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A ROLE FOR CULTURAL TRANSMISSION IN FERTILITY TRANSITIONS

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Cultural variables in economic analysis have recently experienced a strong renewal. This evolution sheds a new light on the old debate between the "Beckerian model" of fertility and the "synthesis model" of fertility. In this paper, I propose a fertility model making the evolution of culture endogenous. The whole population is divided into two cultures corresponding to specific preferences for fertility. Parents decide their fertility rate and try to transmit their culture to their children. Differential fertility between cultures gives rise to an evolutionary process while differential effort to transmit the parental culture gives rise to a cultural process. The long-run distribution of preferences and the average total fertility rate both result from interactions between these processes. As a result, a fertility transition cannot appear without productivity shocks in favor of the culture that is not biased toward quantity of children. However, asymmetric productivity shocks are not always sufficient to cause a fertility transition.

Keywords: Cultural Transmission, Evolutionary Mechanisms, Fertility Transition, Industrial Revolution

1. INTRODUCTION

The consideration of cultural variables in economic analysis has recently experienced a strong renewal coming from the recent availability of rich data sets such as the World Value Survey. These data sets make the concept of culture quantifiable and causality between culture and economic variables testable [see Guiso et al. (2006)]. Among its multiple implications, this evolution sheds new light on the old debate between the "Beckerian model" of fertility and the "synthesis model" of fertility. The first focuses on the economic determinants of fertility. Becker and co-workers (1973, 1976, 1988) propose a framework where parents value

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both the quantity of offspring and their quality (human capital, wealth, etc.). By maximizing their expected utility subject to a nonlinear cost structure, parents face a trade-off between quality and quantity. This fundamental contribution has been followed by major improvements by Galor and Weil (1996, 1999), De la Croix and Doepke (2003), and others. The second approach, by Easterlin (1978) and Easterlin et al. (1980), proposes the "synthesis model" of fertility.[1] In this model, agents are utility maximizers à la Becker but culture and social norms are included as determinants of parental utility. Preferences determine individual demand for commodities and children, whereas social norms determine preferences. However, this second approach failed in making endogenous the long-run evolution of culture and social norms. As a result, it does not provide a better explanation for the long-run evolution of fertility than the Beckerian approach.

In this paper, I argue that interactions between economic and cultural determinants of fertility are at the heart of the long-run decrease in fertility. As in the synthesis model, culture influences rational fertility behavior. However, the evolution of economic conditions endogenously shapes the long-run dynamics of culture. More precisely, I assume the existence of two alternative cultures in the population. Agents of each cultural group are rational utility maximizers à la Becker. Their preferences are determined by the group they belong to. Belonging to a cultural group consists in adopting the fertility norm of this group and its mode of production. Notice that I do not explore the determination of the specific norms within each culture, but I explore the reasons that such norms can persist over time (or disappear) and their impact on demographic dynamics. In other words, the evolution of culture is endogenous at the scale of the society.

The first culture is called the "traditional" culture. "Traditionalists" follow an explicit high norm of fertility[2] and adopt a rural mode of production. The second culture is called the "modern" culture. "Modernists" do not follow any norm of fertility and adopt an industrial mode of production. Historically, this segmentation of the population can be illustrated by religious differences, at least in early Western Europe. This will be discussed in the following section.

The cultural structure of the population results from an endogenous cultural evolution mechanism. This mechanism is based on the theory of endogenous preferences formation and especially follows Bisin and Verdier (2001). Preferences are acquired through a socialization process. During the first stage of this process, parents try to transmit their culture to their children because they prefer their children to resemble them.[3] If this familial socialization fails, children enter a second stage where they adopt the culture of a role model they are randomly matched with. Because parents rationally choose their socialization effort, the cultural heterogeneity characterizing the society crucially depends on economic conditions such as the costs of raising children, parental incomes, and differential productivity between the modes of production.

In this framework, a productivity shock in favor of the industrial mode of production has an evolutionary effect in favor of traditionalists and a cultural effect in favor of modernists. Indeed, this shock implies an increase in the wealth gap between

modernists and traditionalists. Cultural deviation[4] becomes more acceptable for traditionalists because their children will enjoy higher incomes when they adopt the modern culture. Consequently, traditionalist parents reduce their socialization effort. They also increase their fertility because the total expected utility per child is higher. The reverse is true for modernists: an increase in their relative income makes their children's cultural deviation more costly. Thus they tend to increase their socialization effort. Furthermore, because children are time-consuming, they reduce their fertility. So, as traditionalists increase their fertility while modernists decrease theirs, the proportion of traditionalists in the whole population tends to increase: this is called the "evolutionary effect." However, as modernists increase their socialization efforts while Traditionalists decrease their own, the proportion of Modernists also tends to increase: this is called the "cultural effect."

Interactions between evolutionary and cultural effects imply three major results. First, asymmetric technological progress in favor of modernists provokes a fertility transition only when it is combined with a cultural transition making the modernists the majority culture. Second, if traditionalists are strongly attached to their culture, they will be less sensitive to the increase in the wage gap between their mode of production and the modernists' one. As a consequence, facing the asymmetric technological progress, they will maintain relatively high socialization efforts:[5] the cultural effect is weak relative to the evolutionary effect. Thus cultural and demographic transitions will appear later and be achieved more rapidly. Third, in an environment where the modern mode of production is initially weakly productive and does not experience sufficiently strong improvements, the modernist culture can disappear in the long run. Conversely, if there exists strongly biased technological progress in favor of the modern mode of production, the traditionalist culture disappears. Notice that this biased technological progress need not be permanent. It only has to maintain a sufficient wage gap between the two modes of production during a limited period of time. Indeed, the disappearance of a culture is an irreversible event.

The rest of the paper is organized as follows. Section 2 presents the existing explanations for the long-run decrease in fertility and the contribution of the present paper to this literature. It also discusses the main evidence in favor of the model's assumptions. Section 3 presents the model itself, its microeconomic properties, and its long-run dynamics. Section 4 proposes some numerical examples. Section 5 concludes.

2. RELATED LITERATURE AND STYLIZED FACTS

2.1. Related Literature

The existing economics literature provides consistent explanations for the appearance and the pace of the fertility transition. Fertility transition in early developed economies is closely related to the Industrial Revolution and the process of urbanization [see Galor (2005a)]. Two main explanations are relevant regarding empirical evidence on the fertility transition.[6] The first lies in the evolution of

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the wage gap between men and women. Galor and Weil (1996) argue that the great technological progress characterizing the Industrial Revolution reduced the gender wage gap. Higher wages for women increased the opportunity cost of raising children, resulting in lower fertility rates and higher women's working time. The second main explanation lies in the increase in the demand for human capital. Galor and Weil's (1999) model helps explain the emergence of the Industrial Revolution and the demographic transition. The increase in the rate of technological progress induces a rise of both parental wealth and the return to investments in children's human capital. As a result, parents substitute quality for quantity in their demand for children. This major contribution has been followed by papers exploring mechanisms reinforcing the impact of the rise in the demand for human capital on parental fertility. The rise in life expectancy, changes in the marriage market, income inequalities, the decline in child labor, and natural selection[7] are among the most important ones.

The present contribution is more closely related to Galor and Moav (2002). In their evolutionary analysis of the Industrial Revolution and the demographic transition, they also assume the existence of alternative valuations of children's quantity: there exist a group that is quantity-biased and a group that is quality-biased. In the first stage of the evolutionary process, quality-biased agents retain an advantage from their higher investments in human capital. Indeed, economy lies in a Malthusian regime where fertility is positively related to income. Because quality-biased agents are wealthier, they are also more fertile, which implies that their proportion increases. However, some externalities between groups imply that quantity-biased families enjoy the rise in the average return to human capital investment. Thus, they begin to invest in their children's quality and become wealthier. In turn, they increase their fertility, which becomes higher than the quality-biased agents' fertility. They finally become the majority.

In the present paper, cultural transmission is added to purely evolutionary processes. Indeed, contrary to Galor and Moav, I assume that the vertical transmission of preferences from parents to children is not perfect because it is cultural rather than genetic. Furthermore, there also exists an oblique transmission of preferences from the whole society to the children. Thus the model allows mobility between groups. It implies that, when there exists an asymmetric technological progress in favor of modernists (not necessarily a permanent one), the traditionalist group, which is quantity-biased, can disappear despite its natural advantage in the evolutionary process.

