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# SIXTEENTH ASIAN LOGIC CONFERENCE

## AN OFFICIAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

**Nazarbayev University**
**Nur-Sultan, Kazakhstan**
**June 17–21, 2019**

The 16th Asian Logic Conference was held on June 17–21, 2019, at the Nazarbayev University, Nur-Sultan, Kazakhstan. It was the second Asian Logic Conference since its status changed from an ASL-sponsored meeting to an official ASL meeting by ASL Council action in May 2016. The Asian Logic Conference (ALC) is a major international event in mathematical logic. It features the latest scientific developments in the fields in mathematical logic and its applications, logic in computer science, and philosophical logic. The ALC series also aims to promote mathematical logic in the Asia-Pacific region and to bring logicians together both from within Asia and elsewhere to exchange information and ideas.

Funding was provided by Nazarbayev University and the Association for Symbolic Logic. The ALC 2019 conference was co-located with the 14th International Conference on Computability, Complexity, and Randomness (CCR 2019).

The Program Committee consisted of Bektur Baizhanov, Jörg Brendle, Sergey Goncharov (chair), Koichiro Ikeda, Bakhadyr Khoussainov, Andrey Morozov, Hiroakira Ono, Wei Wang, Guohua Wu, Yue Yang, and Liang Yu.

The Local Organizing Committee consisted of Timur Bakibayev, Tussupov Jamalbek, Zhibek Kadyrsizova, Manat Mustafa (Chair), Francesco Sica, Vassilios D. Tourassis (co-chair), Raushan Zhumanova, and Arman Zhumazhanov.

There were eight plenary speakers:

Serikzhan Badaev (Al-Farabi Kazakh National University, Kazakhstan), *Structure of c-degrees of positive equivalences*.

Nikolay Bazhenov (Sobolev Institute of Mathematics, Russia), *Rogers semilattices and their first-order theories*.

Qi Feng (Chinese Academy of Sciences, China), *On a first order theory approximating theoretic economics*.

Sakae Fuchino (Kobe University, Japan), *Strong Löwenheim–Skolem Theorem of stationary logics, game reflection principles, and generically supercompact cardinals*.

Takayuki Kihara (Nagoya University, Japan), *Computability-theoretic methods in descriptive set theory*.

Byunghan Kim (Yonsei University, Korea), *A report on NSOP1 theories*.

Beibut Kulpeshov (IIT University, Kazakhstan), *The countable spectrum of weakly o-minimal theories of finite convexity rank*.

Dietrich Kuske (Technische Universität Ilmenau, Germany), *Automatic structures*: *The complexity of some natural decision problems*.

Dugald Macpherson (University of Leeds, UK), *Definable sets in finite structures*.

Ahti-Veikko Pietarinen (Nazarbayev University, Kazakhstan), *A graphical method for non-classical logics*.


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Zhiwei Sun (Nanjing University, China), *Further results on Hilbert's tenth problem*.
Toshimichi Usuba (Waseda University, Japan), *Choiceless set-theoretic geology*.

Special sessions on the following topics were held (speakers in parentheses): Computability theory (Andrey Frolov, Nurlan Kogabaev, and Rod Downey), Model theory (Sergey Sudoplatov and Viktor Verbovskiy), Logic in Computer Science (Marcin Jurdzinski and Toru Takisaka), Philosophical logic (Katsuhiko Sano and Jiji Zhang), and Set theory (Jin Du, JiaLiang He, and Daisuke Ikegami).

The program also included 45 contributed talks (20 minutes each). There were over 90 attendees including 22 students. Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Organizing Committee
MANAT MUSTAFA

## Abstracts for Plenary Talks

► SERIKZHAN BADAEV, *Structure of $c$-degrees of positive equivalences*.
Department of fundamental mathematics, Al-Farabi Kazakh National University, 71, al-Farabi ave, Almaty 050040, Kazakhstan.
*E-mail*: `Serikzhan.Badaev@kaznu.kz`.

Positive preorders are reflexive and transitive computably enumerable relations on $\omega$. We say that a positive preorder $P$ is computably reducible to a positive preorder $Q$ (symbolically, $P \leqslant_c Q$) if there is a computable function $f$ so that, for every $x, y \in \omega$, $xPy$ if and only if $f(x)Qf(y)$. As usual, we define the $c$-degree of a positive preorder $P$ to be a set $\{Q\colon Q \leqslant_c P \,\&\, P \leqslant_c Q\}$ and consider a partial ordered structure of $c$-degrees induced by $\leqslant_c$.

We will talk on algebraic properties of this structure, definable subsets, types of computable isomorphisms inside a $c$-degree, and other related problems. Besides, we will concern equivalences in the Ershov hierarchy relative to computable reducibility. A survey [1] could be useful as a source of necessary notions and basic facts.

[1] U. ANDREWS, S. BADAEV, and A. SORBI, *A survey on universal computably enumerable equivalence relations*, ***Computability and Complexity*** (A. Day, M. Fellows, et al., editors), Springer, Cham, 2016, pp. 418–451.

► NIKOLAY BAZHENOV, *Rogers semilattices and their first-order theories*.
Sobolev Institute of Mathematics, 4 Acad. Koptyug Ave., Novosibirsk 630090, Russia.
*E-mail*: `bazhenov@math.nsc.ru`.
*URL Address*: `bazhenov.droppages.com`.

A numbering $\nu$ is reducible to a numbering $\mu$ if there is a total computable function $f(x)$ such that $\nu(n) = \mu(f(n))$ for all $n$. The notion of reducibility between numberings gives rise to a class of upper semilattices, which are usually called Rogers semilattices. Goncharov and Sorbi (1997) initiated the systematic study of Rogers semilattices for numberings in various recursion-theoretic hierarchies. We give a review of recent results on Rogers semilattices in hyperarithmetical and analytical hierarchies. Special attention is given to the complexity of first-order theories for Rogers semilattices.

The talk is based on joint works with Mustafa, Ospichev, and Yamaleev.

► QI FENG, *On a first-order theory approximating theoretic economics*.
Chinese Academy of Sciences, China.
*E-mail*: `qifeng@math.ac.cn`.

We attempt to apply the axiomatic method of logic to understanding theoretic economics by establishing appropriate first-order language and abstracting appropriate axioms. The core of the program in such understanding is to provide an intrinsic analysis of value. We shall

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attempt to develop such analysis by isolating certain set of basic concepts and their relations. Interestingly it involves several branches of basic sciences where laws are well established and ready to apply; also axiomatic theory of sets is a part of the development. In this talk, I shall explain an outline of my analysis.

► SAKAÉ FUCHINO, *Strong Löwenheim–Skolem Theorem of stationary logics, game reflection principles and generically supercompact cardinals*.
Graduate School of System Informatics, Kobe University, Rokko-dai 1-1, Nada, Kobe 657-8501, Japan.
*E-mail*: fuchino@diamond.kobe-u.ac.jp.

We consider Strong Löwenheim–Skolem Theorems of variations of stationary logic with the stationarity quantifier over weak second-order variables which run (in case of the standard setting) over countable subsets of the underlying set of a given structure. We single out three natural principles in term of these Löwenheim–Skolem Theorems which imply respectively, that the size of the continuum is $\aleph_1$, $\aleph_2$ or very large (e.g., weakly hyper Mahlo).

Each of these three statements are consequences of three existential statements of generically supercompact cardinals and they are also related to reflection statements in terms of infinitary games and extended notions of generically supercompactness.

Accepting the naturalness of the statements involved, our results strongly suggest that the continuum should be either $\aleph_1$ or $\aleph_2$ or very large.

The resuls presented in this talk are obtained in a joint work with André Ottenbereit and Hiroshi Sakai.

[1] S. Fuchino, A. Ottenbereit, M. Rodriques, and H. Sakai, *Strong downward Löwenheim–Skolem theorems for stationary logics, I*, submitted for publication.

[2] ———, *Strong downward Löwenheim–Skolem theorems for stationary logics, II*, submitted for publication.

[3] ———, *Strong downward Löwenheim–Skolem theorems for stationary logics, III*, in preparation.

► TAKAYUKI KIHARA, *Computability-theoretic methods in descriptive set theory*.
Graduate School of Informatics, Nagoya University, 1 Furo-cho, Chikusa-ku, Nagoya 464-0814, Japan.
*E-mail*: kihara@i.nagoya-u.ac.jp.
*URL Address*: http://www.math.mi.i.nagoya-u.ac.jp/~kihara.

In this talk, we survey our recent work on applications of computability theory to descriptive set theory and related areas. Applications of our method range over the studies on decomposability of Borel functions [1, 2], Borel isomorphism problems related to infinite dimensional topology [4], the Wadge degree structures of BQO-valued Borel functions [3], and so on. We explain how computability-theoretic methods are involved in these studies.

[1] V. Gregoriades, T. Kihara, and K. M. Ng, *Turing degrees in Polish spaces and decomposability of Borel functions*, submitted, 2014, arXiv:1410.1052.

[2] T. Kihara, *Decomposing Borel functions using the Shore–Slaman join theorem*. ***Fundamenta Mathematicae***, vol. 230 (2015), pp. 1–13.

[3] T. Kihara and A. Montalbán, *On the structure of the Wadge degrees of BQO-valued Borel functions*. ***Transactions of the American Mathematical Society***, to appear.

[4] T. Kihara and A. Pauly, *Point degree spectra of represented spaces*, submitted, 2015, arXiv:1405.6866.

► BYUNGHAN KIM, *A report on NSOP$_1$ theories*.
Yonsei University, Korea.
*E-mail*: bkim@yonsei.ac.kr.

SOP$_1$ (i.e., 1-strong order property) is introduced by S. Shelah, and it implies the tree property. Hence any simple theory is NSOP$_1$ (i.e., not having SOP$_1$). The random parametrized equivalence relations, an infinite dimensional vector space with a bilinear map, and an unbounded PAC field are typical examples having nonsimple NSOP$_1$ theories. Recently I. Kaplan and N. Ramsey showed that in any NSOP$_1$ theory, *over models*, Kim-independence

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satisfies all the basic axioms that nonforking satisfies in simple theories (such as symmetry, transitivity, local character, extension, and type-amalgamation), except base monotonicity. (In simple theories, Kim-independence and nonforking independence coincide.)

Now we show that the same holds *over any set* under nonforking existence. Namely, in any $\mathrm{NSOP}_1$ theory with nonforking existence, over any set, Kim-independence satisfies all the mentioned basic axioms (except base monotonicity) including type-amalgamation of Lascar types. I will talk about other related topics/results as well.

*Acknowledgment*. This is a joint work with J. Dobrowolski and N. Ramsey.

▸ B. SH. KULPESHOV, *The countable spectrum of weakly o-minimal theories of finite convexity rank*.
International Information Technology University, Almaty, Kazakhstan.
*E-mail*: b.kulpeshov@iitu.kz.

