THE JOURNAL OF NAVIGATION (2005), **58**, 479–492.
doi:10.1017/S0373463305003395 Printed in the United Kingdom

Gravity Requirements for Compensation of Ultra-Precise Inertial Navigation

Jay Hyoun Kwon[1], Christopher Jekeli[2]

[1] (*University of Seoul*)
(Email: jkwon@uos.ac.kr)

[2] (*The Ohio State University*)

Precision inertial navigation depends not only on the quality of the inertial sensors (accelerometers and gyros), but also on the accuracy of the gravity compensation. With a view toward the next-generation inertial navigation systems, based on sensors whose errors contribute as little as a few metres per hour to the navigation error budget, we have analyzed the required quality of gravity compensation to the navigation solution. The investigation considered a standard compensation method using ground data to predict the gravity vector at altitude for aircraft free-inertial navigation. The navigation effects of the compensation errors were examined using gravity data in two gravimetrically distinct areas and a navigation simulator with parameters such as data noise and resolution, supplemental global gravity model noise, and on-track interpolation method. For a typical flight trajectory at 5 km altitude and 300 km/hr aircraft speed, the error in gravity compensation contributes less than 5 m to the position error after one hour of free-inertial navigation if the ground data are gridded with 2 arcmin resolution and are accurate to better than 5 mGal.

KEY WORDS

1. Inertial Navigation. 2. Gravity compensation. 3. Precision navigation.

1. INTRODUCTION.

Navigation accuracy using an inertial navigation system (INS) depends on the quality of the gravitation compensation as well as the performance of the inertial measurement units (IMU). Here, the performance of the IMU refers mainly to the stability of the biases and scale factor errors of the accelerometers and gyros and the level of white noise; and, the quality of the gravity compensation refers to the accuracy of the gravity vectors that feed into the navigation equation. Gravity compensation is necessary because the accelerometers of an INS do not sense gravitational attraction, whereas the vehicle trajectory is affected significantly by the Earth's gravitational variations, of the order of several hundred metres in horizontal position after one hour of free-inertial (i.e., autonomous, unaided) navigation. Of course, gravity compensation is much less important if the accumulation of gravitational error (and IMU error) can be controlled with sufficiently frequent external position updates, as might be obtained from GPS.

Ultra-high precision IMU's have recently come under consideration based on new technological developments in cold atom interferometry (Kasevich and Chu, 1992; Gustavson et al., 1997). It is predicted that these IMU's have stability of 10^{-8} m/sec^2 and 10^{-6} deg/hr, respectively, in acceleration and rotation rate, which is 100 – 1000 times better than the current navigation-grade IMU. Precision of this kind leads to enhanced applications of free-inertial (i.e., unaided) navigation over long periods in any environment. Indeed, the development of ultra-precise IMU's aims to achieve long-term (one hour) unaided inertial navigation with positioning error of the order of a few metres (DARPA, 2003). The largest potential error source in free-inertial navigation with these IMU's is the gravity compensation, which suggests that a renewed analysis be conducted of the data quality needed to support such systems at the few-metre level of accuracy.

Gravity data archives exist for all regions of the world with varying levels of resolution and accuracy, and the development of global as well as local models and their improvement continue as new data and observations are generated. Gravity compensation using data that refer to ground level requires estimation of the gravitational vector along the vehicle trajectory. Moreover, the data primarily comprise the (scalar) vertical components of the gravitational vector, yet the horizontal (vector) components are needed to compensate the induced navigation error. (However, one might even consider vertical inertial navigation because the ultra-precise IMU's would yield 3-D accuracy at the level of a few metres after one hour.) Thus, in addition to an "upward continuation" of the data (in the case of aircraft navigation) a transformation is required whereby horizontal components are estimated from vertical components.

Both upward continuation and vector estimation from scalar data are founded on well established models in potential theory that solve the classical boundary-value problem. These models are only approximate, however, due to the irregular shape of the Earth's surface (the boundary), and the approximation is ameliorated significantly by introducing a known reference (or *normal*) gravitational field. Therefore, as usual we operate with components of the *gravity disturbance vector*, $\delta\vec{g}$, defined as the difference between the actual gravity and normal gravity vectors. We use gravity anomalies (differing only slightly from the vertical disturbances; Heiskanen and Moritz, 1967) given on the Earth's surface and also approximate this surface as a sphere. Upward continuation and transformation yields components of the gravity disturbance vector at altitude, usually on a grid with pre-defined spacing. Further estimation is required to estimate the gravity disturbances on the vehicle's actual trajectory. Not only the quality of the gravity data in terms of observation accuracy and spatial resolution, but also model error (upward continuation and transformation) affect the final estimation accuracy, which directly affects the accuracy of the navigation solution.