By considering cultural mobility rather than purely evolutionary processes, the present paper makes it possible to consider the major role played, at least in Western Europe, by culture and norms in the relation between industrialization and the long-run decrease of fertility.[8]

2.2. Stylized Facts from Early Western Europe

The study of early fertility transitions in Europe by demographers and historians provides evidence linking the appearance of fertility transitions to urbanization,

industrialization, *and* secularization.[9] Lesthaeghe and Wilson (1986) explore the fertility transition in Western Europe from 1870 to 1930. They find that the more Catholic the population is, the later the fertility transition occurs. Furthermore, a greater extent of the agricultural production sector also delays the appearance of the fertility transition and slackens its pace. They argue that industrialization induces a fertility transition only if, in addition, an ethical transition makes birth control acceptable.

Van Poppel (1985), Sommers and Van Poppel (2003), and Van Bavel and Kok (2005) show that, in the Netherlands, the late fertility transition and the late industrialization are due to the predominance of Catholics and Calvinists who were actively opposed to modern limitation of births. Lesthaeghe (1977) studies the Belgian fertility transition. He distinguishes Walloons and Flemings. In both populations, the more industrialized and urbanized areas were also the more secularized ones. Interestingly, Walloons experienced an earlier fertility transition than Flemings. After controlling for the socioeconomic changes in both populations, Lesthaeghe finds that the remaining differences come from differences in secularization: Flemings were more attached to Catholicism, which was opposed to birth control.

All these studies agree that the dramatic changes in the occupational structure induced by the Industrial Revolution are a very important element in explaining the decrease of fertility in Western Europe. However, they argue that secularization has been necessary to experience the fertility transition.

The present paper proposes a simple model that is able to reproduce this stylized fact. Traditionalists can be identified as Catholics and Calvinists. In compliance with their religious culture, they try to respect a high fertility norm and take part in a familial agricultural mode of production. Modernists are not influenced by religious institutions, their fertility choices are not shaped by explicit norms, and they take part in the industrial sector.

Secularization of the population is represented by the long-run decrease in the proportion of traditionalists. Indeed, it makes the influence of religious norms decrease at the scale of the whole society. When the asymmetric technological progress in favor of industries is sufficiently strong,[10] the population enters secularization and undergoes a fertility transition. However, this mechanism is conditional on the "intolerance" of traditionalists,[11] which partly results from the church ideology. If this intolerance is very high, the population enters secularization and decreases its average fertility rate much later and at a faster pace.[12]

My results crucially come from two assumptions that are cornerstones of the paper: first, there exists a high fertility norm in the traditionalist culture; second, traditionalists are engaged in rural activities whereas modernists are engaged in urban industries.

There exists a large set of evidence in favor of the existence of a high fertility norm in the Catholic and Calvinist cultures, as well as in other major monotheistic religions.[13] Lesthaeghe and Wilson (1986), Sommers and Van Poppel (2003), and Van Poppel (1985) find that practicing Catholics and Calvinists in Western

Europe until the beginning of the Second World War are characterized by higher fertility rates than the rest of the population. Williams and Zimmer (1990), Amin et al. (1997), Adserá (2004), and Baudin (2008) show that religiosity measured by church attendance has a positive and significant impact on fertility. With alternative measures, Janssen and Hauser (1981) and Hacker (1999) find the same result.[14]

Lesthaeghe and Wilson (1986) indicate that high fertility rates in Catholic populations in early Western Europe come, in part, from the fit between the Catholic concept of familial solidarity and the labor-intensive rural mode of production that was glorified by the Catholic Church. In religious families, children are a source of labor; they take part in familial production until they get married and start their own familial production. This fit between the traditionalist culture and the familial mode of production is at the center of my second main assumption. Traditionalists are assumed to adopt a rural activity, namely labor-intensive agriculture or a family protoindustry, and modernists adopt an urban and industrialized activity.[15] For instance, Van Heek (1956) and Neven and Oris (2003) highlight this type of segmentation respectively for eighteenth-century Belgium (especially in the Herve and Tilleur counties) and Holland (during the nineteenth century and the beginning of the twentieth).

3. DESCRIPTION OF THE ECONOMY

3.1. The Model

The model consists of an overlapping-generations economy where there are L_t adult agents who live for two periods. During the first period, they are children and only receive a "social education" from their parent. During the second period they are adults. They choose their optimal level of consumption C_t^i, the number of their children N_t^i, and their social education τ_t^i which is understood as a socialization effort. Families are assumed to be monoparental. Childbearing is costly, each child takes a part $\eta^i > 0$ of its parent's time unit.[16] The cost of one unit of socialization is denoted by $\gamma > 0$. It follows that adults, at period t, have to respect the following budget constraint:

$$C_t^i + \eta^i \Omega_t^i N_t^i + \gamma \tau_t^i N_t^i = \Omega_t^i + \overline{\Omega}. \quad (1)$$

Ω_t^i denotes the labor income of an agent of type i; its labor supply equals its remaining time after childbearing. $\overline{\Omega}$ denotes a nonlabor income that correspond to a minimal domestic production assumed to be the same in all families.[17] Agents are culturally heterogeneous in the sense that they could belong to different cultural groups. There are two cultures in the economy. The first is the *traditional* culture; it is characterized by a high fertility norm. Traditionalists are engaged in the agricultural sector, providing an income Ω_t^T. The second is the *modern* culture; modernists are not influenced by fertility norms. They take part in the industrial sector, providing an income Ω_t^M[18]. The proportion of modernists at period t is denoted q_t; $(1 - q_t)$ is the proportion of traditionalists at that date.

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A modernist parent who has a modernist child enjoys a utility denoted V_t^{MM}; if he has a traditionalist child, he or she enjoys V_t^{MT}. A traditionalist parent who has a traditionalist child enjoys V_t^{TT}, and V_t^{TM} if he or she has a modernist child. All things being equal, parents prefer to have children adopt the same culture (traits) as their own, but they altruistically prefer that their children become rich. Their children's future income is determined by their future culture: their income will be Ω_{t+1}^M if they become modernist, and Ω_{t+1}^T if they become traditionalist. Parents are characterized by static expectations; that is, they expect that their children will enjoy the same income as their own.[19] Thus,

$$\begin{aligned} V_t^{MM} &= \theta^M + \Omega_t^M \quad V_t^{MT} = \Omega_t^T, \\ V_t^{TT} &= \theta^T + \Omega_t^T \quad V_t^{TM} = \Omega_t^M. \end{aligned} \tag{2}$$

$\theta^i > 0$ denotes the supplement of utility a parent of type i enjoys when his or her child adopts the culture i. So θ^i represents the cultural intolerance of parents of type i. $\Delta V_t^i = V_t^{ii} - V_t^{ij} = \theta^i + \Omega_t^i - \Omega_t^j$ represents the loss for a parent of type i who has a child of type j. Thus the loss of a parent, *in the case of cultural deviation*, is equal to his or her cultural intolerance plus the potential loss of income for the child when he or she adopts the alternative culture. If the cultural deviation implies higher incomes, the relative importance of parental intolerance in the choice process decreases. Note that for very high values of Ω_t^j, ΔV_t^i can become negative.

The culture a child will adopt is not exogenously determined; it is the result of a socialization process à la Bisin and Verdier (2001). A child is first exposed to familial socialization. Socialization effort τ_t^i is a pure private good for the family in the sense that one unit of social education benefits only one child. This assumption is a simplification of a more general framework in which socialization efforts can benefit more than one child with decreasing returns, such that total socialization costs would be concave in N_t^i. It simplifies the results without loss of accuracy. Familial socialization succeeds with probability $(\tau_t^i)^{1/2} \in [0, 1]$; the socialization effort exhibits decreasing returns for each child.

