Macroeconomic Dynamics, **23**, 2019, 3099–3139. Printed in the United States of America.
doi:10.1017/S1365100517001067

GROWTH, UNEMPLOYMENT, AND FISCAL POLICY: A POLITICAL ECONOMY ANALYSIS

Tetsuo Ono
Osaka University

This study presents an overlapping-generations model featuring capital accumulation, collective wage-bargaining, and probabilistic voting over fiscal policy. The study characterizes a Markov-perfect political equilibrium of the voting game within and across generations, and it derives the following results. First, the greater bargaining power of unions lowers the capital growth rate and creates a positive correlation between unemployment and public debt. Second, an increase in the political power of elderly persons lowers the growth rate and shifts government expenditure from unemployed persons to elderly ones. Third, prohibiting debt finance increases the growth rate and benefits future generations; however, it worsens the state of present-day employed and unemployed persons.

Keywords: Economic Growth, Fiscal Policy, Government Debt, Unemployment, Voting

1. INTRODUCTION

In most advanced countries in recent decades, public debt and economic growth have been major concerns among policymakers; however, the debt–gross domestic product (GDP) ratio of many Organisation for Economic Co-operation and Development (OECD) countries has increased over the last 20 years. The burden of debt repayment can crowd out private investment and erode economic performance in the long run. As Figure 1 shows, the evidence suggests a negative correlation between public debt and economic growth in advanced economies [see, e.g., Kumar and Woo (2010), Checherita-Westphal and Rother (2012), Reinhart et al. (2012)].

Previous studies suggest the negative growth effect of public debt, many of these studies used a theoretical approach based on neoclassical growth models [Diamond (1965)] or, later, were based on endogenous ones [Saint-Paul (1992), Josten (2000), Bräuninger (2005)]. These studies assume perfectly competitive labor markets with full employment. To consider the effect in a more realistic

I would like to thank the Associate Editor and two anonymous referees for their comments and suggestions. I also thank Yuki Uchida for his research assistance. This work was supported in part by a Grant-in-Aid for Scientific Research (C) from the Ministry of Education, Culture, Sports, Science and Technology of Japan (No. 15K03509).
Address correspondence to: Tetsuo Ono, Graduate School of Economics, Osaka University, 1–7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan; e-mail: tono@econ.osaka-u.ac.jp.

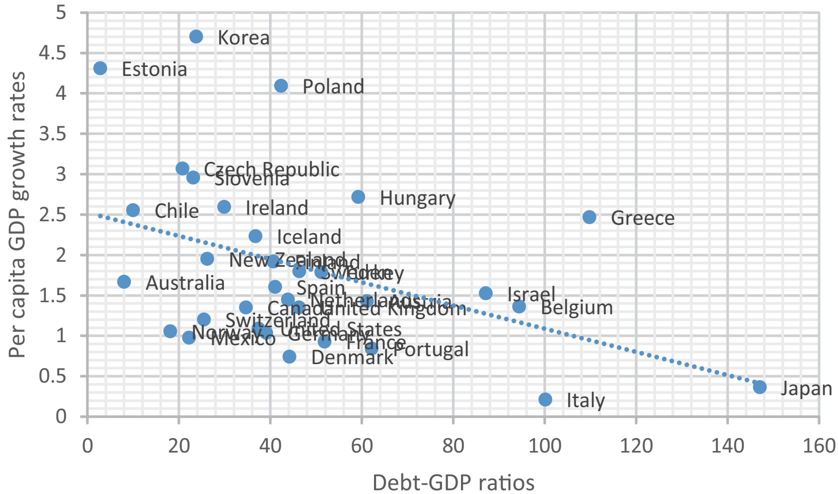

FIGURE 1. Debt–GDP ratio and per-capita GDP growth rate for 32 countries, 1999–2009. *Source:* OECD.Stat (December 17, 2016).

environment, some recent studies extend these models by including unemployment and investigating the effects of fiscal policy on economic growth in the presence of unemployment [Kaas and von Thadden (2004), Josten (2006), Greiner and Flaschel (2010), Yakita (2014)].

However, in these studies, the following issues remain unanswered. First, a high unemployment rate—possibly stemming from the power of trade unions—exerts political pressure on the government to increase spending in favor of unemployed persons. This pressure incentivizes the government to issue more public bonds to finance expanding expenditure, resulting in a crowding out of capital accumulation. That is, trade unions have a political effect on fiscal policy and economic growth through unemployment. As Figure 2 shows, the cross-country evidence shows a slight positive correlation between trade union density and debt–GDP ratios, and a negative correlation between trade union density and per-capita GDP growth rates. The aforementioned studies do not clarify the mechanisms underlying these findings, because they consider fiscal policy exogenously given.

Second, besides unemployment insurance, in advanced countries, there is a large public expenditure pertaining to intergenerational redistribution from young individuals to elderly ones, such as public pensions and health and nursing care systems for elderly individuals. This implies that the greater political power of elderly persons, possibly stemming from an increase in the old-age dependency ratio, exerts pressure on the government to shift fiscal expenditure from young and unemployed persons to elderly and retired ones. However, the cross-country evidence shows that the expenditure on unemployed persons positively correlates with the expenditure on elderly persons (see Figure 3). The evidence suggests that the political effect of elderly persons is outweighed by the political effect of

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

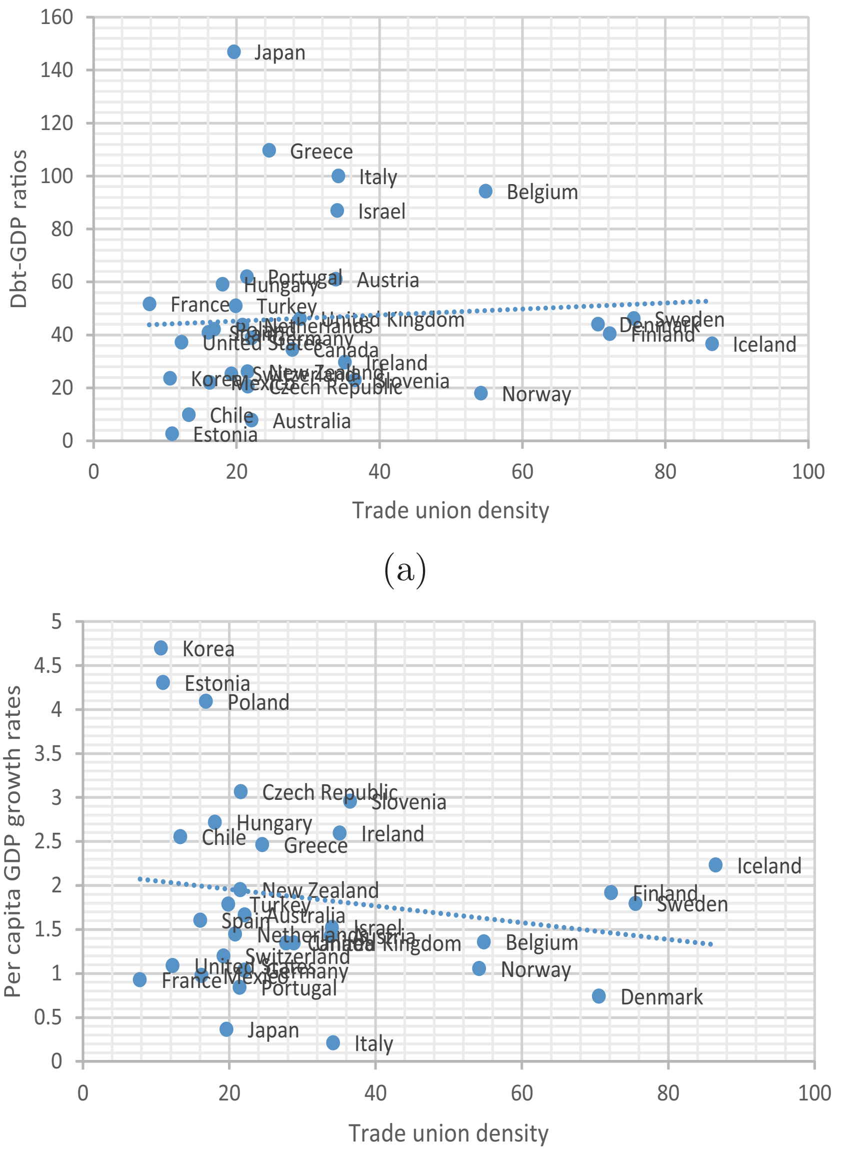

FIGURE 2. Trade union density and debt–GDP ratios [Panel (A)]; trade union density and per-capita GDP growth rates [Panel (B)] for 32 OECD countries, 1999–2009. *Source:* OECD.Stat (December 17, 2016).

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

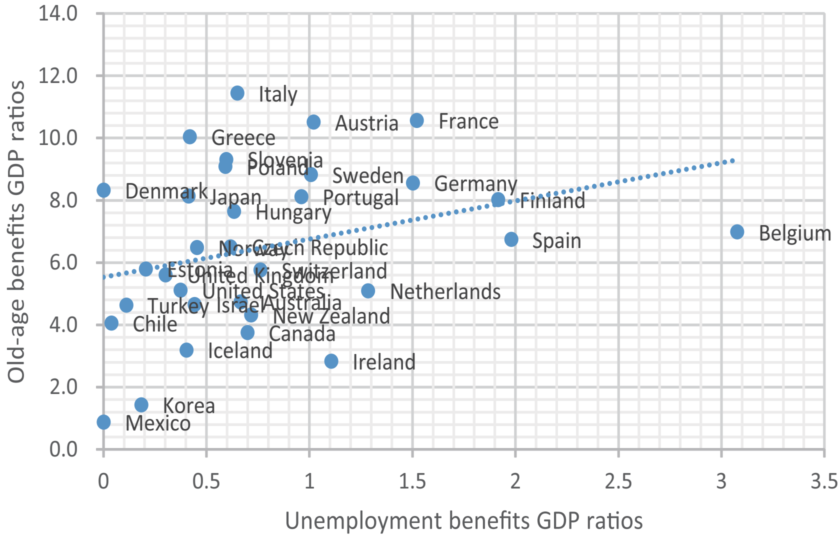

FIGURE 3. Unemployment benefit–GDP ratios and old-age benefit–GDP ratios for 32 OECD countries, 1999–2009. *Source:* OECD.Stat (December 17, 2016).

trade unions, and that an increase in unemployment and old-age benefits would be financed through bond issuance. Corneo and Marquardt (2000), Bräuninger (2005), and Ono (2007) analyze two types of expenditures in a unified framework. However, they assume no public bonds and consider the tax rates for financing these expenditure to be exogenously given; thus, they set aside the political background behind the evidence.

The argument, thus far, suggests that the two issues should be addressed together, because each component influences the other. In addition, the political conflict over the allocation of public budgets should be addressed when analyzing two fiscal expenditures: unemployment and old-age benefits. To address these concerns, we employ a two-period overlapping-generations model with AK technology [Romer (1986)] and collective wage-bargaining [see, e.g., Kaas and von Thadden (2003, 2004), Coimbra et al. (2005), Chang et al. (2007)] to demonstrate capital accumulation and unemployment. Government spending is represented by unemployment-insurance benefits for unemployed persons and public services for elderly persons.[1] Spending is financed by taxing young persons and also by issuing public bonds.

The unemployment-insurance benefit creates a conflict of interest between unemployed and employed persons. Furthermore, spending on services for elderly persons creates a conflict of interest between young persons (either employed or unemployed) and elderly persons. To demonstrate this conflict, we assume probabilistic voting à la Lindbeck and Weibull (1987), where the government objective is to maximize the weighted sum of the utility of young employed persons,

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

FIGURE 4. Unemployment benefit–GDP ratios and debt–GDP ratios for 32 OECD countries, 1999–2009. *Source:* OECD.Stat (December 17, 2016).

young unemployed persons, and retired elderly persons. In particular, we employ a Markov strategy in which policy variables are conditioned on payoff-relevant state variables [Krusell et al. (1997)]. This strategy enables us to demonstrate the forward-looking behavior of individuals who consider intertemporal interaction between current and future policies through capital accumulation [see, e.g., Gonzalez-Eiras and Niepelt (2008), Song (2011, 2012), Kunze (2014), Lancia and Russo (2015)].

Within this framework, we show that the increased power of unions results in higher unemployment and a higher debt–GDP ratio. Thus, a positive correlation holds between unemployment and debt. This result aligns with the empirical evidence observed in advanced countries, as illustrated in Figure 4. The result is also in line with previous studies by Kaas and von Thadden (2004), who show a positive correlation under capital shortages, and by Battaglini and Coate (2016), who show the procyclical behavior of unemployment and debt arising from time-varying productivity. This study presents an alternative approach to explaining the positive correlation.

Following this, we consider the second issue and show that an increase in the political power of elderly persons, possibly stemming from an increase in the old-age dependency ratio, results in a higher ratio of spending to GDP for elderly persons, a lower ratio of unemployment-insurance benefits to GDP, and a lower growth rate of capital. The result suggests that an increase in the political power of elderly persons results in a shift of resources from young unemployed persons to elderly persons, and thus elderly persons harm economic growth through

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

redistributive politics. However, this result should be considered in tandem with the first result, to obtain the model prediction that fits the evidence in Figure 3.

Some OECD countries have introduced budget rules that control public bond issuance, from the perspective of fiscal sustainability. To assess the impact of debt control, we consider an alternative budget scenario that limits public bond issuance: a tax-finance rule where government spending is solely financed by tax. Then, we compare the debt-finance and the tax-finance cases, and obtain the following result. When the government finances its spending by issuing public bonds (i.e., by borrowing in the capital market), the introduction of a tax-finance requirement results in a higher growth rate, and thus benefits future generations; however, it results in a lower unemployment-insurance payment–GDP ratio and a higher tax rate in the initial period. Therefore, the introduction of the tax-finance rule is not Pareto-improving, it benefits future generations at the expense of the current employed and unemployed young.

This study contributes to the following strands of political economy literature. The first is the literature on the positive theory of fiscal policy [see, e.g., Battaglini and Coate (2008), Song et al. (2012), Barseghyan et al. (2013), Arai and Naito (2014), Battaglini (2014), Ono (2015)]. In particular, the framework of this study is based on that of Arai and Naito (2014) and Ono (2015); the present study introduces the managerial trade union as a source of unemployment into their model. This study focuses on the same issue as Battaglini and Coate (2016), who analyzed political decisions on fiscal policy in the presence of unemployment. However, they assume (i) exogenous wage rigidity and time-varying productivity as a source of unemployment, and (ii) no saving behavior—and thus, no capital accumulation. In contrast, this study assumes collective wage-bargaining (to demonstrate the mechanism by which unemployment arises as an equilibrium phenomenon) and the AK technology (to demonstrate the effect of fiscal policy on capital accumulation in the presence of unemployment).

The second strand includes studies on intragenerational and intergenerational redistributive politics in models with physical and/or human capital accumulation [see, e.g., Poutvaara (2006), Bassetto (2008), Gonzalez-Eiras and Niepelt (2008, 2012), Song (2011), Bernasconi and Profeta (2012), Uchida (in press)]. They assume competitive labor markets, and thus no equilibrium unemployment. In contrast, this study presents equilibrium unemployment and demonstrates an intragenerational conflict between employed and unemployed persons, as well as an intergenerational conflict between young persons (either employed or unemployed) and elderly persons. Within this context, we consider redistributive politics regarding unemployment-insurance benefits and redistribution that targets elderly persons. We show the welfare effects of intragenerational and intergenerational conflicts on fiscal policy and capital accumulation, in the presence of unemployment.