Several previous studies exist on the gravitational effect on inertial navigation. For example, Jekeli (1997) analyzed the propagation of unknown gravitation to position error using a statistical model of the gravity field. He focused on the short-term effect (on INS applied to bridging extended GPS outages in precise navigation) and concluded that the position error grows up to several metres within 100 seconds. Similarly, Jordan (1973), Jordan and Center (1986), and Schwarz (1981) discussed the effect of unknown gravity on airborne inertial navigation through a state-space error analysis. These investigations essentially used covariance modelling of the gravity field to determine the effects on the navigation solution in a purely statistical setting, that is, with a covariance propagation analysis. These investigations did not address practical aspects in gravity compensation, including the required spatial resolution of ground data (the data interval), the data observation error, the accuracy of existing global models, and the inherent model errors. An extensive study on gravity modelling and its effect on the navigation solution was undertaken by the Air Force Wright Aeronautical Laboratories (AFWAL, 1986). Focused on the selection of the optimal gravity estimation techniques, the investigation showed through navigation simulations that $5' \times 5'$ gravity data, with accuracy of 0·5 arcsec in the deflection of vertical would support the then current precision navigation. Although this study progressed beyond mere covariance analyses toward practical gravity compensation, some important aspects were not fully examined, such as supplementing the estimation with a global model, and ascertaining necessary levels of ground data noise and resolution. Furthermore, the analysis should now be updated to reflect the potential for vast improvement in IMU precision.

Rather than performing a covariance analysis, we investigate gravity compensation for free-inertial navigation through simulations using actual gravity data and by directly calculating the navigation solution under various scenarios of gravity estimation. The main objective is to identify the principal factors that affect the accuracy of the estimated gravity disturbances, and to assess the requirements of those factors to achieve high accuracy in the navigation solution. For these purposes, actual free-air gravity anomaly data from two areas having characteristically different gravity signatures are taken as the true ground gravity values. Then, adding different levels of noise and sampling the data at different resolutions yields alternative simulated measured data sets. The simulated data are upward continued using the Pizzetti integral and a remove-restore procedure that accounts for an existing global gravity model, resulting in a two dimensional grid of gravity disturbance vectors at flight level. Using this database, simple linear interpolation and an optimal, weighted least-squares interpolation (least square collocation) are applied to estimate the gravity disturbances on the vehicle's trajectory. Comparison of these estimates to the true gravity disturbance vectors yields an assessment of the gravity estimation accuracy. Finally, for each case, the effect of the gravity-related errors on free-inertial navigation is determined with a navigation simulator that compensates for gravity using the estimated gravity disturbances.

This analysis simulates the most straightforward method of gravity compensation, taking advantage of the existing data archives that in many technologically and economically developed areas may require only marginal improvement to support long-term free-inertial navigation. An alternative compensation method relies on the in situ measurement of the gravitational field in the form of gravity gradients. This technique obviates extensive databases and the problems associated with accurate upward continuation modelling. On the other hand, it also requires the addition of a gradiometer to the inertial navigation system. The added expense of hardware and sensor integration would easily be justified for areas where little or no gravity data exist, especially in rugged mountain regions where the gravity signature and the effect on inertial navigation are strongest. Gradiometer-aided inertial navigation has been studied by Britting et al. (1972), Heller and Jordan (1976), and Jekeli (2005); and, the results of the present analysis thus also serve to calibrate the need for such system enhancement.

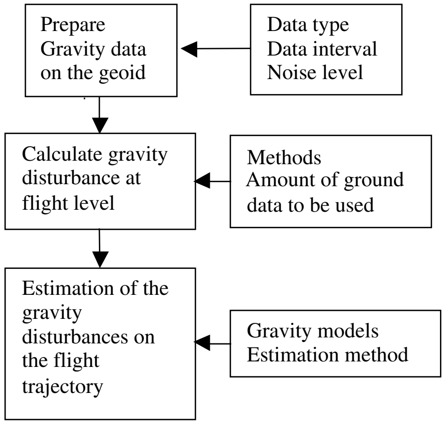

Figure 1. General procedure of traditional gravity compensation.

2. GRAVITY COMPENSATION. The general procedure of gravity compensation is divided into three major steps, shown in Figure 1. The first preparatory step involves acquisition of the gravity data from a gravity survey and reducing these to the form of gravity anomalies. In ocean areas, one might acquire potential differences (derived from satellite altimetry that recovers mean sea level heights) and occasionally on land areas deflections of the vertical are observed using astronomic determinations of the direction of the plumb line. We used only a database of gravity anomalies; and, although they had not been reduced to a reference surface, such as mean sea level (in essence, the *geoid*), we assumed this without loss in generality in order to apply the spherical approximation with greater validity. Also, the data contain unknown observational errors, but we treated them as true quantities, adding our own generated noise to simulate the observational error. We used the highest available spatial resolution ($1' \times 1'$ arcmin grid) and simulated coarser resolution by sampling the data at larger intervals.

Given data on the geoid, the gravity disturbance vector can be estimated at altitude using upward continuation that is rooted in basic potential theory (Heiskanen and Moritz, 1967). In principle, the method requires continuous gravity data over the whole Earth's surface. This is an impractical idealization of the solution, and typically, the gravitational influence of remote zones is captured with an existing global spherical harmonic model, such as EGM96 (Lemoine et al. 1998), and only the local data are integrated within some specified spherical cap centred on the region of interest. Also the global gravity model has some uncertainty, which affects the accuracy of the gravity disturbance estimates and the navigation solution. In this study, the accuracy of EGM96 and that of a more recent, longer-wavelength model, GSM (GRACE Satellite-only Model, July 2003, http://podaac.jpl.nasa.gov/grace/) are used to assess the effect of global gravity model errors.

