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INFLATION AND UNEMPLOYMENT IN COMPETITIVE SEARCH EQUILIBRIUM

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Rocheteau, Rupert, and Wright [*Scandinavian Journal of Economics* 109 (2007), 837–855] show that the relationship between inflation and unemployment can be positive or negative, depending on the primitives of the model. The key features are indivisible labor, nonseparable preferences, and bargaining. Their results are derived only for a special case of bargaining, take-it-or-leave-it offer by buyers. Instead of bargaining, this paper considers competitive search. I show that the results of Rocheteau et al. can be generalized to an environment where both buyers and sellers have nonseparable preferences. In addition, the relationship between inflation and unemployment is robust to allowing free entry by sellers, which cannot be studied in Rocheteau et al. (2007).

Keywords: Inflation, Unemployment, Competitive Search

1. INTRODUCTION

Rocheteau et al. (2007), hereafter RRW, study a model where both unemployment and the role of money have explicit microfoundations. They show that the relationship between anticipated inflation and unemployment need not be zero, even in the long run, as predicted by the theory of the expectations-augmented Phillips curve, but may be positive or negative depending on the utility functions of agents. Unemployment in RRW is due to indivisible labor, as in Rogerson (1988), whereas the role of money is modeled using the search-and-bargaining approach, as in Lagos and Wright (2005).[1] However, RRW are only able to prove their main results for a very special case of the bargaining solution, take-it-or-leave-it offers by buyers. As RRW themselves put it, "we can only prove the main results for $\theta = 1, \ldots$ This is somewhat unfortunate, however, since $\theta = 1$ does preclude many interesting extensions." In particular, when buyers have all the bargaining power, one can never add ex ante investments by sellers, including standard capital accumulation, costly search, or entry-participation decisions by sellers.

This paper develops a similar model, where unemployment is again due to Rogerson's indivisible labor specification, but with a different assumption

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concerning the pricing mechanism in decentralized monetary exchanges—I use competitive search, which combines price posting (instead of bargaining) and directed (instead of random) search.[2] There are several reasons that competitive search is an interesting pricing mechanism. First of all, one can argue that in many situations, price posting is more realistic than bargaining, and directed search is more realistic than random search. At the very least, competitive search avoids some criticism of modern monetary theory by people who dispute the appropriateness of random search and bargaining. Second, it is analytically tractable and often allows one to prove stronger or more general results than bargaining models, as is the case in this paper. Third, it is an efficient pricing mechanism: absent of distortionary policies, competitive search equilibrium generates the first-best allocation, whereas bargaining equilibrium typically does not.

I show that in this model the key results in RRW can be proved without their extreme assumption on bargaining power. As in RRW, each period consists of a centralized market and a decentralized monetary exchange. Employment takes place only in the centralized market. If goods consumed in the centralized market and the decentralized monetary exchange are complements for buyers, inflation reduces consumption in monetary exchange and hence consumption (employment) in the centralized market also decreases. Inflation and unemployment have a positive relationship. However, if goods consumed in the centralized market and the decentralized exchange are substitutes for buyers, inflation reduces unemployment. When the model is generalized to allow both buyers and sellers to have nonseparable preferences, the results in RRW have to be modified. For example, if goods consumed in the centralized market and the decentralized exchange are complements for both buyers and sellers, inflation has opposite effects on buyers and sellers. Therefore, the effect of inflation on aggregate unemployment is ambiguous.

One nice property of competitive search equilibrium is that it endogenously generates implications similar to the ones under the assumption of take-it-or-leave-it offers by buyers, but importantly, it does not preclude ex ante investment by sellers, because sellers do not get zero gains from trade here, as they do in the RRW bargaining model. As an extension, I consider free entry decisions by sellers. It turns out that free entry does not alter the relationship between inflation and unemployment found previously.

The rest of the paper is organized as follows. Section 2 describes the environment. I solve the competitive search equilibrium and discuss the effect of inflation on unemployment in Section 3. Section 4 considers two extensions and Section 5 concludes.

2. ENVIRONMENT

Time is discrete. A continuum of agents with measure 1 live forever. In each period, there are two subperiods. A Walrasian market (hereafter CM, for centralized market) opens in the first subperiod. The second subperiod (hereafter DM, for decentralized market) is characterized by decentralized trades. Agents discount between meetings of the CM at the rate β. There is one nonstorable good in each subperiod—a CM good x and a DM good q.

In the CM, each agent is endowed with one unit of indivisible labor and trades randomized consumption bundles as in the standard Rogerson (1988) model. The production technology of x is such that one unit of labor is converted into one unit of x. As shown in Rocheteau et al. (2008), this indivisible labor specification in the CM can replace the quasilinear preference to make the distribution of money holdings tractable.

In the DM, agents are anonymous. There is no production.[3] Instead, all agents are endowed with $\bar{q}$ units of q. Upon entering into the DM, each agent receives a preference shock. Preference shocks are i.i.d. across agents and across time. With probability σ, an agent has preferences $u^b(q, x, h)$, where because of indivisible labor, $h \in \{0, 1\}$. With probability $1 - \sigma$, an agent has preferences $u^s(q, x, h)$, where $\partial u^b(q, x, h)/\partial q > \partial u^s(q, x, h)/\partial q$. Hence, there are potential gains from trade between these two types of agents. Standard assumptions about lack of commitment and record-keeping [see Kocherlakota (1998)] preclude credit, which implies that money is essential in this subperiod. Those who have $u^b(q, x, h)$ are labeled as buyers and those who have $u^s(q, x, h)$ are labeled as sellers. In aggregate, the measure of buyers is σ and the measure of sellers is $1 - \sigma$.

Throughout this paper, I focus on the case where $u^j(q, x, h)$ is separable in (q, x) and h for $j = b, s$, which is case (ii) in RRW. That is, $u^b(q, x, h) = f(q, x) + v(h)$ and $u^s(q, x, h) = F(q, x) + v(h)$. Assumptions on $f(q, x)$ include $f_q(q, x) > 0$, $f_{qq}(q, x) < 0$, $f_x(q, x) > 0$, $f_{xx}(q, x) < 0$. In terms of the sign of $f_{qx}(q, x)$, $f_{qx}(q, x) > 0$ if q and x are complements, and $f_{qx}(q, x) < 0$ if q and x are substitutes. Similar assumptions apply to $F(q, x)$.