Section 2 of this paper presents the model and characterizes the economic equilibrium. Section 3 characterizes a political equilibrium where government expenditure is financed by tax and the issuance of public bonds. Section 4 considers an alternative scenario that prohibits public bond issuance, and it investigates the effect of this prohibition on fiscal policy, growth, and welfare. Section 5 determines the robustness of the results under alternative assumptions. Section 6 presents some caveats to the analysis, along with our conclusions. Appendix A contains the proofs. Appendix B is available as online supplementary information in the online version of this journal.

2. MODEL AND ECONOMIC EQUILIBRIUM

Consider a two-period-lived overlapping-generations model where the economy comprises perfectly competitive firms, ex ante identical individuals, a trade union, and a government. Time is discrete and denoted by $t = 0, 1, 2, \ldots$. A new generation is born in each period $t = 0, 1, 2, \ldots$, and individuals in each generation live for two periods (i.e., youth and old age). No population growth is assumed, and the population in each generation is normalized to unity.

2.1. Preferences and Utility Maximization

An individual supplies one unit of labor inelastically in youth and retires in old age. The lifetime utility of an individual born in period t is given by

$$U_t^i = \ln c_t^{yi} + \beta \ln c_{t+1}^{oi} + \beta\eta \ln g_{t+1},$$

where c_t^{yi} is consumption in youth, c_{t+1}^{oi} is consumption in old age, g_{t+1} is public services for elderly persons (e.g., medical-care systems and nursing-care insurance systems), $\beta \in (0, 1)$ is a discount factor, and $\eta(> 0)$ captures the preference weight for public services. The subscript t denotes the period of consumption, and the superscript i denotes the status of labor: $i = e$ or $i = u$ if an individual is employed or unemployed, respectively. The status is assigned according to bargaining between the trade union and the firm (described later) at the beginning of each period. The specification of the logarithmic utility function makes the aggregation of the savings functions tractable.

An individual chooses consumption and savings to maximize lifetime utility, under the following budget constraints:

$$\begin{aligned} c_t^{yi} + s_t^i &\leq x_t(1-\tau_t)w_t + (1-x_t)b_t, \ x_t \in \{0, 1\} \\ c_{t+1}^{oi} &\leq R_{t+1}s_t^i, \end{aligned}$$

where $x_t = 1$ and $x_t = 0$ if an individual is employed or unemployed, respectively. w_t is the wage, b_t is the unemployment-insurance benefit, s_t is savings, R_{t+1} is the gross interest rate, and τ_t is the tax on labor income. Unemployment-insurance benefits are assumed to be tax-exempt.

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

By solving the utility-maximization problem, we obtain the savings function of a type-i individual as follows:

$$s_t^i = \frac{\beta}{1+\beta}\left[x_t(1-\tau_t)w_t + (1-x_t)b_t\right].$$

The corresponding consumption functions are

$$c_t^{yi} = \frac{1}{1+\beta}\left[x_t(1-\tau_t)w_t + (1-x_t)b_t\right],$$

$$c_{t+1}^{oi} = \frac{\beta R_{t+1}}{1+\beta}\left[x_t(1-\tau_t)w_t + (1-x_t)b_t\right].$$

These functions state that a higher wage level or unemployment-insurance benefit implies higher savings and consumption, whereas a higher tax rate implies lower savings and consumption. Using these functions, the indirect utility functions of employed, unemployed, and elderly persons are given by

$$V_t^{ye} = (1+\beta)\ln(1-\tau_t)w_t + \beta\ln R_{t+1} + \beta\eta\ln g_{t+1} + \left(\ln\frac{1}{1+\beta} + \beta\ln\frac{\beta}{1+\beta}\right),$$

$$V_t^{yu} = (1+\beta)\ln b_t + \beta\ln R_{t+1} + \beta\eta\ln g_{t+1} + \left(\ln\frac{1}{1+\beta} + \beta\ln\frac{\beta}{1+\beta}\right), \text{ and}$$

$$V_t^{o} = \eta\ln g_t + \ln R_t s_{t-1}^i, \quad i = e, u,$$

respectively.

2.2. Technology and Profit Maximization

There is a continuum of identical firms that are perfectly competitive profit maximizers and which produce the final product Y_t with a constant returns-to-scale Cobb–Douglas production function, $Y_t = A_t\,(K_t)^{\alpha}\,(L_t)^{1-\alpha}$. Here, A_t is the productivity parameter, K_t is aggregate capital, L_t is aggregate labor, and $\alpha \in (0, 1)$ is a constant parameter representing capital share. Capital is assumed to fully depreciate within a period.

In each period t, a firm chooses capital and labor in order to maximize its profit, $A_t\,(K_t)^{\alpha}\,(L_t)^{1-\alpha} - \rho_t K_t - w_t L_t$, where ρ_t is the rental price of capital and w_t is the wage rate. The firm takes these prices as given. The first-order conditions with respect to K_t and L_t are given by

$$K_t : \rho_t = \alpha A_t\,(K_t)^{\alpha-1}\,(L_t)^{1-\alpha},$$

$$L_t : w_t = (1-\alpha)A_t\,(K_t)^{\alpha}\,(L_t)^{-\alpha}.$$

The productivity parameter A_t is assumed to be proportional to the aggregate capital in the overall economy: $A_t = A\,(K_t)^{1-\alpha}$. Thus, capital investment involves a technological externality of the type often used in endogenous-growth theories

[Romer (1986)]. This assumption, called "AK" technology, results in a constant interest rate across periods, as demonstrated below. This approach enables us to obtain an analytical solution for the model.

Under this assumption, the first-order conditions are rewritten as follows:

$$\rho_t = \alpha A (l_t)^{1-\alpha} = R_t, \quad (1)$$

$$w_t = (1-\alpha)\mathrm{AK}_t (l_t)^{-\alpha}, \quad (2)$$

where l_t is the employment rate in the economy, and $l_t = L_t$ holds because the number of people in each generation is unity. The arbitrage condition $\rho_t = R_t$ holds for all t, because the capital market is competitive and capital fully depreciates within a period.

2.3. Government Budget Constraint

Fiscal policy is determined through elections, and public bonds are traded in a domestic capital market. Let D_t denote the aggregate inherited debt. A budget constraint in period t is

$$D_{t+1} + \tau_t l_t w_t = g_t + (1-l_t)b_t + R_t D_t, \quad (3)$$

where D_{t+1} is newly issued public bonds, $\tau_t l_t w_t$ is the labor income-tax revenue, g_t is expenditure for elderly persons, $(1-l_t)b_t$ is unemployment-insurance payments, and $R_t D_t$ is debt repayment. We assume that in each period, the government is committed to not repudiating the debt.

Equation (3) indicates that the government can freely issue public bonds, as long as it satisfies the budget constraint. In Section 3, we demonstrate the political equilibrium outcome of this case. In Section 4, we consider an alternative case: the tax-finance case, in which the government is required to finance its spending solely through taxation.

2.4. Right-to-Manage Model

Following Pemberton (1988), we assume a managerial trade union whose objective is to pursue two targets: a high real wage, w_t, and a high employment rate, l_t. In particular, the trade union's objective function is specified using the following Cobb–Douglas function:

$$(w_t - \bar{w}_t)^{\delta} \cdot (l_t)^{1-\delta},$$

where $\bar{w}_t$ is the reference wage of the trade union, and $\delta \in (0, 1)$ is a parameter capturing the relative intensity of the two targets.

Following Corneo and Marquardt (2000), we assume that the reference wage is the competitive wage, which is calculated by setting $l_t = 1$ in the first-order condition with respect to labor, (2):

$$\bar{w}_t = (1-\alpha)\mathrm{AK}_t.$$

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

Alternatively, we assume that the reference wage is set to the unemployment-insurance benefits, $\bar{w}_t = b_t$ [see, e.g., Chang et al. (2007)], which we discuss in Section 5.

The present study employs the right-to-manage model [see, e.g., Benassy (2011), Chapter 15, and Heijdra (2009), Chapter 7, for an overview of the model]. The union and the firm negotiate wages through a generalized Nash bargaining solution. Given the solution, employment is determined to satisfy the labor demand function of the firm. According to this solution, the wage chosen after bargaining maximizes the geometrically weighted average of the gains to both the union and the firm, subject to the firm's demand for labor. Formally, the problem is as follows:

$$\max_{w_t} \Omega_t = \left[(w_t - \bar{w}_t)^{\delta} (l_t)^{1-\delta} \right]^{\theta} \cdot \left[A_t (K_t)^{\alpha} (l_t)^{1-\alpha} - w_t l_t \right]^{1-\theta}$$

$$\text{s.t. } w_t = (1-\alpha) A_t (K_t)^{\alpha} (l_t)^{-\alpha}$$

$$\text{given } \bar{w}_t,$$

where $\theta \in [0, 1]$ represents the relative strength of the union. The term $(w_t - \bar{w}_t)^{\delta} (l_t)^{1-\delta}$ is the gain to the union, whereas the term $A_t (K_t)^{\alpha} (l_t)^{1-\alpha} - w_t l_t$ is the gain to the firm.

To solve the problem, we impose the following assumption, which ensures the second-order condition for an interior solution.

Assumption 1. $\alpha < \min \left[1, \frac{1-\delta\theta}{(1-\theta)+\delta\theta} \right]$.

Under Assumption 1, the wage determined through bargaining becomes

$$w_t = \phi \bar{w}_t = \phi (1-\alpha) \mathrm{AK}_t, \tag{4}$$

where the second equality comes from $\bar{w}_t = (1-\alpha)\mathrm{AK}_t$, and ϕ is defined by

$$\phi \equiv \frac{(1-\delta)\theta + (1-\alpha)(1-\theta)}{(1-\delta)\theta + (1-\alpha)(1-\theta) - \alpha\delta\theta} (> 1).$$

We provide in Appendix A.1 the derivation of (4).

We substitute (4) into the labor demand function $w_t = (1-\alpha)\mathrm{AK}_t (l_t)^{-\alpha}$ to obtain the employment rate determined through bargaining:

$$l_t = l \equiv (1/\phi)^{1/\alpha}. \tag{5}$$

Given that ϕ is increasing in θ, we immediately understand that higher union power yields lower employment, $\partial l / \partial \theta < 0$.

Equation (5) indicates that the employment rate (or unemployment rate) is independent of fiscal policy and the capital stock. This implies that the present model demonstrates the effect of (un)employment on fiscal policy and capital accumulation; however, it does not show the reverse effect. This property, caused by the specification of collective wage-bargaining, points to some of this study's limitations. However, the present model enables us to consider the interaction between fiscal policy and capital accumulation in the presence of unemployment.

Using (5), we can write the aggregate output and the gross interest rate in terms of the employment rate:

$$Y_t = \mathrm{AK}_t l(l)^{-\alpha} = Al\phi K_t, \tag{6}$$

$$R_t = \alpha Al(l)^{-\alpha} = \alpha Al\phi \equiv R. \tag{7}$$

We use the expressions in (5)–(7) in the following analysis. Hereafter, to simplify the presentation, we often use R instead of $R_t = \alpha Al\phi$.

2.5. Economic Equilibrium

A market-clearing condition for capital is $K_{t+1} + D_{t+1} = s_t$, which expresses the equality of total savings by young agents in generation t, $s_t \equiv l_t s_t^e + (1 - l_t)s_t^u$, to the sum of the stocks of aggregate physical capital and aggregate public debt, $K_{t+1} + D_{t+1}$:

$$D_{t+1} + K_{t+1} = \frac{\beta}{1+\beta}\left[l(1-\tau_t)\phi(1-\alpha)\mathrm{AK}_t + (1-l)b_t\right]. \tag{8}$$

We are now ready to formally define in the present model an economic equilibrium.

DEFINITION 1. *Given a sequence of policy parameters* $\{\tau_t, b_t, g_t, D_{t+1}\}_{t=0}^{\infty}$, *an economic equilibrium is a sequence of prices and allocations,* $\left\{c_t^{yi}, c_{t+1}^{oi}, s_t^i, l_t, K_t, w_t, \bar{w}_t, \rho_t, R_t\right\}_{t=0}^{\infty}$, *with initial conditions* K_0 *and* D_0 *such that the following conditions are satisfied: (i) given* (w_t, R_{t+1}) *and a fiscal policy,* $\left(c_t^{yi}, c_{t+1}^{oi}, s_t^i\right)$ *solves the utility-maximization problem of a type-i agent, (ii) given* (w_t, ρ_t), (l_t, K_t) *solves the profit-maximization problem of a firm, (iii) given* $(\bar{w}_t, K_t)$, w_t *solves the Nash bargaining problem, (iv) the reference wage* $\bar{w}_t$ *is calculated by assuming full employment in the labor market, (v) given* (l_t, w_t, R_t, D_t), $(\tau_t, b_t, g_t, D_{t+1})$ *satisfies the government budget constraint, (vi)* $\rho_t = R_t$ *holds, and (vii) the capital market clears:* $D_{t+1} + K_{t+1} = l_t s_t^e + (1 - l_t)s_t^u$.

In each period, the timing of the events is as follows. First, the government representing young persons and elderly persons decides upon a fiscal policy to maximize its objective function (demonstrated later). Second, the wage is determined by the bargaining process, taking as given that the agents understand how wage affects labor demand. Then, the firm demands capital and labor, and sets employment according to its labor demand curve. Given a fiscal policy, a wage, and an interest rate, each young agent sets savings and consumption to maximize his or her utility. Finally, the capital market clears.

3. THE POLITICS

To consider the government's behavior, we need to determine its objective and the agents' indirect utility functions. Recall that V_t^{ye}, V_t^{yu}, and V_t^o denote the indirect utility of a young employed agent in period t, the indirect utility of a young unemployed agent in period t, and the indirect utility of an elderly agent in period t, respectively. These are expressed as functions of government policy and/or the capital stock, as follows:

$$V_t^{ye} = (1+\beta)\ln(1-\tau_t)\phi(1-\alpha)\mathrm{AK}_t + \beta\eta\ln g_{t+1} + C;$$
$$V_t^{yu} = (1+\beta)\ln b_t + \beta\eta\ln g_{t+1} + C;$$
$$V_t^{o} = \ln R(K_t + D_t) + \eta\ln g_t,$$

where

$$C \equiv \beta\ln R + \left(\ln\frac{1}{1+\beta} + \beta\ln\frac{\beta}{1+\beta}\right).$$

The terms $(1-\tau_t)\phi(1-\alpha)\mathrm{AK}_t$ and b_t correspond to the lifetime income (and thus, lifetime consumption) of employed and unemployed persons, respectively.

This study assumes probabilistic voting à la Lindbeck and Weibull (1987) in the demonstration of the political mechanism [see Acemoglu and Robinson (2005), appendix, and Persson and Tabellini (2000), pp. 54–58, for an overview of this voting mechanism]. In each period, the government in power chooses fiscal policy to maximize its political objective. Formally, the political objective function in period t is given by

$$P_t = \omega V_t^o + (1-\omega)\left[l\cdot V_t^{ye} + (1-l)\cdot V_t^{yu}\right],$$

or

$$P_t = \omega\eta\ln g_t + (1-\omega)(1+\beta)\left[l\cdot\ln(1-\tau_t)\phi(1-\alpha)\mathrm{AK}_t + (1-l)\cdot\ln b_t\right] + (1-\omega)\beta\eta\ln g_{t+1},$$

where ω and $1-\omega$ are the relative weights of elderly and young agents, respectively, and l and $1-l$ are the relative weights of employed and unemployed persons measured as a percentage of the young generation of the population, respectively.[2] Terms unrelated to politics are omitted from the above expression.

