1)$. The binary attack/no attack indicator is then defined as

$$A_i = \begin{cases} 0, & \text{if Attack}_i < 0 \\ 1, & \text{if Attack}_i \geq 0 \end{cases}. \tag{13}$$

The true parameters are therefore equal to 5.0 for X, Z_C, and Z_W, and -5.0 for Z_{SQ}.

We conduct our analyses on five models. The first is a "naive" probit of player 1's decision to attack, of the form:

$$\Pr(A_i = 1) = \Phi[\beta_{N1}X_i + \beta_{N2}Z_{\text{SQ}_i} + \beta_{N3}Z_{C_i} + \beta_{N4}Z_{W_i}]. \tag{14}$$

where we use the subscript N to denote the results from this model. This corresponds to the sort of approach Signorino (rightly) criticizes existing work for adopting; we include it here as a point of reference. A second model is an "interactive probit" of the sort described above (and denoted with the subscript I), in which the variables which impact player 1's utility for war are interacted with that for player 2's utility for war:

$$\Pr(A_i = 1) = \Phi[\beta_{I1}X_i + \beta_{I2}Z_{\text{SQ}_i} + \beta_{I3}Z_{C_i} + \beta_{I4}Z_{W_i} + \beta_{I5}X_iZ_{C_i} + \beta_{I6}X_iZ_{W_i}]. \tag{15}$$

As we mentioned previously, this model should be a relatively good first-order approximation of the true data-generating process in equations (11)–(13).

Yet, a third alternative model is the two-stage approach discussed above, where we include information about the empirical predicted probabilities from the second-stage estimate in the model of player 1's attack behavior:

$$\Pr(A_i = 1) = \Phi[\beta_{T1}Z_{SQ_i} + \beta_{T2}\hat{Z}_{C_i} + \beta_{T3}\hat{Z}_{W_i}]. \tag{16}$$

Here, $\hat{Z}_{W_i} = Z_{W_i} \times \Phi[X_i\hat{\beta}]$ and $\hat{Z}_{C_i} = Z_{W_i} \times \{1 - \Phi[X_i\hat{\beta}]\}$, where $\Phi[X_i\hat{\beta}]$ corresponds to the predicted probabilities of player 2's defending, obtained from estimating equation (11).

A fourth model is what we term the "attack-only" approach (denoted with the subscript A), where we estimate only the influence of the covariates on the probability of player 1 attacking:

$$\Pr(A_i = 1) = \Phi\{\beta_{A2}Z_{SQ_i} + \beta_{A3}Z_{C_i} + [\Phi(\beta_{A1}X_i)]\beta_{A4}Z_{W_i} + [\Phi(\beta_{A1}X_i)\beta_{A3}Z_{C_i}]\}. \tag{17}$$

Note that this model, unlike those in equations (14)–(16), cannot be estimated using standard logit/probit software, requiring instead that the (relatively simple) likelihood

