

2017–2018 WINTER MEETING  
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

**San Diego Convention Center and Marriott Marquis San Diego Marina  
San Diego, CA  
January 12–13, 2018**

The 2017–2018 Winter Meeting of the Association of Symbolic Logic was held on January 12–13, 2018 at the San Diego Convention Center and the San Diego Marriott Hotel and Marina in conjunction with the annual Joint Mathematics Meetings. The members of the Program Committee were Jan Reimann (Chair), Philip Scowcroft, and Anush Tserunyan. The program consisted of seven invited 50-minute talks and eight contributed talks. The ASL hosted a welcoming reception on Friday, January 12th, at the San Diego Marriott Hotel.

An AMS/ASL co-sponsored Special Session on *Set Theory, Logic and Ramsey Theory*, organized by Andrés Caicedo and José Mijares was held on Wednesday and Thursday, January 10th and 11th, in the San Diego Convention Center.

The fifty-minute invited addresses at the ASL meeting were as follows:

Cameron Donnay Hill (Wesleyan University), *0, 1-Laws and pseudofiniteness of  $\aleph_0$ -categorical theories*.

Antonina Kolokolova (Memorial University of Newfoundland), *Power of reasoning over richer domains*.

Aristotelis Panagiotopoulos (California Institute of Technology), *Games orbits play*.

Emily Riehl (Johns Hopkins University), *A synthetic theory of  $\infty$ -categories in homotopy type theory*.

Simon Thomas (Rutgers University), *The isomorphism and bi-embeddability relations for countable torsion abelian groups*.

Sebastien Vasey (Harvard University), *Nonelementary classification theory*.

Keita Yokoyama (JAIST, Japan), *Ramsey's theorem in arithmetic*.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee  
JAN REIMANN

**Abstracts of invited talks**

- ▶ CAMERON DONNAY HILL, *0, 1-Laws and pseudofiniteness of  $\aleph_0$ -categorical theories*.  
Department of Mathematics and Computer Science, Wesleyan University, 265 Church St.,  
Middletown, CT 06459, USA.  
*E-mail*: cdhill@wesleyan.edu.

I will survey some recent results around questions of the form “What are pseudofinite  $\aleph_0$ -categorical theories like?” and “What are almost-sure  $\aleph_0$ -categorical theories like?” Ideally, we would like to answer such questions solely in terms of structural properties the theories’ models—without reference to ultraproducts or probability theory. I will present evidence that certain higher amalgamation properties and model-theoretic properties like super-simplicity are sufficient to answer our questions completely.

- ▶ ANTONINA KOLOKOLOVA, *Power of reasoning over richer domains*.  
Department of Computer Science, Memorial University of Newfoundland, St. John’s, NL A1B 3X5, Canada.  
*E-mail:* antonina.kolokolova@gmail.com.  
*E-mail:* kol@mun.ca.

How does the richness of underlying concepts affects the power of reasoning systems built on them? This question arises in many different settings. Here, we look at it from proof complexity and from bounded arithmetic perspective.

For the former, we focus on proof complexity of satisfiability modulo theories (SMT) framework. There, propositional SAT solvers operate over Boolean combinations of atoms from an underlying theory (such as theory of linear arithmetic or uninterpreted functions with equality), using a dedicated theory solver to analyse viability of supposed satisfying assignments from the theory perspective, and derive new facts by theory reasoning. A natural question is just how much it helps to augment propositional, resolution-based reasoning with the power of the theory. We show that even a theory of uninterpreted functions, decidable in near-linear time, helps enormously: resolution over that theory can simulate a much more powerful Frege (natural deduction) system.

Then, switching to bounded arithmetic setting, we ask whether existence of expander graphs, combinatorial objects that are widely used in derandomization, can be proven without algebraic reasoning. Surprisingly, we show that this is indeed the case: it is possible to prove existence of expanders by purely combinatorial and probabilistic reasoning, using only concepts definable by polynomial-size formulas. An interesting corollary of this result is that monotone Frege reasoning is just as powerful as its nonmonotone counterpart, in stark contrast to circuit complexity.

Based on joint work with Vijay Ganesh and Robert Robere, and Buss, Koucky and Kabanets.

- ▶ ARISTOTELIS PANAGIOTOPOULOS, *Games orbits play*.  
Mathematics Department, California Institute of Technology, 253-37, Pasadena, CA 91125, USA.  
*E-mail:* panagio@caltech.edu.