3.1. Political Equilibrium

Given K_t and D_t, the problem of the government in period t is to choose a set of fiscal policies, $(\tau_t, g_t, b_t, D_{t+1})$, to maximize P_t subject to the period-t government budget constraint. The problem is dynamic in that the values of the next-period state variables, K_{t+1} and D_{t+1}, passed from the current government to the next government, will affect the choice of g_{t+1} by the next government. This choice, in turn, has an effect on the utility of the current young persons, and thus on the current government's objective.

To take account of the above feature, we restrict our attention to a Markov-perfect equilibrium. Markov perfection implies that the outcomes depend only on the payoff-relevant state variables—that is, capital K and public debt D. Therefore, the expected level of public services in the next period, g_{t+1}, is given by a function of the next-period capital stock and public debt, $g_{t+1} = G(K_{t+1}, D_{t+1})$. Using a recursive notation with x' denoting the next period x, we can define a Markov-perfect political equilibrium as follows.

DEFINITION 2. *A Markov-perfect political equilibrium is a set of functions,* $\langle \tilde{T}, \tilde{G}, \tilde{B}, \tilde{D} \rangle$*, where* $\tilde{T} : \Re_{++} \times \Re \to [0, 1]$ *is a tax rule,* $\tau = \tilde{T}(K, D)$*;* $\tilde{G} : \Re_{++} \times \Re \to \Re_{++}$ *is a public expenditure rule,* $g = \tilde{G}(K, D)$*;* $\tilde{B} : \Re_{++} \times \Re \to \Re_{+}$ *is an unemployment-insurance rule,* $b = \tilde{B}(K, D)$*; and* $\tilde{D} : \Re_{++} \times \Re \to \Re$ *is a debt rule,* $D' = \tilde{D}(K, D)$*, such that*

(i) the capital market clears

$$\tilde{D}(K, D) + K' = \frac{\beta}{1+\beta} \left\{ l \left[1 - \tilde{T}(K, D) \right] \phi (1-\alpha) AK + (1-l) \tilde{B}(K, D) \right\}; \quad \mathbf{(9)}$$

(ii) given K and D, $\langle \tilde{T}(K, D), \tilde{G}(K, D), \tilde{B}(K, D), \tilde{D}(K, D) \rangle = \arg\max P$ *subject to* $g' = \tilde{G}(K', D')$*, the capital market-clearing condition in (9), and the government budget constraint*

$$\tilde{G}(K, D) + (1-l)\tilde{B}(K, D) + RD = \tilde{T}(K, D) l \phi (1-\alpha) AK + \tilde{D}(K, D),$$

where P is defined by

$$P(K, g, \tau, b, g') = \omega\eta \ln g + (1-\omega)(1+\beta) \left[l \ln(1-\tau)\phi(1-\alpha)AK + (1-l)\ln b \right] + (1-\omega)\beta\eta \ln g'.$$

The following proposition characterizes the political equilibrium.

PROPOSITION 3. *Denote* $\tilde{\eta} \equiv (1-\alpha)(1+\beta)/(1+\alpha\beta)$*. Given K and D, a Markov-perfect political equilibrium,* $\{\tau, b, g, K', D'\}$ *, is characterized by*

$$b = \tilde{B}(K, D) \equiv \frac{1-\omega}{(1-\omega)\left[1+\beta\left(1+\eta\right)\right]+\omega\eta} \cdot \frac{(1+\beta)^2}{1+\alpha\beta} \cdot \left[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD} \right], \quad \mathbf{(10)}$$

$$g = \tilde{G}(K, D) \equiv \frac{\omega\eta}{(1-\omega)\left[1+\beta\left(1+\eta\right)\right]+\omega\eta} \cdot \left[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD} \right], \quad \mathbf{(11)}$$

$$D' = \tilde{D}(K, D) \equiv \frac{1-\omega}{(1-\omega)\left[1+\beta\left(1+\eta\right)\right]+\omega\eta} \cdot \beta \cdot (\tilde{\eta} - \eta) \cdot \left[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD} \right], \quad \mathbf{(12)}$$

$$\tau = \tilde{T}(K, D) \equiv \Lambda + \frac{\alpha}{1-\alpha} \cdot (1-\Lambda) \cdot \frac{D}{K}, \qquad \textbf{(13)}$$

where

$$\Lambda \equiv 1 - \frac{1-\omega}{(1-\omega)\left[1+\beta\left(1+\eta\right)\right]+\omega\eta} \cdot \frac{l(1+\beta)^2}{1+\alpha\beta};$$

additionally, by the law of motion of capital,

$$\frac{K'}{K} = \frac{1-\omega}{(1-\omega)\left[1+\beta\left(1+\eta\right)\right]+\omega\eta} \cdot \frac{\beta\left[\alpha\left(1+\beta\right)+\left(1+\alpha\beta\right)\eta\right]}{1+\alpha\beta} \cdot \left[l\phi(1-\alpha)A - R\frac{D}{K}\right],$$

where

$$\frac{D}{K} = \begin{cases} \dfrac{D_0}{K_0} & \text{for} \quad t = 0, \\ \dfrac{(1+\alpha\beta)\cdot(\tilde{\eta}-\eta)}{\alpha\left(1+\beta\right)+\left(1+\alpha\beta\right)\eta} & \text{for} \quad t \geq 1. \end{cases}$$

The tax rate is set within the range $(0, 1)$ *for period* $t(\geq 0)$ *if* $\Lambda > 0$ *and* $D_0/K_0 < (1-\alpha)/\alpha$*.*

Proof. See Appendix A.2. ■

Proposition 1 implies that the economy has the following features. First, g and b are linear functions of wage income $l\phi(1-\alpha)$AK minus debt repayment RD. The available resources for the government are $l\phi(1-\alpha)$AK − RD, and it uses them for expenditure on public services for elderly persons and on unemployment benefits for young persons.

Second, the government borrows or lends in the capital market. The state of financial balance depends on η, which captures the preference weight of public services for elderly persons: $D' \gtreqless 0$ if and only if $\eta \lesseqgtr \tilde{\eta} \equiv (1-\alpha)(1+\beta)/(1+\alpha\beta)$. A higher η incentivizes young voters to lower public debt, from the perspective of maintaining public services they will enjoy in old age. This is the disciplined effect produced by young voters [Song et al. (2012)]. In particular, when η is above $\tilde{\eta}$, the disciplinary effect is so large that the government need not issue bonds. Rather, the government lends in the capital market to utilize its surplus funds.

Third, the growth rate is constant across periods, except for the initial period. This is because the model exhibits a constant interest rate inherited from the AK technology. However, the growth rate changes between the first two periods—that is, periods 0 and 1—because the government starts to borrow or lend in the capital market in period 0. In particular, the growth rate decreases when the government borrows in the capital market. The issuance of public bonds by the period-0 government pushes the next-period government to raise taxes to finance debt repayment. In addition, the issuance of public bonds crowds out capital accumulation, and thus impedes economic growth. The opposite result holds when the government lends in the capital market.

3.2. Political Power of Elderly Persons and of Trade Unions

Based on the result in Proposition 1, we now investigate how policies and the growth rate are affected by an increase in the power of elderly persons and of the trade unions. The following proposition summarizes the result.

PROPOSITION 4. *In the political equilibrium presented in Proposition 1, increases in ω and θ cause the following:*

(i) $\partial\left(D/Y\right)/\partial\omega = 0, \partial\tau/\partial\omega > 0, \partial\left(g/Y\right)/\partial\omega > 0, \partial\left[(1-l)b/Y\right]/\partial\omega < 0$, *and* $\partial\left(K'/K\right)/\partial\omega < 0$;
(ii) $\partial\left(D/Y\right)/\partial\theta > 0, \partial\tau/\partial\theta > 0, \partial\left(g/Y\right)/\partial\theta = 0, \partial\left[(1-l)b/Y\right]/\partial\theta > 0$, *and* $\partial\left(K'/K\right)/\partial\theta < 0$.

Proof. See Appendix A.3. ■

To understand the result in Proposition 2, we first consider the D/Y ratio. This is rewritten as $D/Y = (D/K)/A(l)^{1-\alpha}$, where D/K is independent of ω and θ. The employment rate l is independent of the political power of elderly persons, because it is determined to balance the conflicting items between young employed persons and the trade union. However, the employment rate decreases as the political power of the union increases. This in turn reduces the aggregate output, and thus increases the D/Y ratio.

Second, the tax rate increases as the political power of elderly persons increases, because elderly persons owe no tax burden, and thus they want to increase public services at the expense of the financial burden placed on young persons. The tax rate also increases as the bargaining power of the union increases. When the union has greater bargaining power, the employment rate decreases. In response to this change, the government increases the tax rate on employed persons to maintain the tax revenue level.

Third, g/Y increases but $(1-l)b/Y$ decreases as the political weight of elderly persons increases. A larger weight on elderly persons incentivizes the government to shift resources from young persons (including those who are unemployed) to elderly persons, resulting in a higher g/Y and a lower $(1-l)b/Y$. The relative bargaining strength has no effect on g/Y, because its effect on the policy function of g is offset by its effect on the aggregate output Y. However, the relative bargaining strength has a positive effect on $(1-l)b/Y$, because increased union power reduces the number of employed persons, and thus increases aggregate spending on unemployment-insurance payments.

Finally, the capital growth rate decreases as the political weight of elderly persons increases. A larger weight on elderly persons forces the government to focus on elderly persons, and spend more resources on public services, resulting in fewer resources for savings and capital accumulation. The bargaining power of the union has two opposing effects on the growth rate: a higher markup, ϕ, and a lower number of employed persons. The result suggests that the latter negative effect always outweighs the former positive effect, resulting in a lower growth rate in response to an increase in the power of the union.

A noteworthy feature of the result in Proposition 2 is that the greater power of the union leads to a higher unemployment rate and a higher debt–GDP ratio, this suggests a positive correlation between these two variables. This is in line with the predictions of previous studies, such as those of Kaas and von Thadden (2004) and Battaglini and Coate (2016). However, Kaas and von Thadden (2004) assume fixed unemployment benefits and a fixed tax rate, and thus rule out the possibility of changes to these policy variables on account of voting. Battaglini and Coate (2016) overcome this issue by demonstrating voting on fiscal policy; however, they subtract from their model physical capital accumulation. The current study instead demonstrates a politico-economic model with physical capital accumulation, and shows that the government responds to an increase in unemployment by increasing the tax rate and issuing more public bonds; these changes, in turn, reduce the growth rate.

Another noteworthy feature is that an increase in the political power of elderly persons results in a higher ratio of spending for elderly persons to GDP and a lower ratio of unemployment-insurance benefits to GDP. This implies a shift of resources from young unemployed persons to elderly retired persons. However, the evidence in Figure 3 shows that the two expenditures positively correlate. The evidence suggests that the political effect of elderly persons is outweighed by the political effect of the trade union. In addition, increased spending for unemployed persons and elderly persons would be financed by issuing public bonds.[3]

4. DEBT FINANCE VERSUS TAX FINANCE

In the preceding section, we considered fiscal policy and economic growth when the government is able to issue public bonds to finance its expenditure. We assume no constraint on public bond issuance. However, in the real world, many countries have introduced fiscal constitutions that govern the determination of fiscal policies. In particular, constitutional balanced-budget rules are in force in Austria, Germany, Italy, Slovenia, Spain, Switzerland, and the United States [Azzimonti et al. (2016)].

To investigate the role and impact of budget rules, we consider an alternative scenario: the tax-finance case, in which the government is unable to issue public bonds, and so it finances its spending solely through taxation. In particular, throughout this section, we assume $\eta < \tilde{\eta}$. The assumption implies that in the absence of a constraint, the government borrows in the capital market (see Proposition 1). In other words, the government wants to borrow in the capital market, but its borrowing is restricted when a budget rule is introduced. We compare the debt-finance case presented in the preceding section and the tax-finance case presented below, in terms of government expenditure and economic growth, we also investigate the welfare consequences of shifting from debt finance to tax finance.[4]

For comparison, we assume that capital K and debt D are given in the beginning of each period; however, the government is unable to issue new public bonds. Therefore, a *tax-finance Markov-perfect political equilibrium* is a set of functions, $\langle T, G, B\rangle$, where $T : \Re_{++} \times \Re \to [0, 1]$ is a tax rule, $\tau = T(K, D)$; $G : \Re_{++} \times \Re \to \Re_{++}$ is a public services provision rule, $g = G(K, D)$, and $B : \Re_{++} \times \Re \to \Re_{+}$ is an unemployment-insurance rule, $b = B(K, D)$, such that

1. (i) the capital market clears

$$K' = \frac{\beta}{1+\beta}\{l\,[1 - T(K, D)]\,\phi(1-\alpha)\mathrm{AK} + (1-l)B(K, D)\}\,; \qquad \textbf{(14)}$$

2. (ii) given K, $\langle T(K, D), G(K, D), B(K, D)\rangle = \arg\max P(K, D, \tau, g, b, g')$ subject to $g' = G(K', D')$, (14), and the government budget constraint,

$$G(K) + (1-l)B(K) + \mathrm{RD} = T(K)l\phi(1-\alpha)\mathrm{AK}.$$

The following proposition characterizes the tax-finance political equilibrium.

PROPOSITION 5. *Given K and D, a tax-finance Markov-perfect political equilibrium, $\{\tau, b, g, K'\}$, is characterized by the policy functions:*

$$b = \frac{1-\omega}{(1-\omega)[1+\beta(1+\eta)] + \omega\eta}[1+\beta(1+\eta)]\,[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}]\,,$$

$$g = \frac{\omega\eta}{(1-\omega)[1+\beta(1+\eta)] + \omega\eta}\,[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}]\,,$$

$$\tau = \frac{\beta(1+\alpha\beta)}{(1+\beta)^2}\cdot(\tilde{\eta} - \eta) + \frac{(1+\alpha\beta)[1+\beta(1+\eta)]}{(1+\beta)^2}\cdot\left[\Lambda + \frac{\alpha}{1-\alpha}\,(1-\Lambda)\,\frac{D}{K}\right],$$

as well as by the law of motion of capital,

$$\frac{K'}{K} = \frac{1-\omega}{(1-\omega)[1+\beta(1+\eta)] + \omega\eta}\cdot\frac{\beta[1+\beta(1+\eta)]}{1+\beta}\cdot\left[l\phi(1-\alpha)A - R\frac{D}{K}\right],$$

where

$$\frac{D_t}{K_t} = \begin{cases} D_0/K_0 & \textit{for} \quad t = 0, \\ 0 & \textit{for} \quad t \geq 1. \end{cases}$$

The tax rate is set within the range $(0, 1)$ *if* $\Lambda > 0$ *and* $D_0/K_0 < (1-\alpha)/\alpha$.