If the familial process of socialization fails, the child is engaged in a second stage of socialization where he or she is randomly matched with a role model in the society and adopts its traits. With probability q_t the child is matched with a modernist and with probability $1 - q_t$ with a traditionalist. Transition probabilities can be expressed as follows:

$$\begin{aligned} P_t^{MM} &= \left(\tau_t^M\right)^{\frac{1}{2}} + \left[1 - \left(\tau_t^M\right)^{\frac{1}{2}}\right] q_t, & P_t^{MT} &= \left[1 - \left(\tau_t^M\right)^{\frac{1}{2}}\right] [1 - q_t], \\ P_t^{TT} &= \left(\tau_t^T\right)^{\frac{1}{2}} + \left[1 - \left(\tau_t^T\right)^{\frac{1}{2}}\right] [1 - q_t], & P_t^{TM} &= \left[1 - \left(\tau_t^T\right)^{\frac{1}{2}}\right] q_t. \end{aligned} \tag{3}$$

$P_t^{ij} \in [0, 1]$ denotes the probability that a parent of type i has a child of type j. The probability for a child to become modernist (traditionalist) increases with the proportion of modernists (traditionalists) in the economy. Finally, the utility of an

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agent of type i is denoted W_t^i and is described by[20]

$$W_t^i\left(C_t^i, N_t^i, \tau_t^i\right) = C_t^i + \pi^i \left(N_t^i\right)^{\frac{1}{2}} + \left(N_t^i\right)^{\frac{1}{2}} \left[P_t^{ii} V_t^{ii} + P_t^{ij} V_t^{ij}\right]$$
$$\text{with} \quad \pi^i = \begin{cases} 0 & \text{if } i = M \\ \pi & \text{if } i = T \end{cases}. \tag{4}$$

Because traditionalists belong to a culture characterized by high fertility norms, they give a higher value to quantity of children than modernists, who only value quantity through their imperfect altruism. Higher values of π reflects higher fertility norms. There are two instruments for traditionalists and modernists to ensure their reproductive success in the long run: their fertility rate and their socialization effort. With a high fertility rate, a group ensures widespread implementation of its socialization process. So it can make a lower socialization effort per family to ensure the same reproductive success as a group with a low fertility rate. Conversely, a group adopting a high socialization effort per family needs a lower total fertility rate. The cultural and demographic dynamics are expressed, respectively, by the equations

$$q_{t+1} = \frac{q_t N_t^M P_t^{MM} + (1 - q_t) N_t^T P_t^{TM}}{q_t N_t^M + (1 - q_t) N_t^T}, \tag{5}$$

$$\frac{L_{t+1} - L_t}{L_t} = q_t N_t^M + (1 - q_t) N_t^T - 1. \tag{6}$$

The proportion of modernists at period $t + 1$ is equal to the number of children with modernist parents ($q_t N_t^M$) who also become modernists[21] plus the number of children with traditionalist parents ($(1 - q_t) N_t^T$) who become modernists, divided by the number of modernists in t. Equation (6) is simply the weighted average fertility rate minus one. Transition probabilities and fertility levels crucially depend on parental microeconomic choices described in what follows.

3.2. Individual Behavior

Modernists. A modernist parent born in $(t - 1)$ chooses C_t^M, N_t^M, and τ_t^M to maximize (4) subjected to (1), (2), (3), and $i = M$. I obtain the following decision rules:

$$N_t^{M^*} = \begin{cases} \dfrac{1}{\eta^m} & \text{if } \Omega_t^M \leq \widehat{\Omega}_t \\ \left(\dfrac{q_t \Delta V_t^M + V_t^{MT}}{2\eta^M \Omega_t^M}\right)^2 & \text{otherwise} \end{cases}, \tag{7}$$

$$\tau_t^{M^*} = \begin{cases} 0 & \text{if } \Omega_t^M < \Omega_t^T - \theta^M \\ \dfrac{\eta^M}{4\gamma^2}\left[(1 - q_t)\Delta V_t^M\right]^2 & \text{if } \Omega_t^M \in \left[\Omega_t^T - \theta^M, \widehat{\Omega}\right] \\ \left[\dfrac{\eta^M \Omega_t^M}{\gamma} \cdot \dfrac{(1 - q_t)\Delta V_t^M}{q_t \Delta V_t^M + V_t^{MT}}\right]^2 & \text{if } \Omega_t^M > \widehat{\Omega}_t \end{cases}, \tag{8}$$

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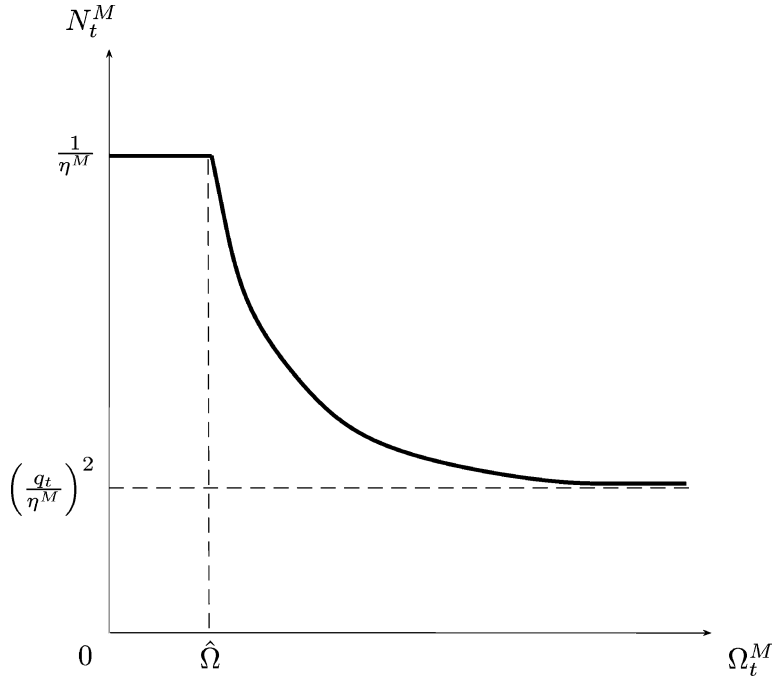

FIGURE 1. Fertility of modernists.

with[22]

$$\widehat{\Omega}_t = \frac{q_t\theta^M + \{1 - q_t\}\,\Omega_t^T}{2\{\eta^M\}^{\frac{1}{2}} - q_t}.$$

The value of $C_t^{M^*}$ is directly deduced from the budget constraint. The optimal fertility choice of a modernist parent can be represented as in Figure 1.

For interior solutions, an increase in modernist earnings incites modernist parents to increase their socialization effort and to decrease their fertility rate. Indeed, a higher value of Ω_t^M increases parental income and children's future income if they become modernists. Thus the expected loss per child born to modernist parents, *in case of cultural deviation*, increases. Thus, modernists tend to implement a higher socialization effort to reduce that expected loss. The increase in the Modernist income has, a priori, a more ambiguous impact on the modernists' fertility. Indeed, when Ω_t^M increases, the *total expected gain per child* increases,[23] this has a positive effect on the parental fertility. However, as in standard endogenous fertility models, the cost of children's quantity increases with incomes. This has a negative impact on the modernists' fertility. It is straightforward that, in the present framework, the negative impact is always the strongest one.[24] Notice that, when $\Omega_t^M \in [0, \widehat{\Omega}_t]$, fertility is constrained and does not decrease; nevertheless socialization efforts increase.

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The modernists' socialization effort decreases with the proportion of modernist parents. The vertical socialization (from parent) and the oblique socialization (from role models) are substitutes. When parental socialization fails, a child with modernist parents still has a chance to become modernist if he or she is matched with a modernist role model in the society. When q_t increases, the probability for any child to be matched with a modernist role model becomes higher. Therefore the expected gain per child born increases and parents can reduce their familial (costly) socialization effort and have more children. Obviously, when q_t equals one, the probability for a child to be matched with a modern role model is one; thus modernist parents stop directly socializing their children, $\tau_t^{M^*} = 0$. They allocate all their income to fertility and consumption.

Traditionalists. Traditionalists born in $(t-1)$ choose C_t^T, N_t^T, and τ_t^T in order to maximize (4) subjected to (1), (2), (3), and $i = T$. The optimal behavior of traditionalist parents is described by[25]

$$N^{T^*} = \begin{cases} \left[\dfrac{(1-q_t)\,\Delta V_t^T + q_t\Omega_t^M + \pi}{2\eta^T\Omega_t^T}\right]^2 & \text{if } \Omega_t^M < \widetilde{\Omega}_t \\ \dfrac{1}{\eta^T} & \text{otherwise} \end{cases}, \tag{9}$$

$$\tau_t^{T^*} = \begin{cases} \left[\dfrac{\eta^T\Omega_t^T}{\gamma}\cdot\dfrac{q_t\Delta V_t^T}{(1-q_t)\,\Delta V_t^T + q_t\Omega_t^M + \pi}\right]^2 & \text{if } \Omega_t^M < \widetilde{\Omega}_t \\ \dfrac{\eta^T}{4\gamma^2}\left[q_t\Delta V_t^T\right]^2 & \text{if } \Omega_t^M \in \left[\widetilde{\Omega}_t,\, \Omega_t^T + \theta^T\right], \\ 0 & \text{if } \Omega_t^M > \Omega_t^T + \theta^T \end{cases} \tag{10}$$

with[26]

$$\widetilde{\Omega}_t \equiv \frac{\left[2(\eta^T)^{\frac{1}{2}} + q_t - 1\right]\Omega_t^T - (1-q_t)\,\theta^T - \pi}{q_t}.$$

Vertical and oblique socialization are still substitutes for traditionalist parents. So an increase in q_t incites them to have fewer children and to implement a higher socialization effort. For interior solutions, an increase in the traditionalists' earnings incites parents to substitute socialization effort for quantity of children. Notice that, because of the fertility norm, even if traditionalists and modernists had the same fertility costs and the same income, traditionalists' fertility would be higher than modernists.