Classification problems occur in all areas of mathematics. Descriptive set theory provides methods to assign complexity to such problems. Using a technique developed by Hjorth, Kechris, and Sofronidis proved for example, that the problem of classifying all unitary operators  $\mathcal{U}(\mathcal{H})$  of an infinite dimensional Hilbert space up to unitary equivalence  $\simeq_U$  is strictly more difficult than classifying graph structures with domain  $\mathbb{N}$  up to isomorphism. We present a game-theoretic approach to anticlassification results for orbit equivalence relations and use this development to reorganize conceptually the proof of Hjorth’s turbulence theorem. We also introduce a dynamical criterion for showing that an orbit equivalence relation is not Borel reducible to the orbit equivalence relation induced by a CLI group action: that is, a group which admits a complete left invariant metric (recall that, by a result of Hjorth and Solecki, solvable groups are CLI). We deduce that  $\simeq_U$  is not classifiable by CLI group actions.

This is a joint work with Martino Lupini.

- ▶ EMILY RIEHL AND MICHAEL SHULMAN, *A synthetic theory of  $\infty$ -categories in homotopy type theory*.  
Department of Mathematics, Johns Hopkins University, 3400 N. Charles Street, Baltimore, MD 21218, USA.

*E-mail:* eriehl@math.jhu.edu.

*URL Address:* <http://www.math.jhu.edu/~eriehl>.

Department of Mathematics, University of San Diego, 5998 Alcalá Park San Diego, CA 92110, USA.

*E-mail:* shulman@sandiego.edu.

*URL Address:* <http://home.sandiego.edu/~shulman/>.

Homotopy type theory is a new field of mathematics that grew out of recently-discovered connections between type theory, homotopy theory, and higher category theory. Voevodsky's discovery of a model for intentional type theory in a category whose objects represent *homotopy types* or  $\infty$ -*groupoids* rather than sets led him to introduce a new axiom asserting that “identity is equivalent to equivalence.” A profound consequence is that all constructions within homotopy type theory are automatically homotopy invariant. For this reason and taking account of the increasing variety of semantic models, homotopy type theory is developing into a viable “implicit foundation” for the unformalized mathematics done by mathematicians working with objects in settings that have intrinsic homotopical content.

In this talk, we propose foundations for a synthetic theory of  $\infty$ -*categories* in homotopy type theory [1], using the common nickname for the weak infinite-dimensional categories more properly referred to as “ $(\infty, 1)$ -categories.” Our work is motivated by a particular model of homotopy type theory [5], which contains a well-known model of  $\infty$ -categories [4] whose category theory can be developed synthetically [2]. We introduce simplices and cofibrations into homotopy type theory to probe the internal categorical structure of types, and define *Segal types*, in which binary composites exist uniquely up to homotopy, and *Rezk types*, in which the categorical isomorphisms are equivalent to the type-theoretic identities—a “local univalence” condition. We then demonstrate that these simple definitions suffice to develop the synthetic theory of  $\infty$ -categories. So far this includes functors, natural transformations, co- and contravariant type families with discrete ( $\infty$ -groupoid) fibers, a “dependent” Yoneda lemma that looks like “directed identity-elimination,” and the theory of coherent adjunctions closely resembling [3].

[1] E. RIEHL and M. SHULMAN, *A type theory for synthetic  $\infty$ -categories*, 2017, pp. 1–75, [arXiv:1705.07442](https://arxiv.org/abs/1705.07442).

[2] E. RIEHL and D. VERITY,  *$\infty$ -category theory from scratch*, 2015, pp. 1–53, [arXiv:1608.05314](https://arxiv.org/abs/1608.05314).

[3] ———, *Homotopy coherent adjunctions and the formal theory of monads*. *Advances in Mathematics*, vol. 286 (2016), no. 2, pp. 802–888.

[4] C. REZK, *A model for the homotopy theory of homotopy theory*. *Transactions of the American Mathematical Society*, vol. 353 (2001), no. 3, pp. 973–1007.

[5] M. SHULMAN, *The univalence axiom for elegant Reedy presheaves*. *Homology, Homotopy, and Applications*, vol. 17 (2015), no. 2, pp. 81–106.

- ▶ SIMON THOMAS, *The isomorphism and bi-embeddability relations for countable torsion abelian groups*.

Mathematics Department, Rutgers University, 110 Frelinghuysen Road, Piscataway, NJ 08854, USA.

