Religious Studies (2024), **60**, 706–716
doi:10.1017/S0034412524000064

BOOK SYMPOSIUM

Falsity and untruth

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(Received 3 February 2024; accepted 3 February 2024; first published online 29 April 2024)

Abstract

Jc Beall's *Divine Contradiction* is a fascinating defence of the idea that contradictions are true of the tri-personal God. This project requires a logic that avoids the consequence that every proposition follows from a contradiction. Beall presents such a logic. This 'gap/glut' logic is the topic of this article. A gap/glut logic presupposes that falsity is not simply the absence of truth – for a proposition that is true may also be false. This article is essentially an examination of the idea that falsity is not simply untruth. The author rejects this position but does not claim to have an argument against it. In lieu of an argument, he presents three 'considerations'. First, it is possible to give an intuitive semantics for the language of sentential logic that yields 'classical' sentential logic (including '$p, \neg p \vdash q$') and which makes no mention of truth-values. Second, it is possible to imagine a race who manage their affairs very well without having the concept 'falsity'. Third, it is possible to construct a semantics that yields a logic identical with the dialetheist logic and which makes no mention of truth-values – and which, far from being plausible, seems pointless.

Keywords: logic; falsity; untruth; glut; contradiction

I will understand the *glut theory*[1] to be the twofold thesis that

(i) Every proposition falls into one of four categories:
'true and not false'
'false and not true'
'true and false'
'not true and not false'

and

(ii) If any of these four categories is empty, that is more than logic (the science of formally valid inference) can tell us. That is, if, for example, no proposition is both true and false, that is not a logical truth but a truth whose foundation lies in some less general science or discipline than logic – physics, perhaps, or mathematics, or metaphysics.[2]

It is important to understand that, from the point of view of the glut theory, a proposition that is both true and false is true without qualification and false without qualification. A proposition that is both true and false is 'just as true' as a proposition that is true and not false and just as false as a proposition that is false and not true. A proposition that is neither true nor false, moreover, is not a proposition that (as it were) lies in the grey area between truth and falsity. A proposition that is neither true nor false is just as untrue as a proposition that is false and not true and just as 'unfalse' as a proposition that is true and not false.

It is widely supposed that if there is a proposition that is both true and false, then all propositions are true. Arguments for this thesis are all variants of one sort or another on the following schematic argument:

1.	The proposition that p is true	*Assumption for conditional proof*
2.	The proposition that p is false	*Assumption for conditional proof*
3.	The proposition that p is true $\rightarrow p$	*Premise*
4.	The proposition that p is false $\rightarrow \neg p$	*Premise*
5.	p	*(1), (3) MP*
6.	$\neg p$	*(2), (4) MP*
7.	$p \vee q$	*(5) Addition*
8.	q	*(6),(7) Disjunctive Syllogism*
9.	$q \rightarrow$ the proposition that q is true	*Premise*
10.	The proposition that q is true	*(8), (9) MP*

11. The proposition that p is true. $\wedge$ the proposition that p is false: $\rightarrow$ the proposition that q is true
(1)–(10) Conditional Proof

(This consequence, or alleged consequence, of there being a proposition that is both true and false is sometimes called 'Explosion'.) Since no glut theorist wishes to accept either of the theses

> For every sentence x and every sentence y, the argument whose premises are x and the denial of x, and whose conclusion is y is logically valid

> For every sentence x and every sentence y, the argument whose premises are ⌜the proposition that x is true⌝ and ⌜the proposition that x is false⌝ and whose conclusion is ⌜the proposition that y is true⌝ is logically valid,

glut theorists require a sentential logic according to which the inference-form

$$p, \neg p \vdash q$$

is invalid. And they also require a logic according to which at least one of the inferences involved in the 'Explosion' argument is invalid.[3]

Jc Beall has devised a semantics for the language of first-order logic that has these features: according to this semantics '$p, \neg p \vdash q$'[4] and Disjunctive Syllogism and Modus Ponens are invalid.[5] (Addition, however, is valid.)