The pricing mechanism in the DM is competitive search. A set of submarkets Ω open in the DM after agents realize their preference shocks. Each submarket $\omega \in \Omega$ is characterized by its posted terms of trade. I adopt the version that market makers design these submarkets and post the terms of trade for each submarket at the beginning of each period.[4] Agents can see the postings and choose which submarket to visit. In each submarket $\omega \in \Omega$, buyers and sellers are matched randomly and trade bilaterally. Let the measure of buyers and the measure of sellers in submarket ω be B_ω and S_ω, respectively. It follows that $\sum_{\omega \in \Omega} B_\omega = \sigma$ and $\sum_{\omega \in \Omega} S_\omega = 1 - \sigma$. The matching function $\mathcal{M}(B_\omega, S_\omega)$ displays constant return to scale.

Money in this economy is supplied by the monetary authority. The growth rate of the money supply is γ and $M_+ = (1 + \gamma)M$. The subscript "+" denotes variables in the next period. New money is injected via lump-sum transfers to all agents by the monetary authority at the beginning of each period.

3. COMPETITIVE SEARCH EQUILIBRIUM

This section begins with solving an agent's problem in the CM and then proceeds to solve an agent's problem in the DM. Finally, I characterize the equilibrium allocation and discuss the relationship between inflation and unemployment.

3.1. Centralized Market

Let $W(m)$ be the value function of an agent in the CM with money holding m. Let $\hat{m}_h$ be the money balance that an agent carries to the DM for $h \in \{0, 1\}$. Suppose that agents are employed and consume x_1 with probability ℓ. With probability $1 - \ell$, agents are unemployed and consume x_0. An agent's value function is

$$W(m) = \max_{\ell, x_1, x_0, \hat{m}_1, \hat{m}_0} \{\ell[v(1) + V(\hat{m}_1, x_1)] + (1-\ell)[v(0) + V(\hat{m}_0, x_0)]\}$$

$$\text{s.t. } \ell(px_1 + \hat{m}_1) + (1-\ell)(px_0 + \hat{m}_0) = p\ell + m + \gamma M, \tag{1}$$

where $V(\hat{m}_h, x_h)$ is the agent's DM value function for $h \in \{0, 1\}$ and p is the price of x in the CM. Let λ denote the Lagrangian multiplier. The Lagrangian is

$$\mathcal{L} = \max_{\ell, x_1, x_0, \hat{m}_1, \hat{m}_0, \lambda} \left\{ [\ell v(1) + (1-\ell)v(0) + \ell V(\hat{m}_1, x_1) + (1-\ell)V(\hat{m}_0, x_0)] + \lambda \left[\ell + \frac{m + \gamma M}{p} - \ell \left(x_1 + \frac{\hat{m}_1}{p} \right) - (1-\ell) \left(x_0 + \frac{\hat{m}_0}{p} \right) \right] \right\}.$$

Assuming that $\ell \in (0, 1)$, the first-order conditions for interior solutions are

$$\ell : V(\hat{m}_1, x_1) + v(1) - V(\hat{m}_0, x_0) - v(0) = \lambda \left[\left(x_1 + \frac{\hat{m}_1}{p} \right) - \left(x_0 + \frac{\hat{m}_0}{p} \right) - 1 \right], \tag{2}$$

$$x_1 : \frac{\partial V(\hat{m}_1, x_1)}{\partial x_1} = \lambda, \tag{3}$$

$$x_0 : \frac{\partial V(\hat{m}_0, x_0)}{\partial x_0} = \lambda, \tag{4}$$

$$\hat{m}_1 : \frac{\partial V(\hat{m}_1, x_1)}{\partial \hat{m}_1} = \frac{\lambda}{p}, \tag{5}$$

$$\hat{m}_0 : \frac{\partial V(\hat{m}_0, x_0)}{\partial \hat{m}_0} = \frac{\lambda}{p}, \tag{6}$$

$$\lambda : \ell + \frac{m + \gamma M}{p} = \ell \left(x_1 + \frac{\hat{m}_1}{p} \right) + (1-\ell) \left(x_0 + \frac{\hat{m}_0}{p} \right). \tag{7}$$

It follows that (2) to (6) determine $(\hat{m}_1, \hat{m}_0, x_1, x_0, \lambda)$ and (7) pins down ℓ. There are several useful observations. First, the choice of $(\hat{m}_1, \hat{m}_0, x_1, x_0, \lambda)$ does not depend on m. Only ℓ depends on m in (7). Second, $W(m)$ is linear in m. In particular,

$$\frac{\partial W(m)}{\partial m} = \frac{\lambda}{p}. \tag{8}$$

Third, it can be shown that $x_1 = x_0$, $\hat{m}_1 = \hat{m}_0$, and $\lambda = v(0) - v(1)$.[5] In general, it is not true that $x_0 = x_1$ and $m_0 = m_1$ in indivisible labor models, but the results are obtained here because I assume preferences are separable in h and (x, q). Let $x = x_1 = x_0$ and $\hat{m} = \hat{m}_1 = \hat{m}_0$. Notice that λ can be interpreted as the value of leisure. Denote the aggregate labor supply by $\bar{\ell}$. From (7), it is immediate that $\bar{\ell} = x$. Furthermore, $W(m)$ is simplified to

$$W(m) = \max_{x,\hat{m}} \left\{ \left(x + \frac{\hat{m} - m - \gamma M}{p} \right) [v(1) - v(0)] + V(\hat{m}, x) + v(0) \right\}. \quad \textbf{(9)}$$

3.2. Decentralized Market

Before the preference shock is realized, an agent's expected value function in the DM is

$$V(m, x) = \sigma V^b(m, x) + (1 - \sigma) V^s(m, x), \quad \textbf{(10)}$$

where $V^b(m, x)$ and $V^s(m, x)$ represent the value functions for a buyer and a seller, respectively.

At the beginning of each period, market makers announce the terms of trade for each submarket ω. The terms of trade (q_ω, d_ω) specify that d_ω units of money can be used to trade for q_ω units of goods in submarket ω. After preference shocks are realized, agents enter the submarkets. Those who direct their search to the same (q, d) form an active submarket. Recall that in submarket ω the measure of buyers is B_ω and the measure of sellers is S_ω. The market tightness in ω is defined as $Q_\omega = B_\omega / S_\omega$. In equilibrium, the actual Q_ω should be consistent with agents' rational expectations. The probability for a buyer to trade with a seller in ω is $\alpha^b(Q_\omega) = \mathcal{M}(B_\omega, S_\omega)/B_\omega = \mathcal{M}(Q_\omega, 1)/Q_\omega$. Similarly, the probability for a seller to trade with a buyer is $\alpha^s(Q_\omega) = \mathcal{M}(B_\omega, S_\omega)/S_\omega = \mathcal{M}(Q_\omega, 1)$. Once a buyer and a seller meet, they trade at the posted terms (q_ω, d_ω). Therefore, the buyer's consumption in the DM is $\bar{q} + q_\omega$ and the seller's consumption is $\bar{q} - q_\omega$. Buyers and sellers have the following value functions:

$$V^b(m, x) = \max_{\omega \in \Omega} \left\{ \alpha^b(Q_\omega)[f(\bar{q} + q_\omega, x) + \beta W_+(m - d_\omega)] \right.$$
$$\left. + [1 - \alpha^b(Q_\omega)][f(\bar{q}, x) + \beta W_+(m)] \right\},$$
$$V^s(m, x) = \max_{\omega \in \Omega} \left\{ \alpha^s(Q_\omega)[F(\bar{q} - q_\omega, x) + \beta W_+(m + d_\omega)] \right.$$
$$\left. + [1 - \alpha^s(Q_\omega)][F(\bar{q}, x) + \beta W_+(m)] \right\}.$$