*E-mail:* [simon.rhys.thomas@gmail.com](mailto:simon.rhys.thomas@gmail.com).

In this talk, I will discuss the isomorphism  $\cong_{TA}$  and bi-embeddability  $\equiv_{TA}$  relations on the space of countable torsion abelian groups. As I will explain, the bi-embeddability relation has a strictly simpler complete invariant than the isomorphism relation. Thus it is somewhat counterintuitive that  $\cong_{TA}$  and  $\equiv_{TA}$  turn out to be incomparable with respect to Borel reducibility. However, under a relatively mild large cardinal assumption, we obtain the intuitively correct result if we replace Borel reducibility by  $\Delta_2^1$  reducibility.

This is joint work with Filippo Calderoni.

- ▶ SEBASTIEN VASEY, *Nonelementary classification theory*.

Department of Mathematics, Harvard University, Cambridge, MA 02138, USA.

*E-mail:* sebv@math.harvard.edu.

*URL Address:* <http://math.harvard.edu/~sebv/>.

The classification theory of elementary classes was started by Michael Morley in the early sixties, when he proved that a countable  $\mathbb{L}_{\omega,\omega}$  theory with a single model in *some* uncountable cardinal has a single model in *all* uncountable cardinals. The proof of this result, now called Morley's categoricity theorem, led to the development of forking, a joint generalization of linear independence in vector spaces and algebraic independence in fields, which is now a central pillar of modern model theory.

Lately, it has become apparent that forking also exists in several nonelementary contexts. Prime among those is the axiomatic framework of abstract elementary classes (AECs), encompassing the class of models of any  $\mathbb{L}_{\infty,\omega}$ -theory and closely connected to the more general accessible categories. A test question to judge progress in this direction is the forty year old eventual categoricity conjecture of Shelah, which says that a version of Morley's categoricity theorem should hold of any AEC. I will survey recent developments, including the connections with category theory and large cardinals, as well as my resolution of the eventual categoricity conjecture for classes of models axiomatized by a *universal*  $\mathbb{L}_{\infty,\omega}$ -theory.

► KEITA YOKOYAMA, *Ramsey's theorem in arithmetic.*

School of Information Science, Japan Advanced Institute of Science and Technology, 1-1 Asahidai, Nomi, Ishikawa 923-1292, Japan.

*E-mail:* y-keita@jaist.ac.jp.

Calibrating the strength of various combinatorial principles is one of the important topics in the study of reverse mathematics. Especially, deciding the strength of Ramsey's theorem for pairs is hard and it is precisely studied from the viewpoints of both of computability theory and proof theory (see, e.g., [3]). In this talk, I will focus on the first-order consequences of infinite Ramsey's theorem and overview the recent developments.

To decide the first-order part of a second-order theory, a standard approach is proving  $\Pi_1^1$ -conservation over some induction or bounding axiom by showing  $\omega$ -extension property. In [1], Cholak/Jockusch/Slaman showed  $\text{WKL}_0 + \text{RT}_2^2 + \text{I}\Sigma_2^0$  is a  $\Pi_1^1$ -conservative extension of  $\text{I}\Sigma_2^0$  and  $\text{WKL}_0 + \text{RT}^2 + \text{I}\Sigma_3^0$  is a  $\Pi_1^1$ -conservative extension of  $\text{I}\Sigma_3^0$ , and they posed whether they are  $\Pi_1^1$ -conservative over  $\text{B}\Sigma_2^0$  and  $\text{B}\Sigma_3^0$ , respectively. For  $\text{RT}^2$ , the answer is yes, which is shown by sharpening the argument in [1] (see [4]). For  $\text{RT}_2^2$ , the question is more difficult, but it is now known that  $\text{WKL}_0 + \text{RT}_2^2$  is actually  $\Pi_3^0$ -conservative over  $\text{B}\Sigma_2^0$  (see [2]). The key idea for this proof is the indicator argument originally introduced by Paris and Kirby in 1970's. Here, we will characterize the first-order part of  $\text{WKL}_0 + \text{RT}_2^2$  by generalizing the indicator argument used in [2] with an idea of forcing.

[1] P. A. CHOLAK, C. G. JOCKUSCH, and T. A. SLAMAN, *On the strength of Ramsey's theorem for pairs.* *The Journal of Symbolic Logic*, vol. 66 (2001), no. 1, pp. 1–55.