Here we discuss the Vaught's problem for weakly o-minimal theories of finite convexity rank. Convexity rank has been introduced in [3]. In particular, a theory has convexity rank 1 if there is no parametrically definable equivalence relation with an infinite number of infinite convex classes. Obviously, any o-minimal theory has convexity rank 1.

As it is known, in [5] the Vaught's conjecture for o-minimal theories was solved. Recently in [4] the Vaught's conjecture for quite o-minimal theories was solved. From the above works it follows that these theories have the same spectrum, namely, such a theory has either continuum of countable models or exactly $6^a 3^b$ countable models for nonnegative integers $a$ and $b$.

In [1] B. S. Baizhanov and A. Alibek have constructed for every ordinal $\kappa$ with $4 \leq \kappa \leq \omega$ examples of weakly o-minimal theories having exactly $\kappa$ countable models. All these examples have convexity rank 1. Recently in [2] the Vaught's conjecture for weakly o-minimal theories of convexity rank 1 was solved. Their countable spectrum differs from the countable spectrum of o-minimal theories. The following theorem describes the countable spectrum of weakly o-minimal theories of finite convexity rank (which coincides with the countable spectrum of weakly o-minimal theories of convexity rank 1):

THEOREM 1. *Let $T$ be a weakly o-minimal theory of finite convexity rank in a countable language. Then exactly one of the following possibilities holds*:

(1) *$T$ is $\aleph_0$-categorical*
(2) *$T$ has $k$ countable models, where $3 \leq k < \omega$*
(3) *$T$ has $\omega$ countable models*
(4) *$T$ has $2^\omega$ countable models.*

[1] A. ALIBEK and B. S. BAIZHANOV, *Examples of countable models of a weakly o-minimal theory*. ***International Journal of Mathematics and Physics***, vol. 3 (2012), no. 2, pp. 1–8.

[2] A. ALIBEK, B. S. BAIZHANOV, B. SH. KULPESHOV, and T. S. ZAMBARNAYA, *Vaught's conjecture for weakly o-minimal theories of convexity rank 1*. ***Annals of Pure and Applied Logic***, vol. 169 (2018), no. 11, pp. 1190–1209.

[3] B. SH. KULPESHOV, *Weakly o-minimal structures and some of their properties*. ***The Journal of Symbolic Logic***, vol. 63 (1998), no. 4, pp. 1511–1528.

[4] B. SH. KULPESHOV and S. V. SUDOPLATOV, *Vaught's conjecture for quite o-minimal theories*. ***Annals of Pure and Applied Logic***, vol. 168 (2017), no. 1, pp. 129–149.

[5] L. L. MAYER, *Vaught's conjecture for o-minimal theories*. ***The Journal of Symbolic Logic***, vol. 53 (1988), no. 1, pp. 146–159.

▸ DIETRICH KUSKE, *Automatic structures: The complexity of some natural decision problems*.
Technische Universität Ilmenau, Germany.
*E-mail*: dietrich.kuske@tu-ilmenau.de.

Computable structures, while being a natural class of relational structures, suffer from the high complexity of all associated decision problems: typically, they are either arithmetical or even in low levels of the analytical hierarchy.

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To overcome this high complexity, we consider structures that are described by finite automata in much the same way that computable structures are described by Turing machines. A typical example of an automatic structure is formed by the additive group of integers (coded in binary). Over the past 20 years, many problems for such automatic structures have been shown decidable. This involves in particular the elementary theory of any automatic structure as well as theories of logics properly between first-order and second-order logics. The computational complexity of these theories spans a wide range of complexity classes.

Other problems, that are undecidable for recursive structures, either become simpler (measured in the arithmetical hierarchy) or preserve their full complexity when considered for automatic structures.

The talk gives an overview over the results, proof techniques, and some open problems in this spectrum.

► H. D. MACPHERSON, *Definable sets in finite structures*.
School of Mathematics, University of Leeds, Leeds LS2 9JT, UK.
*E-mail*: h.d.macpherson@leeds.ac.uk.

A theorem of Chatzidakis, van den Dries, and Macintyre from 1992 ensures that definable sets in finite fields satisfy a strong uniformity condition on their approximate cardinalities (sufficient, for example, to ensure that their ultraproducts have supersimple rank 1 theory. The conclusion of the theorem turned into the definition of an abstract model-theoretic framework (an "asymptotic class" of "finite structures") in work of Elwes, myself, and Steinhorn. I will discuss this and further more recent refinements (the notion of "multidimensional asymptotic class," work in progress with Anscombe, Steinhorn, and Wolf). I will also discuss infinitary analogues: for example, any ultraproduct of a "multidimensional asymptotic class" is "generalised measurable." The focus will be on the wide range of examples from combinatorics and algebra; for example, by work of Ryten, any family of finite simple groups of fixed Lie type forms an asymptotic class.

► AHTI-VEIKKO PIETARINEN, *A graphical method for non-classical logics*.
Nazarbayev University, Kazakhstan.
*E-mail*: ahtiveikko.pietarinen@nu.edu.kz.

The design of graphical languages for different logics dates back to the late 19th century but was subsequently forgotten. By graphical languages we mean those that use nonlinear, nontabular, and occurrence-referential notations. We can redesign such languages for a range of nonclassical logics that preserve the deep-inferential nature of their transformation rules. This talk presents such cases for graphical intuitionistic, bi-intuitionistic, and Brouwer logics.

► ZHI-WEI SUN, *Further results on Hilbert's Tenth Problem*.
Department of Mathematics, Nanjing University, Nanjing 210093, P. R. China.
*E-mail*: zwsun@nju.edu.cn.
*URL Address*: http://math.nju.edu.cn/~zwsun.

Hilbert's Tenth Problem (HTP) asks for an effective algorithm to test whether an arbitrary polynomial equation

$$P(x_1, \dots, x_n) = 0$$

(with integer coefficients) has solutions over the ring $\mathbb{Z}$ of the integers. This was finally solved by Matiyasevich in 1970 negatively.

In this talk we introduce the speaker's further results on HTP. We present a sketch of the proof of the speaker's main result that there is no effective algorithm to determine whether an arbitrary polynomial equation $P(x_1, \dots, x_{11}) = 0$ (with integer coefficients) in 11 unknowns has integral solutions or not. We will also mention some other results of the speaker, for example, there is no algorithm to test for any $P(z_1, \dots, z_{17}) \in \mathbb{Z}[z_1, \dots, z_{17}]$ whether $P(z_1^2, \dots, z_{17}^2) = 0$ has integral solutions, and also there is a polynomial $Q(z_1, \dots, z_{20}) \in$

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$\mathbb{Z}[z_1, \dots, z_{20}]$ such that

$$\{Q(z_1^2, \dots, z_{20}^2) \colon z_1, \dots, z_{20} \in \mathbb{Z}\} \cap \{0, 1, 2, \dots\}$$

coincides with the set of all primes.

▸ TOSHIMICHI USUBA, *Choiceless set-theoretic geology*.
Faculty of Science and Engineering, Waseda University, Tokyo 100-0003, Japan.
*E-mail*: usuba@waseda.jp.

The set-theoretic geology, which was initiated by Fuchs–Hamkins–Reitz, is a study of the structure of all ground models of the universe. The standard geology is assuming the axiom of choice, but in the current set theory, the forcing method over choiceless model become a common tool. In this talk, we try to develop the set-theoretic geology without the axiom of choice. We show that if the universe satisfies some downward Loewenheim–Skolem type property holds, then every ground model is uniformly definable, and such a property follows from large cardinal axioms.

## Abstracts for the Special Session on Computability Theory

▸ ROD DOWNEY, *A Hierarchy of Degrees*.
School of Mathematics, Statistics and Operations Research, Victoria University of Wellington, P.O. Box 600, Wellington, New Zealand.
*E-mail*: rod.downey@vuw.ac.nz.

Noam Greenberg and I have extensively developed a hierarchy of degrees which classify the combinatorics of many constructions. I will discuss this by looking at an example of presentations of halting probabilities.

*Acknowledgment*. This is joint work with Noam Greenberg.

▸ ANDREY FROLOV, *Computable vs low linear orders*.
Department of Mathematics, Kazan (Volga Region) Federal University, Kazan, Russian Federation.
*E-mail*: a.frolov.kpfu@gmail.com.

I will talk about order properties P which guarantees that if a low linear order is such that P(L) then L has a computable copy. I will give the complete review of the known positive and negative results. Also I will present new results and some of their applications.

▸ NURLAN KOGABAEV, *Computable dimension and projective planes*.
Sobolev Institute of Mathematics and Novosibirsk State University, Acad. Koptyug avenue 4, Novosibirsk 630090, Russia.
*E-mail*: kogabaev@math.nsc.ru.

In the present article we investigate the question of possible computable dimensions of countable structures in the following familiar classes of projective planes: free projective planes, freely generated projective planes, pappian projective planes, and desarguesian projective planes.

For free projective planes the following characterization of computable categoricity have been found.

THEOREM 1. *Every free projective plane has computable dimension either* 1 *or* $\omega$. *Futhermore, such a plane is computably categorical if and only if it has finite rank*.

It turns out that the results of Theorem 1 cannot be extended to the case of freely generated projective planes.

In [1] it was shown that the class of symmetric irreflexive graphs is *HKSS-complete* in the following computable-model-theoretic sense: for every countable structure $\mathcal{A}$, there exists a countable symmetric irreflexive graph $\mathcal{G}$ which has the same degree spectrum as $\mathcal{A}$, the same $\mathbf{d}$-computable dimension as $\mathcal{A}$ (for each degree $\mathbf{d}$), the same computable dimension as $\mathcal{A}$

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under expansion by a constant, and which realizes every degree spectrum $\mathrm{DgSp}_{\mathcal{A}}(R)$ (for every relation $R$ on $\mathcal{A}$) as the degree spectrum of some relation on $\mathcal{G}$.

We construct an effective coding of symmetric irreflexive graphs into freely generated projective planes preserving most computable-model-theoretic properties and obtain the following result.

THEOREM 2. *The class of freely generated projective planes is HKSS-complete. In particular, for every $n \in \omega \cup \{\omega\}$ there exists a freely generated projective plane of infinite rank with computable dimension $n$.*

In [2] it was proved that the class of fields is HKSS-complete. We use some natural coding of fields into pappian projective planes to obtain the following theorem.

THEOREM 3. *The class of pappian (desarguesian) projective planes is HKSS-complete. In particular, for every $n \in \omega \cup \{\omega\}$ there exists a pappian (desarguesian) projective plane with computable dimension $n$.*

We also calculate the complexity of the computable categoricity problem for familiar classes of projective planes.