Proof. See Appendix A.4. ■

The result in Proposition 3 indicates that the policy function of the expenditure for elderly persons, g, is identical between the debt-finance and the tax-finance cases, while the other policy functions and the law of motion of capital differ between the two cases. To investigate the differences in detail, we compare the two cases in terms of government expenditure–GDP ratios, g/Y and $(1-l)b/Y$, and economic growth, K'/K. First, we compare those in period 0 and obtain the

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

following result. Variables in the debt-finance and tax-finance cases are denoted with subscripts "debt" and "tax," respectively.

PROPOSITION 6. *Assume* $\eta < \tilde{\eta}$. *Given* $D_0/K_0 < (1-\alpha)/\alpha$, K_1/K_0, g_0/Y_0, $(1-l)b_0/Y_0$, *and* τ_0 *in the debt-finance and tax-finance cases are compared as follows:*

$$\left.\frac{K_1}{K_0}\right|_{debt} < \left.\frac{K_1}{K_0}\right|_{tax}, \quad \left.\frac{g_0}{Y_0}\right|_{debt} = \left.\frac{g_0}{Y_0}\right|_{tax},$$

$$\left.\frac{(1-l)b_0}{Y_0}\right|_{debt} > \left.\frac{(1-l)b_0}{Y_0}\right|_{tax}, \text{ and } \left.\tau_0\right|_{debt} < \left.\tau_0\right|_{tax}.$$

Proof. See Appendix A.5. ■

The result in Proposition 4 indicates that in the initial period, the growth rate in the debt-finance case is lower than that in the tax-finance case, because public debt crowds out private investment, and thus also capital formation. The result also indicates that the tax rate in the debt-finance case is lower than that in the tax-finance case, this is because the government can issue public bonds to finance the tax cut. Thus, prohibiting public bond issuances creates a tax-hike effect.

To understand the effects of fiscal stance on public services for elderly persons and on unemployment-insurance benefits, recall the political objective function in the debt-finance case:

$$\begin{aligned} P = {} & \omega\eta \ln g + (1-\omega)(1+\beta)l \ln\left[(l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD})\right. \\ & \left. - g - (1-l)b + D'\right] \\ & + (1-\omega)(1+\beta)(1-l)\ln b \\ & + (1-\omega)\beta\eta \ln\left\{l\phi(1-\alpha)A\frac{\beta}{1+\beta}\left[(l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}) - g\right]\right. \\ & \left. - \left[\frac{l\phi(1-\alpha)A}{1+\beta} + R\right]D'\right\}. \end{aligned} \tag{15}$$

The objective function indicates that the fiscal stance on the taxpayers generates the following two types of costs in terms of utility. First, the burden results in decreased consumption among employed persons. Second, the burden also results in decreased savings among employed persons, which in turn lowers the level of next-period capital stock—and thus, future public services for elderly persons. However, the unemployment-insurance benefit is irrelevant to the latter, because it is an intragenerational transfer and thus has no effect on aggregate saving.

With this difference in mind, let us consider the effects of issuing public bonds on the two types of costs. First, issuing bonds enables the government to cut the tax rate, thereby reducing the first cost [as observed in the second term in equation (15)]. Second, issuing bonds crowds out capital formation and reduces the future provision of public services for elderly persons, and thus increases the second cost

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

as observed in the fourth term of the political objective function. The provision of public services for elderly persons is not affected by the issuance of public bonds, because the first effect is offset by the second effect. However, bond issuance does affect unemployment-insurance provision, because the second effect is irrelevant. Therefore, the expenditure for elderly persons is independent of fiscal stance, whereas the unemployment-insurance level is higher in the debt-finance case than in the tax-finance case.

Next, we compare the two cases in terms of the growth rate and public-services–GDP ratio, in period $t (\geq 1)$.

PROPOSITION 7. *Assume $\eta < \tilde{\eta}$. Given K_0 and D_0, we compare K_{t+1}/K_t and g_t/Y_t in the debt-finance and the tax-finance cases as follows:*

$$\left.\frac{K_{t+1}}{K_t}\right|_{debt} < \left.\frac{K_{t+1}}{K_t}\right|_{tax} \text{ and } \frac{g_t}{Y_t}_{debt} < \left.\frac{g_t}{Y_t}\right|_{tax}.$$

Proof. See Appendix A.5. ■

The effect on economic growth in period t (≥ 1) is qualitatively similar to that in the initial period. However, the effect on public services for elderly persons in period $t (\geq 1)$ differs from that in the initial period. The public services–GDP ratio is independent of the fiscal stance in the initial period, whereas it is critically affected by the fiscal stance in period $t \geq 1$. To understand this difference, recall that for period $t \geq 1$, the resources available to the government in the debt-finance case are given by $l\phi(1-\alpha)\text{AK} - \text{RD}$, these are smaller than those in the tax-finance case, because the government in the debt-finance case must use part of its resources for debt repayment. Because of this difference in resource availability, the public services–GDP ratio in the debt-finance case is lower than that in the tax-finance case.

The tax rate and unemployment-insurance payments are directly affected by the state of financial balance in the initial period; however, these effects are not straightforward for period $t \geq 1$, as we demonstrate in the following.

PROPOSITION 8. *For period $t \geq 1$, there is a critical value of η, denoted by $\hat{\eta} \equiv \alpha(1+\beta)/\beta(1-\alpha)$, such that*

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{debt} \geq \left.\frac{(1-l)b_t}{Y_t}\right|_{tax} \text{ and } \tau_t|_{debt} \leq \tau_t|_{tax} \text{ if } \alpha < \frac{\beta}{1+2\beta} \text{ and } \eta \in [\hat{\eta}, \tilde{\eta});$$

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{debt} < \left.\frac{(1-l)b_t}{Y_t}\right|_{tax} \text{ and } \tau_t|_{debt} > \tau_t|_{tax}$$

$$\text{if either } \alpha \geq \frac{\beta}{1+2\beta}, \text{ or } \alpha < \frac{\beta}{1+2\beta} \text{ and } \eta \in (0, \hat{\eta}).$$

Proof. See Appendix A.5. ■

Bond issuance produces two opposing effects on the tax rate. The first is a tax-cut effect, as demonstrated in Proposition 4. The second is a tax-hike effect: the bond issuance incurs debt repayment costs from period 1 onward, and the government finances a part of the costs by raising the tax rate. The result in Proposition 6 suggests that if either $\alpha \geq \beta/(1+2\beta)$, or $\alpha < \beta/(1+2\beta)$ and $\eta \in (0, \hat{\eta})$, the tax-hike effect will outweigh the tax-cut effect—that is, prohibiting debt finance will reduce the tax rate. Otherwise, the opposite is true.

The bond issuance also produces two opposing effects on unemployment insurance benefits, these are observed in the second terms of the political objective function in the debt-finance case in equation (15). The positive effect is that the bond issuance increases the resources available to the government, and thus enables the government to increase the unemployment-insurance payments. The negative effect is the incurrence of debt repayment costs from period 1 onward, which reduces the resources available to the government and hence results in a decrease in the unemployment insurance payments.

The result in Proposition 6 suggests that if either $\alpha \geq \beta/(1+2\beta)$, or $\alpha < \beta/(1+2\beta)$ and $\eta \in (0, \hat{\eta})$, the negative effect on unemployment insurance will outweigh the positive effect on it. In other words, the shift from debt finance to tax finance may increase the unemployment-insurance payment–GDP ratio. With the result in Proposition 5, we find that the shift increases the growth rate and spending for the elderly persons–GDP ratio, and may also increase the unemployment-insurance payment–GDP ratio. This result suggests that the balanced-budget requirement, which has been or is being introduced in some countries or states to instill fiscal discipline, may benefit future generations at the expense of young employed and unemployed persons in the initial period. We investigate this welfare implication further in the following.

PROPOSITION 9. *There is a positive integer, $T (\geq 1)$, such that*

$$V_t^{ye}\big|_{debt} \geq V_t^{ye}\big|_{tax} \text{ and } V_t^{yu}\big|_{debt} \geq V_t^{yu}\big|_{tax} \text{ for } t \leq T,$$
$$V_t^{ye}\big|_{debt} < V_t^{ye}\big|_{tax} \text{ and } V_t^{yu}\big|_{debt} < V_t^{yu}\big|_{tax} \text{ for } t > T.$$

Proof. See Appendix A.6. ■

The result in Proposition 7 suggests that tax finance definitely benefits future generations at the expense of current employed and unemployed persons. To understand this, suppose that the tax-finance rule is introduced in period 0. As demonstrated in Proposition 4, in period 0, the government requires a higher tax rate and a lower unemployment-insurance benefit level than in debt finance, because the government is unable to issue public bonds to finance its expenditure. This creates negative income effects on both current employed and unemployed persons, and thus brings about lower utility in the tax-finance case than in the debt-finance case.

Proposition 7 shows that the result is the opposite for future generations. As demonstrated in Proposition 6, future young persons benefit from tax finance if either $\alpha \geq \beta/(1+2\beta)$, or $\alpha < \beta/(1+2\beta)$ and $\eta \in (0, \hat{\eta})$. If this condition fails to hold, future young persons suffer from a negative income effect on account of tax finance. However, the current government can bequeath more capital to future generations in the tax-finance case. This implies that the future government can use more resources to provide public services for elderly persons. This positive effect outweighs the negative income effect. Therefore, future generations will benefit from tax finance, regardless of the parameter values.

5. EXTENSIONS

The main analysis of this study assumes that the reservation wage is the competitive wage. In this section, we attempt to relax this assumption in two ways, and briefly examine how the analysis and the results thereof would change.

5.1. Unemployment-Insurance Benefit as the Reservation Wage

An alternative assumption here is that the reservation wage is given by the level of unemployment-insurance benefits, b_t [see, e.g., Bean and Pissarides (1993), Chang et al. (2007)]. The union's objective under this alternative assumption is $[(1-\tau_t)\, w_t - b_t]^{\delta}\, (l_t)^{1-\delta}$. The objective function in the Nash bargaining problem is now modified as

$$\Omega_t = \left\{[(1-\tau_t)\, w_t - b_t]^{\delta}\, (l_t)^{1-\delta}\right\}^{\theta} \cdot \left[A_t\, (K_t)^{\alpha}\, (l_t)^{1-\alpha} - w_t l_t\right]^{1-\theta},$$

and the solution to the problem becomes

$$(1-\tau_t)\, w_t = \phi b_t,$$

where $\phi(> 1)$ is defined in (4).

The indirect utility functions of employed and unemployed persons are now given by

$$V_t^{ye} = (1+\beta)\ln\phi b_t + \beta\ln R_{t+1} + \beta\eta\ln g_{t+1},$$
$$V_t^{yu} = (1+\beta)\ln b_t + \beta\ln R_{t+1} + \beta\eta\ln g_{t+1},$$

respectively. An increase in b improves the utility of both employed and unemployed persons—that is, there is no conflict over the provision of unemployment-insurance benefits between employed and unemployed persons. This is empirically implausible. Therefore, for the union, this study uses the competitive wage as the reservation wage.

5.2. After-Tax Wage as a Union's Target

In the main analysis, the unemployment rate was endogenous but independent of fiscal policy. This enabled us to solve the model in a tractable way; however, it fails to capture the relationship between unemployment and fiscal policy, as suggested by Battaglini and Coate (2014). To overcome this limitation, we consider the following alternative objective function for the union:

$$[(1-\tau_t)\, w_t - \bar{w}_t]^{\delta} \cdot (l_t)^{1-\delta},$$

where one of the union's targets is the after-tax wage, $(1-\tau_t)\, w_t$, rather than the before-tax wage, w_t.

Solving the Nash bargaining problem with this modification leads to the following employment rate:

$$l_t = l\,(\tau_t) \equiv \left(\frac{1-\tau_t}{\phi}\right)^{1/\alpha}.$$

The unemployment rate now depends on the tax rate. A higher tax rate implies less after-tax wage income. This incentivizes the union to set a higher wage through bargaining. Therefore, the employment rate is lower when the union's target is the after-tax wage than when it is the before-tax wage.

The political objective function, P, is now modified by replacing l with $l\,(\tau_t)$. However, this modification makes it difficult to obtain an analytical solution, because the choice of τ_t affects the employment rate and thus also the optimality condition with respect to τ_t. To overcome this problem, we divide the decision process of fiscal policy into the following two substages: (1) given K and D, the government sets the tax rate to satisfy its constraint, with the expectation of b, g, and D' determined through voting and (2) given τ_t, the government decides a set of (g, b, D') to maximize its objective.

This modification enables us to take $l\,(\tau_t)$ as given in choosing g, b, and D'. This implies that the political equilibrium policy and allocation, denoted by $\{g, \tau, b, D', K'\}$, are characterized by the equations demonstrated in Proposition 1, but the l values in them are replaced by $l(\tau)$. Thus, g/Y and $[1-l(\tau)]\, b/Y$ ratios and the growth rate become

$$\frac{g}{Y} = \frac{\omega\eta}{(1-\omega)\,[1+\beta\,(1+\eta)]+\omega\eta} \cdot \left[(1-\alpha)-\alpha\frac{D}{K}\right],$$

$$\frac{[1-l(\tau)]\, b}{Y} = \frac{[1-l(\tau)]\,(1-\omega)}{(1-\omega)\,[1+\beta\,(1+\eta)]+\omega\eta} \cdot \frac{(1+\beta)^2}{1+\alpha\beta} \cdot \left[(1-\alpha)-\alpha\frac{D}{K}\right],$$

$$\frac{K'}{K} = \frac{(1-\omega)\,\beta\,[\alpha\,(1+\beta)+(1+\alpha\beta)\,\eta]}{\{(1-\omega)\,[1+\beta\,(1+\eta)]+\omega\eta\}\,(1+\alpha\beta)} \cdot \left[l(\tau)\phi(1-\alpha)A - \alpha A l(\tau)\phi\frac{D}{K}\right].$$

The equations suggest that changing the target from w to $(1-\tau)\, w$ creates the following three effects. First, the ratio g/Y remains unchanged, because the negative effects on the wage and GDP are offset by the positive effect of a reduction in the interest rate. Second, the ratio $(1-l)b/Y$ increases because, given b and Y, employment decreases. Third, the growth rate decreases because the negative effect on the wage outweighs the positive effect of a reduction in the interest rate. Therefore, we conclude that the model presented in the main analysis fails to capture the effect of fiscal policy on employment; however, the effect could be easily included in the model.

6. CONCLUDING REMARKS

This study shows a positive correlation between unemployment and the debt–GDP ratio, resulting from the strong political power of a trade union. The study also shows that an increase in the political power of elderly persons results in a higher ratio of spending for elderly persons to GDP, a lower ratio of unemployment-insurance benefits to GDP, and a lower capital growth rate. In addition, the tax-finance requirement shifts resources from the younger to the older generation via fiscal policy, and thus benefits current elderly persons at the expense of current young persons; however, it improves economic growth, and thus the utility of future generations.

The key assumptions in demonstrating these results are (i) the additively separable, logarithmic utility function, (ii) AK technology, and (iii) the union's objective function, which targets the before-tax wage. The first assumption abstracts from the effect of savings on the preferences of public services for elderly persons. If we assume nonseparable preferences for private consumption and public services, individuals could substitute private consumption for public services, and thus prefer fewer public services and the issuance of public bonds.

The second assumption produces a constant interest rate that rules out the effect of fiscal policy on the interest rate through capital accumulation. We can include the interest rate effect by assuming a Cobb–Douglas production function that abstracts from the capital externality. This assumption implies that the interest rate is decreasing in capital, thereby incentivizing individuals to prefer fiscal policy that discourages capital accumulation and economic growth.