[2] C.-T. CHONG, T. A. SLAMAN, and Y. YANG,  *$\Pi_1^1$ -conservation of combinatorial principles weaker than Ramsey's Theorem for pairs.* *Advances in Mathematics*, vol. 230 (2012), pp. 1060–1077.

[3] D. HIRSCHFELDT, *Slicing the Truth*, Lecture Notes Series, Institute for Mathematical Sciences, National University of Singapore, 2014.

[4] L. PATEY and K. YOKOYAMA, *The proof-theoretic strength of Ramsey's theorem for pairs and two colors*, submitted.

[5] T. A. SLAMAN and K. YOKOYAMA, *The strength of Ramsey's theorem for pairs and arbitrary many colors*, submitted.

### Abstracts of contributed talks

► ATHAR ABDUL-QUADER, *Classifying Enayat models of Peano Arithmetic.*

Bronx Community College, 2155 University Avenue, Bronx, NY 10453, USA.

*E-mail:* aabdulquader@gradcenter.cuny.edu.

Simpson, in [2], used arithmetic forcing to show that every countable model  $\mathcal{M} \models \text{PA}$  has an undefinable, inductive subset  $X \subseteq M$  such that the expansion  $(\mathcal{M}, X)$  is pointwise

definable. Enayat later showed, in [1], that there are many models with the property that every expansion upon adding a predicate for an undefinable class is pointwise definable. We refer to models with this property as Enayat models. That is, a model  $\mathcal{M} \models \text{PA}$  is Enayat if for each undefinable class  $X \subseteq M$ , the expansion  $(\mathcal{M}, X)$  is pointwise definable. In this talk we show that a model is Enayat if it is countable, has no proper cofinal submodels and is a conservative extension of each of its elementary cuts.

[1] A. ENAYAT, *Undefinable classes and definable elements in models of set theory and arithmetic. Proceedings of the American Mathematical Society*, vol. 103 (1988), no. 4, pp. 1216–1220.

[2] S. G. SIMPSON, *Forcing and models of arithmetic. Proceedings of the American Mathematical Society*, vol. 43 (1974), no. 1, pp. 193–194

- ▶ FRANCIS ADAMS, *The loose number of graphs on topological spaces*. Mathematics and Statistics, Georgia State University, 25 Park Place, 14th Floor, Atlanta, GA 30303, USA.

*E-mail:* fadams@gsu.edu.

Given a graph  $G$  on a topological space  $X$ , we define the loose number of  $G$ ,  $\lambda(G)$ . This invariant depends on the graph-theoretic properties of  $G$  as well as the topology on the vertex set  $X$ . When  $X$  is separable metric,  $\lambda(G)$  lies between two well-known graph invariants, the chromatic number and the coloring number. Evaluating this cardinal leads to interesting connections with forcing, infinitary combinatorics, descriptive set theory, and topology. We discuss these connections and provide many examples.

Much of this work is joint with Jindrich Zapletal.

- ▶ RACHEL EPSTEIN AND KAREN LANGE, *Computable reducibility and equality on a given set*.

Department of Mathematics, Georgia College, Milledgeville, GA 31061, USA.

*E-mail:* rachel.epstein@gcsu.edu.

Department of Mathematics, Wellesley College, 106 Central St., Wellesley, MA 02481, USA.

*E-mail:* klange2@wellesley.edu.

An equivalence relation  $E$  on the set of all computably enumerable (c.e.) sets is *computably reducible* to an equivalence relation  $F$  on the c.e. sets, written  $E \leq F$ , if there is a computable function  $f$  such that  $W_n E W_m$  if and only if  $W_{f(n)} F W_{f(m)}$ . Coskey, Hamkins, and R. Miller have explored the hierarchy of equivalence relations on the c.e. sets. Here we look at a natural class of equivalence relations and fit them into the hierarchy. The equivalence relation  $E_A$  on the c.e. sets is given by  $W_n E_A W_m$  if and only if  $W_n \cap A = W_m \cap A$ . If  $A$  is c.e., then it is not hard to show that  $E_A$  is computably bireducible to the equality equivalence relation on the class of c.e. sets, which we call  $=^{ce}$ . If  $A$  is co-c.e., then  $E_A \leq =^{ce}$  and the reduction is strict if and only if  $A$  is hyper-hyper-immune. We also construct sets  $A$  and  $B$  such that  $E_A$  and  $E_B$  are incomparable under computable reducibility.