3.3. Equilibrium

When designing submarkets, market makers maximize the expected value of an agent who is a buyer in ω such that an agent who is a seller in ω can get the expected market value $\bar{J}$.[6] Let W_ω^b be the CM value function of an agent who is a buyer in ω and let W_ω^s be the CM value function of an agent who is a seller in ω. The objective of a market maker who designs ω is

$$\max_{q_\omega, d_\omega} W_\omega^b(m) \quad \text{s.t. } W_\omega^s(m) = \bar{J}.$$

Note that market makers announce the terms of trade at the beginning of each period. This implies that agents can take these terms of trade into consideration when they make their choices of money balances in the CM. The following result is straightforward, so the proof is omitted.

LEMMA 1. *In the CM, an agent chooses to bring just enough money balance to make a purchase if the agent becomes a buyer in the DM.*

To simplify notation, I define $u(q, x) = f(\bar{q} + q, x) - f(\bar{q}, x)$ and $c(q, x) = F(\bar{q}, x) - F(\bar{q} - q, x)$. Consider first the value functions of an agent who is a buyer in ω. Before the preference shock is realized, the agent enters into ω as a buyer with probability σ. With probability $1 - \sigma$, the agent becomes a seller and can enter into $\tilde{\omega}$ where $\omega, \tilde{\omega} \in \Omega$. Because $\partial W(m)/\partial m = \lambda/p$, $V^b(m, x)$ and $V^s(m, x)$ are

$$V^b(d_\omega, x) = \alpha^b(Q_\omega)u(q_\omega, x) + f(\bar{q}, x) + [1 - \alpha^b(Q_\omega)]\frac{\beta\lambda d_\omega}{p_+} + \beta W_+(0), \tag{11}$$

$$V^s(d_\omega, x) = \alpha^s(Q_{\tilde{\omega}})\left[-c(q_{\tilde{\omega}}, x) + \frac{\beta\lambda d_{\tilde{\omega}}}{p_+}\right] + F(\bar{q}, x) + \frac{\beta\lambda d_\omega}{p_+} + \beta W_+(0). \tag{12}$$

The agent's value function in the CM is rearranged as

$$\begin{aligned} W_\omega^b(m) = \max_x \Bigg\{ & \left(x + \frac{d_\omega - m - \gamma M}{p}\right)[v(1) - v(0)] + \beta W_+(0) \\ & + \sigma\left\{\alpha^b(Q_\omega)u(q_\omega, x) + f(\bar{q}, x) + [1 - \alpha^b(Q_\omega)]\frac{\beta\lambda d_\omega}{p}\right\} \\ & + (1-\sigma)\left\{\alpha^s(Q_{\tilde{\omega}})\left[-c(q_{\tilde{\omega}}, x) + \frac{\beta\lambda d_{\tilde{\omega}}}{p_+}\right] + F(\bar{q}, x) + \frac{\beta\lambda d_\omega}{p_+}\right\}\Bigg\}. \end{aligned} \tag{13}$$

The first-order condition with respect to x is

$$\begin{aligned} v(0) - v(1) = {} & \sigma[\alpha^b(Q_\omega)u_x(q_\omega, x) + f_x(\bar{q}, x)] \\ & + (1-\sigma)[-\alpha^s(Q_{\tilde{\omega}})c_x(q_{\tilde{\omega}}, x) + F_x(\bar{q}, x)]. \end{aligned} \tag{14}$$

The subscript x represents the partial derivatives with respect to x. Note that the optimal x depends on $(q_\omega, q_{\tilde{\omega}}, Q_\omega, Q_{\tilde{\omega}})$. That is, the choice of x in the CM

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generally depends on the terms of trade as well as the probability of trading in both ω and $\tilde{\omega}$. Let $x^b(\omega, \tilde{\omega})$ be the solution to (14). This is the optimal x for an agent who chooses ω conditional on being a buyer and chooses $\tilde{\omega}$ conditional on being a seller.

Now consider an agent who is a seller in ω. Before the preference shock is realized, the agent can potentially go to submarket $\hat{\omega}$ if he becomes a buyer with probability σ. By similar arguments, the CM value function of an agent who is a seller in ω is rewritten as

$$\begin{aligned} W^s_\omega(m) = \max_x \Bigg\{ & \left(x + \frac{d_{\hat{\omega}} - m - \gamma M}{p}\right)[v(1) - v(0)] + \beta W_+(0) \\ & + \sigma \left\{\alpha^b(Q_{\hat{\omega}})u(q_{\hat{\omega}}, x) + f(\bar{q}, x) + [1 - \alpha^b(Q_{\hat{\omega}})]\frac{\beta\lambda d_{\hat{\omega}}}{p_+}\right\} \\ & + (1-\sigma)\left\{\alpha^s(Q_\omega)\left[-c(q_\omega, x) + \frac{\beta\lambda d_\omega}{p_+}\right] + F(\bar{q}, x) + \frac{\beta\lambda d_{\hat{\omega}}}{p_+}\right\}\Bigg\}, \end{aligned} \quad \mathbf{(15)}$$

and the first-order condition with respect to x is

$$\begin{aligned} v(0) - v(1) = {} & \sigma[\alpha^b(Q_{\hat{\omega}})u_x(q_{\hat{\omega}}, x) + f_x(\bar{q}, x)] \\ & + (1-\sigma)[-\alpha^s(Q_\omega)c_x(q_\omega, x) + F_x(\bar{q}, x)]. \end{aligned} \quad \mathbf{(16)}$$

Let $x^s(\hat{\omega}, \omega)$ be the optimal x for an agent who is a seller in ω. As described earlier, for each $\omega \in \Omega$, market makers maximize the CM value function of a future buyer so that a future seller can get the equilibrium expected value $\bar{J}$. For ease of notations, I define the following two terms.