THEOREM 4. *The computable categoricity problem for the class of free projective planes is $m$-complete $\Sigma^0_3$. For the classes of freely generated, pappian, desarguesian, and arbitrary projective planes, the computable categoricity problem is $m$-complete $\Pi^1_1$.*

*Acknowledgment*. This work was supported by RFBR (grant 17-01-00247) and by the Grants Council under RF President for State Aid of Leading Scientific Schools (grant NSh-5913.2018.1).

[1] D. R. HIRSCHFELDT, B. KHOUSSAINOV, R. A. SHORE, and A. M. SLINKO, *Degree spectra and computable dimensions in algebraic structures*. ***Annals of Pure and Applied Logic***, vol. 115 (2002), no. 1–3, pp. 71–113.

[2] R. MILLER, B. POONEN, H. SCHOUTENS, and A. SHLAPENTOKH, *A computable functor from graphs to fields*. ***The Journal of Symbolic Logic***, vol. 83 (2018), no. 1, pp. 326–348.

## Abstracts for the Special Session on Model Theory

► SERGEY SUDOPLATOV, *Classification of countable models of complete theories and its applications*.

Sobolev Institute of Mathematics, Novosibirsk, Russia; Novosibirsk State Technical University, Novosibirsk, Russia; Novosibirsk State University, Novosibirsk, Russia.
*E-mail*: sudoplat@math.nsc.ru.
*URL Address*: http://math.nsc.ru/~sudoplatov/.

We consider a survey of results on classification of countable models of complete theories [8] in terms of Rudin–Keisler preorders, distributions of limit and other countable models, and their applications to simple theories [1], quite *o*-minimal theories [2, 3], and disjoint unions of theories with few countable models [10].

We use this general approach studying combinations of structures [9], families of theories, their closures [6], *e*-spectra [7], ranks and related characteristics [4, 11], as well as dynamics under natural operations. We apply the general context to the class of theories of abelian groups [5].

*Acknowledgment*. This research was partially supported by Russian Foundation for Basic Researches (Project No. 17-01-00531-a) and Committee of Science in Education and Science Ministry of the Republic of Kazakhstan (Grant No. AP05132546).

[1] B. S. BAIZHANOV, S. V. SUDOPLATOV, and V. V. VERBOVSKIY, *Conditions for non-symmetric relations of semi-isolation*. ***Siberian Electronic Mathematical Reports***, vol. 9 (2012), pp. 161–184.

[2] B. Sh. Kulpeshov and S. V. Sudoplatov, *Vaught's conjecture for quite o-minimal theories*. ***Annals of Pure and Applied Logic***, vol. 168 (2017), no. 1, pp. 129–149.

[3] ———, *Distributions of countable models of quite o-minimal Ehrenfeucht theories*. ***Eurasian Mathematical Journal***, to appear, 2018, arXiv:1802.08078v1 [math.LO].

[4] N. D. Markhabatov and S. V. Sudoplatov, *On ranks for families of all theories of given languages*, ***International Conference "Mal'tsev Meeting", November 19–22, 2018, Collection of Abstracts***, Sobolev Institute of Mathematics, Novosibirsk State University, Novosibirsk, 2018, p. 213.

[5] I. I. Pavlyuk and S. V. Sudoplatov, *Families of theories of abelian groups and their closures*. ***Bulletin of the Karaganda University. Mathematics Series***, vol. 92 (2018), no. 4.

[6] S. V. Sudoplatov, *Closures and generating sets related to combinations of structures*. ***The Bulletin of Irkutsk State University. Series "Mathematics"***, vol. 16 (2016), pp. 131–144.

[7] ———, *Relative e-spectra and relative closures for families of theories*. ***Siberian Electronic Mathematical Reports***, vol. 14 (2017), pp. 296–307.

[8] ———, ***Classification of Countable Models of Complete Theories: Monograph in Two Parts***, NSTU, Novosibirsk, 2018.

[9] ———, *Combinations of structures*. ***The Bulletin of Irkutsk State University. Series "Mathematics"***, vol. 24 (2018), pp. 65–84.

[10] ———, *On distributions of countable models of disjoint unions of Ehrenfeucht theories*. ***Russian Mathematics***, vol. 62 (2018), no. 11, pp. 76–80.

[11] ———, *On ranks for families of theories and their spectra*, ***International Conference "Mal'tsev Meeting", November 19–22, 2018, Collection of Abstracts***, Sobolev Institute of Mathematics, Novosibirsk State University, Novosibirsk, 2018, p. 216.

▸ VIKTOR VERBOVSKIY, *On the sets and functions definable in ordered Abelian groups*.
Kazakh British Technical University, Kazakhstan.
*E-mail*: `viktor.verbovskiy@gmail.com`.

During the last decade Model theory of ordered Abelian groups is developed mainly in two classes of theories: dp-minimal and o-stable. In my talk I review some history of model theory of ordered Abelian groups and focus on subsets and functions which are definable in dp-minimal and o-stable ordered groups. In particular, for a dp-minimal ordered group $G$ with finitely many definable convex subgroups it holds that any definable subset is a Boolean combination of convex sets and cosets of $nG$, and any definable function is locally monotone (the joint result with J. Goodrick). Also I discuss properties of subsets and functions definable in o-stable ordered groups and give similarity and difference of properties of definable subsets in dp-minimal and o-stable ordered groups.

A theory is *not dp-minimal* if there is a model $M$ and formulas $\varphi(x; y)$, $\psi(x; z)$ with $|x| = 1$, and elements $a_{ij}, b_i, c_j$ such that for all $i, j, i', j'$,

$$i = i' \iff M \models \varphi(a_{ij}, b_{i'})$$
$$j = j' \iff M \models \psi(a_{ij}, c_{j'}).$$

Otherwise, it is said to be dp-minimal (S. Shelah, J. Goodrick, A. Onshuus, and A. Usvyatsov).

We say that $G$ contains *boundedly many definable convex subgroups* if there is a cardinal $\lambda$, such that in any group which is elementary equivalent to $G$ the number of convex definable subgroups does not exceed $\lambda$.

Theorem 1 (J. Goodrick and V. Verbovskiy). *Let $G$ be an ordered group whose elementary theory is dp-minimal, and let $G$ has boundedly many definable convex subgroups. Then any definable subset is a finite union of convex sets intersected with cosets of definable subgroups of the form $nG$, that is $G$ is weakly quasi-o-minimal.*

Theorem 2 (J. Goodrick and V. Verbovskiy). *Let $G$ be an ordered groups whose elementary theory is dp-minimal and let $G$ has boundedly many definable convex subgroups. Then for any definable unary function $f$ there is a finite partition $X_1, \dots, X_n$ of its domain, such that the restriction of $f$ to any of $X_i$ is locally monotone.*

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## Abstracts for the Special Session on Set Theory

► JIN DU, *The tree property and its strengthenings at small cardinals*.
University of Illinois at Chicago, USA.
*E-mail*: jdu8@uic.edu.

For $\kappa$ inaccessible, $\kappa$ is weakly compact iff it has the tree property, strongly compact iff it has the strong tree property, and supercompact iff it has the ineffable tree property. The tree property always holds at $\aleph_0$ and fails at $\aleph_1$; but beyond that, even the consistency of the tree property depends on large cardinals. A major open project is obtaining the tree property, or its strengthenings, at many small cardinals; this tests the power of forcing and large cardinals to build universes with a high level of compactness.

In this talk, I will discuss various results in this project, including a article of mine and a recent article due to Cummings, et al.

► JIALIANG HE AND BOAZ TSABAN, *Some covering properties in metrizable groups*.
Department of Mathematics, Sichuan University, Chengdu, Sichuan 610064, China.
*E-mail*: jialianghe@scu.edu.cn.
Department of Mathematics, Bar-Ilan University, Ramat-Gan 5290002, Israel.
*E-mail*: Boaz.Tsaban@biu.ac.il.

In this talk, we will introduce some background of selection principle mathematics and the connection of cardinal invariants with it. In particular, motivated by the open problem whether, consistently, every product of two Menger-bounded subgroups of the Baer–Specker group $\mathbb{Z}^{\omega}$ is Menger-bounded; equivalently, whether Menger-bounded and Scheepers-bounded subgroups of the Baer–Specker group are, consistently, the same. We consider a more general class of Menger-bounded and Scheepers-bounded sets in metrizable groups. We proved the productivity of Menger-bounded set is not hold in ZFC. The productivity of Scheepers-bounded set or group is equivalent to NCF, which is also equivalent to the union of each two Scheepers-bounded set is still Scheepers-bounded set. We also proved that the finite quotient of Scheepers-bounded subset of $[\omega]^{\omega}$ is Scheepers-bounded is equivalent to there is no rapid filter.

[1] M. Machura, S. Shelah, and B. Tsaban, *Squares of Menger-bounded groups*. ***Transactions of the American Mathematical Society***, vol. 362 (1998), no. 4, pp. 1751–1764.

[2] M. Machura and B. Tsaban, *The combinatorics of the Baer–Specker group*. ***Israel Journal of Mathematics***, vol. 168 (2008), no. 1, pp. 125–151.

[3] H. Mildenberger, *Cardinal characteristics for Menger-bounded subgroups*. ***Topology and its Applications***, vol. 156 (2008), no. 1, pp. 130–137.

[4] M. Tkačenko, *Introduction to topological groups*. ***Topology and its Applications***, vol. 86 (1998), no. 3, pp. 179–231.

► DAISUKE IKEGAMI, *On supercompactness of* $\omega_1$.
SIT Research Laboratories, Shibaura Institute of Technology, Japan.
*E-mail*: ikegami@shibaura-it.ac.jp.

In ZFC, all the large cardinals are much bigger than $\omega_1$, the least uncountable cardinal. In ZF without assuming the Axiom of Choice, $\omega_1$ could have large cardinal properties such as measurability and supercompactness by the results of Jech and Takeuti. In this talk, we discuss some consequences of supercompactness of $\omega_1$ on sets of reals in terms of regularity properties and determinacy.

*Acknowledgment*. This is joint work with Nam Trang.

## Abstracts for the Special Session on Logic in Computer Science

▸ MARCIN JURDZINSKI, *Universal trees and quasi-polynomial bounds for games and automata*.
Department of Computer Science, University of Warwick, UK.
*E-mail*: `Marcin.Jurdzinski@warwick.ac.uk`.

Universal trees are very basic combinatorial objects: an ordered tree is $(n, h)$-universal if every ordered tree of height at most $h$ and with at most $n$ leaves can be embedded into it. We give nearly-matching upper and lower bounds on the size of the smallest $(n, h)$-universal trees: if $h$ is asymptotically logarithmic in $n$ then it is polynomial in $n$, and if $h$ is asymptotically super-logarithmic in $n$ then it is quasipolynomial. We then discuss several applications of universal trees:

- to design algorithms for solving parity games [5] and mean-payoff parity games [3], which are quasi-polynomial and pseudo-quasi-polynomial, respectively;
- to show that separating automata (due to Bojańczyk and Czerwiński) and universal trees unify all the currently known quasi-polynomial algorithms for solving parity games [1, 5, 6];
- to prove a quasi-polynomial lower bound on the size of all strongly separating automata [2], hence pinpointing a quasi-polynomial barrier that most existing techniques for solving parity games efficiently are vulnerable to;
- to improve and streamline translations from alternating parity automata on words to alternating weak automata [4].