The third assumption produces an unemployment rate that is independent of fiscal policy. This result enables us to obtain analytically the political-equilibrium solution; however, it rules out the possibility of interaction between fiscal policy choice and unemployment. To overcome this limitation, Section 5.2 considers an alternative assumption—namely, that the union's target is the after-tax wage. Under this assumption, we find that unemployment depends on fiscal policy, and that unemployment-insurance spending is lower but the growth rate is higher in comparison to those in the main analysis.

The mechanism behind the third assumption is briefly revealed by analyzing the political equilibrium under the alternative assumption. However, the roles of the first two assumptions have not been analyzed fully. In particular, relaxing these assumptions would require numerical computation. However, as our aim is to demonstrate definitive results, this task is left to future research.

SUPPLEMENTARY MATERIAL

The supplementary material for this article is available online at www.journals.cambridge.org/jid/10.1017/S1365100517001067.

NOTES

1. In general, there are two types of public expenditures on elderly persons: public pensions, which compensate for the lack of postretirement income, and public services, which improve utility in old age. This study focuses on the latter type of expenditure.

2. Appendix B.1 provides the microfoundation of the political objective function.

3. The result in Proposition 2 suggests that the political power of elderly persons and of the trade union may affect the utility of employed and unemployed persons, through changes to public services for elderly persons, unemployment-insurance benefits, and economic growth. To investigate the welfare effects of those powers on employed and unemployed persons, we focus on the utility gap between them, $V_t^{ye} - V_t^{yu} = (1+\beta)\ln(1-\tau_t)\,w_t/b_t$. By substituting the policy functions τ_t and b_t in Proposition 1 in this gap function, we obtain $V_t^{ye} - V_t^{yu} = (1+\beta)\ln 1 = 0$. Thus, there is no income gap—that is, no utility gap—between employed and employed persons. The political power of elderly persons and of trade unions have no effect on the utility gap.

4. In Appendix A.7, we consider an alternative budget rule—that is to say, a constrained debt-finance requirement, such as that of Azzimonti et al. (2016), and investigate its long-run consequences.

REFERENCES

Acemoglu, Daron and James A. Robinson (2005) *Economic Origins of Dictatorship and Democracy*. New York, NY: Cambridge University Press.

Arai, Real and Katsuyuki Naito (2014) A Politico-Economic Approach on Public Debt in an Endogenous Growth Economy. MPRA working paper 56213, Munich Personal RePEc Archive.

Azzimonti, Marina, Marco Battaglini, and Stephen Coate (2016) The costs and benefits of balanced budget rules: Lessons from a political economy model of fiscal policy. *Journal of Public Economics* 136, 45–61.

Barseghyan, Levon, Marco Battaglini, and Stephen Coate (2013) Fiscal policy over the real business cycle: A positive theory. *Journal of Economic Theory* 148, 2223–2265.

Bassetto, Marco (2008) Political economy of taxation in an overlapping-generations economy. *Review of Economic Dynamics* 11, 18–43.

Battaglini, Marco (2014) A dynamic theory of electoral competition. *Theoretical Economics* 9, 515–554.

Battaglini, Marco and Stephen Coate (2008) A dynamic theory of public spending, taxation, and debt. *American Economic Review* 98, 201–236.

Battaglini, Marco and Stephen Coate (2016) A political economy theory of fiscal policy and unemployment. *Journal of the European Economic Association* 14, 303–337.

Bean, Charles and Christopher Pissarides (1993) Unemployment, consumption and growth. *European Economic Review* 37, 837–854.

Benassy, Jean-Pascal (2011) *Macroeconomic Theory*. New York, NY: Oxford University Press.

Bernasconi, Michele and Paola Profeta (2012) Public education and redistribution when talents are mismatched. *European Economic Review* 56, 84–96.

Bräuninger, Michael (2005) Social security, unemployment, and growth. *International Tax and Public Finance* 12, 423–434.

Chang, Juin-jen, Ming-fu Shaw, and Ching-Chong Lai (2007) A "managerial" trade union and economic growth. *European Economic Review* 51, 365–384.

Checherita-Westphal, Cristina and Philipp Rother (2012) The impact of high government debt on economic growth and its channels: An empirical investigation for the euro area. *European Economic Review* 56, 1392–1405.

Coimbra, Rui, Teresa Lloyd-Braga, and Leonor Modesto (2005) Endogenous fluctuations in unionized economies with productive externalities. *Economic Theory* 26, 629–649.

Corneo, Giacomo and Marko Marquardt (2000) Public pension, unemployment insurance, and growth. *Journal of Public Economics* 75, 293–311.

Diamond, Peter A. (1965) National debt in a neoclassical growth model. *American Economic Review* 55, 1126–1150.

Gonzalez-Eiras, Martin and Dirk Niepelt (2008) The future of social security. *Journal of Monetary Economics* 55, 197–218.

Gonzalez-Eiras, Martin and Dirk Niepelt (2012) Ageing, government budgets, retirement, and growth. *European Economic Review* 56, 97–115.

Greiner, Alfred and Peter Flaschel (2010) Public debt and public investment in an endogenous growth model with real wage rigidities. *Scottish Journal of Political Economy* 57, 68–84.

Heijdra, Ben J. (2009) *Foundations of Modern Macroeconomics*, 2nd ed. Oxford: Oxford University Press.

Josten, Stefan D. (2000) Public debt policy in an endogenous growth model of perpetual youth. *FinanzArchiv/Public Finance Analysis* 57, 197–215.

Josten, Stefan D. (2006) Dynamic fiscal policies and unemployment in a simple endogenous growth model. *International Tax and Public Finance* 13, 701–716.

Kaas, Leo and Leopold von Thadden (2003) Unemployment, factor substitution and capital formation. *German Economic Review* 4, 475–495.

Kaas, Leo and Leopold von Thadden (2004) Budgetary policy and unemployment dynamics in an OLG model with collective bargaining. *Economic Journal* 114, 867–889.

Krusell, Per, Vincenzo Quadrini, and José-Victor Ríos-Rull (1997) Politico-economic equilibrium and economic growth. *Journal of Economic Dynamics and Control* 21, 243–272.

Kumar, Manmohan and Jaejoon Woo (2010) Public Debt and Growth. IMF working paper 10/174, July 2010.

Kunze, Lars (2014) Life expectancy and economic growth. *Journal of Macroeconomics* 39, 54–65.

Lancia, Francesco and Alessia Russo (2015) Public education and pensions in democracy: A political economy theory. *Journal of the European Economic Association* 14, 1038–1073.

Lindbeck, Assar and Jögen W. Weibull (1987) Balanced budget redistribution and the outcome of political competition. *Public Choice* 52, 273–297.

Ono, Tetsuo (2007) Unemployment dynamics in an OLG Economy with public pensions. *Economic Theory* 33, 549–577.

Ono, Testuo (2015) Intergenerational politics, government debt, and economic growth. https://ideas.repec.org/p/osk/wpaper/1423r2.html (accessed August 5, 2015).

Pemberton, James (1988) A "managerial" model of the trade union. *Economic Journal* 98, 755–771.

Persson, Torsten and Guido Tabellini (2000) *Political Economics: Explaining Economic Policy*. Cambridge, MA: MIT Press.

Poutvaara, Panu (2006) On the political economy of social security and public education. *Journal of Population Economics* 19, 345–365.

Reinhart, Carmen M., Vincent R. Reinhart, and Kenneth S. Rogoff (2012) Debt Overhangs: Past and Present. NBER working paper no. 18015.

Romer, Paul M. (1986) Increasing returns and long run growth. *Journal of Political Economy* 94, 1002–1037.

Saint-Paul, Gilles (1992) Fiscal policy in an endogenous growth model. *Quarterly Journal of Economics* 107, 1243–1259.

Song, Zheng (2011) The dynamics of inequality and social security in general equilibrium. *Review of Economic Dynamics* 14, 613–635.

Song, Zheng, Kjetil Storesletten, and Fabrizio Zilibotti (2012) Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt. *Econometrica* 80, 2785–2803.

Uchida, Yuki (in press) Education, social mobility, and talent mismatch. *Economic Theory*.

Yakita, Akira (2014) Involuntary unemployment and sustainability of bond-financed fiscal deficit. *Journal of Macroeconomics* 41, 79–93.

APPENDIX A: PROOFS

A.1. BARGAINING SOLUTION

We substitute the constraint into the objective function to obtain the following unconstrained problem:

$$\max_{w_t} \Omega_t = \left\{[(1-\alpha)A_t]^{(1-\delta)/\alpha} (K_t)^{1-\delta}\right\}^{\theta} \cdot \left\{[(1-\alpha)A_t]^{1/\alpha} [1/(1-\alpha)-1] K_t\right\}^{1-\theta} \cdot \tilde{\Omega}_t,$$

where $\tilde{\Omega}_t \equiv (w_t - \bar{w}_t)^{\delta\theta} \cdot (w_t)^{-[(1-\delta)\theta+(1-\alpha)(1-\theta)]/\alpha}$. Therefore, the problem is reduced to

$$\max_{w_t} \tilde{\Omega}_t \equiv (w_t - \bar{w}_t)^{\delta\theta} \cdot (w_t)^{-\frac{(1-\delta)\theta+(1-\alpha)(1-\theta)}{\alpha}}$$

given $\bar{w}_t$.

The first derivative of $\tilde{\Omega}_t$ with respect to w_t is

$$\frac{\partial \tilde{\Omega}_t}{\partial w_t} = (w_t - \bar{w}_t)^{\delta\theta-1} \cdot (w_t)^{-\frac{(1-\delta)\theta+(1-\alpha)(1-\theta)}{\alpha}-1} \cdot \left[\delta\theta w_t - \frac{(1-\delta)\theta+(1-\alpha)(1-\theta)}{\alpha}(w_t - \bar{w}_t)\right],$$

indicating that $\partial\tilde{\Omega}_t/\partial w_t = 0$ implies $w_t = \phi\bar{w}_t$, where ϕ is defined in Section 2. The second derivative of $\tilde{\Omega}_t$ with respect to w_t, evaluated at $\partial\tilde{\Omega}_t/\partial w_t = 0$, is

$$\left.\frac{\partial^2 \tilde{\Omega}_t}{\partial w_t^2}\right|_{\partial\tilde{\Omega}_t/\partial w_t=0} = (w_t - \bar{w}_t)^{\delta\theta-1} \cdot (w_t)^{-\frac{(1-\delta)\theta+(1-\alpha)(1-\theta)}{\alpha}-1} \cdot \left[\delta\theta - \frac{(1-\delta)\theta+(1-\alpha)(1-\theta)}{\alpha}\right],$$

where $\partial^2\tilde{\Omega}_t/\partial w_t^2|_{\partial\tilde{\Omega}_t/\partial w_t=0} < 0$ holds under Assumption 1. ∎

A.2. PROOF OF PROPOSITION 1

To find a set of policy functions, let us first recall the government budget constraint in Definition 2(ii), which can be rewritten as follows:

$$1-\tau = \frac{l\phi(1-\alpha)\mathrm{AK} - g - (1-l)b - \mathrm{RD} + D'}{l\phi(1-\alpha)\mathrm{AK}}.$$

Using this constraint, the capital market-clearing condition can be rewritten as

$$D' + K' = \frac{\beta}{1+\beta}\left\{[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}] - g + D'\right\}, \tag{A.1}$$

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

where D' on the left-hand side indicates borrowing (lending) by the government if $D' > (<)0$, whereas D' on the right-hand side implies the benefits (costs) arising from the shift of fiscal resources from taxes to public bonds if $D' > (<)0$.

To find the policy functions that maximize the political objective, P, we need to conjecture the future policy function $g' = \tilde{G}(K', D')$. Here, we conjecture that $g' = G_0 \cdot \left[l\phi(1-\alpha)\mathrm{AK}' - \mathrm{RD}'\right]$, where $G_0(> 0)$ is constant. The term $l\phi(1-\alpha)\mathrm{AK}' - \mathrm{RD}'$ in the conjecture shows the aggregate labor income minus debt repayment, and implies the resources available to the government. Plugging the capital market-clearing condition (A.1) into this conjecture, we obtain

$$g' = G_0 \cdot \left(l\phi(1-\alpha)A\frac{\beta}{1+\beta}\{[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}] - g\} - \left[\frac{l\phi(1-\alpha)A}{1+\beta} + R\right]D' \right).$$

Using this guessing function, we can reformulate the political objective function as follows:

$$\begin{aligned} P &= \omega\eta \ln g + (1-\omega)(1+\beta)l \ln\left[l\phi(1-\alpha)\mathrm{AK} - g - (1-l)b - \mathrm{RD} + D'\right] \\ &+ (1-\omega)(1+\beta)(1-l)\ln b \\ &+ (1-\omega)\beta\eta \ln\left(l\phi(1-\alpha)A\frac{\beta}{1+\beta}\{[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}] - g\} \right. \\ &\left. - \left[\frac{l\phi(1-\alpha)A}{1+\beta} + R\right]D' \right), \end{aligned}$$

where the terms unrelated to political decisions are omitted from the expression.

The first-order conditions with respect to g, D', and b are

$$\begin{aligned} g &: \frac{\omega\eta}{g} = \frac{(1-\omega)(1+\beta)l}{\tilde{I}(K, D, D')} + \frac{(1-\omega)\beta\eta l\phi(1-\alpha)A\frac{\beta}{1+\beta}}{\tilde{J}(K, D, D')}, \\ D' &: \frac{(1-\omega)(1+\beta)l}{\tilde{I}(K, D, D')} = \frac{(1-\omega)\beta\eta\left[\frac{l\phi(1-\alpha)A}{1+\beta} + R\right]}{\tilde{J}(K, D, D')}, \\ b &: \frac{(1-\omega)(1+\beta)l(1-l)}{\tilde{I}(K, D, D')} = \frac{(1-\omega)(1+\beta)(1-l)}{b}, \end{aligned}$$

where $\tilde{I}(K, D, D')$ and $\tilde{J}(K, D, D')$ are defined as follows:

$$\begin{aligned} \tilde{I}(K, D, D') &\equiv [l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}] - g - (1-l)b + D', \\ \tilde{J}(K, D, D') &\equiv l\phi(1-\alpha)A\frac{\beta}{1+\beta}\{[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}] - g\} - \left[\frac{l\phi(1-\alpha)A}{1+\beta} + R\right]D'. \end{aligned}$$

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

The first-order conditions with respect to D' are rewritten as

$$D' = \left((1+\beta) l \cdot l\phi(1-\alpha) A \frac{\beta}{1+\beta} \cdot \{[l\phi(1-\alpha)\text{AK} - \text{RD}] - g\} \right.$$
$$\left. -\beta\eta \left[\frac{l\phi(1-\alpha)A}{1+\beta} + R \right] \{[l\phi(1-\alpha)\text{AK} - \text{RD}] - g - (1-l)b\} \right)$$
$$\times \left([(1+\beta)l + \beta\eta] \left[\frac{l\phi(1-\alpha)A}{1+\beta} + R \right] \right)^{-1}. \tag{A.2}$$

Using (A.2), we can reformulate $\tilde{I}(K, D, D')$ and $\tilde{J}(K, D, D')$ as follows:

$$\tilde{I}(K, D, D') = \frac{(1+\beta)l}{(1+\beta)l + \beta\eta} \cdot \left(\frac{(1-\alpha)\beta}{1+\alpha\beta} \{[l\phi(1-\alpha)\text{AK} - \text{RD}] - g\} \right.$$
$$\left. + [l\phi(1-\alpha)\text{AK} - \text{RD}] - g - (1-l)b \right),$$
$$\tilde{J}(K, D, D') = \frac{\beta\eta}{(1+\beta)l + \beta\eta} \cdot \left[\frac{l\phi(1-\alpha)A}{1+\beta} + R \right]$$
$$\times \left(\frac{(1-\alpha)\beta}{1+\alpha\beta} \{[l\phi(1-\alpha)\text{AK} - \text{RD}] - g\} \right.$$
$$\left. + [l\phi(1-\alpha)\text{AK} - \text{RD}] - g - (1-l)b \right).$$

With the use of these expressions, we can rewrite the first-order conditions with respect to g and b as (10) and (11), respectively. (10) and (11) lead to (12). The functions in (10), (11), and (12) constitute a Markov-perfect political equilibrium, as long as $G_0 = \omega\eta \cdot \{(1-\omega)[1+\beta(1+\eta)] + \omega\eta\}^{-1}$.