$$\begin{aligned} \Pi^b(x^b) = {} & x^b[v(1) - v(0)] + \sigma f(\bar{q}, x^b) + (1-\sigma)[-\alpha^s(Q_{\tilde{\omega}})c(q_{\tilde{\omega}}, x^b) \\ & + F(\bar{q}, x^b)], \end{aligned} \quad \mathbf{(17)}$$

$$\begin{aligned} \Pi^s(x^s) = {} & x^s[v(1) - v(0)] + \sigma[\alpha^b(Q_{\hat{\omega}})u(q_{\hat{\omega}}, x^s) \\ & + f(\bar{q}, x^s)] + (1-\sigma)F(\bar{q}, x^s). \end{aligned} \quad \mathbf{(18)}$$

Ignoring the terms that do not depend on (q_ω, d_ω), the problem of a market maker who designs ω is

$$\max_{q_\omega, d_\omega} \left\{-\frac{\lambda d_\omega}{p} + \sigma\alpha^b(Q_\omega)u(q_\omega, x^b) + [1 - \sigma\alpha^b(Q_\omega)]\frac{\beta\lambda d_\omega}{p_+} + \Pi^b(x^b)\right\} \quad \mathbf{(19)}$$

$$\text{s.t. } (1-\sigma)\alpha^s(Q_\omega)\left[-c(q_\omega, x^s) + \frac{\beta\lambda d_\omega}{p_+}\right] + \Pi^s(x^s) = J, \quad \mathbf{(20)}$$

where $J = \bar{J} + \frac{m+\gamma M}{p}[v(1) - v(0)] - \beta W_+(0) - [1 - \sigma\alpha^b(Q_{\hat{\omega}})]\frac{\beta\lambda d_{\hat{\omega}}}{p_+}$. The interpretation of the constraint (20) is that the market tightness Q_ω should adjust to ensure that each seller gets the market value $\bar{J}$.

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Rearranging (20),

$$\frac{\beta\lambda d_\omega}{p_+} = c(q_\omega, x^s) + \frac{J - \Pi^s(x^s)}{(1-\sigma)\alpha^s(Q_\omega)}. \tag{21}$$

In the steady state, the inflation rate is $p_+/p = 1+\gamma$. The nominal interest rate i is implied by the Fisher equation, $1+i = (1+r)\frac{p_+}{p} = \frac{1+\gamma}{\beta}$, where the real interest rate is implicitly given by $1+r = 1/\beta$. Because I focus on the determination of the terms of trade in submarket ω, I now omit the subscript ω without any confusion. The unconstrained problem for the market maker is

$$\max_{q,Q}\left\{[i+\sigma\alpha^b(Q)]\left[c(q,x^s) + \frac{J-\Pi^s(x^s)}{(1-\sigma)\alpha^s(Q)}\right] + \sigma\alpha^b(Q)u(q,x^b) + \Pi^b(x^b)\right\}. \tag{22}$$

Notice that from (14) and (16), $-(1-\sigma)\alpha^s(Q)c_x(q,x^s) + K_x^s(x^s) = 0$ and $\sigma\alpha^b(Q)u_x(q,x^b) + K_x^b(x^b) = 0$. Let $\eta(Q) = \alpha_Q^s(Q)[Q/\alpha^s(Q)]$ and $1-\eta(Q) = -\alpha_Q^b(Q)[Q/\alpha^b(Q)]$. The first-order conditions for interior solutions are

$$q: -c_q(q,x^s) + \frac{\sigma\alpha^b(Q)u_q(q,x^b)}{i+\sigma\alpha^b(Q)} = 0, \tag{23}$$

$$Q: \frac{J-\Pi^s(x^s)}{(1-\sigma)\alpha^s(Q)} + c(q,x^s) = \frac{\eta(Q)u_q(q,x^b)c(q,x^s) + [1-\eta(Q)]c_q(q,x^s)u(q,x^b)}{\eta(Q)u_q(q,x^b) + [1-\eta(Q)]c_q(q,x^s)}. \tag{24}$$

Here the subscript q or Q represents the partial derivatives with respect to q or Q.

DEFINITION 1. *A competitive search equilibrium is a list* $(q_\omega, Q_\omega, d_\omega, S_\omega)$ *and a* $\bar{J} \geq 0$ *such that given* $\bar{J}$, $(q_\omega, Q_\omega, d_\omega, S_\omega)$ *maximize the expected value of a buyer subject to the constraint that a seller gets* $\bar{J}$, *where* $\bar{J}$ *satisfies* $\sum S_\omega = 1-\sigma$ *and* $\sum S_\omega Q_\omega = \sigma$.

LEMMA 2. *Competitive search equilibrium exists.*

Proof. Let (q^*, x^*, ℓ^*) be the first-best allocation, which is derived from the planner's problem in the Appendix. I restrict the attention to $q \in [0, q^*]$, $\alpha^b(Q)$, $\alpha^s(Q) \in [0,1]$, and $\beta\lambda d/p_+ \in [c(q^*), u(q^*)]$, Similarly to the proof of existence in Rocheteau and Wright (2005) or Lagos and Rocheteau (2005), one can show that competitive search equilibrium exists by the Theorem of the Maximum. ■

In what follows, I focus on equilibrium where there is a unique submarket open.[7] This implies that $Q = \frac{\sigma}{1-\sigma}$ and $x^b(q) = x^s(q) = x(q)$. In addition, α^b and α^s are constants where $\alpha^b(Q) = \alpha^b(\frac{\sigma}{1-\sigma})$ and $\alpha^s(Q) = \alpha^s(\frac{\sigma}{1-\sigma})$. For interior solutions, (q, x) are solved from

$$\frac{u_q(q,x)}{c_q(q,x)} - 1 - \frac{i}{\sigma\alpha^b} = 0, \quad (25)$$

$$\sigma[\alpha^b u_x(q,x) + f_x(\bar{q},x)] + (1-\sigma)[-\alpha^s c_x(q,x) + F_x(\bar{q},x)]$$
$$= v(0) - v(1). \quad (26)$$

When designing submarkets, market makers take $\bar{J}$ as given. However, $\bar{J}$ adjusts to clear the market in equilibrium. Mathematically, (24) determines J and hence $\bar{J}$. Finally, $\beta\lambda d/p_+$ is obtained from (21).

PROPOSITION 1. *In competitive search equilibrium, the optimal monetary policy is the Friedman rule.*

Proof. It is easy to check that when $i \to 0$, (q, x) that solve (25) and (26) are the same as the planner's choice. ■

Given that there is no policy distortion, competitive search equilibrium endogenously generates the efficient allocation. In RRW, the Friedman rule is also the optimal monetary policy. The difference between the bargaining equilibrium in RRW and the competitive search equilibrium here is that sellers get 0 trading surplus in RRW, whereas buyers and sellers split the trading surplus in competitive search equilibrium.