[1] C. S. Calude, S. Jain, B. Khoussainov, W. Li, and F. Stephan, *Deciding parity games in quasipolynomial time*, ***49th ACM Symposium on Theory of Computing***, 2017, pp. 252–263.

[2] W. Czerwiński, L. Daviaud, N. Fijalkow, M. Jurdziński, R. Lazić, and P. Parys, *Universal trees grow inside separating automata*: *Quasi-polynomial lower bounds for parity games*, ***30th ACM-SIAM Symposium on Discrete Algorithms***, 2019, pp. 2333–2349.

[3] L. Daviaud, M. Jurdziński, and R. Lazić, *A pseudo-quasi-polynomial algorithm for solving mean-payoff parity games*, ***33rd ACM/IEEE Symposium on Logic in Computer Science***, 2018, pp. 325–334.

[4] L. Daviaud, M. Jurdziński, and K. Lehtinen, *Alternating weak automata from universal trees*, ***30th International Conference on Concurrency Theory***, Leibniz International Proceedings in Informatics, vol. 140, 2019, to appear.

[5] M. Jurdziński and R. Lazić, *Succinct progress measures for solving parity games*, ***32nd ACM/IEEE Symposium on Logic in Computer Science***, 2017, pp. 1–9.

[6] K. Lehtinen, *A modal $\mu$ perspective on solving parity games in quasi-polynomial time*, ***33rd ACM/IEEE Symposium on Logic in Computer Science***, 2018, pp. 639–648.

▸ TORU TAKISAKA, *Large Scale Geometries of Infinite Strings*.
National Institute of Informatics, Tokyo 100-0003, Japan.
*E-mail*: `takisaka@nii.ac.jp`.

Bi-Lipschitz maps between metric spaces with a bounded distortion are called Quasi-isometries. Informally, two metric spaces are quasi-isometric if these spaces look the same from far away. Quasi-isometry relation forms an equivalence relation on the class of all metric spaces, and quasi-isometry invariants are known to be crucial in group theory. Meanwhile, infinite string over finite alphabets is one of the main objects in discrete mathematics, such as geometric group theory, formal language theory, or logic.

In the talk we introduce a notion of quasi-isometry for infinite strings, with interest to their large scale geometries; it is a quasi-isometry between strings that is also color-preserving, where the metric over a string is defined by the difference of positions of letters. It turns out that this "isometry" is not a symmetric relation, and we instead have a partial order on the quotient set of strings by mutual quasi-isometry. We look into several questions that naturally

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arise, e.g., the structure of the partial order, language equivalence problem over the quotient set, and complexity of deciding the existence of quasi-isometry, and show related results.

## Abstracts for the Special Session on Philosophical Logic

▶ KATSUHIKO SANO, *Goldblatt–Thomason theorems for non-classical logics*.
Department of Philosophy and Ethics, School of Humanities and Human Sciences, Hokkaido University, Nishi 7 Chome, Kita 10 Jo, Kita-ku, Sapporo, Hokkaido 060-0810, Japan.
*E-mail*: v-sano@let.hokudai.ac.jp.

Many nonclassical logics have Kripke semantics based on the notion of a graph or a Kripke frame $(W, R)$ where $W$ is a nonempty set and $R$ is a binary relation on $W$. It is well known that each of $R$'s reflexivity, transitivity, symmetry, and seriality is definable by a modal formula, whereas there is an undefinable property of $R$ (e.g., irreflexivity) in terms of sets of modal formulas. Given a first-order property of Kripke frames, when is such a property definable by a set of modal formulas? The Goldblatt–Thomason theorem [3] for modal logic answers this question as follows: given a first-order definable class $\mathbb{F}$ of frames, the class $\mathbb{F}$ is definable by a set of modal formulas, if and only if $\mathbb{F}$ is closed under taking generated subframes, disjoint unions and bounded morphic images and the complement of the class $\mathbb{F}$ is closed under taking ultrafilter extensions. That is, the modal definability of a first-order definable property of Kripke frames is characterized in terms of "nice" frame constructions. This talk overviews Goldblatt–Thomason-style characterizations for nonclassical logics beyond modal logic, some of which are the author's own contributions [6, 7] with collaborators. Such examples of nonclassical logics may include graded modal logic [2], modal logic with the universal modality [4], modal dependence logic [8], intuitionistic logic [5], and intuitionistic inquisitive logic [1].

[1] I. Ciardelli, R. Iemhoff, and F. Yang, *Questions and dependency in intuitionistic logic*. ***Notre Dame Journal of Formal Logic***, in press, 2019.

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[4] V. Goranko and S. Passy, *Using the universal modality: Gains and questions*. ***Journal of Logic and Computation***, vol. 2 (1992), no. 1, pp. 5–30.

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[6] K. Sano and M. Ma, *Goldblatt–Thomason-style theorems for graded modal language*, ***Advances in Modal Logic*** (L. Beklemishev, V. Goranko, and V. Shehtman, editors), College Publications, vol. 8, no. 1, 2010, pp. 330–349.

[7] K. Sano and J. Virtema, *Characterising modal definability of team-based logics via the universal modality*. ***Annals of Pure and Applied Logic***, available online 11 April 2019.

[8] J. Väänänen, *Modal dependence logic*, ***New Perspectives on Games and Interaction*** (K. R. Apt and R. van Rooij, editors), Texts in Logic and Games, vol. 4, 2008, pp. 237–254.

▶ JIJI ZHANG, *A characterization of (weakly centered) Lewisian causal models*.
Department of Philosophy, Lingnan University, Hong Kong, China.
*E-mail*: jijizhang@ln.edu.hk.

In the interventionist account of causation and causal explanation, subjunctive or counterfactual conditionals are interpreted as statements about consequences of hypothetical interventions. This semantics of conditionals is formally developed in terms of functional causal models or structural equation models, and a natural question is how the causal-model-based semantics is related to the possible worlds semantics. In this talk, I present several results pertaining to this question, which greatly extend Joseph Halpern's result that every recursive causal model is Stalnakerian (in the sense that for every recursive causal

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model, there is a possible worlds model that satisfies Robert Stalnaker's constraint and validates the exact same formulas as the causal model does, in a language that does not allow nested conditionals or conditionals with disjunctive antecedents). Specifically, I will precisely characterize the class of Stalnakerian causal models, the class of Lewisian causal models (models that satisfy David Lewis's constraints), and the class of weakly centered Lewisian causal models (models that satisfy David Lewis's constraints except that the requirement of centering is replaced by weak centering). I will discuss some philosophical implications of these results and relate in particular to a recent literature on the causal exclusion problem in the philosophy of mind.

## Abstracts for Contributed Talks

▸ SERIKZHAN BADAEV AND BIRZHAN KALMURZAYEV, *Remarks on positive preorders*.
Department of Fundamental Mathematics, Al-Farabi Kazakh National University, Al-Farabi ave. 71, Almaty 050040, Kazakhstan.
*E-mail*: Serikzhan.Badaev@kaznu.kz, birzhan.kalmirzayev@gmail.com.

We study positive (c.e.) preorders on natural numbers relative to the computable reducibility. We say that a preorder $P$ is computably reducible to a preorder $Q$ if there is a computable function $f$ such that, for all $x, y \in \omega$, $xPy$ if and only if $f(x)Qf(y)$. The notion of computable reducibility of preorders is a natural extension of such a notion for computable enumerable equivalence relations (ceers). We refer to the survey [1] for undefined notions and necessary results on ceers.

For a preorder $P$, we denote the equivalence relation $\{(x, y)\colon xPy \,\&\, yPx\}$ by $\mathrm{supp}(P)$. A preorder $P$ is called precomplete (weakly precomplete, uniformly finitely precomplete, $e$-complete, or effectively inseparable) if $\mathrm{supp}(P)$ is precomplete (respectively, weakly precomplete, uniformly finitely precomplete, $e$-complete, or effectively inseparable) equivalence relation.

Theorem 1. *There is a universal positive preoder that is precomplete* (*weakly precomplete, uniformly finitely precomplete, e-complete, or effectively inseparable*).

Theorem 2. *There exist infinitely many pairwise incomparable positive preorders that are precomplete* (*weakly precomplete, uniformly finitely precomplete, e-complete, or effectively inseparable*).

Remind that any two nontrivial precomplete ceers are computably isomorphic.

[1] U. Andrews, S. Badaev, and A. Sorbi, *A survey on universal computably enumerable equivalence relations*, ***Computability and Complexity*** (A. Day, M. Fellows, et al., editors), Springer, Cham, 2016, pp. 418–451.

▸ B. S. BAIZHANOV AND A. MUKANKYZY, *Dp-rank in different classes of theories*.
Institute of Mathematics and Mathematical Modeling, Pushkin street 125, Almaty 050010, Kazakhstan.
*E-mail*: baizhanov@math.kz.
Al-Farabi Kazakh National University, al-Farabi Ave 71, Almaty 050040, Kazakhstan.
*E-mail*: amukankyzy@gmail.com.

In this article we will give a notion of a family of relations of equivalence of depth n and consider theories of dp-rank $\omega$ and infinity. We will use the properties of superstability and independence introduced by S. Shelah [1] and dp-rank, from P. Simon's article [2].

Definition ([1]). A formula $\varphi(\bar{x}, \bar{y})$ has the independence property if for every $n < \omega$ there are sequences $\bar{a}_l (l < n)$ such that for every $\omega \subseteq n$,

$$\models (\exists \bar{x})[\bigwedge_{l<n} \varphi(\bar{x}, \bar{a}_l)_{l\in\omega}].$$

$T$ has Independence property (IP property) if some formula $\varphi(x, \bar{y})$ has independence Property.

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Theorem ([1]). *The following are equivalent.*

(1) *$T$ is superstable, i.e., stable in every large enough $\lambda$ (in fact $\lambda \geq 2^{|T|}$).*
(2) *$T$ is stable in some $\lambda$ for which $\lambda^{\aleph_0} > \lambda$.*
(3) $R^1[x = x, L, (2^{|T|})^{++}] < |T|^+$.
(4) *For some $m < \omega$, $R^m(\bar{x} = \bar{x}, L, \infty) < \infty$.*
(5) *$T$ is stable and $D^1(x = x, L, |T|^{++}) < |T|^+$.*
(6) *$T$ is stable and for some $m < \omega$, $D^m(\bar{x} = \bar{x}, L, \infty) < \infty$.*

*From this theorem it follows that, if the countable theory $T$ is superstable, then in $T$ does not exist inifinitly branching tree of formulas with one formula in each level.*

Definition ([2]). A theory T has dp-rank $\geq n$, if there are formulas $\varphi_1(x, \bar{y}), \varphi_2(x, \bar{y}), \dots, \varphi_n(x, \bar{y})$ and mutually indescernible sequences $(\bar{a}_i^1)_{i<\omega}, (\bar{a}_i^2)_{i<\omega}, \dots, (\bar{a}_i^n)_{i<\omega}$, such that for any function $\sigma\colon \{1, \dots, n\} \to \omega$ the type

$$\{\varphi_k(x, \bar{a}^k_{\sigma(k)})\colon k \leq n\} \cup \{\neg\varphi_k(x, \bar{a}_i^k)\colon i \neq \sigma(k), k \leq n\}$$

is consistent.

