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Magnetic island evolution in rotating plasmas

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Abstract. The time evolution of the magnetic island formed at the tearing stable rational surface by the external magnetic flux perturbation in the plasma with poloidal flow is investigated numerically by using the resistive magnetohydrodynamic (MHD) model and the magnetic island is found to show rapid growth after the reduced growth phase due to the plasma rotation. It was also found that the onset condition of this rapid growth depends on the resistivity, but does not depend on the viscosity, in the parameter regime used in this study. On the other hand, the time constant of the rapid growth phase is almost independent of both the plasma resistivity and the viscosity. This rapid growth of magnetic island is the possible candidate for the trigger problem of the neoclassical tearing mode in a tokamak.

1. Introduction

It has been well understood that, in tokamak plasmas, magnetic islands resonant with the low q rational surface deteriorate the plasma confinement. In particular, in a high-performance tokamak aiming at the steady state fusion reactor, island formation is the critical issue. Hence, the suppression and control of magnetic islands are urgent subjects in tokamak fusion research. There are two origins of the magnetic island. One is due to the unstable tearing mode and the other is the forced magnetic reconnection. The latter process is also the origin of the seed island, which is important to the neoclassical tearing mode (NTM). For example, magnetohydrodynamic (MHD) instability such as the sawtooth oscillation acts as the external perturbation for the mode with different helicity. However, the relationship between the externally applied perturbation and the onset of the driven magnetic island is not clear. In this paper, we study the time evolution of the magnetic island due to the growing external perturbation in rotating tokamak plasmas.

To study the magnetic island evolution in a tokamak, we employ the reduced set of resistive MHD equations for a low β in cylindrical coordinate (r, θ, φ) [1] as

$$\frac{\partial U}{\partial t} = \frac{1}{r}[U, \phi] + \frac{1}{r}[\psi, j] + \frac{B_0}{R_0}\frac{\partial j}{\partial \varphi} + \nu \nabla_\perp^2 (U - U_0)$$

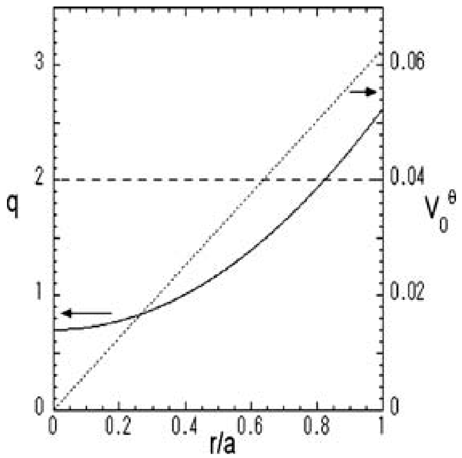

Figure 1. Safety factor (solid curve) and the poloidal background flow profile (dotted curve).

$$\frac{\partial \psi}{\partial t} = \frac{1}{r}[\psi, \phi] + \frac{B_0}{R_0}\frac{\partial \phi}{\partial \varphi} + \eta j - E,$$

where

$$j = \frac{\partial^2}{\partial r^2}\psi + \frac{1}{r}\frac{\partial}{\partial r}\psi + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\psi, \quad U = \frac{\partial^2}{\partial r^2}\phi + \frac{1}{r}\frac{\partial}{\partial r}\phi + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\phi,$$

$$[a, b] = \frac{\partial a}{\partial r}\frac{\partial b}{\partial \theta} - \frac{\partial b}{\partial r}\frac{\partial a}{\partial \theta}.$$

Here, ψ is the poloidal flux function, ϕ is the flow potential, η is the resistivity, ν is the viscosity, j is the toroidal current density, u is the vorticity, E is the electric field at the wall, B_0 is the toroidal magnetic field, and R_0 is the major radius. In this study, the time t is normalized by the poloidal Alfvén transit time and the length r is normalised by the plasma minor radius. In the following sections, we consider only the MHD activity with helical symmetry. In this study, the background plasma flow is considered by including the flow potential of

$$\phi_{0/0} = -\frac{2\pi}{\lambda\tau}(1 - r^\lambda), \quad \lambda = 2.$$

This flow potential profile corresponds to the rigid rotation. The background flow, V_0^θ profile and the safety factor, q, profile are shown in Fig. 1. We consider only the $q = 2$ rational surface, where the tearing mode is stable. The external magnetic perturbation is applied at the plasma edge in the following form:

$$\psi_{2/1}(r = a) = \dot{\psi} \cdot (t - t_0) \cdot \psi_{0/0}(r = a), \tag{1.1}$$

where $\dot{\psi}$ is constant and set to 10^{-6} in this study, $\psi_{0/0}(r = a)$ is the $m/n = 0/0$ harmonics of ψ at $r = a$ and $t = 0$.