PROPOSITION 2. *Monetary equilibrium exists when the inflation rate is not too high. In addition,* $\frac{dq}{di} < 0$.

Proof. Define $\psi(q) = \frac{u_q(q,x(q))}{c_q(q,x(q))} - 1 - \frac{i}{\sigma\alpha^b}$, where $x(q)$ is given by (26). The solution of q is given by $\psi(q) = 0$. Note that when $i \to 0, q \to q^*$. It follows that $\psi(q^*) < 0$ for $i > 0$. Because I restrict to $q \in [0, q^*]$ and the solution is unique, it must be true that $\psi'(q) < 0$ at the solution $\psi(q) = 0$. It follows that $\frac{dq}{di} < 0$. When $q = 0$, $\frac{u_q(0,x(0))}{c_q(0,x(0))} = \frac{f_q(\bar{q},x(0))}{F_q(\bar{q},x(0))} > 1$ by definition. Because $\frac{f_q(\bar{q},x(0))}{F_q(\bar{q},x(0))} - 1$ and $\sigma\alpha^b(\frac{\sigma}{1-\sigma})$ are finite, there exists an $\bar{\imath}$ such that $\frac{f_q(\bar{q},x(0))}{F_q(\bar{q},x(0))} = 1 + \frac{\bar{\imath}}{\sigma\alpha^b(\frac{\sigma}{1-\sigma})}$. When i exceeds $\bar{\imath}$, $q = 0$ and monetary equilibrium does not exist. ■

Having defined monetary equilibrium, I proceed to find the conditions that guarantee that $\ell \in (0, 1)$. Because utility is separable in (q, x) and h, from (7),

$$\ell(m) = x + \frac{\hat{m}}{(1+\gamma)p} - \frac{m}{p}.$$

It is obvious that ℓ decreases in m. This means that agents entering into the CM with greater money balances work with lower probability. When a unique submarket opens in the DM, agents' money balances can take three possible values upon exiting the DM. For unmatched agents, they still hold $\hat{m}$. For matched buyers, they end up with 0 unit of money, whereas for matched sellers, they accumulate $2\hat{m}$

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units of money. In the steady state, $\hat{m} = (1+\gamma)M$. It follows that

$$\ell_{\max} = \ell(0) = x + \frac{M}{p} \text{ and } \ell_{\min} = \ell(2\hat{m}_-) = x - \frac{M}{p},$$

where the subscript "−" represents variables in the previous period. To ensure that $\ell \in (0, 1)$, one needs $\ell_{\max} < 1$ and $\ell_{\min} > 0$. The condition reduces to

$$\frac{M}{p} < x < 1 - \frac{M}{p}.$$

Notice that M/p is endogenously determined in (21). As discussed in RRW, it should not be hard to find parameters such that ℓ is interior. So the rest of the paper assumes that $\ell \in (0, 1)$.

3.4. Inflation and Unemployment

Given the equilibrium conditions, this subsection examines how inflation affects unemployment. When the inflation rate increases, it usually distorts transactions in the DM where money is used in exchange. With nonseparable preferences, however, it is not obvious how inflation affects activity in the CM.

I first assume that only buyers have nonseparable preferences in (q, x), as in RRW, i.e., $F_{qx}(q, x) = 0$. Differentiating (25) and (26) with respect to i,

$$[F_q(\bar{q}-q,x)f_{qq}(\bar{q}+q,x) + f_q(\bar{q}+q,x)F_{qq}(\bar{q}-q,x)]\frac{dq}{di} = \frac{[F_q(\bar{q}-q,x)]^2}{\mathcal{M}} - F_q(\bar{q}-q,x)f_{qx}(\bar{q}+q,x)\frac{dx}{di}, \quad (27)$$

$$\mathcal{M}f_{qx}(\bar{q}+q,x)\frac{dq}{di} = -[\mathcal{M}f_{xx}(\bar{q}+q,x) + (\sigma-\mathcal{M})f_{xx}(\bar{q},x) + \mathcal{M}F_{xx}(\bar{q}-q,x) + (1-\sigma-\mathcal{M})F_{xx}(\bar{q},x)]\frac{dx}{di}, \quad (28)$$

where I use the definition of $u(q, x)$ and $c(q, x)$. Let $a \simeq b$ denote a and b are equal in sign.

PROPOSITION 3. *In competitive search equilibrium,* $\frac{d(1-\bar{\ell})}{di} \simeq -\frac{dx}{di} \simeq f_{qx}(\bar{q}+q,x)$.

Proof. From (28), $\frac{dx}{di} \simeq -f_{qx}(\bar{q}+q,x)$. Recall that from (7), $d\bar{\ell} = dx$. Hence, $\frac{d(1-\bar{\ell})}{di} = -\frac{dx}{di}$. ■

It is common that inflation reduces the consumption of the DM good in monetary search models. One interesting result is that inflation may increase or decrease the consumption of the CM good or unemployment depending on the sign of $f_{qx}(\bar{q}+q,x)$. As in RRW, $\frac{d(1-\bar{\ell})}{di} > 0$ if q and x are complements. When q

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decreases, x also decreases. Because x is produced in the CM, inflation increases unemployment. If q and x are substitutes, $\frac{d(1-\bar{\ell})}{di} < 0$ and hence inflation reduces unemployment. It is straightforward that if buyers' preferences are also separable in q and x, inflation does not affect unemployment.

In competitive search equilibrium, inflation increases unemployment if x and q are complements and inflation reduces unemployment if x and q are substitutes. The result is the same as in RRW and it holds true in competitive search equilibrium. There is no need to resort to an extreme case, as in bargaining equilibrium.

4. EXTENSIONS

4.1. Nonseparable Preferences for Sellers

In RRW, only buyers are assumed to have nonseparable preferences. It is easy to allow both buyers and sellers to have nonseparable preferences in competitive search equilibrium. It turns out that the main results in RRW have to be modified.

PROPOSITION 4. *When both buyers and sellers have nonseparable preferences in* (q, x), $\frac{dq}{di} < 0$ *and* $\frac{dx}{di} \simeq F_{qx}(\bar{q} - q, x) - f_{qx}(\bar{q} + q, x)$. *Moreover,* $\frac{d(1-\bar{\ell})}{di} \simeq -\frac{dx}{di} \simeq f_{qx}(\bar{q} + q, x) - F_{qx}(\bar{q} - q, x)$.