Using these policy functions, we calculate the tax rate. Recall the government budget constraint,

$$\tau = \frac{g + (1-l)b + \text{RD} - D'}{l\phi(1-\alpha)\text{AK}}.$$

Plugging (10)–(12) into this expression and rearranging the terms, we obtain the following:

$$\tau = \Lambda + \frac{(1-\Lambda)R}{l\phi(1-\alpha)A} \cdot \frac{D}{K}, \tag{A.3}$$

where

$$\Lambda \equiv 1 - \frac{(1-\omega)l(1+\beta)[l\phi(1-\alpha)A + R]}{\left[\frac{l\phi(1-\alpha)A}{1+\beta} + R\right]\{(1-\omega)[1+\beta(1+\eta)] + \omega\eta\}}$$
$$= 1 - \frac{1-\omega}{(1-\omega)[1+\beta(1+\eta)] + \omega\eta} \cdot \frac{l(1+\beta)^2}{1+\alpha\beta} \in (0, 1).$$

To determine the tax rate, we need to calculate the ratio D/K. This is done using the policy function $D' = D(K, D)$ and the capital market-clearing condition $K' = K(K, D)$. We substitute the policy functions of D' and g—given by (12) and (11), respectively—into the capital market-clearing condition (A.1), and rearrange the terms to obtain

$$\frac{K'}{K} = \frac{1-\omega}{(1-\omega)\,[1+\beta\,(1+\eta)]+\omega\eta} \times \frac{\beta\cdot[\alpha(1+\beta)+(1+\alpha\beta)\eta]}{1+\alpha\beta}\cdot\left[l\phi(1-\alpha)A-R\frac{D}{K}\right]. \quad \textbf{(A.4)}$$

Using (12) and (A.4), we can calculate the ratio D/K as follows:

$$\frac{D}{K} = \begin{cases} D_0/K_0 & \text{for} \quad t=0, \\ \frac{(1+\alpha\beta)(\tilde{\eta}-\eta)}{\alpha(1+\beta)+(1+\alpha\beta)\eta} & \text{for} \quad t\geq 1, \end{cases} \quad \textbf{(A.5)}$$

where $\tilde{\eta} \equiv (1-\alpha)(1+\beta)/(1+\alpha\beta)$, and D_0/K_0 is an initial condition and is taken as given.

To find the conditions that ensure $\tau_t \in (0,1)$ for all $t \geq 0$, consider first the period-0 tax rate:

$$\tau_0 = \Lambda + \frac{\alpha}{1-\alpha}\,(1-\Lambda)\,\frac{D_0}{K_0}.$$

Given that $\Lambda < 1$, we obtain

$$\tau_0 < 1 \Leftrightarrow \frac{\alpha}{1-\alpha}\,(1-\Lambda)\,\frac{D_0}{K_0} < 1-\Lambda \Leftrightarrow \frac{D_0}{K_0} < \frac{1-\alpha}{\alpha},$$

$$\tau_0 > 0 \Leftrightarrow \Lambda > 0.$$

Therefore, $\tau_0 \in (0,1)$ holds if $\Lambda > 0$ and $D_0/K_0 < (1-\alpha)/\alpha$.

Next, consider the period-$t\,(\geq 1)$ tax rate. Using the ratio D/K in (A.5), we can write τ for period $t \geq 1$ as

$$\tau = \Lambda + \frac{\alpha}{1-\alpha}\cdot(1-\Lambda)\cdot\frac{(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{\alpha(1+\beta)+(1+\alpha\beta)\eta},$$

where $\tau > 0$ if $\Lambda > 0$. We also have

$$\tau < 1 \Leftrightarrow (1-\Lambda)\cdot\left[1-\frac{\alpha}{1-\alpha}\cdot\frac{(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{\alpha(1+\beta)+(1+\alpha\beta)\eta}\right] > 0$$

$$\Leftrightarrow 1 > \frac{\alpha}{1-\alpha}\cdot\frac{(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{\alpha(1+\beta)+(1+\alpha\beta)\eta}, \text{ since } \Lambda < 1$$

$$\Leftrightarrow (1-\alpha)\,(1+\alpha\beta)\,\eta > -\alpha\,(1+\alpha\beta)\,\eta,$$

where the final expression always holds. Therefore, $\tau > 0$ for $t \geq 1$ if $\Lambda > 0$. ■

A.3. PROOF OF PROPOSITION 2

A.3.1. Effect of ω and θ on D/Y.

Given $Y_t = Al\phi K_t$, D_t/Y_t can be rewritten as

$$\frac{D_t}{Y_t} = \frac{D_t}{K_t}\cdot\frac{1}{Al\phi}.$$

The ratio D/K can be rewritten as

$$\frac{D_t}{K_t} = \begin{cases} D_0/K_0 \text{ for } t = 0, \\ \frac{\frac{1+\beta-\eta}{1+\beta} l\phi(1-\alpha)A - \eta R}{\frac{\eta}{1+\beta} l\phi(1-\alpha)A + (1+\eta)R} = \frac{\frac{1+\beta-\eta}{1+\beta}(1-\alpha) - \eta\alpha}{\frac{\eta}{1+\beta}(1-\alpha) + (1+\eta)\alpha} \text{ for } t \geq 1. \end{cases}$$

The ratio D_t/K_t is independent of ω and θ for $t \geq 0$, as observed in the previous expression: $\partial(D_t/K_t)/\partial\omega = 0$ and $\partial(D_t/K_t)/\partial\theta = 0$ for $t \geq 0$.

Next, consider the term $1/Al\phi$ that appeared on the right-hand side of the expression $D/Y = (D/K) \cdot (1/Al\phi)$. The term can be rewritten as

$$\frac{1}{Al\phi} = \frac{1}{A}(\phi)^{1/\alpha - 1},$$

where the equality arises from $l = (1/\phi)^{1/\alpha}$. Given $\partial\phi/\partial\omega = 0$ and $\partial\phi/\partial\theta > 0$, we obtain $\partial(D/Y)/\partial\omega = 0$ and $\partial(D/Y)/\partial\theta > 0$ for $t \geq 0$.

A.3.2. Effects of ω and θ on τ.

The tax rate in (13) is rewritten as follows:

$$\tau = 1 - \frac{(1-\omega)l(1+\beta)}{\left[\frac{(1-\alpha)}{1+\beta} + \alpha\right]\{(1-\omega)[1+\beta(1+\eta)] + \omega\eta\}} \cdot \left[1 - \frac{\alpha}{(1-\alpha)} \cdot \frac{D}{K}\right].$$

Given that $\partial(D/Y)/\partial\omega = 0$ and $\partial(D/Y)/\partial\theta > 0$ as demonstrated in Section A.3.1, and $\partial l/\partial\omega = 0$ and $\partial l/\partial\theta < 0$, we obtain $\partial\tau/\partial\omega > 0$ and $\partial\tau/\partial\theta > 0$ for $t \geq 0$.

A.3.3. Effects of ω and θ on g/Y.

Using the policy function $g = \tilde{G}(K, D)$ in (11), we can write g/Y as follows:

$$\frac{g}{Y} = \frac{\omega\eta}{(1-\omega)[1+\beta(1+\eta)] + \omega\eta} \cdot \left[(1-\alpha) - \alpha\frac{D}{K}\right].$$

Given $\partial(D/K)/\partial\omega = 0$ and $\partial(D/K)/\partial\theta = 0$ (in Section A.3.1), we obtain $\partial(g/Y)/\partial\omega > 0$ and $\partial(g/Y)/\partial\theta = 0$ for $t \geq 0$.

A.3.4. Effects of ω and θ on $(1-l)b/Y$.

Using the policy function $b = \tilde{B}(K, D)$ in equation (10), we can write $(1-l)b/Y$ as follows:

$$\frac{(1-l)b}{Y} = \frac{(1-l)(1-\omega)(1+\beta)}{\left(\frac{1-\alpha}{1+\beta} + \alpha\right)\{(1-\omega)[1+\beta(1+\eta)] + \omega\eta\}} \cdot \left[(1-\alpha) - \alpha\frac{D}{K}\right].$$

Given that $\partial l/\partial\omega = 0$ and $\partial l/\partial\theta < 0$, and that $\partial(D/K)/\partial\omega = 0$ and $\partial(D/K)/\partial\theta > 0$ as demonstrated in Section A.3.1, we obtain $\partial[(1-l)b/Y]/\partial\omega < 0$ and $\partial[(1-l)b/Y]/\partial\theta > 0$ for $t \geq 0$.

A.3.5. Effect on K'/K.

For $t = 0$, the growth rate demonstrated in Proposition 1 can be rewritten as

$$\frac{K_1}{K_0} = \frac{1-\omega}{(1-\omega)\left[1+\beta\left(1+\eta\right)\right]+\omega\eta} \cdot \frac{\beta\left[\alpha\left(1+\beta\right)+\left(1+\alpha\beta\right)\eta\right]}{1+\alpha\beta}$$
$$\times\left(\phi\right)^{1-1/\alpha} A\left[\left(1-\alpha\right)-\alpha\frac{D_0}{K_0}\right].$$

Given $\partial\phi/\partial\omega = 0$ and $\partial\phi/\partial\theta > 0$, we obtain $\partial\left(K_1/K_0\right)/\partial\omega < 0$ and $\partial\left(K_1/K_0\right)/\partial\theta < 0$. For $t \geq 1$, the growth rate can be rewritten as

$$\frac{K_{t+1}}{K_t} = \frac{(1-\omega)\beta\eta}{(1-\omega)\left[1+\beta\left(1+\eta\right)\right]+\omega\eta}\left(\phi\right)^{1-1/\alpha} A,$$

indicating that $\partial\left(K_{t+1}/K_t\right)/\partial\omega < 0$ and $\partial\left(K_{t+1}/K_t\right)/\partial\theta < 0$. ■

A.4. PROOF OF PROPOSITION 3

To find a set of policy functions, let us first recall the government budget constraint in (3), which can be rewritten as

$$1-\tau = \frac{l\phi(1-\alpha)\mathrm{AK} - g - (1-l)b - \mathrm{RD} - D'}{l\phi(1-\alpha)\mathrm{AK}}, \qquad \textbf{(A.6)}$$

where $D' = 0$ in the tax-finance case. Plugging (A.6) into the capital market-clearing condition (14), we obtain

$$K' = \frac{\beta}{1+\beta}\left\{\left[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}\right] - g\right\}.$$

Conjecture a linear policy function of public services in the next period as $g' = G_0 \cdot \left[l\phi(1-\alpha)\mathrm{AK}' - \mathrm{RD}'\right]$, or

$$g' = G_0 \cdot l\phi(1-\alpha)A \cdot \frac{\beta}{1+\beta}\left\{\left[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}\right] - g\right\},$$

where $G_0(> 0)$ is a constant parameter and $D' = 0$. Given this conjecture and the government budget constraint in (A.6), we can write the political objective function as follows:

$$P = \omega\eta\ln g + (1-\omega)(1+\beta)l\ln\left\{\left[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}\right] - g - (1-l)b\right\}$$
$$+ (1-\omega)(1+\beta)(1-l)\ln b + (1-\omega)\beta\eta\ln\left\{\left[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}\right] - g\right\},$$

where the terms unrelated to policy are omitted from the expression.

The first-order conditions with respect to g and b are summarized as

$$g = G(K) \equiv \frac{\omega\eta}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\left[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}\right], \qquad \textbf{(A.7)}$$

$$b = B(K) \equiv \frac{(1-\omega)[1+\beta(1+\eta)]}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\left[l\phi(1-\alpha)\mathrm{AK} - \mathrm{RD}\right]. \qquad \textbf{(A.8)}$$

These functions constitute a stationary Markov-perfect political equilibrium, as long as $G_0 = \omega\eta / \{(1-\omega)[1+\beta(1+\eta)] + \omega\eta\}$ holds.

We substitute the policy functions (A.7) and (A.8) into the government budget constraint in (A.6) to obtain

$$\tau = \frac{(1-l)(1-\omega)[1+\beta(1+\eta)]+\omega\eta}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta} + \frac{l(1-\omega)[1+\beta(1+\eta)]}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D}{K}, \tag{A.9}$$

or

$$\begin{aligned}\tau &= \left(1 - \frac{l(1-\omega)[1+\beta(1+\eta)]}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\right) + \frac{l(1-\omega)[1+\beta(1+\eta)]}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D}{K}\\ &= \left(1 - \frac{1+\alpha\beta}{(1+\beta)^2}\cdot[1+\beta(1+\eta)]\cdot(1-\Lambda)\right) + \frac{1+\alpha\beta}{(1+\beta)^2}\cdot[1+\beta(1+\eta)]\cdot(1-\Lambda)\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D}{K},\end{aligned}$$

where Λ is defined in Proposition 1 and the third equality comes from

$$1-\Lambda = \frac{1-\omega}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\cdot\frac{l\,(1+\beta)^2}{1+\alpha\beta}.$$

The above expression is reformulated as follows:

$$\tau = \frac{\beta\,(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{(1+\beta)^2} + \frac{(1+\alpha\beta)\,[1+\beta(1+\eta)]}{(1+\beta)^2}\cdot\left[\Lambda + (1-\Lambda)\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D}{K}\right],$$

where $D/K = D_0/K_0$ for period 0 and $D/K = 0$ for period $t \geq 1$.

To find the conditions that ensure $\tau_t \in (0,1)$ for all t, consider first the period-0 tax rate:

$$\tau_0 = \frac{\beta\,(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{(1+\beta)^2} + \frac{(1+\alpha\beta)\,[1+\beta(1+\eta)]}{(1+\beta)^2}\cdot\left[\Lambda + (1-\Lambda)\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D_0}{K_0}\right],$$

where $\tilde{\eta}-\eta > 0$ by the assumption and $1-\Lambda > 0$ by the definition of Λ. From this expression, we find that $\tau_0 < 1$ if

$$\begin{aligned}&\frac{(1+\alpha\beta)\,[1+\beta(1+\eta)]}{(1+\beta)^2}\cdot\left[\Lambda + (1-\Lambda)\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D_0}{K_0}\right] < 1 - \frac{\beta\,(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{(1+\beta)^2}\\ &= \frac{(1+\alpha\beta)\,[1+\beta(1+\eta)]}{(1+\beta)^2} \Leftrightarrow \Lambda + (1-\Lambda)\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D_0}{K_0} < 1 \Leftrightarrow \frac{D_0}{K_0} < \frac{1-\alpha}{\alpha}.\end{aligned}$$

Therefore, we have $\tau_0 \in (0,1)$ if $\Lambda > 0$ and $D_0/K_0 < (1-\alpha)/\alpha$.