- ▶ SHAY LOGAN, *Constant domain semantics for contractionless first-order relevance logics*. Department of Philosophy and Religious Studies, North Carolina State University, Raleigh NC 27607, USA.

*E-mail:* salogan@ncsu.edu.

Kit Fine showed (see [1]) that quantified relevance logics are incomplete for the naïve constant-domain semantics. So in order to give a semantics for which quantified relevance logics are complete, he resorted in [2] to a varying-domain semantic theory known as *stratified semantics*.

In this talk I give a *constant-domain* stratified semantics for contractionless first-order relevance logics. Roughly, I do so by blending together Fine’s semantics and the four-valued semantics for contractionless propositional relevance logics given in [3]. In the resulting semantic theory, the domain of a model comes in two pieces:  $D$  and  $\Omega$ .  $D$  contains ‘ordinary’ objects—those that can be named by individual constants.  $\Omega$  contains countably many objects that are ‘arbitrary’ in the following two senses:

- First, at any level  $X$  of the stratification, almost every  $\omega \in \Omega$  is featureless in all the  $X$ -setups.
- Second, if  $\omega \in \Omega$  is featureless at level  $X$ , there is another level  $Y$  where, for any  $d \in D \cup \Omega$  that *isn't* featureless at  $X$ ,  $\omega$  is extensionally indistinguishable from  $d$  throughout some fragment of the level- $Y$  model.

The semantics then says  $\forall x\phi(x)$  is true in a setup  $s$  at a level  $X$  just when there is an arbitrary object  $\omega$  and a level  $Y$  such that  $\phi(\omega)$  is true in all situations at level  $Y$  that are extensions of  $s$ .

[1] K. FINE, *Incompleteness for quantified relevance logics*, *Directions in Relevant Logic* (J. Norman and R. Sylvan, editors), Springer, Netherlands, 1989, pp. 205–225.

[2] ———, *Semantics for quantified relevance logic*, *Journal of Philosophical Logic*, vol. 17 (1988), no. 1, pp. 27–59.

[3] G. RESTALL, *Four-valued semantics for relevant logics (and some of their rivals)*, *Journal of Philosophical Logic*, vol. 24 (1995), no. 2, pp. 139–160.

- ▶ MICHAEL MCGRADY, *The molecular structure of mathematical proof*. Mica Scientific, Ltd., 223 Taylor Avenue South, North Bend, WA 98045, USA.  
E-mail: michael.mcgrady@gmail.com.

Finite classes are decidable. All mathematical logic formulas are members of finite classes ordered by their number of dyadic predicates. There is a mathematical *proof procedure* in the first-order logic that allows for a semi-automated, partly heuristic *discovery procedure* for the *decision procedure* for *finite sets* of formulas that are defined by the number  $i$  of dyadic predicates  $P_{\alpha\beta}^1 \dots P_{\alpha\beta}^i$  in a reduction class for validity. Hence, this proof procedure, based on Herbrand's fundamental theorem of logic, is itself at least a heuristic solution to *an alternative version of the Entscheidungsproblem (decision problem) for finite sets*. The keystone of the proof procedure is that the dyadic predicate atomic formulas  $P_{\alpha\beta}^1 \dots P_{\alpha\beta}^i$  have a ubiquitous *molecular structure*  $\Sigma = (P_{m-1,m}^1 \vee P_{m-2,m}^1 \vee \dots \vee P_{0,m}^1) \vee \dots \vee (P_{m-1,m}^i \vee P_{m-2,m}^i \vee \dots \vee P_{0,m}^i)$  such that  $m, i \in \mathbb{Z}^+$  underlying first-order logic proofs seen through the lens of a closed, prenex normal form, highly restricted reduction class for validity  $\exists x \exists y M_{xx'y}$  where  $x' = x + 1$ , and which allows the discovery procedure to find the decision procedure. This presentation demonstrates such a discovery procedure with a proof of a decision procedure.

- ▶ JOACHIM MUELLER-THEYS, *The total unprovability of unprovability*. Kurpfalzstr. 53, 69226 Nußloch bei Heidelberg, Germany.  
E-mail: mueller-theys@gmx.de.