Proof. Modifying (27) and (28) as

$$[F_q(\bar{q} - q, x) f_{qq}(\bar{q} + q, x) + f_q(\bar{q} + q, x) F_{qq}(\bar{q} - q, x)] \frac{dq}{di}$$
$$= \frac{[F_q(\bar{q} - q, x)]^2}{\mathcal{M}} - [F_q(\bar{q} - q, x) f_{qx}(\bar{q} + q, x)$$
$$- f_q(\bar{q} + q, x) F_{qx}(\bar{q} - q, x)] \frac{dx}{di}, \tag{29}$$

$$\mathcal{M}[f_{qx}(\bar{q} + q, x) - F_{qx}(\bar{q} - q, x)] \frac{dq}{di}$$
$$= -[\mathcal{M} f_{xx}(\bar{q} + q, x) + (\sigma - \mathcal{M}) f_{xx}(\bar{q}, x) + \mathcal{M} F_{xx}(\bar{q} - q, x)$$
$$+ (1 - \sigma - \mathcal{M}) F_{xx}(\bar{q}, x)] \frac{dx}{di}. \tag{30}$$

Similarly to the proof of proposition 2, $\frac{dq}{di} < 0$ and $\frac{d(1-\bar{\ell})}{di} \simeq -\frac{dx}{di}$. From (30), $\frac{dx}{di} \simeq F_{qx}(\bar{q} - q, x) - f_{qx}(\bar{q} + q, x)$. ∎

When both buyers and sellers have nonseparable preferences, there are several cases to consider. First, if q and x are complements for both buyers and sellers, then the sign of $\frac{d(1-\bar{\ell})}{di}$ depends on $f_{qx}(\bar{q}+q, x) - F_{qx}(\bar{q}-q, x)$. The effect of inflation on unemployment is ambiguous. Second, if q and x are substitutes for both buyers and sellers, the sign of $\frac{d(1-\bar{\ell})}{di}$ is also ambiguous. Third, if q and x are complements for buyers but substitutes for sellers, I have $\frac{d(1-\bar{\ell})}{di} > 0$ and inflation decreases unemployment. Finally, if q and x are substitutes for buyers and complements for sellers, $\frac{d(1-\bar{\ell})}{di} < 0$ and inflation reduces unemployment. It is trivial to check that if only sellers have nonseparable preferences in q and x, inflation increases unemployment if q and x are substitutes, and inflation decreases unemployment if q and x are complements. If only buyers have nonseparable preferences in q and x, this is the case discussed in the preceding section.

The above results are very intuitive. Take the case where q and x are substitutes for both buyers and sellers as an example. Inflation reduces q, which means that the consumption of q decreases for buyers, but increases for sellers. Because q and x are substitutes, the consumption of x should increase for buyers, but should decrease for sellers. As agents do not know whether they become buyers or sellers in the CM, the overall effect of inflation on x is ambiguous. Hence, the overall effect of inflation on unemployment is ambiguous. However, if q and x are substitutes for buyers and complements for sellers, a lower q implies a higher x for both buyers and sellers. Therefore, inflation must reduce unemployment. Allowing sellers to have nonseparable preferences alters the results in RRW, but the intuition remains the same. The key point is that inflation may have different effects on buyers and sellers in the sector where money is essential. In the case where both buyers and sellers consume (x, q) as complements (or substitutes), inflation has opposite effects on buyers and sellers.

4.2. Free Entry by Sellers

As discussed previously, competitive search equilibrium endogenizes how buyers and sellers split the trading surplus. It further allows one to add ex ante investment or entry/exit decisions by sellers. In this subsection, I extend the environment to allow free entry by sellers. Market makers still post terms of trade at the beginning of each period. After seeing the postings, an agent can make a decision as follows. He can choose a submarket ω if he becomes a buyer in the DM. If he is a seller in the DM, he can choose to go to submarket $\hat{\omega}$ or not to go to any submarket at all. The cost of entry into any submarket for sellers is k.

To facilitate comparisons with RRW, I allow only buyers to have nonseparable preferences.[8] Sellers' preferences are $u^s(q, x, h) = F(q) + G(x) + v(h)$, where standard assumptions of utility functions apply to $F(q)$ and $G(x)$. I assume that σ is such that Q_ω is not constrained for any ω. In aggregate, the measure of sellers is endogenously determined. Based on this modification, a market maker designs submarket ω so that it maximizes the surplus of a buyer who enters this submarket subject to the constraint that a seller always gets surplus k from entering into the submarket. Formally, the market maker's problem is

$$\max_{q,d,Q} \left\{ -\frac{\lambda d}{p} + \sigma\alpha^b(Q)u(q, x^b) + [1 - \sigma\alpha^b(Q)]\frac{\beta\lambda d}{p_+} + \Pi^b(x^b) \right\}$$

$$\text{s.t. } \alpha^s(Q)\left[-c(q) + \frac{\beta\lambda d}{p_+}\right] = k.$$

One can follow steps similar to those in the previous section to solve the unconstrained maximization problem by substituting $\beta\lambda d/p_+$ from the constraint into the objective function. The equilibrium (q, Q) are characterized by

$$\frac{u_q(q, x^b)}{c_q(q)} = 1 + \frac{i}{\sigma\alpha^b(Q)}, \tag{31}$$

$$\frac{k}{\alpha^s(Q)} + c(q) = g(q, Q, x^b), \tag{32}$$

where x^b is from

$$\sigma[\alpha^b(Q)u_x(q, x^b) + f_x(\bar{q}, x^b)] + (1-\sigma)G_x(x^b) = v(0) - v(1)$$

and

$$g(q, Q, x^b) = \frac{\eta(Q)u_q(q, x^b)c(q) + [1-\eta(Q)]c_q(q)u(q, x^b)}{\eta(Q)u_q(q, x^b) + [1-\eta(Q)]c_q(q)}. \tag{33}$$

Again, I focus on equilibrium where there is a unique submarket open.

PROPOSITION 5. *In competitive search equilibrium with free entry,* $\frac{d(1-\bar{\ell})}{di} \simeq -\frac{dx}{di} \simeq f_{qx}(\bar{q}+q, x)$.