2. Numerical results

Figure 2 shows the time evolution of magnetic island for the cases with and without the background flow. In the case without background flow, the magnetic island

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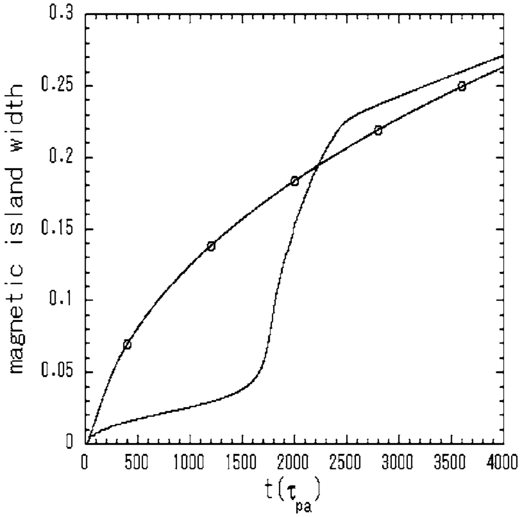

Figure 2. Time evolution of the magnetic island width for the cases with flow, $\tau = 100$ (curve with points and without flow) (curve without points).

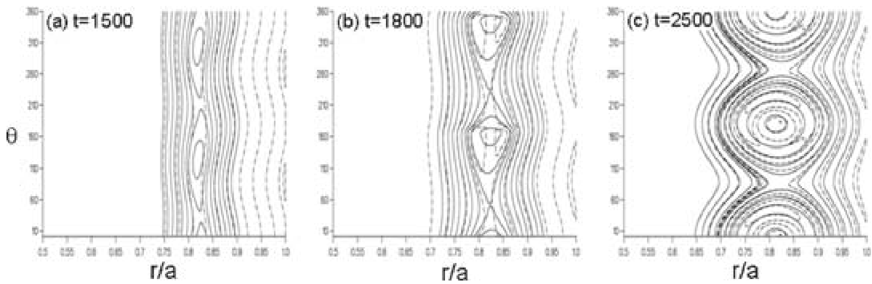

Figure 3. Contour plots of magnetic flux function (solid lines) and flow potential (dotted lines) at (a) $t = 1500$, (b) $t = 1800$ and (c) $t = 2500$ for $\eta = 10^{-5}$.

grows monotonically with the increasing externally applied magnetic flux, as shown in Fig. 2. The island width increases with the square root of the time, that is, the amplitude of the external applied field. This feature resembles the Rutherford type regime of the unstable tearing mode [2]. On the other hand, in the case with the background flow, the growth of magnetic island is typically divided into three phases. In the first phase, the magnetic island growth is reduced by the background flow until $t \approx 1500$; then, in the second phase, the magnetic island grows rapidly from $w_{\text{island}} \approx 0.04$ to $w_{\text{island}} \approx 0.22$ during the short time interval of $\Delta t \approx 900$. The third phase after this rapid growth is almost the same as in the case without the background flow. The suppression of island width in the first phase is roughly consistent with the previous theoretical works [3, 4]. In the previous work, the stabilization effect was studied from the view point of the island deformation. Figure 3 shows the contour plots of the poloidal flux function and the flow potential in the case with the background flow. At $t = 1500$ just before the rapid magnetic island growth, the magnetic island is deformed with island axis shifted to the

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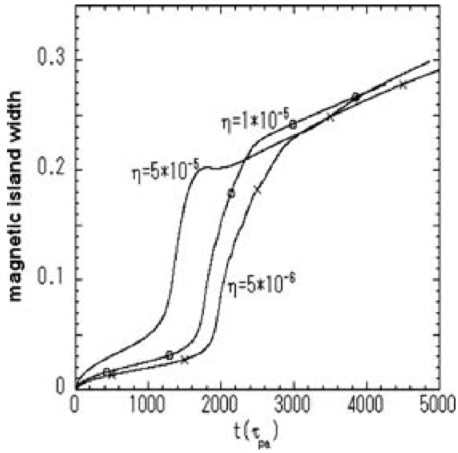

Figure 4. Time evolution of the magnetic island width for different resistivities, $\eta = 5 \times 10^{-5}$, 1×10^{-5} and 5×10^{-6}.