Table 1 Monte Carlo results

Model parameter	*n = 250*	*n = 1000*	*n = 10,000*
Naive probit			
β_{N1} (X)	−0.01 [−0.21, 0.20]	−0.002 [−0.10, 0.10]	−0.001 [−0.03, 0.03]
β_{N2} (Z_{SQ})	−1.56 [−1.95, −1.27]	−1.50 [−1.69, −1.36]	−1.49 [−1.54, −1.45]
β_{N3} (Z_C)	0.78 [0.53, 1.06]	0.75 [0.64, 0.88]	0.75 [0.71, 0.79]
β_{N4} (Z_W)	0.78 [0.56, 1.04]	0.75 [0.65, 0.88]	0.75 [0.71, 0.79]
Interactive probit			
β_{I1} (X)	−0.01 [−0.38, 0.40]	0.002 [−0.16, 0.15]	−0.002 [−0.05, 0.05]
β_{I2} (Z_{SQ})	−3.10 [−4.52, −2.37]	−2.83 [−3.28, −2.50]	−2.75 [−2.87, −2.64]
β_{I3} (Z_C)	1.55 [1.10, 2.34]	1.41 [1.20, 1.70]	1.38 [1.31, 1.45]
β_{I4} (Z_W)	1.57 [1.11, 2.29]	1.41 [1.21, 1.69]	1.37 [1.30, 1.45]
β_{I5} ($X \times Z_C$)	−1.44 [−2.28, −0.90]	−1.24 [−1.56, −0.97]	−1.18 [−1.27, −1.09]
β_{I6} ($X \times Z_W$)	1.44 [0.86, 2.26]	1.24 [0.97, 1.57]	1.18 [1.09, 1.27]
Two-stage probit			
β_{T1} (Z_{SQ})	−5.23 [−8.05, −3.92]	−5.05 [−6.03, −4.39]	−5.00 [−5.28, −4.77]
β_{T2} ($\hat{Z}_C$)	5.22 [3.95, 8.14]	5.06 [4.40, 6.02]	5.01 [4.78, 5.29]
β_{T3} ($\hat{Z}_W$)	5.25 [3.87, 8.19]	5.07 [4.37, 6.06]	5.01 [4.76, 5.29]
Attack only			
β_{A1} (X)	4.99 [3.10, 10.20]	5.00 [3.97, 6.52]	4.99 [4.66, 5.43]
β_{A2} (Z_{SQ})	−5.50 [−8.38, −4.13]	−5.13 [−6.09, −4.42]	−5.01 [−5.27, −4.78]
β_{A3} (Z_C)	5.56 [4.12, 8.49]	5.13 [4.41, 6.12]	5.01 [4.77, 5.29]
β_{A4} (Z_W)	5.54 [4.00, 8.69]	5.14 [4.43, 6.04]	5.01 [4.78, 5.29]
Simultaneous model			
β_{S1} (X)	5.01 [3.83, 7.07]	5.03 [4.38, 5.84]	5.00 [4.78, 5.24]
β_{S2} (Z_{SQ})	−5.28 [−8.27, −4.04]	−5.08 [−6.07, −4.40]	−5.00 [−5.24, −4.78]
β_{S3} (Z_C)	5.37 [3.98, 8.15]	5.09 [4.38, 6.08]	5.01 [4.78, 5.26]
β_{S4} (Z_W)	5.34 [3.92, 8.32]	5.08 [4.37, 6.10]	5.00 [4.78, 5.25]

Note. Cell entries are median coefficient estimates (1000 iterations); numbers in brackets are empirical 5th and 95th percentiles of the distributions. See text for details.

be programmed and estimated. Finally, our fifth model simultaneously estimates $\Pr(A_i)$ and $\Pr(D_i)$ as part of a single likelihood. As we note above, this is equivalent to Signorino's strategic model and should recover the underlying parameters perfectly.

We begin by simulating 1000 data sets with N = 250, 1000, and 10,000 each, with variables corresponding to the data-generating process outlined above. We then estimate each of the five models and retain the recovered parameter values for each.[14] Table 1 reports the median values of the recovered parameters across the 1000 data sets, along with their empirical 5th and 95th percentile values, for each of the five models. From the Monte Carlo results, several things are apparent. First, as in Signorino (1999), the naive model does a particularly poor job of recovering the correct parameter values. Interpretation of the results for the interactive probit model are complicated by the fact that they include interactive terms; we therefore return to them below.

All three subsequent models—the two-stage probit, the "attack-only" model, and the simultaneous model—accurately recover the true parameter values; moreover, all three do

[14]The *Stata* code for each of these estimators is available in a Web appendix.