Let us consider that traditionalists' income is high enough so that, when the modernists' income is low, their fertility and socialization choices are interior. When the modernists' income increases, traditionalists reduce their socialization effort and increase their fertility. Indeed, the loss resulting from cultural deviation is smaller and the overall expected utility per child higher. When Ω_t^M reaches the threshold $\widetilde{\Omega}_t$, traditionalists cannot increase their fertility any more because they

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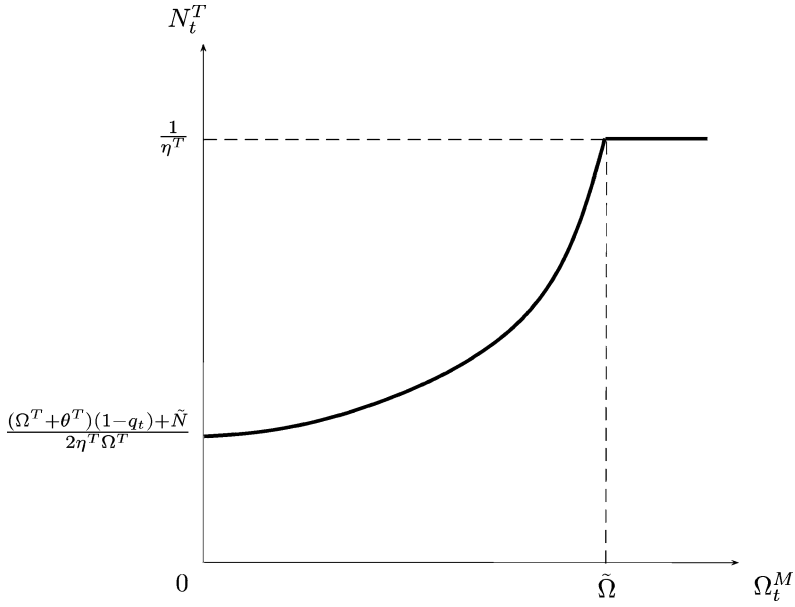

FIGURE 2. Fertility of traditionalists.

have reached their maximum fertility rate. Thus, they decrease their socialization effort without increasing their fertility. Finally, when Ω_t^M reaches $\Omega_t^T + \theta^T$, ΔV_t^T becomes negative and then traditionalists stop socializing their children. Indeed, despite their cultural intolerance, they forecast that their children will be wealthier if they become modernists. The evolution of the traditionalists' socialization effort and fertility is described by Figures 2 and 3.

Following these microeconomic results, the cultural and demographic dynamics of the economy are analyzed in the next sections.

3.3. Cultural Dynamics

Multiple Equilibria and Cultural Heterogeneity. The cultural dynamics of the population is given by equations (2), (3), (5), (7), (8), (9), and (10). The presence of corner solutions depending on the value of Ω_t^M implies the existence of multiple regimes. The main properties of this dynamics are described in the following proposition.

PROPOSITION 1. (i) *When* $\Omega_t^M \leq \Omega_t^T - \theta^M$, $q_t = \{0, 1\}$ *are the only existing steady states, and* $q_t = 0$ *is globally stable while* $q_t = 1$ *is unstable.* (ii) *When* $\Omega_t^M \geq \Omega_t^T + \theta^T$, $q_t = \{0, 1\}$ *are also the only existing steady states; however,* $q_t = 0$ *is unstable while* $q_t = 1$ *is globally stable.* (iii) *When* Ω_t^M *takes intermediary values such that* $\Omega_t^M \in [\Omega_t^T - \theta^M, \Omega_t^T + \theta^T]$, $q_t = \{0, \overline{q}, 1\}$ *are the only existing steady states.* $q_t = \{0, 1\}$ *are unstable, whereas the only interior steady state* $\overline{q}$ *is globally stable and allows cultural heterogeneity.*

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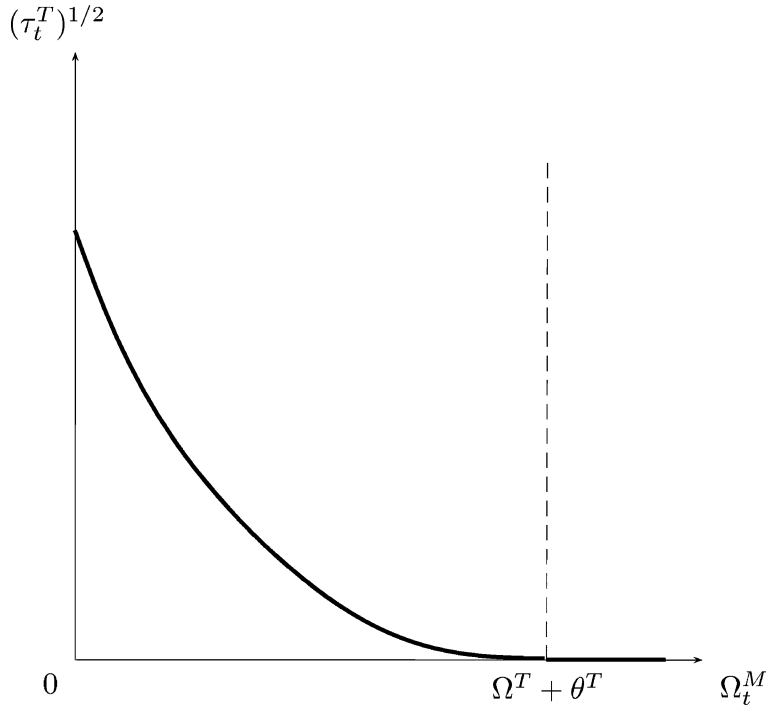

FIGURE 3. Socialization effort of traditionalists.

Proof. See Appendix B. ■

Stability of the interior solution crucially comes from substitutability between vertical socialization (from parents) and oblique socialization (from the whole society). All other things being equal, parents in the majority culture tend to make a smaller socialization effort than parents in the minority culture. This means that, for intermediate levels of inequality between incomes of modernists and traditionalists, society is characterized by long-run cultural heterogeneity.

Notice that, in the interior regime (when $\overline{q}$ does exist), when Ω_t^M increases, the traditionalist mode of production becomes inefficient relative to the modernist mode of production. However, the traditionalist culture does not disappear. This culture will disappear only when the inefficiency of its mode of production is very high ($\Omega_t^M \geq \Omega_t^T + \theta^T$) so that members of this culture will choose stop transmitting their culture to their children. The reverse is also true; if the productivity of the modernist mode of production is very low ($\Omega_t^M \leq \Omega_t^T - \theta^M$), the modernist culture disappears in the long run.

Comparative Statics. As a result, a rise in modernist productivity does not always increase the long-run proportion of modernists in the population. Indeed, it can easily be shown that the long-run proportion of modernists will increase after

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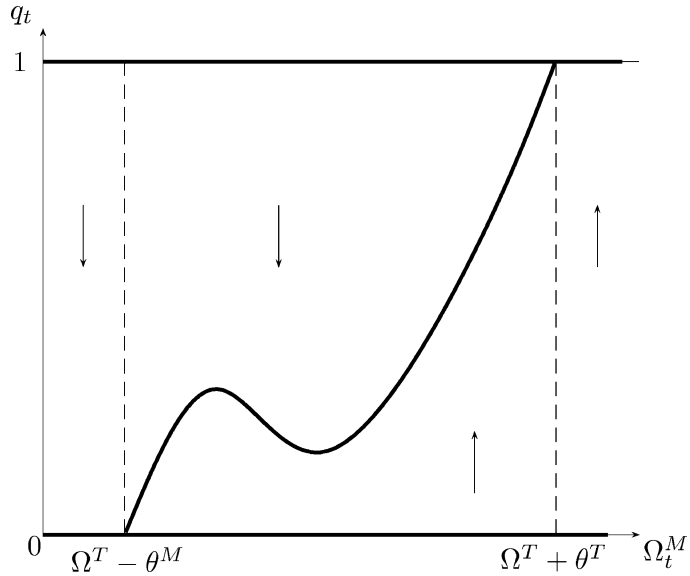

FIGURE 4. $\Omega^T > \theta^M$ and temporary domination of the evolutionary effect.