In the absence of an exact, planned trajectory, one usually prepares a grid of gravity disturbance estimates at one or more flight altitudes. During the flight, the gravity disturbances are interpolated from the grid onto the trajectory using some gravity modelling and estimation techniques. Gravity modelling refers in part to the

https://doi.org/10.1017/S0373463305003395 Published online by Cambridge University Press

characterization of the gravity field as deterministic, stochastic, or of hybrid type. For example, the direct estimation of the gravity values from potential theory models and the spherical harmonic representation are deterministic; least-squares estimation using covariance functions to characterize the stochastic nature of the gravity disturbances represents a statistical approach; and the combination of the potential theory model and least-square estimation is a hybrid method. Differences among these techniques reflect levels of theoretical rigour, estimation accuracy, and computational efficiency. In the deterministic approach, the gravity field is represented by a set of functions or models and applied directly to the navigation equations. For example, the gravity disturbance data are stored in 3-D lattices and the INS-indicated position is used to extract the gravity compensation by 3-D linear interpolation. Higher order interpolation techniques, such as bicubic splines, can be applied where the data embody a high degree of continuity.

On the other hand, statistical estimation methods optimally account for the noise in the gravity data. Conceptually more appropriate in that respect, these computational tools such as Kalman filtering and least-squares collocation nevertheless require a tremendous effort in developing reasonable statistical models for the local gravity field. Such modelling may need to be performed for every flight and thus is computationally expensive. Moreover, care must be exercised since the quality of the statistical models affects the performance of estimation. For these reasons, the deterministic method is simpler and easier to implement than the statistical approach. However, particularly in the interpolation from the grid to the trajectory, a deterministic method may be significantly poorer in accuracy if the grid has large mesh size and/or if the data noise levels are high.

3. SIMULATING THE GRAVITY FIELD. The investigation of the trade-off between gravity database parameters and gravity compensation efficacy was conducted by simulation using actual gravity anomaly data in two gravitationally dissimilar regions. Treating the available gravity anomaly data as true quantities, artificial data sets were constructed depending on two parameters: simulated noise with given amplitude and sampling interval with given grid size. The constructed data sets were upward continued using approximate models derived from potential theory; and the gravity disturbance vector was estimated along a simulated trajectory using either simple or statistically optimal interpolation. A navigation simulator then used these estimated gravity disturbances to determine a compensated trajectory that was compared to the compensated trajectory implied by the original true gravity data. The following describes the analysis procedures in detail.

The true gravity field was defined by a set of $1' \times 1'$ free-air gravity anomalies in two areas (Figure 2). The first area, Area 1, covers latitude 37°–44° North and longitude 103°–110° West, representing rough terrain and gravity signature in the Rocky Mountains over Wyoming and Colorado. Area 2, covers latitude 38°–42° North and longitude 80°–88° West, representing relatively plain terrain and moderate gravity signature in Ohio. In each area, an E-W trajectory about 280 km in length was generated with endpoints on the 40th parallel (see Figure 2). The trajectories were generated by a navigation simulator, assuming constant vehicle altitude (5 km) and velocity (300 km/hr) along a great circle path. Both an error-free trajectory and one corrupted by IMU and gravity errors were generated. IMU sensor errors included

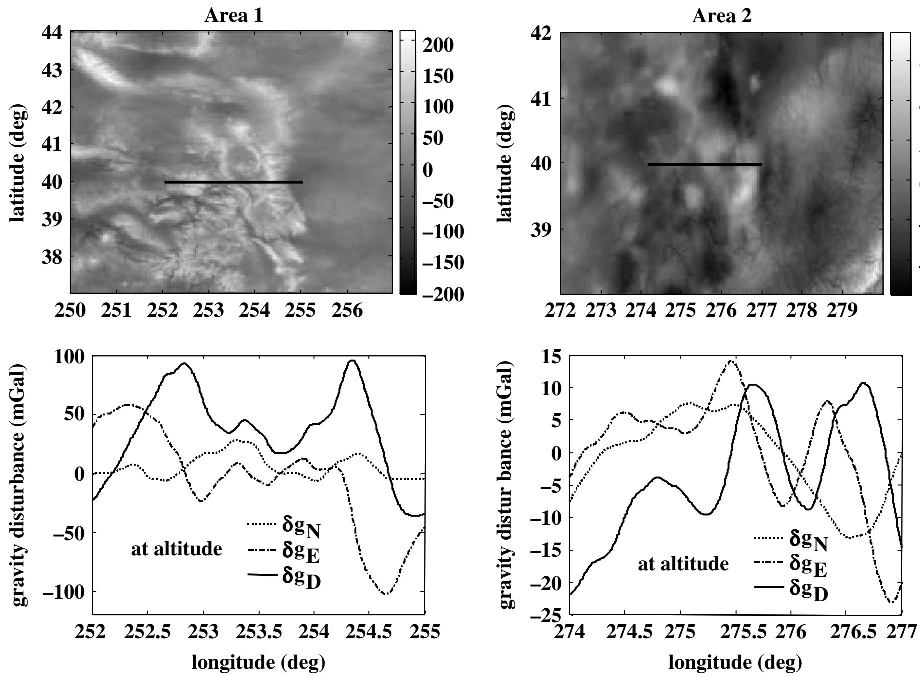

Figure 2. Gravity fields defined by given $1' \times 1'$ gravity anomalies assumed to reside on the geoid (top) and gravity disturbances at altitude along the simulated trajectories (bottom), for rough and moderate areas (left and right, respectively). (Note: Longitude is expressed as degrees East of Greenwich.)

constant biases and scale factor errors, as well as simulated white noise processes and the gravity disturbances. The sensor error parameters refer to future ultra-precise IMU's that contribute no more than 5 m position error to the navigation solution after one hour (Table 1).