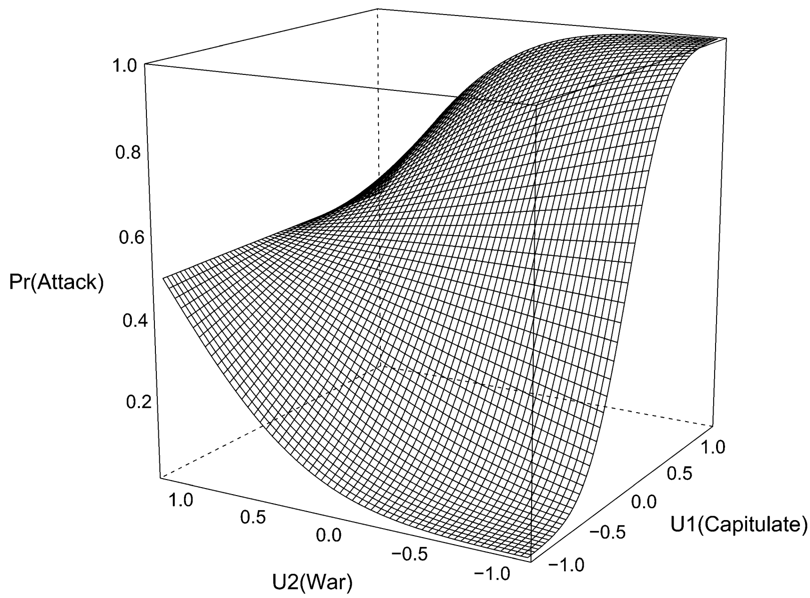

Fig. 7 Simulated predicted probabilities of attack, interactive model.

so with an increasing degree of reliability as the sample size increases. As a general rule, the accuracy of the simultaneous model is greater than that for the attack-only model, and the ranges between the 5th and 95th percentiles are slightly smaller in the simultaneous model as well; this likely reflects the latter's greater accuracy vis-à-vis the data-generating process. It is notable, however, that the two-stage model yields results which are generally as accurate as those for the simultaneous model; this, combined with the fact that it can be estimated using standard statistical software, confirms the fact that there is nothing about the error structure arising in strategic models of this sort that requires anything more complicated than a series of sequential binary-choice models.

Additionally, the findings from the interactive probit model deserve further mention. The interactive specification constitutes a first-order approximation of the true data-generating process. Because the strategic component of the model is captured in the interaction terms, direct recovery of the parameters of interest is not possible (cf. Friedrich 1982). However, it is instructive to consider the predicted probabilities that arise from the model. Figure 7 plots predicted values of the probability of attack against values of X and Z_C with values in the range $[-1, 1]$, using the median estimated parameter values for the $N = 10{,}000$ analyses reported in Table 1. Those predictions closely mirror the values for the theoretical probabilities of an attack plotted in Fig. 3, suggesting that the interactive model offers a reasonable empirical approximation of the random-utility stochastic model described above. To the extent that the interactive model requires nothing more than incorporation of multiplicative interaction terms into a standard binary-response model, this represents a significantly more tractable approach than derivation and estimation of the model's full set of structural parameters.

4 Conclusion

Recent developments in integrating theoretical and empirical models have unquestionably improved scholars' ability to specify and test models of strategic interaction. Beyond international security, recent examples include work in American politics (e.g., Carson

2003) and political economy (e.g., Leblang 2003). Like these scholars and others, we recognize the importance of incorporating insights from models of strategic interaction into empirical models of political phenomena. In this respect, then, Signorino (1999) is exactly correct, in that one must carefully operationalize predictions generated by strategic behavior. Specifying naive models that atheoretically include a host of covariates and loosely operationalizing predictions predicated on strategic models can both lead to inaccurate inferences. Thus, we completely agree that to correctly test a theory, one must correctly operationalize those tests.

That said, a number of approaches exist for deriving and operationalizing strategic models that do not entail the complex machinery endorsed by Signorino and his coauthors. For example, as previously demonstrated, a deterministic version of Signorino's crisis bargaining model predicts exactly the same relationships as the stochastic version, the only difference being that the deterministic version does not smooth the cut points. Thus, as long as the researcher recognizes that the deterministic version predicts interaction effects, where the weight to be assigned to the payoff from any given state depends on the probability of that state arising, the researcher will generate the same functional form for the empirical estimator as he/she would if he/she derived a stochastic formal model.[15]

Similarly, we believe that correctly specifying comparative statics, even from simple deterministic models, and testing those predictions using a logit or probit is a perfectly appropriate alternative strategy to deriving and programming a full model-based likelihood. Correctly derived comparative statics can and do successfully capture all the nonlinear and conditionally monotonic relationships that Signorino uses to highlight the failure of traditional logit analyses, and the error structures generated in simple strategic games are appropriately modeled in standard logits and probits. Particularly promising in this light are two-step estimators, which include (correctly specified) functions of empirical second-stage predictions in first-stage equations of interest. Such an approach has the advantage of corresponding to a simple equilibrium concept (subgame perfection) as well as being substantively easy to understand and extremely simple to implement.[16] Likewise, models incorporating linear-interactive effects can also provide very good first-order approximations of more complicated strategic models and require nothing more than theoretically informed inclusion of conditioning variables.