a positive shock on Ω_t^M if the following condition is fulfilled:[27]

$$\left[\frac{1}{2}\left(\tau_t^M\right)^{-1/2} N_t^M \frac{\partial \tau_t^M}{\partial \Omega_t^M} - \frac{1}{2}\left(\tau_t^T\right)^{-1/2} N_t^T \frac{\partial \tau_t^T}{\partial \Omega_t^M}\right] + \left[\left(\tau_t^M\right)^{1/2} \frac{\partial N_t^M}{\partial \Omega_t^M} - \left(\tau_t^T\right)^{1/2} \frac{\partial N_t^T}{\partial \Omega_t^M}\right] > 0. \quad (\mathbf{11})$$

The first term between parentheses consists in the cultural effect and is positive, whereas the second term between parentheses consists in the evolutionary effect and is negative or equal to zero. Indeed, when modernist income increases, modernists provide a greater socialization effort, whereas traditionalists reduce their own. However, when not constrained, traditionalists increase their fertility whereas modernists reduce their own. In other words, when Ω_t^M increases, traditionalists get an advantage in the evolutionary process (the evolutionary effect) and modernists get an advantage in the cultural transmission process (the cultural effect). The bifurcation diagrams in Figures 4–7 represent the evolution of cultural steady states.[28]

As mentioned in Proposition 1, $q_t = \{0, 1\}$ are always cultural steady states. Notice that when $\theta^M > \Omega^T$ (Figures 6 and 7), the modernist culture will never disappear because modernists will always prefer having modernist children ($\Delta V^M > 0$). In $\Omega_t^M = \{\Omega_t^T - \theta^M, \Omega_t^T + \theta^T\}$, the cultural dynamics enters into bifurcations.[29]

A rise in Ω_t^M implies an opposition between evolutionary and cultural processes. Nevertheless, it is intuitive that in the neighborhood of $\Delta V_t^M = 0$ and $\Delta V_t^T = 0$,

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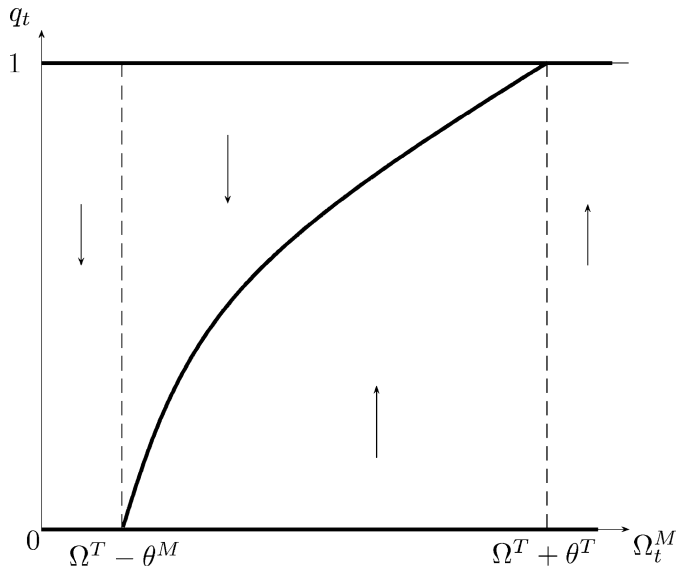

FIGURE 5. $\Omega^T > \theta^M$ and complete domination of the cultural effect.

the cultural effect always dominates the evolutionary effect. Indeed, when Ω_t^M becomes closed from $\Omega_t^T + \theta^T$, τ_t^T converges to zero because the loss of traditionalists in the case of cultural deviation (ΔV^T) will be close to zero. Furthermore, the modernists' fertility decreases, but very slowly (see Figure 1). So, for high values

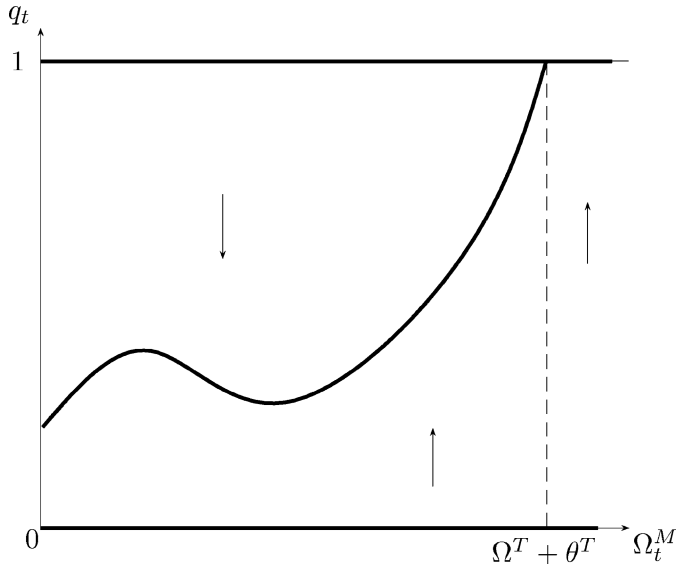

FIGURE 6. $\Omega^T \leq \theta^M$ and temporary domination of the evolutionary effect.

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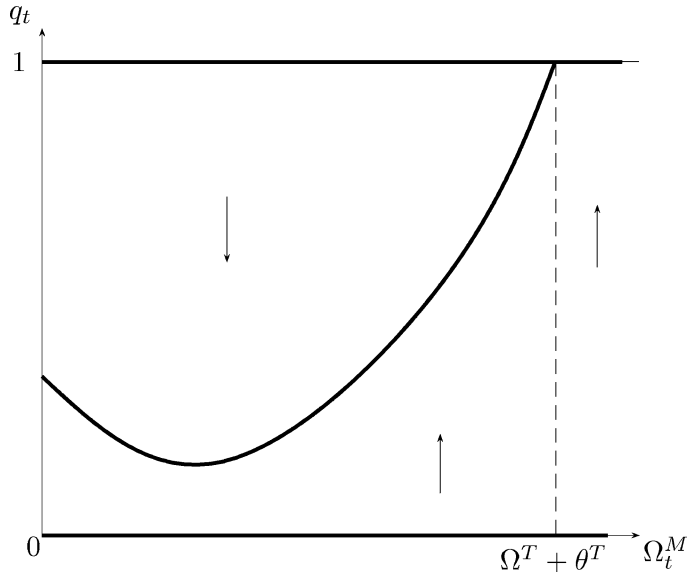

FIGURE 7. $\Omega^T \leq \theta^M$ and initial domination of the evolutionary effect.

of modernist income, the evolutionary effect does not play a role any more [see (11)]. In the same way, when Ω_t^M is in the neighborhood of $\theta^M - \Omega_t^T$, ΔV_t^M tends to zero. So τ_t^M also tends to zero, and decreasing returns to familial socialization imply that the cultural effect is strong. Furthermore, for low values of the Ω_t^M, the modernists' fertility is constrained (see Figure 1): $\partial N_t^M / \partial \Omega_t^M = 0$.

However, for intermediate values of Ω_t^M, the evolutionary process can dominate the cultural process. In this case, an income shock in favor of modernists may finally reduce the long-run proportion of modernists.

Cultural Dynamics after a Productivity Shock in Favor of Modernists. This section illustrates the impact of a biased technological shock on the cultural dynamics. I show how an improvement in the modernists' wealth does not always increase their proportion in the population. In Figures 8 and 9, I represent the evolution of q_t given its initial value q_0 and the interplay between evolutionary and cultural processes after an income shock.

In this example, the biased productivity shock in favor of modernists arises when q_t equals q_1. Three shock's magnitudes are proposed. For a "small shock" increasing Ω_t^M from Ω_A^M to Ω_B^M, the long-run cultural dynamics is dominated by evolutionary effects. In other words, the rise in the fertility differential in favor of traditionalists more than compensates for the rise in the socialization differential in favor of modernists. So, after the biased income shock, the proportion of modernists decreases toward its low long-run level. For an intermediate shock (from Ω_A^M to Ω_C^M), the cultural effect dominates the evolutionary effect. Thus, q_t

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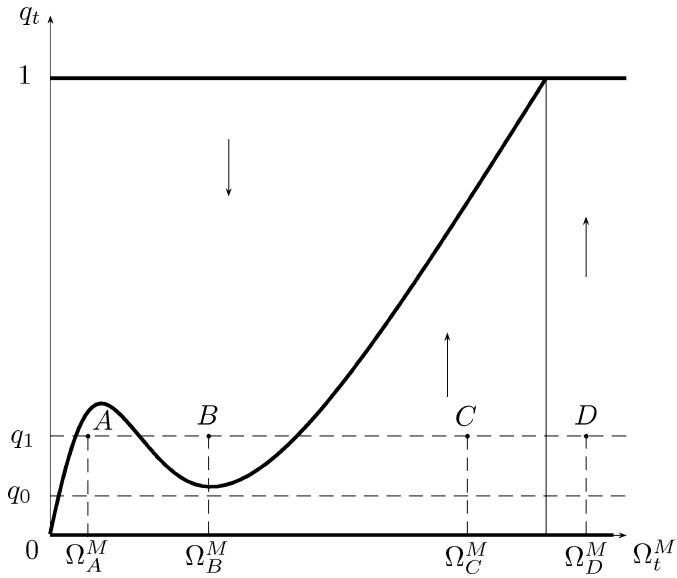