I. Let  $\Sigma \vdash Q$  (minimal arithmetic), and  $\Sigma \vdash \sigma$  imply  $\Sigma \vdash \iota(\ulcorner \sigma \urcorner)$  (1),  $\Sigma \vdash \iota(\ulcorner \sigma \urcorner \rightarrow \ulcorner \tau \urcorner) \rightarrow (\ulcorner \sigma \urcorner \rightarrow \ulcorner \tau \urcorner)$  (2),  $\Sigma \vdash \iota(\ulcorner \sigma \urcorner) \rightarrow \iota(\ulcorner \iota(\ulcorner \sigma \urcorner) \urcorner)$  (3).  $\kappa_i^\sigma := \neg \iota(\ulcorner \sigma \urcorner)$ . If  $\Sigma$  is consistent,  $\Sigma \not\vdash \kappa_i^{\sigma_0}$ , like  $\sigma_0 = \perp$ ,  $\theta \doteq I$ .

UNIVERSALISATION LEMMA.  $\Sigma \not\vdash \neg \iota(\ulcorner \perp \urcorner)$  implies  $\Sigma \not\vdash \neg \iota(\ulcorner \sigma \urcorner)$ .

Proof. Since  $\Sigma \vdash \perp \rightarrow \sigma$ , by (1),  $\Sigma \vdash \iota(\ulcorner \perp \urcorner \rightarrow \ulcorner \sigma \urcorner)$ , whereby, by (2),  $\Sigma \vdash \iota(\ulcorner \perp \urcorner) \rightarrow \iota(\ulcorner \sigma \urcorner)$ , whence  $\Sigma \vdash \neg \iota(\ulcorner \sigma \urcorner) \rightarrow \neg \iota(\ulcorner \perp \urcorner)$ . So  $\Sigma \vdash \neg \iota(\ulcorner \sigma \urcorner)$  implies  $\Sigma \vdash \neg \iota(\ulcorner \perp \urcorner)$ , whereby the claim.

THEOREM (unprovability of unprovability). *If  $\Sigma$  is consistent,  $\Sigma \not\vdash \neg \iota(\ulcorner \sigma \urcorner)$  for all  $\sigma$ .*

PROOF. By Löb's Theorem,  $\Sigma \not\vdash \neg \iota(\ulcorner \perp \urcorner)$ , whence, by the lemma, the claim.

II. Certain issues become trivial now, as the conclusion of the theorem is implied by any proposition.

COROLLARY 1 (total negative self-irrepresentability).  $\Sigma \not\vdash \sigma$  implies  $\text{non } \Sigma \vdash \neg \iota(\ulcorner \sigma \urcorner)$ .

COROLLARY 2. *If  $\kappa_i^\sigma$  states consistency in whatever sense &  $\Sigma$  is consistent,  $\Sigma \not\vdash \kappa_i^\sigma$ .*

III. Now let  $\iota(x)$  express  $\Sigma$ -provability in the standard model, viz.  $\mathcal{N} \models \iota(\ulcorner \sigma \urcorner)$  iff  $\Sigma \vdash \sigma$ . Then  $\mathcal{N} \models \kappa_i^\sigma$  implies  $\Sigma$  consistent, and  $\kappa_i^\sigma$  is a *consistency sentence* :iff  $\Sigma$  consistent implies  $\mathcal{N} \models \kappa_i^\sigma$ .

PROPOSITION. (a1) *If  $\Sigma \not\vdash \sigma$ ,  $\kappa_i^\sigma$  is a consistency sentence;*

(a2) *If  $\Sigma \vdash \neg \sigma$ ,  $\kappa_i^\sigma$  is a consistency sentence;*

(b) *If  $\Sigma$  is consistent and  $\kappa_i^\sigma$  is a consistency sentence,  $\Sigma \not\vdash \sigma$ .*

COROLLARY 3 (unprovability of all consistency sentences). *If  $\Sigma$  is consistent and  $\kappa_i^\sigma$  is a consistency sentence,  $\Sigma \not\vdash \kappa_i^\sigma$ .*

COROLLARY 4 (Gödel's second incompleteness theorem). *If  $\Sigma$  is consistent and  $\kappa_i^\sigma$  is a consistency sentence induced by  $\Sigma \vdash \neg\sigma$ ,  $\Sigma \not\vdash \kappa_i^\sigma$ .*

REMARK. If  $\Sigma \vdash \text{PA}$  is decidable, the results become true for  $\iota$  being the provability predicate  $\text{Prov}_\Sigma$  and  $\text{Con}_\Sigma^\sigma := \kappa_{\text{Prov}_\Sigma}^\sigma$ .