In the case of sentential logic, Beall's semantics is based on the assignments of, so to call them, 'truth-value combinations' – 'true and not false', 'false and not true', 'true and false', and 'not true and not false' – to the sentence-letters that occur in formulas. (I shall find it convenient to refer to each such assignment as a *model*, although Beall does not use the term.) I will present a formulation of this semantics, but I will call the semantics I present 'the Glut-theoretical Semantics', rather than (say) 'Beall's semantics', since I

https://doi.org/10.1017/S0034412524000064 Published online by Cambridge University Press

have formulated it using language that is not his (and also because the semantics that follows treats only of sentential logic).[6]

The Glut-theoretical Semantics

The following rules of the familiar, orthodox 'glut free' semantics for sentential logic are also rules of the Glut-theoretical Semantics:

- the negation of a formula is true if and only if that formula is false
- the negation of a formula is false if and only if that formula is true
- a disjunction is true if and only if at least one of its disjuncts is true
- a disjunction is false if and only if both its disjuncts are false
- a conjunction is true if and only if both its conjuncts are true
- a conjunction is false if and only if at least one its conjuncts is false.

And the validity and invalidity of inference-forms are defined by this rule:

> If the premises of an inference-form are true on a certain model, and its conclusion is not true,[7] that model is a *countermodel* to that inference-form. An inference-form is valid if none of its models are countermodels, and invalid otherwise.[8]

Consider, for example, the inference-form '$p, \neg p \vdash q$'. Suppose we assign 'true and false' to 'p' and 'untrue and false' (that is, false and *not* also true) to 'q' – a model we may represent in this manner:

$$p^{\mathrm{tf}}, \neg\, p^{\mathrm{tf}} \vdash q^{\times\mathrm{f}}.$$

The first of our two negation rules tells us that '$\neg p$' is true if and only if 'p' is false; and, therefore, since 'p' is both true and false, '$\neg p$' is true. The second of our two negation rules tells us that '$\neg p$' is false if and only if 'p' is true; and, therefore, since 'p' is both true and false, '$\neg p$' is false. '$\neg p$' is therefore both true and false, and we may write

$$p^{\mathrm{tf}}, \neg^{\mathrm{tf}}\, p^{\mathrm{tf}} \vdash q^{\times\mathrm{f}}$$

Our model is therefore a countermodel: both the premises of '$p, \neg p \vdash q$' are true and false, and its conclusion is false and not true – which implies that both the premises of '$p, \neg p \vdash q$' are true, and its conclusion is not true. '$p, \neg p \vdash q$' is therefore invalid.

Consider next Disjunctive Syllogism:

$$p \vee q, \neg\, p \vdash q.$$

This model of Disjunctive Syllogism

$$p^{\mathrm{tf}} \vee q^{\times\mathrm{f}}, \neg\, p^{\mathrm{tf}} \vdash q^{\times\mathrm{f}}$$

is a countermodel:

$$p^{\mathrm{tf}} \vee^{\mathrm{tf}} q^{\times\mathrm{f}}, \neg^{\mathrm{tf}}\, p^{\mathrm{tf}} \vdash q^{\times\mathrm{f}}$$

and Disjunctive Syllogism is invalid.
Consider next Modus Ponens:

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$p, p \rightarrow q \vdash q.$

(We shall regard '$p \rightarrow q$' as a notational variant on '$\neg\, p. \vee q$'.) The model

$p^{\text{tf}}, \neg\, p^{\text{tf}}. \vee q^{\text{xx}} \vdash q^{\text{xx}}$

is a countermodel:

$p^{\text{tf}}, \neg^{\text{tf}}\, p^{\text{tf}}. \vee^{\text{tx}}\, q^{\text{xx}} \vdash q^{\text{xx}}.$

Now one might protest that a 'logic' that offers its users neither Disjunctive Syllogism nor Modus Ponens is too meagre a system of inference to deserve the name 'logic'. To this the dialetheist will reply that (a) there are fewer logical validities than you had thought there were; get over it, and (b) . . . well, (b) is a little complicated and I'd better give it a paragraph all to itself.