FIGURE 8. Effect of a shock on Ω_t^M.

converges to a long-run value that is higher than q_1. Notice that, in this case, long-run cultural heterogeneity is ensured because the income shock has not been very strong. However, when Ω_t^M increases from Ω_A^M to Ω_D^M, the wealth gap between the two groups is so high ($\Omega_t^M - \Omega_t^T > \theta^T$) that traditionalists stop directly

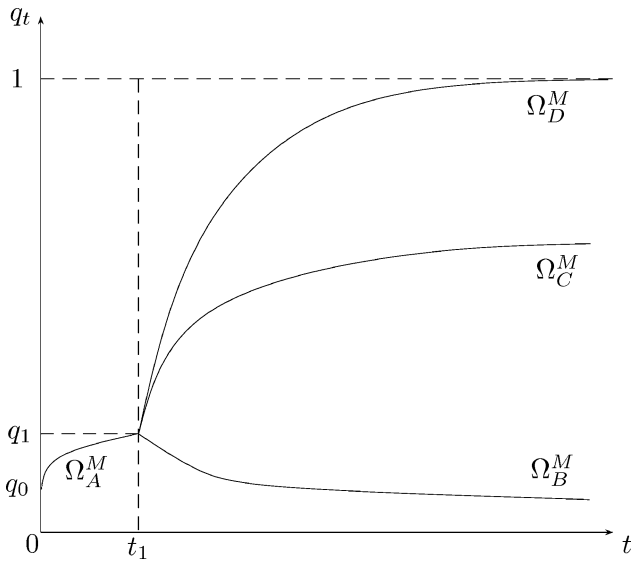

FIGURE 9. Cultural dynamics as a function of the shock's magnitude.

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socializing their children. Thus, q_t converges to 1 and there is no long-run cultural heterogeneity in the population.

It finally appears that a sufficiently strong asymmetric technological progress ensures the cultural homogenization of the population. Such biased technological progress need not be permanent; it only has to be such that $\Omega_t^M - \Omega_t^T > \theta^T$ until q_t converges to one. At this time, traditionalism has definitely disappeared. It is also intuitive that a stronger attachment of traditionalists to their culture will make traditionalism survive higher income shocks. This will be further discussed in the following sections, but it is obvious that, if θ^T takes higher values, the wealth gap between the two modes of production ($\Omega_t^M - \Omega_t^T$) has to be higher.

3.4. Population Dynamics: Scenarios for a Fertility Transition

In this section, I propose some scenarios that could occur after a rise in the wealth gap between modernists and traditionalists. To do so, rather than assuming a single discrete shock on Ω_t^M, I assume a progressive adjustment. In other words, I assume that there exists a transitory biased technological progress in favor of modernists. Based on this, the description of the fertility rate's evolution will be more precise.

It is intuitive that, if the biased technological progress is sufficiently strong, a fertility transition is inevitable. Indeed, as shown in Figures 4–8, a strong increase in Ω_t^M finally raises the long-run proportion of modernists, who reduce their fertility, whereas it reduces the proportion of traditionalists, who cannot indefinitely increase their fertility (see Figure 2). The decrease in total fertility rate occurs even if traditionalism does not completely disappear and well before the disappearance of traditionalists if Ω_t^M becomes higher than $\Omega_t^T + \theta^T$. Indeed, at the latest, when the traditionalists' fertility becomes constrained because of the income gap (see Figure 2), the total fertility rate unambiguously decreases. Furthermore, the convex relation between N_t^M and Ω_t^M implies that the effect of the reduction in the modernists' fertility is initially strong.

Empirical evidence [see Galor (2005b)] indicate that, at the beginning of the demographic transition, total fertility rate can increase. This stylized fact can easily be reproduced by the model, but with a different mechanism than in the usual literature. Indeed, if the fertility of modernists is initially constrained because of their low income (see Figure 1), the increase in their income will not initially incite them to reduce their fertility. However, traditionalists increase their fertility, because their total expected utility per child increases. Then, as long as the modernists' fertility remains constrained, the asymmetric technological progress make the average total fertility rate increase. When the modernists' fertility is no longer constrained, two polar scenarios can be envisaged. In the first one, the income converges to a relatively low value where the evolutionary effect dominates the cultural effect (as B in Figure 8). Thus, the economy remains trapped in a traditionalist regime where the average total fertility rate is high. In the second case, Ω_t^M converges to a relatively high value (as in D); thus the average total

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fertility rate will unambiguously decrease. Indeed, traditionalism progressively disappears and modernist fertility decreases.

The model also generates situations where the total fertility rate decreases as soon as asymmetric technological progress appears. Indeed, when the modernists' fertility is not initially constrained and the income shocks leads to situation where the cultural process dominates (such as C or D), the reduction in the modernists' fertility can immediately overwhelm the increase of the traditionalists'.

For higher values of θ^T, the homogenization of the society ($q_t = 1$) will require stronger asymmetric income differences. For a given technological progress, the rise in the long-run proportion of modernists will be slower. Indeed, when traditionalists are more intolerant with regard to their children's cultural deviation, they are less sensitive to the improvement of wealth their children could enjoy if they became modernists. Thus, when Ω_t^M increases, they reduce their socialization efforts less. The completion of the fertility transition will be longer.

Describing the exact evolution of the total fertility rate requires a numerical example. Indeed, total fertility rate depends on Ω_t^M in a complex way because it directly depends on Ω_t^M but also on the cultural dynamic path, which also depends on the evolution of Ω_t^M.

4. NUMERICAL EXAMPLE

This numerical example aims at illustrating the impact of exogenous growth of modernist income Ω_t^M and the influence of traditionalism on the long-run population dynamics. It will appear that the long-run decrease of fertility is the by-product of two phenomena: the long-run disappearance of traditionalists and the decrease in modernist fertility. Furthermore, a high degree of traditionalism can delay the appearance of the fertility transition but accelerate its pace once it is engaged.

4.1. On the Cultural and Demographic Transitions

Two main numerical examples are proposed in this section. In the first one, $\theta^M \leq \Omega^T$, which implies that for $\Omega_t^M < \Omega^T - \theta^M$, ΔV_t^M will be negative. In the second numerical example, $\theta^M \geq \Omega^T$, so that ΔV_t^M will never be negative; furthermore, $\Omega_0^M > \widetilde{\Omega}_0$. These two exercises hold the parametrization in Table 1.

$g^M = \frac{\Omega_{t+1}^M - \Omega_t^M}{\Omega_t^M}$ denotes the exogenous asymmetric technological progress in favor of the industrial mode of production. The value of 0.2 is close to the average annual output growth in Western Europe since 1820 [see Bairoch (1997)]. For simplicity, this technological progress is assumed to be not transitory but permanent. In other words, given g^M and Ω_0^M, the homogenization of the population is inevitable. For $\eta^T = 0.2$ and $\eta^M = 0.35$, the maximal number of children per family is 10 for traditionalists and close to 6 for modernists. As mentioned in the first sections, this difference comes from the alternative status of children in the two mode of production: children are more costly in urban areas than in rural

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TABLE 1. Values of parameters for the alternative exercises

Parameter	Case 1: $\theta^M \leq \Omega^T$	Case 2: $\theta^M > \Omega^T$
Ω^T	160	80
Ω_0^M	40	40
θ^T	100	100
θ^M	100	100
$\overline{N}$	10	10
$\overline{\Omega}$	15	15
γ	60	60
η^T	0.2	0.2
η^M	0.35	0.35
g^M	0.2	0.2
q_0	0.41	0.2

areas. γ is calibrated so that socialization probabilities are in the range [0, 1]. The initial income of modernists is chosen so that, when $\theta^M \leq \Omega^T$, ΔV_t^M can be negative in the beginning of the growth process of Ω_t^M. The two exercises lead to the cultural and demographic dynamics in Figures 10 and 11.[30]

In the first exercise (Figure 10), ΔV_t^M is initially lower than zero. Thus, until Ω_t^M reaches $\widetilde{\Omega}_t$ (in approximately one period), the total fertility rate increases because the modernists' fertility remains constant while the traditionalists' increases (for $\Omega^T = 160$, it is not initially constrained). This effect is reinforced by the rise in the proportion of traditionalists in the whole population until ΔV_t^M becomes positive.