$T$ has Independence property (IP property) if some formula $\varphi(x, \bar{y})$ has independence Property.

Proposition. *There exists an $\omega$-stable theory with dp-rank $\omega$.*

Theorem. *If theory $T$ has dp-rank infinity, then $T$ is nonsuperstable.*

Proposition. *Model of theory with a family of relations of equivalences of depth $\geq n$ has dp-rank $\geq n$.*

Definition. We say that family of equivalence relations $F$ of depth $\omega$ is uniformly if there is a formula $E(x, y, \bar{z})$ such that for any $i \in I$ there is $\bar{\alpha}_i$ and $E_i(x, y) = E(x, y, \bar{\alpha}_i)$.

Corollary. *Model of theory with a family of relations of equivalences of depth $\omega$ is nonsuper stable.*

Theorem. *Any of theory with uniform family of equivalences of depth $\omega$ has IP property.*

*Acknowledgment*. This research was partially supported by Committee of Science in Education and Science Ministry of the Republic of Kazakhstan (Grant No. AP05132546).

[1] S. Shelah, ***Classification Theory and the Number of Non-Isomorphis Models***, 1978.
[2] P. Simon, *Dp-minimality: Invariant types and dp-rank*. ***The Journal of Symbolic Logic***, vol. 79 (2014), no. 4, pp. 1025–1045.

► BEKTUR BAIZHANOV, OLZHAS UMBETBAYEV, AND TATYANA ZAMBARNAYA,
*Omitting and realizing countable sequences of types*.
Institute of Mathematics and Mathematical Modeling, 125 Pushkin str., Almaty, Kazakhstan.
*E-mail*: `baizhanov@math.kz`.
Institute of Mathematics and Mathematical Modeling, 125 Pushkin str., Almaty, Kazakhstan; Kazakh-British Technical University, Tole bi st. 59, Almaty, Kazakhstan.
*E-mail*: `umbetbayev@math.kz`.
Institute of Mathematics and Mathematical Modeling, 125 Pushkin str., Almaty, Kazakhstan; Al-Farabi Kazakh National University, 71 al-Farabi Ave., Almaty, Kazakhstan.
*E-mail*: `zambarnaya@math.kz`.

Let $\{p_i\}$ and $\{q_i\}$ be two countable sequences of complete nonisolated types in a small theory, such that for every natural number $n$ there is a model which realizes the first $n$ $p_i$'s and omits the first $n$ $q_i$'s. We study the question of whether there exists a countable model which realizes every $p_i$ and at the same time omits every $q_i$ for $i < \omega$.

In this thesis we present a necessary and sufficient condition for existence of such a model, but this is not a complete answer to the question.

*Acknowledgment*. The authors were supported by the grant AP05134992 of SC of the MES of RK.

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[3] D. Marker, ***Model Theory: An Introduction***, Graduate Texts in Mathematics, Springer-Verlag New York, 2002.

► BEKTUR BAIZHANOV AND TATYANA ZAMBARNAYA, *Properties of 2-formulas acting on a set of realizations of 1-type in a small ordered theory*.

Institute of Mathematics and Mathematical Modelling, 125 Pushkin str., Almaty, Kazakhstan.
*E-mail*: baizhanov@math.kz.
Institute of Mathematics and Mathematical Modelling, 125 Pushkin str., Almaty, Kazakhstan; Al-Farabi Kazakh National University, 71 al-Farabi Ave., Almaty, Kazakhstan.
*E-mail*: zambarnaya@math.kz.

In the work we introduce a notion of a type-preserving convex to the right (left) 2-formula. There are three kinds of such formulas, all connected with some properties of the theory. In particular, a linearly ordered theory, which has the so-called quasi-successor formula on a type, has $2^{\omega}$ countable nonisomorphic models. The second kind, an equivalence-generating formula, creates an equivalence relation which partitions the realization set of a type into equivalence classes. And if there is a formula of the third kind, a decreasing family of nested convex definable sets appears.

*Acknowledgment*. The authors were supported by the grant AP05134992 of SC MES RK.

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[2] B. S. Baizhanov, *One-types in weakly o-minimal theories*, ***Proceedings of Informatics and Control Problems Institute***, Almaty, Kazakhstan, 1996, pp. 75–88.

[3] B. S. Baizhanov and B. Sh. Kulpeshov, *On behaviour of 2-formulas in weakly o-minimal theories*, ***Mathematical Logic in Asia—Proceedings of the 9th Asian Logic Conference*** (R. Downey, S. S. Goncharov, and H. Ono, editors), World Scientific, Singapore; Novosibirsk, Russia, 2006, pp. 31–40.

► SAYAN BAIZHANOV AND BEIBUT KULPESHOV, *$V_p$-independence*.

Institute of Mathematics and Mathematical Modelling, 125 Pushkin str., Almaty, Kazakhstan; Al-Farabi Kazakh National University, 71 al-Farabi Ave., Almaty, Kazakhstan.
*E-mail*: sayan-5225@mail.ru.
International Information Technology University, 050040, Manas str. 34/1, Almaty, Kazakhstan.
*E-mail*: b.kulpeshov@iitu.kz.

In the work notions of $V_p$-independence and $V_p$-$n$-gon are introduced, and their properties are studied. In particular, properties of $V_p$-$n$-gons in pseudogeometric theories are considered. Pseudogeometric theories are theories in which the condition of non-almost orthogonality of one types is symmetric.

*Acknowledgment*. The second author was partially supported by Grant of Science Committee of Ministry of Education and Science of the Republic of Kazakhstan (AP05134992).

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▸ ANDREY FROLOV, ISKANDER KALIMULLIN, ALEXANDER MELNIKOV, AND MAXIM ZUBKOV, *Punctually computable copies of linear orders*.
Kazan Federal University, Kazakhstan.
*E-mail*: a.frolov.kpfu@gmail.com, ikalimul@gmail.com.
Massey University, New Zealand, and Kazan Federal University, Kazakhstan.
*E-mail*: alexander.g.melnikov@gmail.com.
Kazan Federal University, Kazakhstan.
*E-mail*: maxim.zubkov@kpfu.ru.

A countable structure is *punctually computable* if its domain is $\mathbb{N}$ and the operations and predicates of the structure are (uniformly) primitive recursive. Also, I. Kalimullin, A. Melnikov and K. M. Ng [2] called punctually computable structures "fully primitive recursive." We study punctually computable copies of linear orders. S. Grigorieff [1] proved that every computable linear order has a punctually computable copy. The Grigorieff's proof is nonuniform in the following sense: he considered four cases and the oracle $0^{(4)}$ is needed to decide which case is actual for a given linear order. We improved this bound and show that the new bound is optimal. Namely, we proved the following theorem.

THEOREM 1.

(1) *There exists a* $0^{(3)}$*-uniform procedure which by index of computable linear order gives the index of its punctual copy. Moreover, there is no* $0^{(3)}$*-uniform procedure with this property.*
(2) *Every computable linear order* $\mathcal{L}$ *is* $0''$*-isomorphic to punctual one. Moreover,* $0''$ *is the optimal bound.*

We note the unexpected connection of obtained results with the study of copied and diagonalizable classes of countable algebraic structures introduced by A. Montalban in [3]. Montalban proved that the linear orders are 4-copyable and 2-diagonalizable, and also noted that it is unknown whether the linear orders are 3-copyable or 3-diagonalizable.

From the proof of (1) of the theorem 1, it follows that linear orders are 3-copyable.

*Acknowledgments.* The second author was supported by RFBR grant No. 17-51-18083. The last author was supported by RFBR grant No. 18-31-00174.

[1] S. GRIGORIEFF, *Every recursive linear orderings has a copy in DTIME-SPACE*$(n, \log n)$. ***The Journal of Symbolic Logic***, vol. 55 (1990), no. 1, pp. 260–276.

[2] I. SH. KALIMULLIN, A. G. MEL'NIKOV, and K. M. NG, *Algebraic structures computable without delay*. ***Theoretical Computer Science***, vol. 674 (2017), pp. 73–98.

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▸ ASSYLBEK ISSAKHOV AND FARIZA RAKYMZHANKYZY, *On principal numberings of a family of total functions*.
School of Mathematics and Cybernetics, Kazakh-British Technical University, 59 Tole bi st., Almaty 050000, Kazakhstan.
*E-mail*: a.isakhov@kbtu.kz, asylissakhov@gmail.com.
Department of Mechanics and Mathematics, Al-Farabi Kazakh National University, 71 Al-Farabi Ave., Almaty 050040, Kazakhstan.
*E-mail*: fariza.rakymzhankyzy@gmail.com.

If a family $\mathcal{F}$ of total functions is $A$-computable for arbitrary set $A$, then a numbering $\alpha$ of the family $\mathcal{F}$ is called $A$-computable while $\alpha(n)(x)$ is $A$-computable binary function, [1]. If a degree $a$ contains a hyperimmune set, then $a$ is hyperimmune. Otherwise, $a$ is hyperimmune-free.

It is known that if an $A$-computable family $\mathcal{F}$ of total functions contains at least two elements, where $A$ is a hyperimmune set (for $A \geq \emptyset'$ see [2]), then $\mathcal{F}$ has no $A$-computable principal numbering, [3]. It was also proved that if $\mathcal{F}$ is an arbitrary finite $A$-computable family of total functions, where Turing degree of the set $A$ is hyperimmune-free, then $\mathcal{F}$ has an $A$-computable principal numbering, [4].

THEOREM 1. *There exists an infinite $A$-computable family $\mathcal{F}$ of total functions, where Turing degree of the set $A$ is hyperimmune-free, such that $\mathcal{F}$ has an $A$-computable principal numbering.*

[1] S. A. BADAEV and S. S. GONCHAROV, *Generalized computable universal numberings*. ***Algebra and Logic***, vol. 53 (2014), no. 5, pp. 355–364.

[2] A. A. ISSAKHOV, *Ideals without minimal elements in Rogers semilattices*. ***Algebra and Logic***, vol. 54 (2015), no. 3, pp. 197–203.