> Consider those counter-models to an inference-form that assign only 't×' or '×f' to its constituent sentence-letters. Let's call them *strict* counter-models to that inference-form. If an inference-form has no strict countermodels, say that it is widely valid. (The widely valid inference forms are precisely those that are valid according to the standard or classical truth-tabular semantics for sentential logic.) Let us say that 'wide' logic is the science of widely valid reasoning. Anyone who believes that every proposition is either true or false and that no proposition is both is free to use wide logic in the place of logic – of logic in the strict and philosophical sense. Such a reasoner, however, is (in effect) reasoning from an extra-logical premise: that every proposition is either true or false, and that no proposition is both.

Who is right? The glut theorist or that motley whom Gertrude Stein referred to as Most People?

I myself am a Most Person, but I know of no way to demonstrate that Most People are right. But one thing at least is clearly true: the glut theory can be right only if the idea of a proposition that is both true and false makes sense.

It seems to me, Most Person that I am, that this idea doesn't make sense. I have to admit, however, that that's not an argument – strongly tempted to think that it is though I am. I have to admit that I have no argument. All I have are, well, considerations. I'll do my best to attempt to articulate them, to try to frame an explicit statement of what lies behind my conviction that the idea of something that is true being also false makes no sense.

One way to do this is to present a plausible, intuitive semantics that confers validity on the same inference-forms as the Received Semantics (the logic-text truth-tabular semantics), but doesn't proceed by assigning single, 'exclusive' truth-values to sentence letters – and therefore, at least, doesn't *begin* by contradicting the glut theory. I'll present such a semantics, which I'll call the Received Semantics Variant – Rvar for short.

The Received Semantics Variant

We have a (non-empty) domain S.

A *model* for a formula in the language of sentential logic assigns a subset of S (it may be empty) to each sentence-letter of that formula.

If a model assigns a set to a formula, it assigns that set's complement on S to the negation of that formula.

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A model assigns to a disjunction the union of the sets it assigns to its disjuncts.

A model assigns to a conjunction the intersection of the sets it assigns to its conjuncts.

A *model for the inference-form f* assigns a subset of S to each sentence-letter in the premises and the conclusion of *f* (consistently across its premises and conclusion).

A *countermodel* to an inference-form is a model of that inference-form such that for some *x*, *x* is a member of every set the model assigns to a premise of the inference-form and *x* is not a member of the set the model assigns to its conclusion – that is, if the intersection of the sets the model assigns to the premises of the inference-form is not a subset of the set it assigns to its conclusion.

An inference-form is *Rvar-valid* if it has no countermodels, and *Rvar-invalid* if it has countermodels.

Rvar is equivalent to the Received Semantics. For example, Disjunctive Syllogism

$$p \vee q, \neg p \vdash q$$

is an RVar-valid inference-form. For, obviously, if $\boldsymbol{\alpha}$ is the set a model assigns to '*p*', and $\boldsymbol{\beta}$ is the set it assigns to '*q*', any item that belongs to the union of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ and to the complement (on S) of $\boldsymbol{\alpha}$ will belong to $\boldsymbol{\beta}$. It is easy to verify that Modus Ponens and Addition ($p \vdash p \vee q$) are also RVar-valid. The RVar-validity of the inference-form

$$p \wedge \neg p \vdash q$$

is also obvious, for if $\boldsymbol{\alpha}$ is the set a model assigns to '*p*', and $\boldsymbol{\beta}$ is the set it assigns to '*q*', then every item that belongs both to $\boldsymbol{\alpha}$ and to its complement on S (that is, every item that belongs to the empty set) will be a member of $\boldsymbol{\beta}$. It should be intuitively evident that RVar and the standard, truth-tabular semantics for sentential logic confer validity on the same formulas.