The true and estimated gravity disturbances at altitude were constructed as shown in Figure 3. The gradients of the Pizzetti integrals represent the upward-continuation and transformation models according to potential theory, which yield gravity disturbance components at altitude from gravity anomaly data on the geoid (approximated as a sphere) (Heiskanen and Moritz, 1967, p. 234):

$$\begin{aligned}\delta g_r &= \frac{R}{4\pi}\iint_{\sigma} \Delta g \frac{\partial S(r,\psi)}{\partial r} d\sigma \\ &\cong \delta g_r^{EGM96} + \frac{R}{4\pi}\iint_{\sigma_0} (\Delta g - \Delta g^{EGM96}) \frac{\partial S(r,\psi)}{\partial r} d\sigma_0\end{aligned} \tag{1}$$

$$\begin{aligned}\delta g_\phi &= \frac{R}{4\pi r}\iint_{\sigma} \Delta g \frac{\partial S(r,\psi)}{\partial \phi} d\sigma \\ &\cong \delta g_\phi^{EGM96} + \frac{R}{4\pi r}\iint_{\sigma_0} (\Delta g - \Delta g^{EGM96}) \frac{\partial S(r,\psi)}{\partial \phi} d\sigma_0\end{aligned} \tag{2}$$

Table 1. The IMU error parameters used in the simulation.

Error Parameter	Value
accel. bias	1×10^{-8} m/s²
accel. scale factor error	1×10^{-6} ppm
accel. white noise	1×10^{-8} m/s²/$\sqrt{\text{Hz}}$
gyro bias	1×10^{-5} deg/hr
gyro scale factor error	5×10^{-4} ppm
gyro white noise	$1{\cdot}2\times10^{-4}$ deg/hr/$\sqrt{\text{Hz}}$

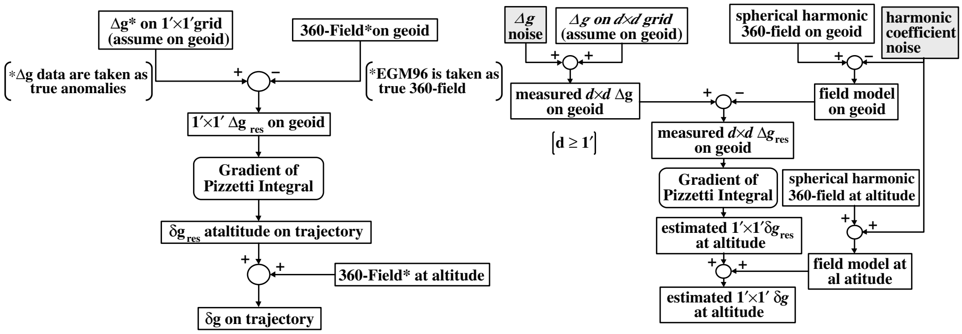

Figure 3. Construction of the true (left) and estimated (right) gravity disturbances at altitude.

$$\begin{aligned}\delta g_\lambda &= \frac{R}{4\pi r\cos\phi}\iint_\sigma \Delta g\,\frac{\partial S(r,\psi)}{\partial\lambda}\,d\sigma \\ &\cong \delta g_\lambda^{EGM96} + \frac{R}{4\pi r\cos\phi}\iint_{\sigma_0}(\Delta g - \Delta g^{EGM96})\frac{\partial S(r,\psi)}{\partial\lambda}\,d\sigma_0\end{aligned} \tag{3}$$

where $\delta g_r, \delta g_\phi, \delta g_\lambda$ are gravity disturbances in radial, latitudinal, and longitudinal directions, R is the mean radius of the Earth, Δg is the free-air gravity anomaly on the geoid, S is Stokes's function, r is the radial coordinate of the computation point, and ψ is the spherical angle between the computation point and the integration point. The global gravity model, EGM96, was first removed from the gravity anomalies and subsequently restored in the form of the gravity disturbances at altitude. This procedure of separating out the longer-wavelength components improves the numerical accuracy of the integrals that were truncated to spherical caps of $\sigma_0 = 1.5^0$ radius. This approximation was found to contribute less than 1 mGal error (1 mGal $= 10^{-5}$ m/s²).