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▶ A. K. ISSAYEVA AND A. R. YESHKEYEV, *On* $(\nabla_1, \nabla_2)$-*cl atomic sets*.
Faculty of Mathematics and Information Technologies, Karaganda State University, University str., 28, building 2, Kazakhstan.
*E-mail*: aibat.kz@gmail.com, isa_aiga@mail.ru.

The study of properties of so-called small models for various subclasses of inductive theories is connecting with given work. Each time the difference in these subclasses always depends on the special conditions imposed on some inductive theory. These conditions are of the following type: the joint embedding property, the amalgam property, convexity, an existential primeness, a certain type of completeness, and perfectness in the sense of Jonsson's theory, if the considered inductive theory is such. All of these conditions are connected with considered theory, but are not purely syntactic, since the abovementioned definitions of conditions relate to the class of models of considered theory, for example, convexity and an existential primeness. On the other hand, the main accent of this work is oriented on the syntactic properties of special subsets of the semantic model of some Jonsson theory.

Let $AAPC \in \{$atomic, algebraically prime, core$\}$. Thus, by specifying the set $X$ as $(\nabla_1, \nabla_2) - cl - AAPC$, (where $AAPC$ is a semantic property), we can also specify atomicity in the sense [1] in relation to atomicity in the sense of [2]. And accordingly, using of the principle of "rheostat" after the $AAPC$ property is defined, we obtain the corresponding concepts of atomic models, the role of which is played $A_2$ from the principle of "rheostat."

And we have regarding considerations those notions some new results. One of the obtained results is the following theorem.

THEOREM 1. *Let $T$ be complete for $\exists$-sentences a strongly convex Jonsson perfect theory and let $M$ is $(\nabla_1, \nabla_2) - cl$-atomic model in $T$.*

(a) *Then $(i) \Rightarrow (ii) \Rightarrow (iii)$ and $(ii) \Rightarrow (ii)^*$ where:*

$(i)$ *$M$ is $(\Sigma, \Sigma) - cl$-atomic model in theory $T$,*

$(ii)$ *$M$ is $(\nabla_1, \nabla_2) - cl$-$\Sigma^*$-nice-model in theory $T$,*

$(ii)^*$ *$M$ is e.c. and $(\nabla_1, \nabla_2) - cl$-$\Sigma$-nice-model in theory $T$,*

$(iii)$ *$M$ is weak $(\Sigma, \Pi) - cl$-atomic model in theory $T$,*

[1] J. T. BALDWIN and D. W. KUEKER, *Algebraically prime models*. ***Annals of Mathematical Logic***, (1981), pp. 289–330.

[2] R. VAUGHT, ***Denumerable Models of Complete Theories in Infinitistic Methods***, Pergamon, London, 1961, pp. 303–321.

▶ SANTIAGO JOCKWICH AND GIORGIO VENTURI, *Reasonable implicative algebra and set theory*.
Institute of Philosophy and Human Sciences, Unicamp, Rua Cora Coralina 100, Brazil.
*E-mail*: gio.venturi@gmail.com.
*E-mail*: santijoxi@hotmail.com.

This talk contributes to the study of algebra-valued models of set theory by providing a new infinite class of models based on algebras associated to nonclassical logics. The idea of replacing Boolean algebras with other kinds of algebraic structures is not new in the literature. Heyting algebras or more complicated lattices have already been proposed to build models of intuitionistic set theory and quantum, or fuzzy set theory.

Recently, a new approach has emerged. Instead of adapting the axioms of Zermelo–Fraenkel (ZF) to a nonclassical setting, in [1] the authors searched for nonclassical models able to validate the classical axioms of ZF. To this aim they define a class of algebras, that we call reasonably implicative. This new approach, however, is still in its infancy. On the one hand, it still needs a strong theoretical motivation for blending classical axioms, like those of ZF, with a nonclassical interpretation; on the other hand, it still does not offer enough examples to provide applications.

In this talk we tackle the second issue by defining an infinite class of algebras that give rise to infinitely many new models of the negation-free fragment of ZF. To do so we study a type of algebraic structures, called reasonably implicative, and use them to construct algebra-valued models of set theory. We show that the expressive resources of meet complemented meet semilattices are enough for the definition of reasonably implicative algebras. Finally, the linear algebra, that form an infinite subclass, allows the definition of a paraconsistent negation and the construction of infinitely many paraconsistent models of ZF.

[1] B. Löwe and S. Tarafder, *Generalized algebra- valued models of set theory*. ***The Review of Symbolic Logic***, vol. 8 (2015), no. 1, pp. 192–205.

► MAIRA KASSYMETOVA, *Uncountable categorical central type of Robinson spectrum*.
Faculty of Mathematics and Information Technologies, Buketov Karaganda State University, University str., 28, building 2, Kazakhstan.
*E-mail*: mairushaasd@mail.ru.

Let theory $T$ be the Robinson theory and $\mathcal{A}$ be an arbitrary model of signature $\sigma$. We call the *Robinson spectrum* of model $\mathcal{A}$ the set:

$$RSp(\mathcal{A}) = \{T \mid T - \text{the Robinson theory in language } \sigma \text{ and } \mathcal{A} \in Mod\, T\}.$$

We say that a Jonsson theory $T_1$ is cosemantic to Jonsson theory $T_2$ ($T_1 \bowtie T_2$) if $\mathcal{C}_{T_1} = \mathcal{C}_{T_2}$, where $\mathcal{C}_{T_i}$ is semantic model of $T_i$, $i = 1, 2$. The relation of cosemanticness on the set of Jonsson theories is an equivalence relation. Then, so any Robinson theory is Jonsson theory, we can consider factor set $RSp(\mathcal{A})/_{\bowtie}$ of Robinson spectrum of model $\mathcal{A}$ by relation $\bowtie$. If $T$ is an arbitrary Robinson theory of signature $\sigma$, then by $E_T$ we denote the class of all existentially closed models of $T$. Let $[T] \in RSp(\mathcal{A})/_{\bowtie}$, then $E_{[T]} = \bigcup\limits_{\Delta \in [T]} E_\Delta$.

For an arbitrary Jonsson theory, we have some scheme ($\sharp$) for obtaining a central type.

Consider the class $[T] \in RSp(A)/\bowtie$. Let $Th(C_{[T]}) = [T]^*$. For every $\Delta \in [T]$ we denote by $\bar{\Delta}$ the theory obtained by the scheme ($\sharp$). Now we can consider the class $[\overline{T}]$ and then the class $[\overline{T}]^*$ after restriction by the scheme ($\sharp$) becomes the central type of the class $[T]$ and denote it by $P^C_{[T]}$.

Definition ([2]). Let $\mathcal{A}, \mathcal{B} \in E_T$ and $\mathcal{A} \underset{\neq}{\subset} \mathcal{B}$. Then $\mathcal{B}$ is called algebraically prime model extension $\mathcal{A}$ in $E_T$ if for any model $\mathcal{C} \in E_T$ from the fact that $\mathcal{A}$ is isomorphically embedded in $\mathcal{C}$ it follows that $\mathcal{B}$ is isomorphically embedded into $\mathcal{C}$.

Let us consider an arbitrary model $\mathcal{A}$ of arbitrary signature $\sigma$. Then we have the following result regarding above mentioned notions [1].

Theorem. *Let* $[T]$ *be a hereditary class from* $RSp(\mathcal{A})/_{\bowtie}$, $[T]$ *is* $\omega$*-categorical, then the following conditions are equivalent*:

(1) *any countable model from* $E_{[T]}$ *has an algebraically prime model extension in* $E_{[T]}$;
(2) $P^C_{[T]}$ *is strongly minimal, where* $P^C_{[T]}$ *is central type of* $[T]$.

[1] J. Barwise (ed.), ***Handbook of Mathematical Logic***, North-Holland, Amsterdam, 1977.

[2] A. Yeshkeyev, ***Jonsson Theories***, KarGU, Karaganda, 2009.

► BORIBAI KASYMKHANULY AND ANDREY MOROZOV, *On holographic structures*.
L. N. Gumilyov Eurasian National University, Kazhimukan str. 13, Nur-Sultan 010008, Kazakhstan.
*E-mail*: boribay@mail.ru.

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Sobolev institute of mathematics SB RAS, Koptyug Ave. 4 and Novosibirsk state university, Pirogova str. 1, Novosibirsk 630090, Russia.
*E-mail*: morozov@math.nsc.ru.

Informally, a structure is called holographic if it can be obtained by copying some of its finite part by means of its automorphisms.

The least upper bound of the numbers of arguments of a signature $\sigma$ is called its *height* and is denoted as $\|\sigma\|$. We consider signatures of finite heights only.

Definition. A structure $\mathfrak{M}$ of predicate signature $\sigma$ of finite height is called holographic if there exists a finite set $S \subseteq \mathfrak{M}$ such that for any $A \subseteq \mathfrak{M}$ of cardinality at most $\|\sigma\|$ there exists a $\varphi \in \mathtt{Aut}\,\mathfrak{M}$ with the property $\varphi(A) \subseteq S$.

We prove the class of countable holographic structures to be different from the class of countably categorical models and give characterizations of the classes of holographic Boolean algebras, Abelian groups, linear orderings, fields, and equivalences.

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► ANDREY MOROZOV AND JAMALBEK TUSSUPOV, *On Δ-definability on families of predicates*.

Sobolev institute of mathematics SB RAS, Koptyug Ave. 4 and Novosibirsk state university, Pirogova str. 1, Novosibirsk 630090, Russia.
*E-mail*: morozov@math.nsc.ru.
L. N. Gumilyov Eurasian National University, ul. Satpaeva 2, Astana 010008, Kazakhstan.
*E-mail*: tussupov@mail.ru.

Fix some countable set $U$. By *predicate* here we mean an arbitrary subset of an arbitrary finite Cartesian power of $U$.

We say that a predicate $R$ is *Δ–definable over the predicates* $P_1, \dots, P_k$ if $R$ itself and its complement can be defined in the structure $\langle U; P_1, \dots, P_k\rangle$ by means of $\exists$-formulas with parameters.

Let $S_0$ and $S_1$ be two finite families of predicates. We say that $S_0$ *is Δ–definable in* $S_1$, if all the predicates in $S_0$ are Δ-definable in $S_1$ and we denote this fact as $S_0 \leqslant^0_\Delta S_1$. If $S_0 \leqslant^0_\Delta S_1$ and $S_1 \leqslant^0_\Delta S_0$ then we denote this fact as $S_0 \equiv^0_\Delta S_1$. If we factorize the relation $\leqslant^0_\Delta$ by isomorphism on families of predicates then we arrive at the notion of Δ-reducibility on families of predicates, which is denoted by $\leqslant_\Delta$. Degrees of reducibilities defined by preorderings $\leqslant^0_\Delta$ and $\leqslant_\Delta$ are defined in a standard way. The ordered structures of such degrees are denoted by $D^0_\Delta$ and $D_\Delta$, respectively.