*Acknowledgment.* The central insight developed from the ASL Spring Meeting 2017 and was outlined at Logic Colloquium 2017: “On the Provability of Consistency” (complementary abstract/handout: The Unprovability of All Consistency Formulæ).

- ▶ ERMEK NURKHAIDAROV, *The automorphism group of a recursively saturated model of Peano Arithmetic.*

Department of Mathematics, Pennsylvania State University Mont Alto, 1 Campus Dr., State College, PA 17237, USA.

Department of Mathematics and Statistics, James Madison University, 60 Bluestone Drive, Harrisonburg, VA 22807, USA.

*E-mail:* esn1@psu.edu.

Let  $M$  to be a countable recursively saturated model of Peano Arithmetic. If  $M$  has an element which is bigger than the standard cut  $\omega$  but smaller than any nonstandard definable elements, we call such  $M$  *wide*. Pointwise stabilizers are the basic open subgroups of the automorphism group of  $M$ .

A countable recursively saturated model of Peano Arithmetic is characterized by two invariants: its first order theory and its standard system. We show that the automorphism group of a wide, countable recursively saturated models of Peano Arithmetic codes its standard system. From which we obtain:

THEOREM 1. *Suppose that  $M_1$  and  $M_2$  are wide, countable, recursively saturated models of Peano Arithmetic such that their automorphism groups are topologically isomorphic. Then  $\text{SSy}(M_1) = \text{SSy}(M_2)$ .*

This theorem is an improvement on a result from [1], who proved it for arithmetically saturated models.

[1] R. KOSSAK and J. H. SCHMERL, *The automorphism group of an arithmetically saturated model of Peano Arithmetic.* **Journal of the London Mathematical Society**, vol. 52 (1992), pp. 235–244.

[2] E. NURKHAIDAROV, *Automorphism groups of arithmetically saturated models.* **The Journal of Symbolic Logic**, vol. 71 (2006), pp. 203–216.

[3] ———, *Decoding in the automorphism group of a recursively saturated model of arithmetic.* **Mathematical Logic Quarterly**, vol. 61 (2015), pp. 179–188.

- ▶ JAKE PARDO, *Reverse mathematics of hypergraph colorings.*

Department of Mathematics, Walker Hall, Appalachian State University, Boone, NC 28607, USA.

*E-mail:* pardojj@appstate.edu.

Reverse mathematics essentially seeks to break results from all areas of mathematics down into their most basic axioms. Graph theory has proven to be a topic replete with fascinating reverse mathematical results, and so it only makes sense to ask similar questions about what happens when we extend our scope to include hypergraphs. I will discuss a bit of what is currently known about the reverse mathematics of hypergraph colorings as well as explain some recent joint work with Davis, Hirst, and Ransom.

[1] C. BERGE, **Hypergraphs: Combinatorics of Finite Sets**, North Holland, 1989.

[2] J. HIRST, *Reverse mathematics and rank functions for directed graphs.* **Archive for Mathematical Logic**, vol. 39 (2000), pp. 569–579.

### Abstracts of papers submitted by title

► JOHN CORCORAN, *Truth-value making relations*.

Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.

E-mail: corcoran@buffalo.edu.

We study several relations expressed by the two-place relational verb-phrases ‘X makes Y true’ and ‘X makes Y false’, or synonyms such as ‘X verifies Y’ and ‘X falsifies Y’ [3, pp. 180, 283]. This abstract gives three examples. Notation and terminology follow [2].

Some usages mislead because the object “made true” or “made false” already had the truth-value in question. Other usages mislead because the object “made true” or “made false” doesn’t—and by nature can’t—have a truth-value. The object “made true” or “made false” isn’t true or false. Aristotle discussed related confusions in *Categories* xii, 14b16.

In one usage, the number two makes the proposition “some number is even” true; three makes “every number is even” false. These have been called, respectively, proexample and counterexample relations [1]. † Two is a proexample for “some number is even”; three is a counterexample for “every number is even”. Here we have propositions which by nature have truth-values; those “made true (or false)” didn’t *come to have* that truth-value.

In another usage, the intended interpretation of number-theoretic English makes the sentence ‘some number is even’ true and it makes ‘every number is even’ false. Here we have uninterpreted English sentences which by nature cannot have truth-values: they are *true (false) under* the interpretations that “make them true (false)” [3, passim].

In still another usage, the fact that some number is even makes the proposition “some number is even” true; the fact that not every number is even makes “every number is even” false. Compare Aristotle, *ibid*.