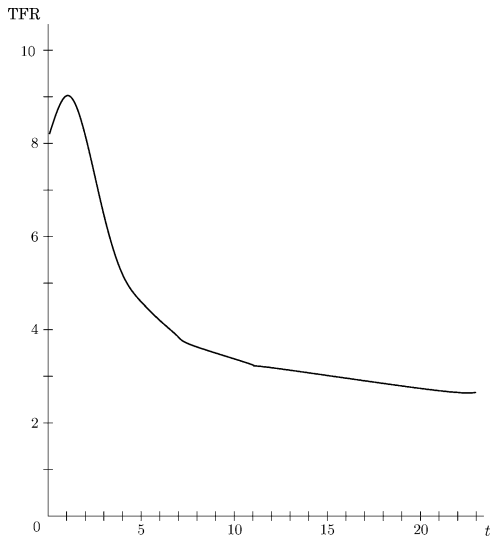

FIGURE 10. Dynamics of fertility in case 1.

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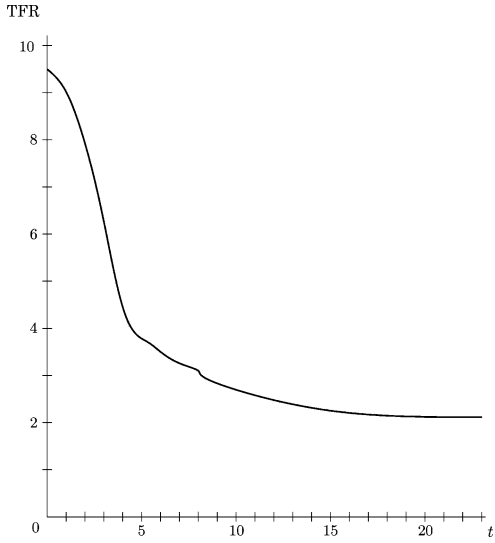

FIGURE 11. Dynamics of fertility in case 2.

When modernists engage in socialization, their proportion increases, whereas their fertility begins to decrease. Thus, a fertility transition does appear.

In the second exercise (Figure 11), Ω^T is such that ΔV_t^M can never be negative. Furthermore, the initial values of q_t and Ω_t^M implies that the traditionalists' fertility is always constrained ($\Omega_0^M > \widetilde{\Omega}_0$ when $q_0 = 0.41$). As mentioned in the preceding sections, in this case, they cannot increase their fertility when the modernists' income increases. Thus, they only decrease their socialization effort. Furthermore, as ΔV_t^M is always positive, modernists increase their socialization effort and decrease their fertility (once Ω_t^M reaches $\widehat{\Omega}_t$). As q_0 is low and the evolutionary process never dominates the cultural process, the proportion of modernists is always increasing and the total fertility rate always decreasing.

4.2. Impact of Traditionalism

It finally appears that, in this model, fertility transition results from two phenomena: a cultural transition making the long-run proportion of modernists grow and a decrease in the modernists' fertility because of the improvement in their income. A central result of the present paper lies in the fact that the cultural transition is a necessary condition for fertility transition. The growth of productivity and income (of modernists) is not sufficient. Indeed, in the present exercise, I propose to simulate the demographic dynamics of the economy for different values of the traditionalists' attachment to their culture, namely θ^T, for case 1 of the preceding section.

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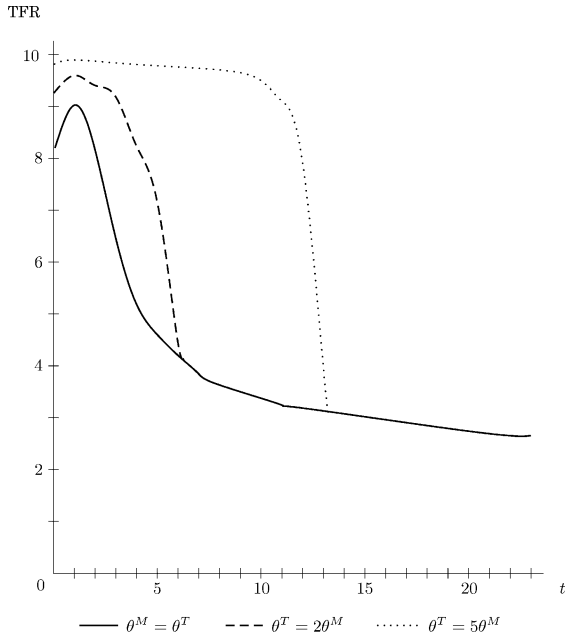

FIGURE 12. Impact of θ^T on the fertility dynamics.

As a general result, a higher degree of traditionalism implies a higher initial total fertility rate and a later but faster fertility transition. As shown in Section 3, when θ^T is strong, the marginal return of the quantity of children is higher. It implies that, for the same initial values of Ω_t^M and q_t, the initial total fertility rate is higher. Furthermore, a higher θ^T implies that traditionalists are less sensitive to the wealth improvement their children could enjoy if they became modernists. Then, when Ω_t^M increases, they reduce their socialization effort less than for low values of θ^T. This implies that the proportion of traditionalists in the population remains high during the beginning of the income growth process. In other words, the cultural effect is weaker when the traditionalists' intolerance is higher.

Finally, traditionalism induces a delayed cultural transition and so a delayed fertility transition (see Figure 12). An initially more traditionalist society needs more favorable economic conditions in the modernist mode of production to enter into the long-run reduction of fertility.

Furthermore, once fertility begins to decrease, societies with a higher degree of traditionalism experience a faster decrease of total fertility rate. This comes simply from the fact that the cultural transition is delayed. Indeed, it appears for higher values of the modernists' income. Thus, when modernists become the majority, their fertility is already very low. Thus, for a similar increase in q_t, the total fertility rate decreases more rapidly.

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Notice that the decrease of fertility in modernists families comes from the rise in industrial productivity and so in their income. Introducing a standard quality–quantity trade-off would have led to the same results: a rise in the marginal return of the modernists' education investment would incite them to substitute quality to quantity. The future income of modernists would be increasing, which incites modernist parents to increase their socialization efforts.[31]

5. CONCLUSIONS

In this paper, I propose a model that enriches the economic analysis of the fertility transition by integrating some cultural aspects of the process. I show that a fertility transition results from an asymmetric technological progress in favor of the industrial sector and a cultural transition making cultures that limit births the majority. Cultural transition will occur because cultural deviation from traditional to modern groups is more enjoyable when asymmetric technological progress takes place. As a result, if traditionalist agents are widely attached to their culture, they will be less sensitive to these asymmetric shocks and maintain strong efforts to make their culture survive despite its growing inefficiency. This mechanism makes it possible to explain the deletion of a fertility transition in more traditionalist countries such early Belgian Flanders and Holland.

The consideration of cultural aspects in the dynamics of reproductive behavior begins to greatly benefit from the more general renewal of cultural analysis in economics. In order to continue the rehabilitation of the synthesis model of fertility, it will be crucial, in future work, to make the long-run evolution of social norms (at least regarding fertility) themselves endogenous, in a quantifiable and therefore testable manner.

NOTES

1. Birdsall (1988) provides an enlightening presentation of Easterlin's contributions.

2. Traditionalism can also correspond to cultures and groups characterized by low fertility norms. For example, hunter and gatherer societies do not exhibit high fertility norms despite their evident traditionalism.

3. Bisin and Verdier (2001) argue that parents prefer to have children who adopt the same preferences as their own using the paternalistic altruism theory. Bergstrom and Stark (1993) give some anthropological foundation to explain imperfect empathy from parents to children.

4. A cultural deviation occurs when a child adopts a culture different from the parental one.

5. I assume that facing asymmetric technological progress in favor of modernists, traditionalists do not abandon their mode of production despite its growing inefficiency. The persistence of inefficient economic behaviors is reported and explained in many papers such as Grusec and Kuckzynski (1997) and Guiso et al. (2006). For instance, Salamon (1992) provides the example of German Catholics in the 1840 United States. They adopted a less profitable way to exploit crops than Yankees and had more children on average.

6. Other explanations challenge these two theories. The decline in infant and child mortality has been a major argument of demographers. Becker (1981) proposes that the increase of income is at the origin of the decrease in fertility. However, these theories appear to be counter factual [see Galor (2005b)].

7. Galor (2005a, 2005b) provides a very enlightening review of this literature.

8. Recent and enlightening papers study the co-determination of culture and industrialization without considering fertility. Doepke and Zilibotti (2008) relate the Industrial Revolution to the transmission of patience among families and the development of financial markets. Ashraf and Galor (2007) propose a model of cultural assimilation and cultural diffusion to explain differences in the timing of the Industrial Revolution.