The main concern of the talk is the study of the structures $D^0_\Delta$ and $D_\Delta$ and preorders $\leqslant^0_\Delta$ and $\leqslant_\Delta$.

Proposition. *For any finite family of predicates $S_0$ there exists a family $S_1$ consisting of a single predicate such that $S_0 \equiv^0_\Delta S_1$.*

Theorem.

(1) *The families consisting of unary predicates define in $D_\Delta$ an ideal of order type $\omega$.*
(2) *The structure $D_\Delta$ fails to be an upper semilattice.*
(3) *The structure $D_\Delta$ contains $2^\omega$ minimal nonzero elements.*

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► N. M. MUSSINA AND A. R. YESHKEYEV, *Hybrid's core models of Jonsson theories*.

Faculty of Mathematics and Information Technologies, Karaganda State University, University str., 28, building 2, Kazakhstan.
*E-mail*: aibat.kz@gmail.com, nazerke170493@mail.ru.

Let $T_1, T_2$ be an arbitrary Jonsson theories in the first-order language of the same signature $\sigma$ and $C_1, C_2$ be a semantic models of the theories $T_1, T_2$, correspondingly.

Let us define the essence of the operation of the symbol $\boxdot$ for algebraic construction of models, which will be playing an important role in the definition of hybrids. Let $\boxdot \in \{\cup, \cap, \times, +, \oplus, \prod_F, \prod_U\}$, where $\cup$-union, $\cap$-intersection, $\times$-Cartesian product, $+$-sum and $\oplus$-direct sum, $\prod_F$-filtered product, and $\prod_U$-ultraproduct.

Definition 1. A hybrid $H(T_1, T_2)$ of Jonsson theories $T_1, T_2$ is said to be the theory $Th_{\forall\exists}(C_1 \boxdot C_2)$, if it is Jonsson. Herewith, the algebraic construction $(C_1 \boxdot C_2)$ is said to be a semantic hybrid of the theories $T_1, T_2$.

Note the following fact:

Fact 1. *For the theory $H(T_1, T_2)$ in order to be Jonsson enough to be that $(C_1 \boxdot C_2) \in E_{H(T_1,T_2)}$.*

Definition 2. The theory is called existentially core if this theory is existentially prime and among existentially closed algebraically prime models there is at least one core model.

The main result is the following theorem.

Let $\nabla \in \{\forall, \exists, \forall\exists\}$.

Theorem 1. *Let $T$ be perfect, strongly convex, existentially core, complete for $\forall\exists$-sentences Jonsson theory. $X_1, X_2$ are $(\nabla, \nabla)$-$cl$-atomic sets of the theory $T$, where $dcl(X_1) = M_1$, $dcl(X_2) = M_2$, $M_1, M_2 \in E_T$. $Th_{\forall\exists}(M_1) = T_1, Th_{\forall\exists}(M_2) = T_2$. $C_1, C_2$ are semantic models of the theories $T_1, T_2$, correspondingly. $T_1$ is model consistent with $T_2$ and with $T$. Let $H(T_1, T_2) = Th_{\forall\exists}(C_1 \times C_2)$. If $M_1, M_2$ are $(\Sigma, \Sigma)$-$cl$-atomic models, then there exist $M \in E_{H(T_1,T_2)}$ such that $M$ is countable model and $M \in AP_{H(T_1,T_2)}$ and $M$ is $(\nabla, \nabla)$-$cl$-algebraically prime model.*

All concepts that are not defined in this abstract can be extracted from [1].

[1] N. M. Mussina and A. R. Yeshkeyev, *Properties of hybrids of Jonsson theories*. ***Bulletin of the Karaganda University, Seria of "Mathematics", Karaganda***, vol. 92 (2018), no. 4, pp. 99–104.

[2] ———, *Properties of hybrids of Jonsson theories*. ***Bulletin of the Karaganda University, Mathematics Series, Karaganda***, vol. 92 (2018), no. 4, pp. 99–104.

▶ MANAT MUSTAFA AND SERGEY OSPICHEV, *Notes on principal numberings in the Ershov hierarchy*.

Department of Mathematics, Nazarbayev University, Kabanbay batyr 53, Nur-Sultan, Kazakhstan.

*E-mail*: manat.mustafa@nu.edu.kz.

Sobolev Institute of Mathematics and Novosibirsk State University, Novosibirsk, Russia.

*E-mail*: ospichev@math.nsc.ru.

Study the properties of extremal elements of Rogers semilattices of families of various objects is one of main questions in numbering theory. In [2], Lachlan shows that every finite family of c.e. sets has a principal numbering, in contrast to the classical case, in [1], K. Abeshev constructed a finite family of disjoint sets on the second level of Ershov hierarchy without computable principal numbering. In this work we try to investigate more about the difference of principle numberings in the setting of Ershov hierarchy.

Let $\mathcal{P} = \langle P, \leq_P \rangle$ be a finite partially ordered set and $\check{p} = \{x | p \leq_P x\}$. We will call a family $\{F_p\}_{p \in \mathcal{P}}$ of nonempty $\Sigma^{-1}_a$-sets *acceptable* if $F_{p_1} \bigcap F_{p_2} = \bigcup_{q \in \check{p}_1 \cap \check{p}_2} F_q$ for any $p_1, p_2 \in \mathcal{P}$. The main result is:

Theorem. *Let $a$ be the ordinal notation of nonzero ordinal. For any finite partially ordered set $\mathcal{P}$ and any acceptable family $\{F_p\}_{p \in \mathcal{P}}$, there is $\Sigma^{-1}_a$-computable principal numbering of family $\{F_p\}_{p \in \mathcal{P}} \bigcup \{\emptyset\}$*

Corollary. *Any finite family of disjoint $\Sigma^{-1}_a$-sets with $\emptyset$ has $\Sigma^{-1}_a$-computable principal numbering.*

*Acknowledgment*. Second author was partially supported by the grant of the President of the Russian Federation (No. MK-1214.2019.1).

[1] K. Abeshev, *On the existence of universal numberings for finite families of d.c.e. sets*. ***Mathematical Logic Quarterly***, vol. 60 (2014), no. 3, pp. 161–167.

[2] A. H. Lachlan, *Standard classes of recursively enumerable sets*. ***Zeitschrift für mathematische Logik und Grundlagen der Mathematik***, vol. 10 (1964), no. 2–3, pp. 23–42.

▸ M. T. OMAROVA AND G. E. ZHUMABEKOVA, *Strongly minimal central types of the class $[T] \in JSp(A)$ which has essential geometric base.*
Faculty of Mathematics and Information Technologies, Karaganda State University, University str., 28, building 2, Kazakhstan.
*E-mail*: omarovamt_963@mail.ru.
*E-mail*: galkatai@mail.ru.

Let $S^{(1)}_{\nabla}(X)$ be set of all complete $\nabla$-1-types over the set $X$. Let $X \subseteq M$, $M \in E_T$.

Definition 1. Type $p \in S^{(1)}_{\nabla}(X)$ is called essential if for any set $Y$, $Y \subseteq N$, $N \in E_T$, such that $X \subseteq Y$ in $T$ exists only unique type $q \in S^{(1)}_{\nabla}(Y)$ and the type $q$ is a $J$-nonforking extension of type $p$.

Definition 2. The set $B = \{[p_i] \in S^{(1)}_{\nabla}[X] | i \in I\}$ is called base for $S^{(1)}_{\nabla}[X]$ if:
(1) $[p_i]$ and $[q_j]$ independent for $i \neq j$;
(2) for any $[q] \in S^{(1)}_{\nabla}[X]$ and any $\mathfrak{A} \in E_T$, $X \subseteq A$, exists such $i \in I$, than $[p_i] \leq_A [q]$.

Definition 3. The base of the theory $T$ is the base for $S^{(1)}_{\nabla}[\varnothing]$ (if it exists). The base $B$ of theory $T$ is called essential if for any $[p] \in B$ exists an essential type $q \in [p]$.

Definition 4. We will call the essential base of the types of Jonsson theory $T$ geometric if the following conditions are satisfied:
(1) $\forall p \in S^{(1)}_{\nabla}(X)$, where $X \subseteq C$, $C$ is semantic model and $(C, cl)$ is $J$-geometry;
(2) The concept of independence in the sense of geometry generated by a $J$-strongly minimal central type will coincide with the concept of independence $(C, cl)$ is $J$-geometry (coincidence of the concept of a base in terms of $J$-strongly minimality).

The class $[T]$ has essential base, its mean that the set of all central types $\Delta^c$ forms essential base. Furthermore, all considered classes have an essential geometric base.

Theorem 1. *For any $\Delta \in [T]$ the following are equivalent*:
(a) *$\psi(A)$ is $J$-strongly minimal, $A \in E_{\Delta}$;*
(b) *For every existentially closed model $B \in E_{\Delta}$, $\psi(B)$ is $J$-minimal in $B$;*
(c) *$\psi(C)$ is $J$-minimal in $C$.*

Theorem 2. *For every $\Delta \in [T]$ if $[\overline{T}]^{*c}$ is $J$-strongly minimal nonalgebraic type, then each $\Delta^c$ will be $J$-minimal nonalgebraic type.*

All concepts that are not defined in this abstract can be extracted from [2, 1].

[1] W. Hodges, ***A Shorter Model Theory***, Publisher, 1997.

[2] M. T. Kassymetova and A. R. Yeshkeyev, ***Jonsson Theories and their Classes of Models***, KarGU, Karaganda, 2016, in Russian.

▸ LUCA SAN MAURO, *The complexity of homomorphisms between groups.*
Institute of Discrete Mathematics and Geometry, Vienna University of Technology, Wiedner Hauptstraße 8–10/104 1040 Vienna, Austria.
*E-mail*: luca.san.mauro@tuwien.ac.at.

In computable structure theory, in addition to studying the complexity of structures, we are often also interested in the complexity of maps that naturally arise. In this work, we investigate the complexity of homomorphisms between computable groups. We define the homomorphism spectrum of computable groups $G$ and $H$ as the set of degrees that computes nontrivial homomorphisms from $G$ to $H$.

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We prove the following:

- any homomorphism spectrum is either meager or it coincides with all degrees;
- every degree of homomorphism is hyperarithmetic but there is a homomorphism spectrum of torsion free abelian groups that contains no hyperarithmetic degrees;
- there is a $\Delta^0_3$ degree that is not a degree of homomorphism;
- for every computable $\alpha \geq 2$, $\mathbf{0}^{(\alpha)}$ is a degree of homomorphism of torsion free abelian groups.

*Acknowledgment*. This is joint work with Meng Che Ho and Dino Rossegger.

► TINKO TINCHEV, *Contact logics with qualitative measure*.
Faculty of Mathematics and Informatics, Sofia University St. Kliment Ohridski, 5 James Bourchier Blvd., 1164 Sofia, Bulgaria.
*E-mail*: `tinko@fmi.uni-sofia.bg`.