[1] J. CORCORAN, *Counterexamples and proexamples*, this BULLETIN, vol. 11 (2005), p. 460.

[2] ———, *Sentence, proposition, judgment, statement, and fact*, *Many Sides of Logic*, (W. Carnielli et al., editors), College Publications, 2009.

[3] W. GOLDFARB, *Deductive Logic*, Hackett, 2003.

► JOHN CORCORAN AND JOHN GLANVILLE, *The Aristotelian dictum de omni?*

Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.

E-mail: corcoran@buffalo.edu.

Much has been written about “the” so-called *dictum de omni* DDO. Many different propositions have been identified as the *dictum de omni*.

Some forms of DDO concern both universal and individual attribution, e.g., *every term belonging universally to a given term belongs individually to each thing coming under the given term*. Some logicians find resonance between certain forms of *universal-individual* DDOs and the rule of universal instantiation in first-order logic.

Other forms concern three universal attributions, e.g., *every term belonging universally to a given term belonging universally to a third term belongs universally to the third term*. Some logicians find resonance between certain forms of *three-universal* DDOs and either the derived rule in first-order logic called *sylogism* or the law of inclusion-transitivity in the theory of classes. Besides universal-individual and three-universal forms of DDO, there are several others.

Ironically, despite a perplexing lack of agreement as to what DDO is there is near unanimity as to the exact passage in Aristotle where it is supposedly found: **Prior Analytics** 24b26-30.

For one thing to be in another as a whole and for the other to be predicated of all of the one is the same thing. We say that it is predicated of every when you can’t take anything of which the other will not be said.—Translation Barnes [1, p. 387].

There is also wide agreement not only that Aristotle took DDO to “justify” or “ground” the “validity” or “self-evidence” of † a first-figure syllogism he took evident but also that Aristotle was right to have done so.

This lecture argues that no recognized form of DDO found or implied by 24b26-30, that Aristotle cited no such proposition in support of claims he made about his deductive system, and that no such proposition could ever be coherently enlisted in such a role.



[1] J. BARNES, *Truth, etc.: Six Lectures on Ancient Logic*. Oxford University Press, Oxford, 2007.

- ▶ JOHN CORCORAN AND JOSÉ MIGUEL SAGÜILLO, *Teaching paradoxes and orthodoxes*.

Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.

*E-mail:* corcoran@buffalo.edu.

Logic, University of Santiago de Compostela, Santiago 15782, Spain.

*E-mail:* josemiguel.saguillo@usc.es.

Following [1, pp. 21f], an argumentation *is a paradox* to persons who (1) believe its reasoning shows its conclusion follows from its premises, (2) believe its premises, and (3) disbelieve its conclusion. Obviously, for a given argumentation to be a paradox to a given person that person must have a mistaken belief. A *paradox* is an argumentation that is a paradox to someone. Paradoxes are participant-relative: only a participant's beliefs can make an argumentation a paradox. Mathematical writing often presumes as participant a community of mathematicians [2].

Ideally, a paradox is *resolved* by a participant through *either* discovering a logical fallacy in the argumentation's reasoning *or* discovering the conclusion isn't false *or* discovering a premise isn't true.

Resolving is thus also participant-relative: only a change in a participant's beliefs can resolve that participant's paradox.

Building on this BULLETIN, vol. 23 (2017), pp. 261–262, we teach this concept of paradox in juxtaposition with the new “opposite” concept of *orthodox*.

An argumentation *is an orthodox* to persons who (1) believe its reasoning shows its conclusion follows from its premises, (2) believe its premises, and (3) believe its conclusion. An *orthodox* is an orthodox to someone.

Many orthodoxes are demonstrations. Indeed, every argumentation believed to be a demonstration is an orthodox to every person with that belief.

In some cases a participant resolves a paradox by changing the disbelief of the conclusion into a belief, thus the very same argumentation that had been a paradox for a participant becomes an orthodox for that same participant. Conversely, a participant *dissolves* an orthodox by changing the belief of the conclusion into a disbelief, thus the very same argumentation that had been an orthodox becomes a paradox.

[1] J. CORCORAN, *Argumentations and logic*. *Argumentation*, vol. 3 (1989), pp. 17–43.

[2] A. GARCADIAGO, *The emergence nonlogical paradoxes of the theory of sets, 1903–1908*. *Historia Mathematica*, vol. 12 (1985), pp. 337–351.