poloidal direction, as shown in Fig. 3(a). The second phase of the rapid growth of magnetic island may correspond to the abrupt change, or the bifurcation, of the island width shown in Fig. 2. In the second phase, the magnetic island is further deformed with increasing island size, while the poloidal phase of magnetic island comes back to that of the external field, as shown in Fig. 3(b). In the third phase, the magnetic island becomes almost symmetric in the poloidal direction and is in the phase of the external field. During the magnetic island growth, the flow potential, which is localized within the magnetic island, changes from the dipole profile (Fig. 3(b)) to almost the flux function (Fig. 3(c)), as the background flow decreases around the $q = 2$ rational surface. This relation between the magnetic island and the flow potential profile in the third phase is consistent with the Rutherford-type evolution. As for the transition from the first to the second phase, or the destabilization of magnetic island, the previous theoretical works [3, 4] show that the critical value of the island width depends not only on the resistivity, η, but also on the viscosity, ν. To investigate this feature, we have performed a series of numerical calculations for different values of η and ν. In these simulations, the increasing rate of the externally applied magnetic flux is kept constant from $t = 10$ to $10\,000$. Hence, the trigger timing of the rapid growth of the magnetic island is proportional to the critical magnetic flux. Figure 4 shows the time evolution of the magnetic island width for the different resistivities, $\eta = 5 \times 10^{-5}$, 1×10^{-5} and 5×10^{-6}. As shown in Fig. 4, the trigger timing to the second phase is delayed as η decreases.

Hence, the critical value is inversely proportional to the resistivity, $\psi_c \sim \eta^{-\alpha}$. By changing the resistivity, the growth rate of the magnetic island, γ_w, in the first phase is also changed, where γ_w is approximately proportional to the resistivity. During the second and also the third phases, however, γ_w does not clearly depend on η. By assuming $\gamma_w \sim \eta^{\beta}$, $\beta \approx 0.408$ in the first phase and $\beta \approx 0.053$, which is estimated by the maximum value of the temporal growth rate, in the second phase. This result shows that during the first phase, the magnetic island gradually grows, keeping the force balance with the background flow, but in the rapid growth phase, the force balance is lost and the magnetic island grows almost independently of the

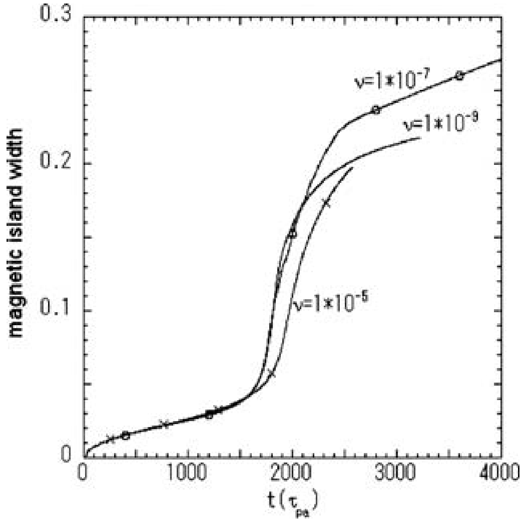

Figure 5. Time evolution of the magnetic island width for different viscosities, $\nu = 10^{-9}$, 10^{-7} and 10^{-5}.

resistivity, η. Figure 5 shows the time evolution of the magnetic island width for the different viscosities, $\nu = 1 \times 10^{-5}$, 1×10^{-7} and 1×10^{-9}. As shown in Fig. 5, the trigger timing and also the growth rate does not change by much even though the viscosity is changed by four orders of magnitude from $\nu = 1 \times 10^{-5}$ to 1×10^{-9}. This means that, in this parameter region, the critical value ψ_c depends very weakly on the viscosity ν. Even in these cases, γ_w does not clearly depend on the viscosity during the second phase.

3. Summary

In this paper, we investigated numerically the time evolution of the magnetic island, which is induced at the tearing stable resonant surface by applying the external magnetic perturbation at the plasma edge, in the rotating plasma. We confirmed the stabilization effects of the background flow on the magnetic island evolution and also the existence of the critical value, beyond which the magnetic island grows rapidly. Simulation results show that the critical value depends on the resistivity, $\psi_c \sim \eta^{-\alpha}$, but does not depend on the viscosity, ν, in the parameter region used in this study. In the rapid growth phase (the second phase), the magnetic island growth rate depends weakly on both the resistivity η and the viscosity ν. In order to clarify the time scales in each phase and its mechanisms, it is necessary to perform the numerical simulation in wider parameter regime and to reconsider the stabilization mechanism of the background flow on the magnetic island evolution, especially in their time evolution.

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