Proof. See the Appendix. ■

It appears that allowing free entry by sellers does not alter the qualitative relationship between inflation and unemployment. When Q is affected by inflation, the choice of x depends on (q, Q). As usual, inflation reduces q in the DM, which lowers per trade surplus in the DM. This intensive margin effect may increase or decrease x depending on whether q and x are complements or substitutes in much the same way as before. Free entry by sellers generates the extensive margin effect on x as follows. When there are fewer sellers (i.e., Q increases), the number of trades decreases in the DM. Recall that $u_x = f_x(q+\bar{q}, x) - f_x(\bar{q}, x)$. If q and x are complements, having more x is beneficial for buyers and hence fewer trades in the DM reduces the marginal benefit of x. A higher Q leads to a lower x. If q and x are substitutes, more x reduces a buyer's utility and hence less trades in the DM raises the marginal benefit of x. A higher Q leads to a higher x. Mathematically, the extensive margin effect does not change how x depends on i.

5. CONCLUSION

This paper is in the recent tradition of studying unemployment in models where money and the effects of inflation are modeled using relatively explicit microfoundations. Following RRW, I use the Rogerson indivisible labor model of unemployment. Because RRW use bargaining, their results are proved only in the very special case of take-it-or-leave-it offer by buyers. Using competitive search

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equilibrium, I find similar results without any such special restriction. This is important because take-it-or-leave-it offer does preclude many interesting extensions like entry/participation decisions by sellers.

In this more general framework, I consider two extensions. The first extension allows both buyers and sellers to have nonseparable preferences. Inflation may have opposite effects on buyers and sellers. Therefore, the results in RRW have to be modified although the basic economic intuition remains the same. The second extension incorporates free entry decisions by sellers, which cannot be studied in RRW. How the relationship between inflation and unemployment depends on the primitives of the model still holds with free entry.

Recent papers such as Berentsen et al. (2011) and Liu (2009) use the Mortensen–Pissarides model of unemployment to study inflation and unemployment. There are some advantages of their models of unemployment, but there are also some disadvantages, including the fact that the other models rely on linear utility and hence do not allow one to prove anything like the propositions presented here concerning complements and substitutes of the utility function in the indivisible labor framework. Both of these two approaches are useful to understand the relationship between inflation and unemployment in models with microfoundations.

NOTES

1. Cooley and Hansen (1989) earlier study a model with indivisible labor and inflation, but money is introduced via a cash-in-advance constraint, and not with explicit microfoundations. Moreover, they consider only price taking, which is perhaps less natural (than price posting and bargaining) once one tries to consider microfoundations. They focus on a specific parametric utility function, and indeed they do not attempt to prove general theorems, and instead present numerical results.

2. Competitive search is first introduced in labor economics by Moen (1997). It has been used in monetary economics since Rocheteau and Wright (2005). See Lagos and Rocheteau (2005), Faig and Huangfu (2007), and Dong (2010) for references.

3. As pointed out in RRW, the main purpose of having no production in the DM is to have unemployment unambiguously determined in the CM.

4. Market makers represent a third party that is not involved in actual trading. Free entry of market makers makes them earn zero profit.

5. Rocheteau et al. (2008) show that the solution to (1) is unqiue and the second order condition holds.

6. Market makers take the market value of a potential seller $\bar{J}$ as given. Because there is a continuum of agents, deviation of one agent will not alter $\bar{J}$. In equilibrium, $\bar{J}$ will adjust so that market makers earn zero profit. Similar arguments have been used by Acemoglu and Shimer (1999). Burdett et al. (2001) show that this method can be justified by considering equilibria in a version of the model with finite numbers of buyers and sellers and then taking the limit as the economy gets large.

7. Let $Q(J)$ be the solution of Q as a function of J and define $\tilde{Q} = 1/Q$. Here $\tilde{Q}(J)$ is decreasing in J. In equilibrium, $\frac{\sigma}{1-\sigma}$ belongs to the convex hull of $\tilde{Q}(J)$. In general, one $\frac{1-\sigma}{\sigma}$ may admit multiple J. When $\tilde{Q}(J)$ is strictly decreasing in J, there is a unique J in equilibrium. In this case, one J may correspond to multiple $\tilde{Q}$ and hence multiple Q in competitive search equilibrium. There could be multiple submarkets open. However, one can add assumptions to ensure that there is a unique submarket. See Lagos and Rocheteau (2005) and Dong (2010) for examples.

8. In the DM, an agent can decide not to visit any submarket if he becomes a seller. After preference shocks are realized, there exist two types of sellers in the economy—those who decide to participate in the DM and those who decide not to participate in the DM. Hence, when both buyers and sellers have nonseparable preferences, an agent's choice of x depends on his entry decision. This setup is slightly more complicated than allowing only buyers to have nonseparable preferences.

REFERENCES

Acemoglu, Daron and Robert Shimer (1999) Holdups and efficiency with search frictions. *International Economic Review* 40(4), 827–849.

Berentsen, Aleksander, Guido Menzio, and Randall Wright (2011) Inflation and unemployment in the long run. *American Economic Review* 101(1), 371–398.

Burdett, Ken, Shouyong Shi, and Randall Wright (2001) Pricing and matching with frictions. *Journal of Political Economy* 109(5), 1060–1085.

Cooley, Thomas F. and Gary D. Hansen (1989) The inflation tax in a real business cycle model. *American Economic Review* 79(4), 733–748.

Dong, Mei (2010) Inflation and variety. *International Economic Review* 51(2), 401–420.

Faig, Miguel and Xiuhua Huangfu (2007) Competitive search equilibrium in monetary economies. *Journal of Economic Theory* 127(1), 709–718.

Kocherlakota, Narayana (1998) Money is memory. *Journal of Economic Theory* 81(2), 232–251.

Lagos, Ricardo and Guillaume Rocheteau (2005) Inflation, output, and welfare. *International Economic Review* 46(2), 495–522.

Lagos, Ricardo and Randall Wright (2005) A unified framework for monetary theory and policy analysis. *Journal of Political Economy* 113(3), 463–484.

Liu, Qian (2009) Inflation and Unemployment: The Roles of Goods and Labor Markets Institutions. Working paper, Queen's University.

Moen, Espen (1997) Competitive search equilibrium. *Journal of Political Economy* 105(2), 385–411.

Rocheteau, Guillaume, Peter Rupert, Karl Shell, and Randall Wright (2008) General equilibrium with nonconvexities and money. *Journal of Economic Theory* 142(1), 294–317.

Rocheteau, Guillaume, Peter Rupert, and Randall Wright (2007) Inflation and unemployment in general equilibrium. *Scandinavian Journal of Economics* 109(4), 837–855.

Rocheteau, Guillaume and Randall Wright (2005) Money in search equilibrium, in competitive equilibrium and in competitive search equilibrium. *Econometrica* 73(1), 175–202.

Rogerson, Richard (1988) Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21 (1), 3–16.