Next, consider the period-$t\,(\geq 1)$ tax rate:

$$\tau = \frac{\beta\,(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{(1+\beta)^2} + \frac{(1+\alpha\beta)\,[1+\beta(1+\eta)]}{(1+\beta)^2}\cdot\Lambda.$$

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

The expression suggests that $\tau > 0$ if $\Lambda > 0$, and that $\tau < 1$ if

$$\frac{\beta\,(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{(1+\beta)^2}+\frac{(1+\alpha\beta)\,[1+\beta(1+\eta)]}{(1+\beta)^2}\cdot\Lambda<1,$$

or $\Lambda < 1$, which holds by the definition of Λ. Therefore, we have $\tau_t \in (0,1)$ if $\Lambda > 0$.

We substitute (A.7) and (A.9) into the capital market-clearing condition to obtain the law of motion of capital:

$$\frac{K'}{K}=\frac{(1-\omega)}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\cdot\frac{\beta\cdot[1+\beta\,(1+\eta)]}{1+\beta}\cdot\left[l\phi(1-\alpha)A-R\frac{D}{K}\right]. \tag{A.10}$$

■

A.5. PROOFS OF PROPOSITIONS 4–6

First, consider the capital growth rate. Recall the laws of motion of capital, as demonstrated in Propositions 1 and 3:

$$\left.\frac{K_{t+1}}{K_t}\right|_{\text{debt}}=\frac{1-\omega}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\cdot\frac{\beta\,[\alpha\,(1+\beta)+(1+\alpha\beta)\,\eta]}{1+\alpha\beta}\cdot\left[l\phi\,(1-\alpha)\,A-R\,\left.\frac{D_t}{K_t}\right|_{\text{debt}}\right],$$

$$\left.\frac{K_{t+1}}{K_t}\right|_{\text{tax}}=\frac{1-\omega}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\cdot\frac{\beta\,[1+\beta\,(1+\eta)]}{1+\beta}\cdot\left[l\phi\,(1-\alpha)\,A-R\,\left.\frac{D_t}{K_t}\right|_{\text{tax}}\right].$$

For period 0, we find that, by direct comparison,

$$\left.\frac{K_1}{K_0}\right|_{\text{debt}}<\left.\frac{K_1}{K_0}\right|_{\text{tax}}\Leftrightarrow\eta<\tilde{\eta}\equiv\frac{(1-\alpha)\,(1+\beta)}{1+\alpha\beta},$$

which holds under the assumption of $\eta < \tilde{\eta}$.

For period $t \geq 1$, we have

$$\left.\frac{D_t}{K_t}\right|_{\text{debt}}=\frac{(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{(1+\alpha\beta)\,\eta+\alpha\,(1+\beta)}\text{ and }\left.\frac{D_t}{K_t}\right|_{\text{tax}}=0.$$

We substitute this into the expression of K_{t+1}/K_t and obtain

$$\left.\frac{K_{t+1}}{K_t}\right|_{\text{debt}}<\left.\frac{K_{t+1}}{K_t}\right|_{\text{tax}}\Leftrightarrow 0<1+\alpha\beta,$$

which holds for any $\alpha > 0$ and $\beta > 0$.

Next, consider the spending for the public services–GDP ratio. From the results in Propositions 1 and 3 and $Y_t = l\phi \mathrm{AK}_t$, $g/Y|_{\mathrm{debt}}$ and $g/Y|_{\mathrm{tax}}$ are calculated as

$$\left.\frac{g}{Y}\right|_{\mathrm{unbalanced}} = \frac{\omega\eta}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\left[(1-\alpha)-\alpha\left.\frac{D}{K}\right|_{\mathrm{debt}}\right],$$
$$\left.\frac{g}{Y}\right|_{\mathrm{unbalanced}} = \frac{\omega\eta}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\left[(1-\alpha)-\alpha\left.\frac{D}{K}\right|_{\mathrm{tax}}\right],$$

respectively. Given K_0 and D_0, we obtain $g/Y|_{\mathrm{debt}} = g/Y|_{\mathrm{tax}}$ in period 0. For period $t \geq 1$, we obtain $g/Y|_{\mathrm{debt}} < g/Y|_{\mathrm{tax}}$ because $D/K|_{\mathrm{debt}} > 0$ holds in the debt-finance case, whereas $D/K|_{\mathrm{tax}} = 0$ in the tax-finance case.

Third, consider the unemployment-insurance payments–GDP ratio. Using the results in Propositions 1 and 3 and $Y_t = l\phi \mathrm{AK}_t$, $g/Y|_{\mathrm{debt}}$, we can calculate $(1-l)b/Y|_{\mathrm{debt}}$ and $(1-l)b/Y|_{\mathrm{tax}}$ as

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\mathrm{debt}} = \frac{(1-l)(1-\omega)}{(1-\omega)\left[1+\beta\left(1+\eta\right)\right]+\omega\eta}\cdot\frac{(1+\beta)^2}{1+\alpha\beta}\cdot\left[(1-\alpha)-\alpha\left.\frac{D}{K}\right|_{\mathrm{debt}}\right],$$
$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\mathrm{tax}} = \frac{(1-l)(1-\omega)}{(1-\omega)\left[1+\beta\left(1+\eta\right)\right]+\omega\eta}\cdot\left[1+\beta\left(1+\eta\right)\right]\cdot\left[(1-\alpha)-\alpha\left.\frac{D}{K}\right|_{\mathrm{tax}}\right],$$

respectively.

For period 0, given K_0 and D_0, we directly compare $(1-l)b_0/Y_0|_{\mathrm{debt}}$ and $(1-l)b_0/Y_0|_{\mathrm{tax}}$ and obtain

$$\left.\frac{(1-l)b_0}{Y_0}\right|_{\mathrm{debt}} \gtrless \left.\frac{(1-l)b_0}{Y_0}\right|_{\mathrm{tax}} \Leftrightarrow \frac{(1+\beta)^2}{1+\alpha\beta} \gtrless \left[1+\beta\left(1+\eta\right)\right]$$
$$\Leftrightarrow \eta \lessgtr \tilde{\eta} \equiv \frac{(1-\alpha)\left(1+\beta\right)}{1+\alpha\beta}.$$

Given the assumption of $\eta < \tilde{\eta}$, we have $(1-l)b_0/Y_0|_{\mathrm{debt}} > (1-l)b_0/Y_0|_{\mathrm{tax}}$.

For period $t \geq 1$, given $D_t/K_t|_{\mathrm{tax}} = 0$, we obtain

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\mathrm{debt}} \gtrless \left.\frac{(1-l)b_t}{Y_t}\right|_{\mathrm{tax}}$$
$$\Leftrightarrow \frac{(1+\beta)^2}{1+\alpha\beta}\cdot\left[(1-\alpha)-\alpha\left.\frac{D}{K}\right|_{\mathrm{debt}}\right] \gtrless \left[1+\beta\left(1+\eta\right)\right]\cdot(1-\alpha)$$
$$\Leftrightarrow \frac{(1+\beta)^2}{1+\alpha\beta}\cdot\left[(1-\alpha)-\alpha\cdot\frac{(1+\alpha\beta)\left(\tilde{\eta}-\eta\right)}{(1+\alpha\beta)\,\eta+\alpha\left(1+\beta\right)}\right] \gtrless \left[1+\beta\left(1+\eta\right)\right]\cdot(1-\alpha)$$
$$\Leftrightarrow \eta \gtrless \hat{\eta} \equiv \frac{(1+\beta)\,\alpha}{\beta\left(1-\alpha\right)},$$

where the second line comes from

$$\left.\frac{D_t}{K_t}\right|_{\mathrm{debt}} = \frac{(1+\alpha\beta)\left(\tilde{\eta}-\eta\right)}{(1+\alpha\beta)\,\eta+\alpha\left(1+\beta\right)}.$$

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

We compare $\tilde{\eta}$ and $\hat{\eta}$, and obtain

$$\hat{\eta} \gtrless \tilde{\eta} \Leftrightarrow \alpha \gtrless \frac{\beta}{1+2\beta}.$$

If $\alpha \geq \beta/(1+2\beta)$, then $\tilde{\eta} \leq \hat{\eta}$, so $\eta < \hat{\eta}$ holds under the assumption of $\eta < \tilde{\eta}$. We have $(1-l)b_t/Y_t|_{\text{debt}} < (1-l)b_t/Y_t|_{\text{tax}}$ for $t \geq 1$ if $\alpha \geq \beta/(1+2\beta)$. However, if $\alpha < \beta/(1+2\beta)$, then $\hat{\eta} < \tilde{\eta}$. We have

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} < \left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} \text{ if } \eta < \hat{\eta},$$
$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} \geq \left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} \text{ if } \hat{\eta} \leq \eta < \tilde{\eta}.$$

Summarizing the results, we obtain

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} < \left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} \text{ if either } \alpha \geq \frac{\beta}{1+2\beta}, \text{ or } \alpha < \frac{\beta}{1+2\beta} \text{ and } \eta < \hat{\eta},$$
$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} \geq \left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} \text{ if } \alpha < \frac{\beta}{1+2\beta} \text{ and } \hat{\eta} \leq \eta < \tilde{\eta}.$$

Finally, consider the tax rate. For period 0, $\tau_0|_{\text{debt}}$ and $\tau_0|_{\text{tax}}$ are compared as follows:

$$\begin{aligned} &\tau_0|_{\text{debt}} \gtrless \tau_0|_{\text{tax}} \\ &\Leftrightarrow \Lambda + \frac{\alpha}{1-\alpha}\Lambda\frac{D_0}{K_0} \gtrless \frac{\beta(1+\alpha\beta)}{(1+\beta)^2}\cdot(\tilde{\eta}-\eta) + \frac{(1+\alpha\beta)[1+\beta(1+\eta)]}{(1+\beta)^2} \\ &\quad \cdot\left[\Lambda + \frac{\alpha}{1-\alpha}(1-\Lambda)\frac{D_0}{K_0}\right] \\ &\Leftrightarrow \frac{\beta(1+\alpha\beta)}{(1+\beta)^2}\cdot(\tilde{\eta}-\eta)\cdot\left[\Lambda + \frac{\alpha}{1-\alpha}(1-\Lambda)\frac{D_0}{K_0}\right] \gtrless \frac{\beta(1+\alpha\beta)}{(1+\beta)^2}\cdot(\tilde{\eta}-\eta). \end{aligned}$$

Dividing both sides by $\beta(1+\alpha\beta)(\tilde{\eta}-\eta)/(1+\beta)^2$, we obtain

$$\begin{aligned} \tau_0|_{\text{debt}} \gtrless \tau_0|_{\text{tax}} &\Leftrightarrow \Lambda + \frac{\alpha}{1-\alpha}(1-\Lambda)\frac{D_0}{K_0} \gtrless 1 \\ &\Leftrightarrow \frac{D_0}{K_0} \gtrless \frac{1-\alpha}{\alpha}. \end{aligned}$$

Under the assumption of $D_0/K_0 < (1-\alpha)/\alpha$, we obtain $\tau_0|_{\text{debt}} < \tau_0|_{\text{tax}}$.

For period $t \geq 1$, $\tau_t|_{\text{debt}}$ and $\tau_t|_{\text{tax}}$ are compared as follows:

$$\begin{aligned} \tau_t|_{\text{debt}} \gtrless \tau_t|_{\text{tax}} &\Leftrightarrow \Lambda + \frac{\alpha}{1-\alpha}(1-\Lambda)\left.\frac{D_t}{K_t}\right|_{\text{debt}} \gtrless \frac{\beta(1+\alpha\beta)}{(1+\beta)^2}\cdot(\tilde{\eta}-\eta) \\ &+ \frac{(1+\alpha\beta)[1+\beta(1+\eta)]}{(1+\beta)^2}\cdot\Lambda. \end{aligned}$$

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

Plugging $D_t/K_t|_{\text{debt}} = (1+\alpha\beta)(\tilde{\eta}-\eta)/[(1+\alpha\beta)\eta+\alpha(1+\beta)]$ into the above expression and rearranging the terms, we obtain

$$\tau_t|_{\text{debt}} \gtrless \tau_t|_{\text{tax}} \Leftrightarrow \eta \lesseqgtr \hat{\eta} \equiv \frac{\alpha(1+\beta)}{\beta(1-\alpha)}.$$

Following the argument above, we can conclude as follows: for $t \geq 1$,

$$\tau_t|_{\text{debt}} > \tau_t|_{\text{tax}} \text{ if either } \alpha \geq \frac{\beta}{1+2\beta}, \text{ or } \alpha < \frac{\beta}{1+2\beta} \text{ and } \eta < \hat{\eta},$$
$$\tau_t|_{\text{debt}} \leq \tau_t|_{\text{tax}} \text{ if } \alpha < \frac{\beta}{1+2\beta} \text{ and } \hat{\eta} \leq \eta < \tilde{\eta}.$$

■

A.6. PROOF OF PROPOSITION 7

Recall that the indirect utility of employed persons in generation t is given by

$$V_t^{ye} = (1+\beta)\ln(1-\tau_t)\phi(1-\alpha)\text{AK}_t + \beta\eta\ln g_{t+1} + C,$$

where $C \equiv \beta\ln R + \ln[1/(1+\beta)] + \beta\ln[\beta/(1+\beta)]$ includes terms irrelevant to political decisions. Substituting the policy functions demonstrated in Proposition 1, we can write the indirect utility function in the debt-finance case as

$$\begin{aligned} V_t^{ye}\Big|_{\text{debt}} &= (1+\beta)\ln(1-\Lambda)\phi A \\ &+ \beta\eta\ln\frac{(1-\omega)\omega\eta\beta\eta}{\{(1-\omega)[1+\beta(1+\eta)]+\omega\eta\}^2}\cdot(l\phi A)^2 + C \\ &+ [1+\beta(1+\eta)]\ln\left[(1-\alpha)\ K_t|_{\text{debt}} - \alpha\ D_t|\right]. \end{aligned} \tag{A.11}$$

The term $(1-\alpha)\ K_t|_{\text{debt}} - \alpha\ D_t|$ in (A.11) is reformulated as follows:

$$\begin{aligned} &(1-\alpha)\ K_t|_{\text{debt}} - \alpha\ D_t| \\ &= (1-\alpha)\cdot\frac{K_t}{K_{t-1}}\bigg|_{\text{debt}}\cdot\frac{K_{t-1}}{K_{t-2}}\bigg|_{\text{debt}}\cdot\cdots\cdot\frac{K_2}{K_1}\bigg|_{\text{debt}}\cdot\frac{K_1}{K_0}\bigg|_{\text{debt}}\cdot K_0\cdot\left(1-\frac{\alpha}{1-\alpha}\cdot\frac{D_0}{K_0}\right) \\ &= (1-\alpha)\cdot\left(\frac{K'}{K}\bigg|_{\text{debt}}\right)^t\cdot[(1-\alpha)K_0 - \alpha D_0], \end{aligned} \tag{A.12}$$

where the second equality comes from the fact that the growth rate is constant along the equilibrium path, as demonstrated in Proposition 1.