In sum, the presence of strategic behavior does not imply that one must necessarily adopt Signorino's approach to testing models of that behavior.[17] Writing down a deterministic model, carefully deriving predictions from that model, and recognizing that the world is not deterministic when generating the empirical estimator is enough to ensure consistency between one's theory and one's test. Thus, one does not have to write a stochastic formal model or derive one's estimator directly from that stochastic model to appropriately test strategic theories of politics. At the same time, we believe our study underscores the significance of Signorino's core insight: that, in the presence of strategic behavior, researchers must work hard to ensure that their empirical model accurately tests the predictions generated by the theory. By providing a more tractable method for estimating

[15]By implication, there is no zero probability event problem for the estimator, since the prediction generated by the deterministic model is being treated "as if" it is probabilistic, and the empirical estimator correctly captures that intuition.

[16]Note that, as a practical matter, analysts adopting such methods should adjust the standard error estimates obtained to account for the introduction of stochastic variation into right-hand–side variables. This can be done in a number of ways; see Murphy and Topel (1985) and Bas, Signorino, and Walker (2006) for discussions.

[17]Moreover, recent work by Haile, Hortaçsu, and Kosenok (2006) has demonstrated the manifold difficulties with empirically testing predictions derived from quantal response models.

such models, we hope to facilitate incorporation of Signorino's key theoretical contributions in the discipline.

Appendix: Comparative Statics of a Simple Crisis Game (from Signorino 1999)

Using backward induction, the cut points in the deterministic model can be derived at each node of the game, fully characterizing the relationship between the probability of war and player 1's military capability. The first cut point (termed M_1^ℓ) corresponds to the last decision node in which player 1 decides to fight, given that he/she chose not to fight at the first node but then was challenged by player 2 at the second node:

$$u_1(\text{War}) > u_1(\text{Cap}) \Leftrightarrow \frac{M_1}{M_1+M_2}A_2 + \frac{M_2}{M_1+M_2}(-A_1 - M_1) > -A_1$$

$$\Rightarrow M_1(A_1 + A_2 - M_2) > 0.$$

Substituting $A_1 = A_2 = 40$ and $M_2 = 20$, we get:

$$M_1^\ell > 0. \tag{A1}$$

The second cut point (denoted M_1^c) marks the upper bound on M_1 at which player 2 will issue a challenge, given that player 1 did not challenge at the first node:

$$u_2(\text{War}) > u_2(\text{SQ}) \Leftrightarrow \frac{M_2}{M_1+M_2}A_1 + \frac{M_1}{M_1+M_2}(-A_2 - M_2) > D = 0$$

$$\Rightarrow M_1^c < \frac{M_2 A_1}{A_2 + M_2}.$$

Substituting yields:

$$M_1^c < \frac{800}{60} = 13.33. \tag{A2}$$

The third cut point (M_1^r) marks where player 2 would decide to fight, given that player 1 challenged at the first node. This sets the upper bound on war for M_1:

$$u_2(\text{War}) > u_2(\text{Cap}_2) \Leftrightarrow \frac{M_2}{M_1+M_2}A_1 + \frac{M_1}{M_1+M_2}(-A_2 - M_2) > -A_2$$

$$\Rightarrow M_1^r < A_1 + A_2.$$

Substituting yields:

$$M_1^r < 80. \tag{A3}$$

The final cut point (M_1^m), denotes the amount of military capability player 1 would require to challenge at the first node:

$$u_1(\text{War}) > u_1(\text{SQ}) \Leftrightarrow \frac{M_1}{M_1+M_2}A_2 + \frac{M_2}{M_1+M_2}(-A_1 - M_1) > D = 0$$

$$\Rightarrow M_1^m > \frac{M_2 A_1}{A_2 - M_2}.$$

Substituting yields:

$$M_1^m > 40. \tag{A4}$$

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