As to invalidity, consider the notoriously invalid inference-form 'Affirming the Consequent':

$$p \rightarrow q, q \vdash p.$$

(That is, '$\neg p \vee q, q \vdash p$)'. Any model that assigns a non-empty set to '*q*' and a proper subset of that set to '*p*' is a counter-model to '$p \rightarrow q, q \vdash p$'.

The intuitive idea behind RVar is a simple, and, to me, compelling one. Think of S as the set of all possible worlds. Identify propositions and sets of possible worlds. Define negation, disjunction, and conjunction in the obvious ways in terms of complement, union, and intersection. Say that a true proposition is one that has the actual world as a member. A proposition *P* will then entail a proposition *Q* just in the case that *P* is a subset of *Q* – for in that case, if *P* is true, *Q* must also be true no matter which world the actual world is. Say that an inference or argument is valid just in the case that the conjunction of its premises entails its conclusion.

'But your statement of the "intuitive idea behind RVar" doesn't include a definition of falsity. And I'll need to see one. For I'm inclined to respond to your statement of the intuitive idea in this way:

> Suppose that – although the worlds in which the proposition *P* is true are a subset of the worlds in which the proposition *Q* is true – some of the worlds in which *Q* is true are also worlds in which *Q* is false. And suppose that some of those worlds, the worlds in which *Q* is both true and false, are worlds in which *P* is true. Then, according to the intuitive idea, *P* entails *Q* – albeit the truth of *P* is consistent with the falsity of *Q*.

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To evaluate this objection, we must know whether the "intuitive idea" allows a proposition to be both true and false. And to do that, we must supplement your statement of the intuitive idea with a definition of falsity. And what could it be but this: a proposition is false if it does not include the actual world? But then every proposition is either true and not false or false and not true. These reflections – surely? – show that, although you made no mention of truth-values when you were setting out RVar, Rvar *presupposes* that no proposition is both true and false.'

Well, that is not surprising. After all, RVar is inconsistent with the Glut-theoretical Semantics (in the sense that it draws the bounds of logical validity in a different place). But I wonder . . . suppose I were simply to drop the word 'false' from my metalinguistic vocabulary. I don't really need it, for I have 'not true' and 'untrue'. If, in fact, I were asked to define 'false' (in circumstances in which I did not feel it necessary to respect the scruples of the glut theorists), I'd define 'false' as 'not true'.

I do not, I say, need 'false'. But the proponent of the Glut-theoretical Semantics certainly *does* need 'false'. That this is so can be seen by examining these two statements:

> If the premises of an inference-form are true on a certain model, and its conclusion is not true, that model is a *countermodel* to that inference-form.

> Let X and A be a set of sentences and, respectively, a sentence from a given language. A counterexample to the pair ⟨X, A⟩ is a relevant possibility in which every sentence in X is true but A is untrue.[9]

The first is from my exposition of the Glut-theoretical Semantics. The second is the corresponding statement in Beall's actual text.

The difference between these two statements, on the one hand, and, on the other, the statements that would result from replacing 'not true' in the former or 'untrue' in the latter with 'false', is enormous. The semantics that would result from that replacement would be very different from the Glut-theoretical Semantics. The vocabulary of the Glut-theoretical Semantics must contain both 'true' and 'false'. The vocabulary of RVar contains neither 'true' nor 'false'. (And the statement of the 'intuitive idea behind RVar' contains only 'true'.)

In defence of the identification of falsity with untruth, I might invoke the Martians – those Martians whose linguistic habits I have more than once appealed to.