APPENDIX

A.1. THE PLANNER'S PROBLEM

Consider a planner who is subject to search frictions and information frictions. Let ℓ be the probability of employment. x_h is the consumption of the CM good for $h \in \{0, 1\}$. Given that agents trade bilaterally, q_h is the quantity per trade in the DM. The planner maximizes the social welfare weighting all agents equally:

$$\max_{x_1, x_0, \ell, q_1, q_0} \ \ell \mathcal{W}_1 + (1-\ell)\mathcal{W}_2$$

$$\text{s.t.} : \ell x_1 + (1-\ell)x_0 = \ell, \tag{A.1}$$

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where $\mathcal{W}_h$ represents the welfare of an agent with $h \in \{0, 1\}$. Denote μ as the Lagrangian multiplier associated with the constraint

$$\mathcal{W}_h = v(h) + \mathcal{M}(\sigma, 1-\sigma) f(\bar{q} + q_h, x_h) + [\sigma - \mathcal{M}(\sigma, 1-\sigma)] f(\bar{q}, x_h) + \mathcal{M}(\sigma, 1-\sigma) F(\bar{q} - q_h, x_h) + [1 - \sigma - \mathcal{M}(\sigma, 1-\sigma)] F(\bar{q}, x_h).$$

The first-order conditions for interior solutions are

$$x_h : \mu = [\sigma - \mathcal{M}(\sigma, 1-\sigma)] f_x(\bar{q}, x_h) + [1 - \sigma - \mathcal{M}(\sigma, 1-\sigma)] F_x(\bar{q}, x_h) + \mathcal{M}(\sigma, 1-\sigma)[f_x(\bar{q} + q_h, x_h) + F_x(\bar{q} - q_h, x_h)], \quad \textbf{(A.2)}$$

$$q_h : \mathcal{M}(\sigma, 1-\sigma)[f_q(\bar{q} + q_h, x_h) - F_q(\bar{q} - q_h, x_h)] = 0, \quad \textbf{(A.3)}$$

$$\ell : \mu(x_1 - x_0 - 1) = [\sigma - \mathcal{M}(\sigma, 1-\sigma)][f(\bar{q}, x_1) - f(\bar{q}, x_0)] + \mathcal{M}(\sigma, 1-\sigma)[f(\bar{q} + q_1, x_1) - f(\bar{q} + q_0, x_0)] + [1 - \sigma - \mathcal{M}(\sigma, 1-\sigma)][F(\bar{q}, x_1) - F(\bar{q}, x_0)] + \mathcal{M}(\sigma, 1-\sigma)[F(\bar{q} - q_1, x_1) - F(\bar{q} - q_0, x_0)] + [v(1) - v(0)], \quad \textbf{(A.4)}$$

$$\mu : \ell = \ell x_1 + (1 - \ell) x_0. \quad \textbf{(A.5)}$$

for $h \in \{0, 1\}$. Notice that (A.2) and (A.3) determine (x_h, q_h). One can prove that the planner's problem is concave. It follows that the solution must be unique, which implies that $x_1 = x_0$ and $q_1 = q_0$. Denoting $x_1 = x_0 = x$ and $q_1 = q_0 = q$, (A.4) and (A.5) are simplified to $\mu = v(0) - v(1)$ and $\ell = x$. The benchmark allocation $(q^*, x^*, \ell^*, \mu^*)$ is characterized by

$$\mu = [\sigma - \mathcal{M}(\sigma, 1-\sigma)] f_x(\bar{q}, x) + [1 - \sigma - \mathcal{M}(\sigma, 1-\sigma)] F_x(\bar{q}, x) + \mathcal{M}(\sigma, 1-\sigma)[f_x(\bar{q} + q, x) + F_x(\bar{q} - q, x)], \quad \textbf{(A.6)}$$

$$0 = f_q(\bar{q} + q, x) - F_q(\bar{q} - q, x), \quad \textbf{(A.7)}$$

$$\mu = v(0) - v(1), \quad \textbf{(A.8)}$$

$$\ell = x. \quad \textbf{(A.9)}$$

A.2. PROOF OF PROPOSITION 5

To study how inflation affects q and x, I differentiate (31) to (33) with respect to i:

$$A_{11}\frac{dq}{di} + A_{12}\frac{dQ}{di} + A_{13}\frac{dx}{di} = \frac{1}{\sigma\alpha^b(Q)},$$

$$A_{21}\frac{dq}{di} + A_{22}\frac{dQ}{di} + A_{23}\frac{dx}{di} = 0,$$

$$A_{31}\frac{dq}{di} + A_{32}\frac{dQ}{di} + A_{33}\frac{dx}{di} = 0,$$

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where

$$A_{11} = \frac{u_{qq}(q,x)c_q(q) - c_{qq}(q)u_q(q,x)}{c_q^2(q)}, \quad A_{12} = \frac{i\alpha_Q^b(Q)}{\sigma[\alpha^b(Q)]^2}, \quad A_{13} = \frac{u_{qx}(q,x)}{c_q(q)};$$

$$A_{21} = g_q(q,Q,x) - c_q(q),\ A_{22} = g_Q(q,Q,x) + \frac{k\alpha_Q^s(Q)}{[\alpha^s(Q)]^2};$$

$$A_{23} = g_x(q,Q,x);\ A_{31} = \sigma\alpha^b(Q)u_{xq}(q,x),\ A_{32} = \sigma\alpha_Q^b(Q)u_{xq}(q,x);$$

$$A_{33} = \sigma[\alpha^b(Q)u_{xx}(q,x) + f_{xx}(\bar{q},x)] + (1-\sigma)G_{xx}(x).$$

Once there is a unique submarket open in the DM, $\frac{dq}{di} < 0$ can be derived following arguments similar to those in the previous sections. From the above equation system,

$$\frac{dx}{di} = \frac{A_{31}A_{22} - A_{21}A_{32}}{A_{32}A_{23} - A_{33}A_{22}}\frac{dq}{di}.$$

One can show that $A_{31}A_{22} - A_{21}A_{32} \simeq u_{xq}(q,x)$. It remains to check the sign of $A_{32}A_{23} - A_{33}A_{22}$.

Consider the unconstrained problem of market makers. The second-order condition with respect to Q is $A_{23}A_{32}/A_{33} - A_{22}$. Given that the optimal Q should be interior, $A_{23}A_{32}/A_{33} - A_{22} < 0$ at the optimal solution and hence $A_{32}A_{23} - A_{33}A_{22} > 0$. To summarize, $\frac{dx}{di} \simeq -u_{xq}(q,x) \simeq -f_{xq}(\bar{q}+q,x)$. So $\frac{d(1-\bar{\ell})}{di} \simeq -\frac{dx}{di} \simeq f_{qx}(\bar{q}+q,x)$.

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