The true gravity disturbances were computed directly from the given $1'\times1'$ ground data and EGM96 (both assumed error free) on the trajectory using the procedure outlined in Figure 3 and equations (1)–(3). A straightforward numerical integration based on the $1'\times1'$ data grid approximated the analytic integrals. We generated the simulated measured field similarly through Pizzetti's integral, but added noise intentionally to both the gravity anomalies on the geoid and the spherical harmonic coefficients of EGM96. The noise process of the observed ground data was generated

https://doi.org/10.1017/S0373463305003395 Published online by Cambridge University Press

from a random variable uniformly distributed in the interval $[-\sqrt{3}, \sqrt{3}]$ (so that its variance is unity) and then scaled by a specified standard deviation. The errors in the spherical harmonic coefficients were similarly simulated by scaling a random variable to the standard deviation of each coefficients, as given for a particular global model. The estimated gravity disturbances were first computed on a $1' \times 1'$ grid at altitude regardless of the resolution of the data and then interpolated onto the trajectory, as might be done with an actual compensation procedure. Note that increasing the resolution to a $1' \times 1'$ grid at altitude from a lower-resolution ground grid is already a preliminary form of interpolation.

Among various interpolation techniques, we tested the direct linear interpolation (DLI), based on the nearest four grid-corner values, and a least-squares collocation (LSC) method. LSC requires covariance functions for the gravity disturbances (Moritz, 1980), and we used a reciprocal distance model (RDM) as described by Jekeli (2003). The RDM is frequently used because it is possible to incorporate harmonic extension into exterior free space (Moritz, 1980; however, we did not need this feature for the assumed level trajectory). We chose the following form:

$$C(s) = \frac{\sigma^2 \beta}{\sqrt{\beta^2 + s^2}} \tag{4}$$

where the covariance, C, depends only on the horizontal distance, s, between two points (it is stationary and isotropic); β is the correlation distance; and s^2 is the variance. These parameters can be determined from a fit to an empirical covariance function with values calculated from the ground data. A separate RDM was developed for each gravity disturbance component, which thus neglects cross correlations among the components. With these models the interpolation was conducted according to standard collocation theory:

$$x = C_s (C_l + D)^{-1} l \tag{5}$$

where x is the vector of estimated gravity disturbances on the trajectory, C_s is the matrix of cross-covariances between gravity disturbances at trajectory points and at grid points, C_l is the matrix of auto-covariances of gravity disturbances at the grid points, and l is the vector of gridded disturbances at altitude whose errors are characterized by the diagonal dispersion matrix, D. The error variance (on the diagonal of D) was assumed to be that of the ground data.

Estimation performance in LSC is affected by the validity of the covariance function model and the spatial extent of the data used. The covariance model depends on the chosen variance and correlation distance. Values for these parameters were determined from the $1' \times 1'$ grid of true gravity disturbances at altitude. We found that a fifty per cent mismodelling of the correlation distance caused gravity disturbance errors of about ± 3 mGal, while mismodelling the variance by 50% caused less than $\pm 0{\cdot}8$ mGal errors. Concerning the data extent, we used a $10' \times 10'$ area centred on each estimation point, which was deemed sufficient since an increase to $20' \times 20'$ changed the estimates by less than 1 mGal.

The simulation of the actual trajectory consisted of three parts (see also Figure 4). First, error-free IMU (accelerometer, gyro) data were generated based on motion of a local-level platform along a predefined great-circle route. Second, simulated IMU errors (biases, scale factor errors, and white noise) were added to these data to

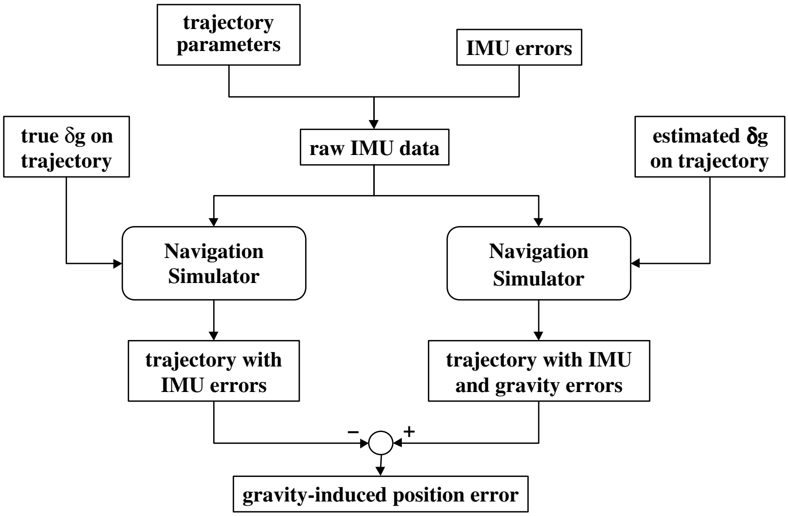

Figure 4. Scheme of free inertial navigation simulator.

simulate raw IMU output. Third, the navigation simulator used this IMU output and the gravity disturbance components along the trajectory to generate the indicated trajectory. Using the true gravity disturbances, the computed trajectory reflects only the effect of IMU errors; while with the estimated gravity disturbances it includes also the effect of gravity estimation error. Comparing the two navigation solutions yields an assessment of the quality of gravity compensation based on the given ground data and estimation procedure. It is important to include the IMU errors in the simulation since they couple non-linearly with the gravity estimation errors.