A Martian scholar who was asked by a Terrestrial linguist to explain the meaning of '*batanaga*', a word of Argyre Basin Recent, the principal Martian language, responded as follows (a translation of his AB Recent statement):

> The word is an adjective that applies to statements or propositions or beliefs or theories. It cannot be given a general definition, for the idea it expresses is so fundamental and pervades all other ideas to such an extent that there are no words that can be used to provide it with a non-trivial and non-circular definition. But one *can* say some useful things about its meaning. One can say, for example, that the proposition that Mars is smaller than the earth is *batanaga* because Mars *is* smaller than the earth. And one can say that the belief – far too common among my fellow Argyre Basiners – that the crimson fever is caused by the consumption of *arrara* mould is not *batanaga* because the crimson fever is *not* in fact caused by the consumption of *arrara* mould. Will you understand me if I present no further examples but say only 'and so on'? And the purpose of the word? Why, it enables one to make *general* statements like, 'Everything said by Kaa-vinagbiiyaan in his *Discourses* is

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> *batanaga*' and 'Many of the things people believed in the 9884th century[10] were not *batanaga*.'

It is clear that '*batanaga*' means 'true'. All eleven of the surviving Martian languages – eight of them apparently historically unrelated to Argyre Basin Recent – have a similar word. And yet, it transpires, neither AB Recent nor any of the other Martian languages has a word that means 'false' if 'false' means anything other than 'untrue'. In AB Recent, for example, one can say '*batanaga kan*' (not true) or '*batanaga-ka*' (untrue), but that is the closest one can come to saying 'false'. 'True and not false' can be translated into a Martian language only as 'true and not not true', 'false and not true' only as 'not true and not true', 'neither true or false' only as 'not true and not not true', and 'both true and false' only as 'true and not true'.

And what is it – if anything – that the Martians lose by not having a word that means 'false' (assuming that the *English* word 'false' does not mean simply 'untrue')? Whatever it may be (if it exists), it is of no *practical* import. The absence of a word that means 'false' is no burden to Martian practice, whether in the ordinary business of life, or in the sciences. It may, of course, have some baleful effect on Martian philosophy. It certainly renders the glut theory difficult for the Martians to grasp. The following exchange was recorded at a joint Terrestrial–Martian philosophical congress. (In this exchange, a human philosopher, speaking in English, speaks first. Then a Martian philosopher who speaks only the Martian language Tharsis Rise East II responds to this statement. She is able to respond because her DeeperL earpiece has translated the speaker's English into ThREII. (However, DeeperL can do no better with 'false' than '*kinbanaca*' – that is, 'not true' or 'untrue'.) The Martian philosopher's response is, of course, in ThREII – which the human speaker's DeeperL earpiece translates into English for him.

The human philosopher's English statement

> The proposition that the moons Titan and Ganymede are larger than the planet Mercury is true and not false.

The DeeperL English translation of the Martian
Philosopher's response in Tharsis Rise East II

> I don't understand why anyone would say 'true and not not true'. If a proposition is true, then of course it isn't not true.

The Martians complain that they simply cannot see what the Terrestrial glut theorists think they're saying – for everything they say (everything they say that isn't a thing anyone might say) seems to be either self-contradictory or to contain pointless and inexplicable redundancies. Matters aren't helped when the dialetheists reply that there's nothing *per se* wrong with saying things that are self-contradictory.

It's a matter of choosing sides, I suppose. Which side are you on? Are you with the dialetheists or the Martians?

I quote remarks that I have made in the course of two previous appeals to the linguistic practices of the Martians (the first altered so as to apply to the present case).

> Such perversity led to much head-shaking among Terrestrial logicians of a glut-theoretical persuasion over the philosophical limitations of the Martian mind. Fortunately, however, it had no untoward practical consequences, since the Martians and the Terrestrials always agreed about which arguments had 'true and not false' conclusions if all their premises were 'true and not false'.[11]

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> Is a Martian language a sort of logical Newspeak? Is a Martian language a language in which certain thoughts simply cannot be expressed (and no wonder, for Martian languages are languages imagined by someone – myself – who thinks there are no such thoughts), or is it a language whose logical clarity makes certain semantical delusions impossible for its speakers? This is a question that any philosophers who are wondering whether to accept the glut theory must answer for themselves. I have to say that I'm a Martian – by philosophical conviction if not by biological ancestry.[12]

I will close by pointing out that the fact that a non-trivial logic with a glut-theoretical semantics is possible is not an argument for the truth of the glut theory. This is because the *meanings* stipulated for '(t×)', '(×f)', (tf), and '(××)' – 'true and not false', and so on – really play no part in the work that the semantics does (the work, that is, of separating inference-forms into two classes, one of which will be labelled 'valid' and the other 'invalid').