Whiteheadean approach to point-free space (or mereotopology) is based on the primitive notion "region" and two binary relations between regions, "part-of" and "contact." Usually they are modeled by the regular closed sets algebra of a topological space and the predicate "nonempty set-theoretic intersection" wherefrom the contact algebras stem. Contact logics (CL) [2] are propositional logics naturally interpreted over contact algebras and they have relational and topological semantics too. Their language contains a set of Boolean terms representing regions and a set of formulas which is the closure with respect to the propositional connectives of the set of atomic formulas. The atomic formulas are expressions of the type $(a \leq b)$ or $C(a, b)$ with the intended meaning "the region $a$ is part-of $b$" and "the region $a$ is in contact with $b$," respectively.

Another approach to point-free space relies on the Boolean algebra of the Borel sets modulo null sets of Lebesgue measure zero, [1]. Some contact logics complete with respect to this kind of algebras and interconnections between both approaches are studied in [4].

In the present talk are proposed contact logics with qualitative measure (CLQM) that bring together both kind of information, topological and size information. Remark that this combination has multifarious applications in the qualitative spatial reasoning [3]. The language of CLQM extends the language of CL with a new kind of atomic formulas: $(a \leq_m b)$ with intended meaning "the size of the region $a$ is less or equal than the size of $b$." The natural semantics of CQML are tuples $(B, \mu)$, where $B$ is a contact algebra and $\mu$ is a positive probabilistic measure on the carrier Boolean algebra. For several classes of important structures the complete axiomatizations are found and the decidability results are proven. For example, among them is the logic of the contact algebra $\mathbf{Pol}([0;1])$ of all polytops over the real interval $[0;1]$ with Lebesgue measure, where $\mathbf{Pol}([0;1])$ is the subalgebra of the contact algebra $RC([0;1])$ of all regular closed subsets of $[0;1]$ generated by the finite unions of closed intervals.

*Acknowledgment*. The research was supported by the contract DN02/15/2016 with the Bulgarian SF.

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[2] P. Balbiani, T. Tinchev, and D. Vakarelov, *Modal logics for region-based theory of space*. ***Fundamenta Informaticæ***, vol. 81 (2007), no. 1–3, pp. 29–82.

[3] A. Gerevini and J. Renz, *Combining topological and size information for spatial reasoning*. ***Artificial Intelligence***, vol. 137 (2002), pp. 1–42.

[4] T. Lando, *Topology and measure in logics for region-based theories of space*. ***Annals of Pure and Applied Logic***, vol. 169 (2018), no. 4, pp. 277–311.

► OLGA ULBRIKHT, *Jonsson independence for $JSp(A)$*.
Faculty of Mathematics and Information Technologies, Buketov Karaganda State University, University str., 28, building 2, Kazakhstan.
*E-mail*: `ulbrikht@mail.ru`.

Let $\mathcal{A}$ be an arbitrary model of signature $\sigma$. Let us call *the Jonsson spectrum* of model $\mathcal{A}$ a set: $JSp(\mathcal{A}) = \{T \mid T$ is the Jonsson theory in a language $\sigma$ and $\mathcal{A} \in Mod\ T\}$. We say that the Jonsson theory $T_1$ is *cosemantic* to the Jonsson theory $T_2$ $(T_1 \bowtie T_2)$ if $C_{T_1} = C_{T_2}$, where $C_{T_i}$ is a semantic model of $T_i$, $i = 1, 2$. The relation of cosemantic on a set of theories is an equivalence relation. Then $JSp(\mathcal{A})/_{\bowtie}$ is the factor set of the Jonsson spectrum of the model $\mathcal{A}$ with respect to $\bowtie$.

Let $X \subseteq C$, where $C$ is a semantic model of the Jonsson theory $T$. We will say that a set $X$ is a $\nabla$-*cl-subset* of $C$, if $X$ satisfies the following conditions: (1) $X$ is a $\nabla$-definable set, where $\nabla$ is a view of formula, for example $\exists, \forall, \forall\exists$;

(2) $cl(X) = M$, $M \in E_T$, where $cl$ is some closure operator defining a pregeometry [2] over $C$.

Let $[T] \in JSp(\mathcal{A})/_{\bowtie}$, $\mathcal{X}$ is the class of all $\nabla$-$cl$-subsets of the semantic model $C_{[T]}$ and $\mathcal{P}$ is the class of all $\nabla$-types (not necessarily complete), let $J^{\nabla}NF$ (Jonsson $\nabla$-nonforking) $\subseteq \mathcal{P} \times \mathcal{X}$ be some binary relation. There is the list of the Axioms 1–7 which defined Jonsson $\nabla$-nonforking notion $J^{\nabla}NF$.

Let $[T] \in JSp(\mathcal{A})/_{\bowtie}$. We will denote $[T_{\nabla}] = \{\Delta \in [T] \colon \Delta$ is $\nabla$-$\lambda$-stable, $\nabla$-complete$\}$.

Theorem 1. *Let $JSp(\mathcal{A})/_{\bowtie}$ be a homogeneous factor spectrum, $[T] \in JSp(\mathcal{A})/_{\bowtie}$, then in class $[T_{\nabla}]$ relation $J^{\nabla}NF$ satisfies Axioms* 1–7.

*Let $[T] \in JSp(\mathcal{A})/_{\bowtie}$, $I = \{(\bar{a}, Y, X) \mid \bar{a} \in C_{[T]}, X, Y$ is $\nabla$-$cl$-subsets of the semantic model $C_{[T]}$ such that $X \subseteq Y\}$. We call the set $I$ a Jonsson (good) independence system if it satisfies some list of axioms.*

*We say that $\bar{a} \in C_{[T]}$ is $J$-independent from $Y$ over $X$ and write $\bar{a} \mathop{\smile\!\!\!\!\!\!|}^{J}_{X} Y$ if $(\bar{a}, Y, X) \in I$, where $I$ is the Jonsson (good) independence system.*

In the study of a $J$-simple perfect class $[T]$ we have the following result.

Theorem 2 (Jonsson variant of the KP theorem [1] for $JSp(\mathcal{A})$). *Let the class $[T] \in JSp(\mathcal{A})/_{\bowtie}$ be $J$-simple, perfect, $\nabla$-complete. Then for every tuple $\bar{a} \in C_{[T]}$ and $\nabla$-$cl$-Jonsson sets $X \subseteq Y$ of a model $C_{[T]}$ the following conditions are equivalent*:

(1) *$\bar{a} \mathop{\smile\!\!\!\!\!\!|}^{J}_{X} Y$ in the language of theory $\Delta$ for each $\Delta \in [T]$;*

(2) *$(tp(\bar{a}/Y), X) \in J^{\nabla}NF$ in the language of the theory $\Delta$ for each $\Delta \in [T]$;*

(3) *for all $p \in \mathcal{P}$ consistent with $[T]^*$, $X \in \mathcal{X}$ the type $p$ not forks over $X$ (in the classical meaning of S. Shelah), where $[T]^*$ is the center of the class $[T]$.*

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[2] D. Marker, ***Model Theory: An Introduction***, Springer-Verlag, New York, 2002.

▸ G. A. URKEN AND A. R. YESHKEYEV, *Similarity of classes from Jonsson spectrum*.
Faculty of Mathematics and Information Technologies, Buketov Karaganda State University, University str., 28, building 2, Kazakhstan.
*E-mail*: aibat.kz@gmail.com, guli_1008@mail.ru.

The notion of similarity between two complete theories was introduced in [2]. For Jonsson theories the similarity between two Jonsson theories was introduced in [1]. In both works were obtained some results which described syntactic and semantic similarity in both cases. In this abstract we going to expand the notion of similarity up classes of Jonsson spectrum for models of an arbitrary signature.

Let $A$ be an arbitrary model of countable language. $JSp(A) = \{T/T$ is Jonsson theory in this language and $A \in ModT\}$ and $JSp(A)$ is said to be the Jonsson spectrum of the model $A$.

Definition 1. We say that the Jonsson theory $T_1$ is cosemantic to the Jonsson theory $T_2$ $(T_1 \bowtie T_2)$ if $C_{T_1} = C_{T_2}$, where $C_{T_i}$ are semantic model of $T_i$, $i = 1, 2$.

The relation of cosemanticness on a set of theories is an equivalence relation. Then $JSp(A)/_{\bowtie}$ is the factor set of the Jonsson spectrum of the model $A$ with respect to $\bowtie$.

Definition 2. Let $[T]_1, [T]_2 \in JSp(A)/_{\bowtie}$. We say that $[T]_1$ is J-syntactically similar to $[T]_2$ if for any $\Delta \in [T]_1$ there is $\Delta' \in [T]_2$ such that $\Delta$ and $\Delta'$ are J-syntactically similar each other as Jonsson theories (in the meaning of [1]).

Definition 3. The pure triple $\langle C, AutC, \overline{E}_{[T]}\rangle$ is called the $J$-semantic triple for class $[T] \in JSp(A)/_{\bowtie}$, where $C$ is the semantic model of $[T]$, $AutC$ is the group of all automorphisms of $C$, $\overline{E}_{[T]}$ is the class of isomorphically images of all existentially closed models of $[T]$.

Definition 4. Let $[T]_1, [T]_2 \in JSp(A)/_{\bowtie}$. We say that $[T]_1$ is J-semantically similar to $[T]_2$ if their semantically triples are isomorphic as pure triples (in the meaning of [1]).

Lemma. *From syntactic similarity of two classes of Jonsson spectrum follows their semantic similarity. Converse statement does not true.*

In the frame of above mentioned notions we have the following results.

Let $\nabla \subseteq L$, if $\varphi \in \nabla$ then $\varphi \in \{\forall, \exists, \forall\exists\}$. All considered theories in further will be complete for $\nabla$-sentences and perfect.

Theorem 1. *Let $[T]_1, [T]_2 \in JSp(A)/_{\bowtie}$. Then $[T]_1$ and $[T]_2$ are J-syntactically similar iff $[T]_1^*$ and $[T]_2^*$ are syntactically similar in the frame of* [2].

Theorem 2. *Let $B$ be a model of the signature of $S - act$, where $S$ is monoid. For any $[T] \in JSp(A)/_{\bowtie}$ there is the class $[T]^{act} \in JSp(B)/_{\bowtie}$ such that this classes are J-syntactically similar (in the meaning of above Definition* 2).

All concepts that are not defined in this abstract can be extracted from [2, 1].

[1] M. T. Kassymetova and A. R. Yeshkeyev, ***Jonsson Theories and their Classes of Models***, KarGU, Karaganda, 2016, in Russian.

[2] T. G. Mustafin, *On similarities of complete theories*, ***Logic Colloquium '90: Proceedings of the Annual European Summer Meeting of the Association for Symbolic Logic***, Helsinki, 1990, pp. 259–265.