4. TESTS AND RESULTS.

Three noise levels were considered for the ground anomaly data and two noise levels for the harmonic coefficients. The EGM96 coefficient standard deviations up to degree 360 served as amplitude for one, and for the other we used the GSM coefficient standard deviation up to degree 120 and the EGM96 standard deviations from degrees 121 to 360. The ground data had one of four levels of resolution as defined by an equiangular grid on the sphere, and interpolation from the grid of computed values at altitude to the trajectory was performed using either DLI or LSC. Therefore, the five parameters of the analysis were resolution and noise of the observed ground data, noise in the global gravity field model, type of interpolation onto the trajectory, and type of gravity signature as exemplified by Areas 1 and 2. The parameter values are summarized in Table 2.

Figures 5 and 6 represent the errors in the gravity disturbance estimates for the various combinations of parameters. Each indicated value is the root-sum-of squares of the individual component errors, which are roughly commensurate. Comparing Figures 5 with 6 shows that the long-wavelength errors in the global model affect primarily the estimation with the coarsest data resolution ($10' \times 10'$). Interestingly, the supposedly more accurate long-wavelength model, GSM, yields poorer gravity estimates. This is because GSM, though far superior at the very long wavelengths,

https://doi.org/10.1017/S0373463305003395 Published online by Cambridge University Press

Table 2. Parameters of the analysis.

Parameter	Values
resolution of ground data	$1' \times 1'$, $2' \times 2'$, $5' \times 5'$, $10' \times 10'$
noise of observed ground data	1 mGal, 5 mGal, 10 mGal
global harmonic model errors	■ EGM96 coefficient standard deviations, degrees 0–360
	■ GSM coefficient standard deviations, degrees 0–120 & EGM96 coefficient standard deviations, degrees 121–360
Interpolation	■ DLI – simple linear interpolation
	■ LSC – optimally weighted interpolation
gravity signature	■ Area 1 (rough)
	■ Area 2 (moderate)

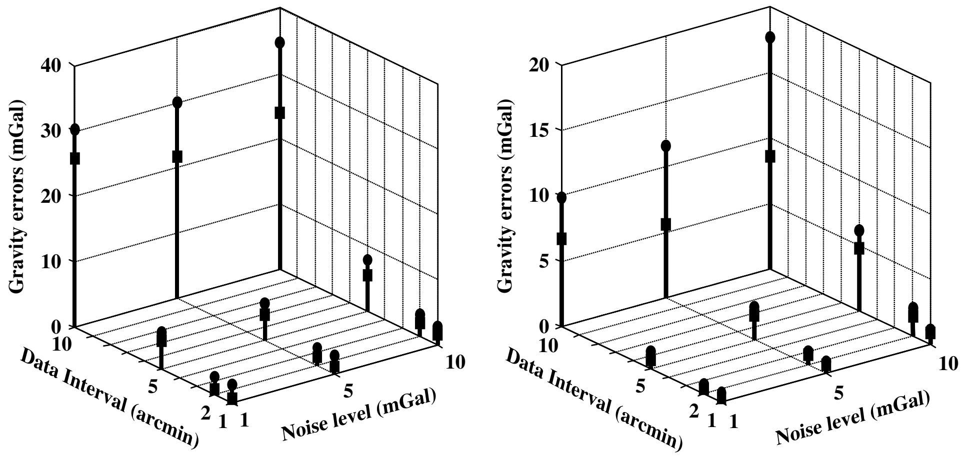

Figure 5. Three-dimensional gravity disturbance error in Areas 1 (left) and 2 (right) using EGM96 and interpolation by DLI (circle) or LSC (square).

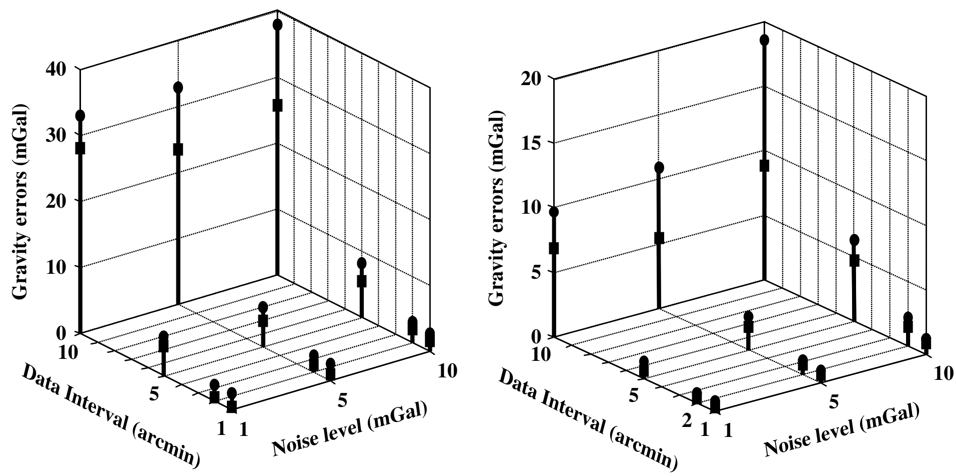

Figure 6. Three-dimensional gravity disturbance error in Areas 1 (left) and 2 (right) using GSM and interpolation by DLI (circle) or LSC (square).

https://doi.org/10.1017/S0373463305003395 Published online by Cambridge University Press