The following semantics for the language of sentential logic partitions the inference-forms expressible in that language exactly as the Glut-theoretical Semantics does.

The pointless semantics

An assignment of one of the four numbers

1 2 3 6

to each sentence-letter that occurs in a formula of the language of sentential logic is a *model* for that formula. Thus

$$(p^6 \wedge \neg q^2) \vee (\neg p^6 \wedge q^2).$$

is a model for '$(p \wedge \neg q) \vee (\neg p \wedge q)$'.

A model for a formula assigns one of the numbers 1, 2, 3, and 6 to that formula according to the following rules.

The number of the negation of a formula is even
if and only if
the number of that formula is evenly divisible by 3 (or 'is threven')

The number of the negation of a formula is threven
if and only if
the number of that formula is even

The number of a disjunction is even
if and only if
the number of at least one of its disjuncts is even

The number of a disjunction is threven
if and only if
the number of each of its disjuncts is threven

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The number of a conjunction is even
if and only if
the number of both its conjuncts is even

The number of a conjunction is threven
if and only if
the number of at least one of its conjuncts is threven

Consider, for example, the formula '$\neg p$'. If a model assigns 1 to the sentence letter 'p', it assigns 1 to '$\neg p$' – for 1, the number assigned to 'p', is neither even nor threven, and the number assigned to '$\neg p$' is therefore neither threven nor even. And 1 is the only one of 1, 2, 3, and 6 that is neither threven nor even. If a model assigns 2 to the sentence letter 'p', it assigns 3 to '$\neg p$' – for in that case, the number assigned to 'p' is even and not threven, and the number assigned to '$\neg p$' is therefore threven and not even. And 3 is the only one of 1, 2, 3, and 6 that is threven and not even. We have:

$$\neg^1 p^1 \qquad \neg^3 p^2 \qquad \neg^2 p^3 \qquad \neg^6 p^6$$

Consider now the model for '$(p \wedge \neg q) \vee (\neg p \wedge q)$' that was briefly mentioned above:

$$(p^6 \wedge \neg q^2) \vee (\neg p^6 \wedge q^2).$$

Using our rules, we may, starting with the numbers of the sentence-letters 'p' and 'q', work our way 'outward' through the sub-formulas of '$(p \wedge \neg q) \vee (\neg p \wedge q)$', finally to discover the number the model assigns to '$(p \wedge \neg q) \vee (\neg p \wedge q)$':

$$(p^6 \wedge \neg^3 q^2) \vee (\neg^6 p^6 \wedge q^2)$$
$$(p^6 \wedge^3 \neg^3 q^2) \vee (\neg^6 p^6 \wedge^6 q^2)$$
$$(p^6 \wedge^3 \neg^3 q^2) \vee^6 (\neg^6 p^6 \wedge^6 q^2).$$

And so for any model of any formula.

A model for a *sequence* of formulas is an assignment of one of 1, 2, 3, and 6 to each sentence-letter that occurs in the sequence. For example:

$$(p^6 \wedge \neg q^2) \vee (\neg p^6 \wedge q^2) \vdash \neg (p^6 \wedge q^2).$$

A model for a sequence thus assigns a number to each of its terms. This model for this inference-form, for example, assigns 6 to its only premise (as we have just now seen) and assigns 6 to its conclusion:

$$(p^6 \wedge^3 \neg^3 q^2) \vee^6 (\neg^6 p^6 \wedge^6 q^2) \vdash \neg^6 (p^6 \wedge^6 q^2).$$

A model is a *counter-model* to an inference-form x just in the case that it assigns an even number to each of the premises of x and does not assign an even number to x's conclusion. An inference-form is *valid* if it has no counter-model. An inference-form that is not valid is *invalid*.