Substitution of (A.12) into (A.11) leads to

$$V_t^{ye}\Big|_{\text{debt}} = V_0^{ye}\Big|_{\text{debt}} + t\,[1+\beta(1+\eta)]\ln\left(\frac{K'}{K}\bigg|_{\text{debt}}\right), \tag{A.13}$$

where

$$V_0^{ye}\big|_{\text{debt}} = (1+\beta)\ln(1-\Lambda)\,\phi A\,[(1-\alpha)K_0 - \alpha D_0] + \beta\eta\ln\frac{(1-\omega)\omega\eta\beta\eta}{\{(1-\omega)[1+\beta(1+\eta)]+\omega\eta\}^2}\cdot(l\phi A)^2\cdot[(1-\alpha)K_0-\alpha D_0]+C. \quad \textbf{(A.14)}$$

In the same manner, $V_t^{ye}\big|_{\text{tax}}$, $V_t^{yu}\big|_{\text{debt}}$, and $V_t^{yu}\big|_{\text{tax}}$ are calculated as follows:

$$V_t^{ye}\big|_{\text{tax}} = V_0^{ye}\big|_{\text{tax}} + t\,[1+\beta(1+\eta)]\ln\left(\frac{K'}{K}\bigg|_{\text{tax}}\right), \quad \textbf{(A.15)}$$

$$V_t^{yu}\big|_{\text{debt}} = V_0^{yu}\big|_{\text{debt}} + t\,[1+\beta(1+\eta)]\ln\left(\frac{K'}{K}\bigg|_{\text{debt}}\right), \quad \textbf{(A.16)}$$

$$V_t^{yu}\big|_{\text{tax}} = V_0^{yu}\big|_{\text{tax}} + t\,[1+\beta(1+\eta)]\ln\left(\frac{K'}{K}\bigg|_{\text{tax}}\right), \quad \textbf{(A.17)}$$

where

$$V_0^{ye}\big|_{\text{tax}} = (1+\beta)\ln\frac{(1-\omega)[1+\beta(1+\eta)]}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\cdot l\phi A\cdot[(1-\alpha)K_0-\alpha D_0]$$
$$+\beta\eta\ln\frac{(1-\omega)\omega\eta(1-\alpha)}{\{(1-\omega)[1+\beta(1+\eta)]+\omega\eta\}^2}\cdot(l\phi A)^2$$
$$\times\frac{\beta[1+\beta(1+\eta)]}{1+\beta}\cdot[(1-\alpha)K_0-\alpha D_0]+C,$$

$$V_0^{yu}\big|_{\text{debt}} = (1+\beta)\ln\frac{(1-\omega)}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}$$
$$\cdot\frac{(1+\beta)^2}{1+\alpha\beta}\cdot l\phi A\cdot[(1-\alpha)K_0-\alpha D_0]$$
$$+\beta\eta\ln\frac{(1-\omega)\omega\eta\beta\eta}{\{(1-\omega)[1+\beta(1+\eta)]+\omega\eta\}^2}\cdot(l\phi A)^2\cdot[(1-\alpha)K_0-\alpha D_0]+C,$$

$$V_0^{yu}\big|_{\text{tax}} = (1+\beta)\ln\frac{(1-\omega)[1+\beta(1+\eta)]}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta}\cdot l\phi A\cdot[(1-\alpha)K_0-\alpha D_0]$$
$$+\beta\eta\ln\frac{(1-\omega)\omega\eta(1-\alpha)}{\{(1-\omega)[1+\beta(1+\eta)]+\omega\eta\}^2}\cdot(l\phi A)^2$$
$$\times\frac{\beta[1+\beta(1+\eta)]}{1+\beta}\cdot[(1-\alpha)K_0-\alpha D_0]+C.$$

Let us first compare $V_0^{ye}\big|_{\text{debt}}$ and $V_0^{ye}\big|_{\text{tax}}$. Direct calculation leads to

$$V_0^{ye}\big|_{\text{debt}} \gtrless V_0^{ye}\big|_{\text{tax}} \Leftrightarrow \phi(\eta) \equiv (1+\beta)\ln\frac{(1+\beta)^2}{1+\alpha\beta}$$
$$+\beta\eta\ln\frac{1+\beta}{1-\alpha}\eta - [1+\beta(1+\eta)]\ln[1+\beta(1+\eta)] \gtrless 0,$$

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

where $\phi(\cdot)$ has the following properties:

$$\phi(0) = (1+\beta)\ln\frac{(1+\beta)}{1+\alpha\beta} > 0,$$

$$\phi(\tilde{\eta}) = 0,$$

$$\begin{aligned}\frac{\partial\phi}{\partial\eta} &= \beta\ln\frac{\eta(1+\beta)}{(1-\alpha)\left[1+\beta(1+\eta)\right]}\\ &< \beta\ln\frac{\tilde{\eta}(1+\beta)}{(1-\alpha)\left[1+\beta(1+\eta)\right]};\ \text{since } \eta<\tilde{\eta}\\ &= 0.\end{aligned}$$

Therefore, we obtain $\phi > 0\forall\eta \in (0, \tilde{\eta})$—that is,

$$V_0^{ye}\big|_{\text{debt}} > V_0^{ye}\big|_{\text{tax}}.$$

Next, we compare $V_t^{ye}\big|_{\text{debt}}$ and $V_t^{ye}\big|_{\text{tax}}$ as follows:

$$V_t^{ye}\big|_{\text{tax}} - V_t^{ye}\big|_{\text{debt}} = \left(V_0^{ye}\big|_{\text{tax}} - V_0^{ye}\big|_{\text{debt}}\right) + t\left[1+\beta(1+\eta)\right]\ln\left(\frac{K'/K|_{\text{tax}}}{K'/K|_{\text{debt}}}\right).$$

The term $\left(V_0^{ye}\big|_{\text{tax}} - V_0^{ye}\big|_{\text{debt}}\right)$ is negative and constant. The term $t\left[1+\beta(1+\eta)\right]\ln\left(K'/K|_{\text{tax}} / K'/K|_{\text{debt}}\right)$ is positive and increasing in t since $K'/K|_{\text{tax}} > K'/K|_{\text{debt}}$. Therefore, there is a positive integer $T(\geq 1)$ such that $V_t^{ye}\big|_{\text{tax}} \leq V_t^{ye}\big|_{\text{debt}}$ for $t \leq T$ and $V_t^{ye}\big|_{\text{tax}} > V_t^{ye}\big|_{\text{debt}}$ for $t > T$.

Following the same procedure, we compare $V_t^{yu}\big|_{\text{debt}}$ and $V_t^{yu}\big|_{\text{tax}}$, and obtain

$$V_0^{yu}\big|_{\text{debt}} > V_0^{yu}\big|_{\text{tax}} \Leftrightarrow \phi > 0\forall\eta \in (0, \tilde{\eta}),$$

$$V_t^{yu}\big|_{\text{tax}} - V_t^{yu}\big|_{\text{debt}} = \left(V_0^{yu}\big|_{\text{tax}} - V_0^{yu}\big|_{\text{debt}}\right) + t\left[1+\beta(1+\eta)\right]\ln\left(\frac{K'/K|_{\text{tax}}}{K'/K|_{\text{debt}}}\right).$$

These imply that there is a positive integer $T(\geq 1)$ such that $V_t^{yu}\big|_{\text{tax}} \leq V_t^{yu}\big|_{\text{debt}}$ for $t \leq T$ and $V_t^{yu}\big|_{\text{tax}} > V_t^{yu}\big|_{\text{debt}}$ for $t > T$. ■

A.7. CONSTRAINED DEBT FINANCE

The analysis of tax finance and comparing it to debt finance—a found in the main text—enables us to offer an insight into the political economy of fiscal policy. However, the requirement for tax finance is somewhat extreme, because in reality, the government is allowed to issue public bonds as long as it is below the debt ceiling. To investigate the effect of the debt ceiling, we introduce the following debt constraint:

$$D' \leq \mu\cdot(K+D) + Al\phi D,$$

where $\mu \in \Re$. Appendix B.2 shows that we can obtain a Markov-perfect equilibrium in the presence of the debt ceiling, as long as the ceiling is given by the above condition.

https://doi.org/10.1017/S1365100517001067 Published online by Cambridge University Press

In the following analysis, for simplicity of analysis, we set $\mu = 0$, we then characterize the debt-finance political equilibrium in the presence of the constraint $D' \leq Al\phi D$. If A is normalized to satisfy $Al\phi = 1$, the constraint is reduced to $D' \leq D$. This corresponds to the balanced-budget rule investigated by Azzimonti et al. (2016). The constraint $D' \leq D$ is equivalent to $\tau l w \geq g + (1-l)b + (R-1)D$, implying that tax revenues are sufficient to cover spending, $g + (1-l)b$, and the costs of servicing the debt, $(R-1)D$.

The problem of the government is now modified by adding the constraint $D' \leq Al\phi D$ into the problem in Definition 2(ii). When the constraint is nonbinding, the political equilibrium allocation matches that in Proposition 1. We consider a political equilibrium when the constraint is binding, and obtain the following result.

Proposition A.1. *Consider a political equilibrium with the debt constraint $D' \leq Al\phi D$. Let d denote a threshold ratio of D/K defined by*

$$d \equiv \frac{(1-\omega)(1-\alpha)\beta(\tilde{\eta}-\eta)}{\{(1-\omega)[1+\beta(1+\eta)]+\omega\eta\}+(1-\omega)\beta\alpha(\tilde{\eta}-\eta)}.$$

If $D/K > d$, then the debt constraint is nonbinding and the political equilibrium is characterized as in Proposition 1. If $D/K \leq d$, then the debt constraint is binding and the political equilibrium is characterized by the following:

$$b = \frac{(1-\omega)[1+\beta(1+\eta)]}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta} \cdot l\phi(1-\alpha)A(K+D),$$

$$g = \frac{\omega\eta}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta} \cdot l\phi(1-\alpha)A(K+D),$$

$$\tau = 1 - \frac{l(1-\omega)[1+\beta(1+\eta)]}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta} \cdot \left(1+\frac{D}{K}\right),$$

$$D' = Al\phi D,$$

$$K' = \frac{\beta}{1+\beta} \cdot \frac{(1-\omega)[1+\beta(1+\eta)]}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta} \cdot l\phi(1-\alpha)A(K+D) - Al\phi D,$$

and

$$\frac{D'}{K'} = f\left(\frac{D}{K}\right) \equiv \left[\chi \cdot \left(\frac{1}{D/K}+1\right) - 1\right]^{-1},$$

where

$$\chi \equiv \frac{\beta}{1+\beta} \cdot \frac{(1-\omega)[1+\beta(1+\eta)]}{(1-\omega)[1+\beta(1+\eta)]+\omega\eta} \cdot (1-\alpha) \in (0,1).$$

Proof. See Appendix B.3.

The results in Propositions 1 and A.1 indicate that for period $t \geq 1$, the debt–capital ratio satisfies the following equation:

$$\frac{D'}{K'} = \begin{cases} f\left(\frac{D}{K}\right) & \text{if } \frac{D}{K} \leq d, \\ \frac{(1+\alpha\beta)\cdot(\tilde{\eta}-\eta)}{\alpha(1+\beta)+(1+\alpha\beta)\eta} & \text{if } \frac{D}{K} > d, \end{cases}$$

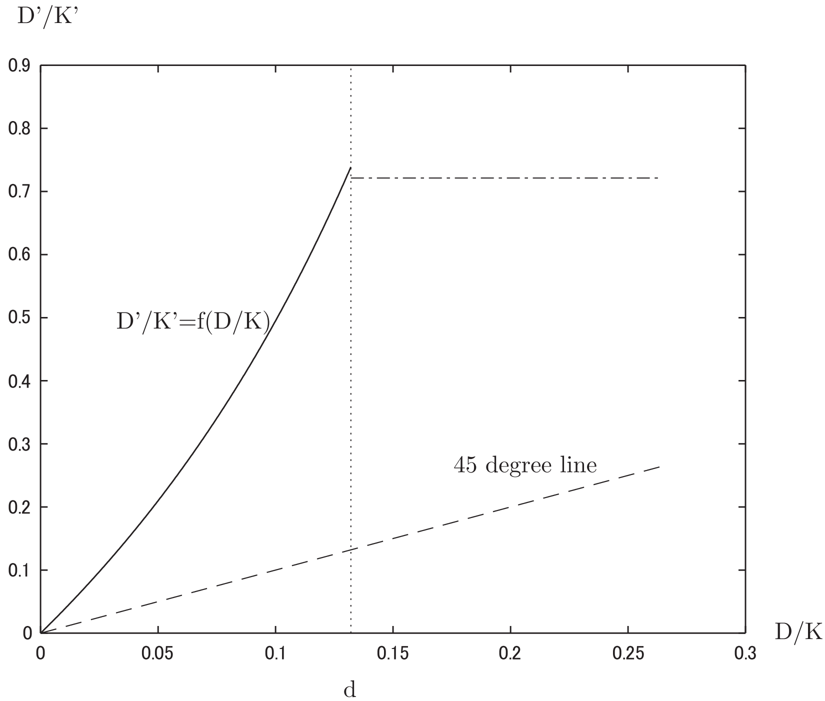

FIGURE 5. A numerical example of the D/K ratio. Parameters are set to $\beta = (0.99)^{30}$, $\delta = 0.2$, $\theta = 0.2$, $\alpha = 0.3$, $\eta = 0.4$, and $\omega = 0.3$ to satisfy Assumption 1 and $\eta < \tilde{\eta}$.

where $f(\cdot)$ is increasing and convex in D/K with $f'(0) = 1/\chi > 1$. The properties of $f(\cdot)$ suggest that given $D_0/K_0 < d$, the debt–capital ratio increases over time while the constraint continues to be binding, but moves into a state where the debt constraint becomes nonbinding and the ratio remains constant through time, as illustrated in Figure 5.

To understand the movement of the debt–capital ratio, consider a situation in which the initial condition D_0/K_0 is lower than the threshold value d. Less public debt today implies that the government can utilize tax revenue for its expenditure. However, this incentivizes the government to issue more bonds, because the government can afford to repay its debt using the tax revenue. Thus, the debt accumulates and reaches its ceiling, $Al\phi D$.

When the debt constraint is binding, the households' tax burden is less compensated by public bond issuance. This implies a negative income effect on households, which in turn implies negative effects on savings and capital accumulation. Because of this negative effect on capital, the debt–capital ratio increases along the equilibrium path with $D' = Al\phi D$. At some future date, the ratio will exceed the threshold value d. Then, the ratio will continue to be below the threshold, and the debt and capital will grow at the same constant rate. Therefore, the economy experiences a decrease in the growth rate, followed by its permanent increase.

The present result is qualitatively different from that of Azzimonti et al. (2016), who demonstrate that debt accumulates initially and then reaches its ceiling. The present study, meanwhile, shows that debt is initially constrained by its ceiling, but then moves into increasing debt accumulation. The difference arises because, in the model used by Azzimonti et al. (2016), there is no capital accumulation and resources are limited across periods, while in the current model, there is capital accumulation that increases over time the income, and thus also the tax base. Therefore, the results of the current study demonstrate an alternative view of the role of debt constraint in the political economy. ■