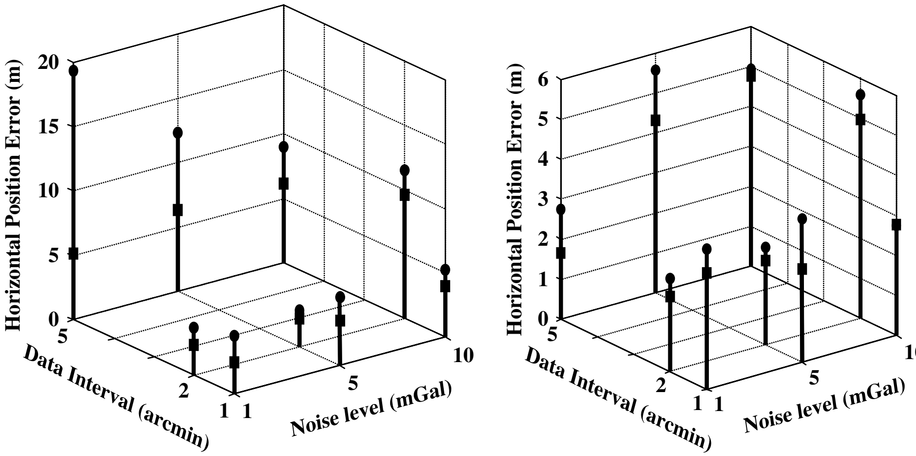

Figure 7. Horizontal position error in Areas 1 (left) and 2 (right) induced by gravity estimation error using GSM and interpolation by DLI (circle) and LSC (square).

is estimated to be less accurate at the higher harmonic degrees, 116–120, than EGM96. The gravity estimation is thus much more sensitive to these wavelengths (of the order of 180 arcmin) when the data resolution is low. Indeed, for the other data resolutions, the difference implied by the EGM96 and GSM errors is less than 1 mGal. As expected, the estimates in Area 1 are much poorer than those in Area 2 because of the rougher gravity signature (e.g., compare the left side of Figures 5 and 6). Examining the results in terms of noise level and data resolution, the errors tend to increase more rapidly with a degradation in resolution than in noise. In all cases, LSC performs better than DLI, and differences between these two methods of interpolation are significant especially when the noise levels and data intervals are large. This conforms to the fact that LSC is an optimal estimator that also uses more data around the estimation points than DLI does. In summary, these simulations show that $2' \times 2'$ ground data with noise level less than 5 mGal are required to achieve on-track estimation accuracy better than 2 mGal for Area 1 (rough gravity signature), while $5' \times 5'$ data resolution with the same noise level are sufficient to achieve this accuracy in Area 2.

The estimated gravity disturbances were subsequently used by the free-inertial navigation simulator to generate the navigation solutions. The differences between solutions with the true and estimated gravity disturbances indicate the effect of the gravity estimation error. Only the GSM global errors were applied since the effect of degrading these to the long-wavelength EGM96 errors was mostly insignificant. The left and right sides of Figure 7 represent the horizontal position errors in Areas 1 and 2, respectively, using the gravity estimates obtained by DLI and LSC. The depicted values are the *maximum* root-sum-of-squares of horizontal component errors, being commensurate, during approximately one hour (53 minutes) of free-inertial navigation, where it is noted that the position errors due to erroneous gravity compensation do not increase monotonically during this period. As a reference, horizontal position errors reached a maximum of 3·1 m with error-free gravity compensation, thus due only to IMU errrors.

https://doi.org/10.1017/S0373463305003395 Published online by Cambridge University Press

Recalling the behaviour of gravity estimation errors under various parameter options, the horizontal position errors would be expected to follow the same pattern. However, the results, generated by simulations, do not always correspond with those expectations. For example, the horizontal position errors for $2' \times 2'$ data with 1 and 5 mGal noise levels are slightly lower than for $1' \times 1'$ data in both areas. This is due to the nonlinear relationship between the gravity errors, the IMU sensor errors, and the dynamics of the vehicle. Therefore, smaller gravity errors do not always guarantee smaller position errors, especially in the presence of IMU biases. Otherwise however, the general pattern of reduced gravity estimation error with increased data resolution and accuracy was reproduced for the horizontal position error. Again, LSC-estimated gravity generated a better navigation solution in all cases. In addition, the position errors in Area 1 are much larger than in Area 2 because of the rougher gravity signature (due primarily to the rough terrain). It was found that $5' \times 5'$ data with better than 10 mGal noise level are sufficient for position accuracy better than 5 m in Area 2 (within approximately one hour of free-inertial navigation), while $2' \times 2'$ data with better than 5 mGal noise are required to yield the same position accuracy in Area 1 using LSC.

5. CONCLUSIONS.

A traditional gravity compensation method for free-inertial navigation was analyzed in view of next-generation inertial measurement units that promise free-inertial navigation accuracy of the order of a few metres of horizontal position error over one hour of operation. The method of investigation utilized ground gravity data supplemented by a global gravity model in an analytical upward continuation based on potential theory. Simulating a practical approach to in-flight gravity compensation, the gravity components were then interpolated from a grid at altitude onto a given trajectory. A number of parameters in this process were considered, including data resolution, data noise, interpolation method, global model accuracy, and magnitude of the gravity signature, to determine the data requirements that would serve an autonomous navigation system based on the ultra-high accuracy IMU's. The following conclusions were drawn from the results of the simulations:

(1) The long-wavelength accuracy of the global gravity model was not a significant factor in the estimation of the gravity disturbances. Although the current EGM96 model was adequate in these simulations, the newer GSM model would be preferred, especially in regions where EGM96 is known to be of poorer quality, such as in polar regions and in alpine regions with poor gravity data coverage.