Here, for example, are counter-models to three inference forms that are classically valid, but invalid in the Glut-theoretical Semantics:

$$p^6 \wedge^6 \neg^6 p^6 \vdash q^3$$
$$p^6 \vee^6 q^3; \neg^6 p^6 \vdash q^3$$
$$p^2 \vdash q^1 \vee^1 \neg^1 q^1.$$

In the case of '$p \wedge \neg p \vdash q$', the display represents the model that assigns 6 to 'p' and 3 to 'q'. Since the model assigns an even number to 'p', it assigns a threven number to the negation of 'p'. Since the model assigns a threven number to 'p', it assigns an even number to the negation of 'p'. The only one of 1, 2, 3, and 6 that is both threven and even is 6. Therefore, the model assigns 6 to the negation of 'p'. The model therefore assigns 6 to both conjuncts of '$p \wedge \neg p$'. The number of both conjuncts is thus even, and the number of the conjunction is even. And number of both (and hence the number of at least one of) the conjuncts is threven, and the number of the conjunction is threven. The only one of 1, 2, 3, and 6 that is both even and threven is 6. Therefore, the model assigns 6 to the conjunction of 'p' and its negation, and assigns an even number to the premise of the argument and a number that is not even to its conclusion. This model is therefore a counter-model, and the inference is invalid.[13]

Notes

1 In the earliest drafts of this article, I gave the name 'dialetheism' to the thesis I am now calling the glut theory. I have changed 'dialetheism' to 'the glut theory' at Professor Beall's request. For the reason why Professor Beall does not wish his theory to be called 'dialetheism', see Beall (2022).

2 For an excellent brief introduction to a logic that permits 'gluts' (as well as a very interesting application of this logic), see Priest (2019).

3 Assuming, that is, that they accept its three premises. What glut theories should say about 'Convention T' sentences is an interesting question that I choose not to discuss in the present article.

4 If '$p, \neg p \vdash q$' is invalid, it follows that at least one of the two inference-forms that occur in the annotations to lines 7 and 8 of the Explosion argument is invalid. But the logic the glut theorists require should tell us in each case whether a given inference-form is valid.

5 See chapter 2, 'Logical and Extra-logical Entailment' of Beall (2023). This book, like the earlier Beall (2021), is an essay in theology, but the present article addresses only the logic of which the theological portion of the book is an application.

6 I chose the name 'the Glut-theoretical Semantics' simply to provide the semantics with a name. I don't mean the name to imply that there could not be other glut-theoretical semantics.

7 Note that the rule says 'not true' and not 'false': if the premises of an inference-form on a model m are (say) true and not false, and its conclusion is true and false, m is not a counter-model to that inference-form.

8 If a valid inference-form has a single premise, and if that premise is a single 'stand alone' sentence-letter that does not occur in the conclusion of the inference, that conclusion is called *a theorem of logic*.

9 Beall (2023, 26–27).

10 The word translated 'century' actually denotes 144 Martian years – a bit more than 270 Terrestrial years.

11 Cf. van Inwagen (2014, 22).

12 Cf. van Inwagen (2023, 9).

13 It would of course be easy enough to construct (using essentially the same technique) a pointless semantics that yielded a logic identical with classical logic. That shows that the fact that a non-trivial logic with a classical semantics is possible is not an argument for the validity of the classical truth-table semantics for sentential logic – a thesis I accept.

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Cite this article: van Inwagen P (2024). Falsity and untruth. *Religious Studies* **60**, 706–716. https://doi.org/10.1017/S0034412524000064