(2) LSC generated better gravity disturbance estimates and consequently better navigation solutions than did DLI in both rough and moderate areas. However, it should be noted that this interpolation method depends on reasonable covariance models for the gravity disturbances (which can require a substantial analysis effort) and involves considerably more real-time computations. With improved ground data resolution and accuracy, the benefits of LSC diminish.

(3) As expected, larger data intervals and larger noise levels generate larger errors in the gravity disturbance estimates, though not necessarily larger

position errors. To achieve gravity disturbance estimates at 5 km altitude accurate to better than 2 mGal in mountainous regions requires $2' \times 2'$ ground data with better than 5 mGal noise.

(4) Ground data resolution better than $2' \times 2'$ and noise level lower than 5 mGal are required to yield position accuracy better than 5 m using next-generation, ultra-precise IMU's (Table 1).

The analysis included several simplifying assumptions. The upward continuation, performed by numerical integration of an approximate integral, was assumed to be error free. A more advanced analysis would assess the accuracy of this upward continuation model, especially in rugged terrain, where gravity data are not readily gridded and do not reside on a level surface. Furthermore, the present analysis was restricted to horizontal trajectories. An extension to varying altitude would require multiple grids of estimated gravity disturbances at different altitudes and three-dimensional interpolation. Neither of these extensions in the analysis, obviously, would predict a relaxation of data quality; however, nor would they necessarily point to a significant increase in the requirements found here. Rather they would suggest improvements in methodology and modelling. Therefore, the present analysis on data quality may be considered relatively robust and complete.

ACKNOWLEDGMENTS

This work was supported by DARPA through a sub-contract with Stanford University, subaward no. 29348-C/Req. 11797980.

REFERENCES

Air Force Wright Aeronautical Laboratories (1986). *Gravity Compensation for INS Demonstration Program*. AFWAL TR-85-1156, Wright Patterson Air Force base, Ohio.

Britting, K. R., S. J. Madden, and R. A. Hildebrant (1972). The impact of gradiometer techniques on the performance of inertial navigation systems. AIAA Paper No. 72–850. Aug. 1972, Stanford, CA.

DARPA (2003). Precision Inertial Navigation Systems (PINS). Technology Thrust, Defense Sciences Office, http://www.darpa.mil/dso/thrust/matdev/pins.htm.

Gustavson, T. L., Bouyer, P. and Kasevich M. A., (1997). Precision Rotation Measurement with an Atom Interferometer Gyroscope. *Physical Review Letters*, **78**, 2046–2049.

Heiskanen, W. and Moritz, H. (1967). *Physical Geodesy*; (reprint 1987). Institute of Physical Geodesy; Technical University; Graz, Austria.

Heller, W. G. and S. K. Jordan (1976). Error analysis of two new gradiometer-aided inertial navigation systems. *J. Spacecraft*, **13**, 340–347.

Jekeli, C. (1997). The Effect of Earth's Gravity on Precise, Short-Term, 3-D Free-Inertial Navigation. *Navigation*, **44**, 347–357.

Jekeli, C. (2005). Precision free-inertial navigation with gravity compensation by an on-board gradiometer. Submitted to *J. Guidance, Control and Dynamics*.

Jordan. S. K. (1973). Effects of geodetic uncertainties on a damped inertial navigation system. *IEEE Trans. Aerospace and Electronic Systems*, **AES-9**(5), 741–752.

Jordan, S. K. and J. L. Center (1986). Establishing requirements for gravity surveys for very accurate inertial navigation. *Navigation, Journal of the Institute of Navigation*, **33**, 90–108.

Kasevich, M. and Chu, S. (1992). Measurement of the Gravitational Acceleration of an Atom with a Light-Pulse Atom Interferometer. *Applied Physics B*, **B54**, 321–332.

Lemoine, F. G., Kenyon, S. C., Factor, J. K., Trimmer, R. G., Pavlis, N. K.,. Chinn, D. S., Cox, C. M., Klosko, S. M., Luthcke, S. B., Torrence, M. H., Wang, Y. M., Williamson, R. G., Pavlis, E. C., Rapp, R. H., and Olson, T. R. (1998). *The development of the joint NASA GSFC and NIMA geopotential model EGM96*. NASA Goddard Space Flight Center, Greenbelt, Maryland,USA, July 1998.

Moritz, H. (1976). *Covariance Functions in Least-Squares Collocation*. Report No. 240, Ohio State University.

Moritz, H. (1980). *Advanced Physical Geodesy*, Herbert Wichman Verlag and Abacus Press.

Schwarz, K. P. (1981). *Gravity Induced Position Errors in Airborne Inertial Navigation*. Report No. 326, Ohio State University.

https://doi.org/10.1017/S0373463305003395 Published online by Cambridge University Press