9. In line with Berger (1973) and Lesthaeghe and Wilson (1986), secularization is defined as a process depriving some aspects of the social and cultural life from the religious authorities.

10. The industrial bias of technological progress during the Industrial Revolution is well documented. See, for instance, Bairoch (1997).

11. "Intolerance" has to be understood as the attachment of an agent to the perpetuation of its culture in its own dynasty. In this paper, modernists also exhibit intolerance.

12. See Van Heek (1956) for Holland. Appendix A provides evidence for Belgium, where Flemish provinces are described as more attached to Catholic values that were opposed to birth control.

13. Evidence in favor of high fertility norms can also be found in Marxist ideologies as in China under Mao [see, for instance, Fan and Zhang (2004)] and in nonreligious states such as France [Spengler (1954)]. Fernández and Fogli (2007) show that culture is important to the understanding of female work and fertility without approximating culture by religion.

14. I assume that culture is a direct determinant of fertility. This is a simplification of a more complex phenomenon. The studies I mention highlight a positive reduced-form relationship between fertility and traditionalism (in its present definition). In reality, culture and fertility are observable behaviors that can be jointly determined by deeper variables such as the socioeconomic structure. For example, in hunters–gatherer societies, the origin of low fertility norms lies in the low productivity of their production technology, which can only support a small population.

15. Alesina and Giuliano (2007) find that strong family ties are associated with home production and higher fertility.

16. Hence, an agent of type i can have, at most, $1/\eta^i$ children. The cost of childbearing are different in the two cultures because of their specific modes of production. In compliance with the empirical evidence of the preceding sections, children are less costly in the rural agricultural production system than in the industrial sector. It follows that $\eta^M \geq \eta^T$.

17. It ensures that a parent giving birth to the maximal number of children can consume a positive amount of good.

18. Note that Ω_t^T and Ω_t^M are exogenous.

19. This simplification does not alter the results but make them more tractable. Indeed, the problem could be analyzed with rational (perfect) expectations. In this case, $\Delta V_t^i = \theta^i + E_t[\Omega_{t+1}^M - \Omega_{t+1}^M] = \theta^i + (1 + g_t^M)\Omega_t^M - (1 + g_t^T)\Omega_t^T$, with g_t^i the expected growth in sector i during period t.

20. As in Becker and Barro (1988), the parental utility function exhibits constant elasticity with regard to the quantity of children. Here, for tractability, I assume that this elasticity equals one-half. The linearity of utility with regard to consumption also consists in a simplification. It makes possible simple and tractable results that are in line with the usual results of endogenous fertility models [see Galor (2005a)] and cultural transmission models [see Bisin and Verdier (2001)].

21. The law of large numbers does apply. So the proportion of children with parents of type i who finaly become adults of type j is equal to P_t^{ij}.

22. Notice that, if $\Omega_t^T < \frac{2\eta^M}{2\eta^M - 1}\theta^M$, then $\widehat{\Omega}_t < \Omega_t^T - \theta^M \ \forall q_t$. This implies that the optimal socialization choice is $\tau_t^M = (\frac{\eta^M \Omega_t^M}{\gamma} \cdot \frac{(1-q_t)\Delta V_t^M}{q_t \Delta V_t^M + V_t^{MT}})^2$ if $\Omega_t^M > \Omega_t^T - \theta^M$ and 0 otherwise. Furthermore, if $\Omega_t^T < \theta^M$, ΔV_t^M can never be negative: thus $\tau_t^M = 0$ in (8) never happens.

23. Indeed, the expected utility of a child for a parent of type M equals $P_t^{MM} V_t^{MM} + P_t^{MT} V_t^{MT}$. When Ω_t^M increases, the utility of the child if he becomes modern, (V_t^{MM}), will be higher. As I previously mentioned, $\Delta V_t^M = \theta^M + \Omega_t^M - \Omega_t^T$ will also be higher.

24. Formally, $\frac{\partial N_t^m}{\partial \Omega_t^M} = -\frac{q_t \theta^M + (1-q_t)\Omega^T}{(\Omega_t^M)^2} < 0$.

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25. Results are displayed as a function of modernist income in order to simplify future reasoning. A more usual presentation would have consisted in presenting the results as a function of the traditionalists' income. These results would have been symmetric to the modernists' ones.

26. Notice that $\widetilde{\Omega}_t < \Omega_t^T + \theta^T \ \forall q_t \in [0, 1]$ if $\eta^T < \frac{1}{4}$, which is assumed for the rest of the paper. This assumption fits the facts [see, for instance, De la Croix and Doepke (2003)].

27. A proof is provided in Appendix C.

28. As shown in Appendix B, whatever the values of $\widehat{\Omega}_t$ and $\widetilde{\Omega}_t$, the equation ensuring that $q_{t+1} - q_t = 0$ is cubic in Ω_t^M. So the variation of $\overline{q}$ can at most also be cubic. A last case has not been represented; it simply consists in the case where $\overline{q}$ is always increasing in Ω_t^M and $\Omega^T > \theta^M$.

29. Indeed, when $\Omega_t^M < \Omega_t^T - \theta^M$, $q_t = 0$ is a stable steady state, whereas it becomes unstable when $\Omega_t^M > \Omega_t^T - \theta^M$. In the same way, when $\Omega_t^M < \Omega_t^T + \theta^T$, $q_t = 1$ is an unstable steady state, whereas it becomes stable when $\Omega_t^M > \Omega_t^T + \theta^T$.

30. The model being formulated in discrete time, the evolution of the total fertility rate has been artificially smoothed.

31. No dynastic analysis would be possible, because each individual would be characterized by a specific situation depending on its familial cultural and economic history and on his own cultural choice. Cultural and economic heterogeneity would make analytical analysis intractable. Thus, a rigorous numerical methodology would be essential to understand the model's main implications.

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APPENDIX A

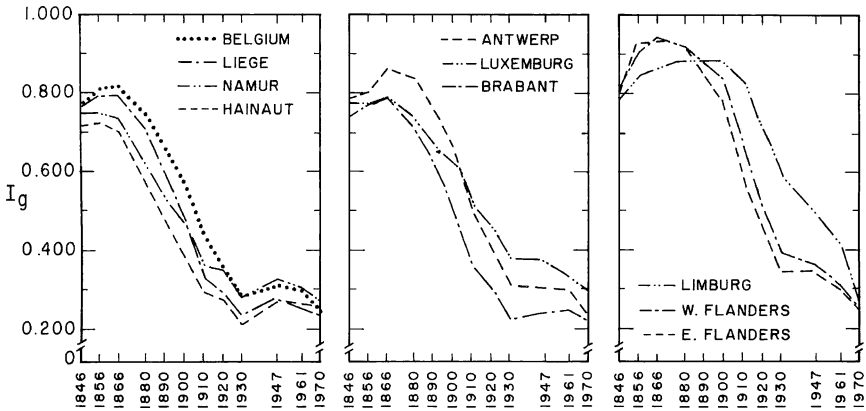

FIGURE A.1. Marital fertility by province in Belgium from 1846 to 1970 from Lesthaeghe (1977). *Source*: Office of Population Research, Princeton University.

APPENDIX B

To prove Proposition 1, I propose four lemmas. Lemma A.1 aims at proving that noninterior steady states are unstable when there exist interior steady states. Lemma A.2 shows that only one noninterior steady state is stable when there is no interior one. Lemmas A.3 and A.4 show that there exists, at most, one interior steady state. These four lemmas, combined with properties of the model, will make it possible to prove Proposition 1. As shown in Section 3, τ_t^M,τ_t^T, N_t^M, and N_t^T are all functions of q_t. They are now respectively denoted by $\tau_t^M(q_t)$,$\tau_t^T(q_t)$, $N_t^M(q_t)$, and $N_t^T(q_t)$.

LEMMA A.1. *If* $\tau_t^M(1) = 0$, $\tau_t^T(1) > 0$, $N_t^M(1) \geq 0$, $N_t^T(1) > 0$ *and* $\tau_t^M(0) > 0$, $\tau_t^T(0) = 0$, $N_t^M(0) > 0$, $N_t^T(0) \geq 0$, *then* $q_t = \{0, 1\}$ *are both unstable steady states of the cultural dynamics at the competitive equilibrium and there exists, at least one interior and stable cultural steady state if* $q_{t+1} - q_t$ *is continuous in* q_t.

Proof. It follows from (3) and (5) that

$$\frac{\partial [q_{t+1} - q_t]}{\partial q_t} = \frac{\left[q_t N_t^M + (1-q_t) N_t^T\right]\left[(1-2q_t) A_t^{MT} + q_t (1-q_t) \frac{\partial A_t^{MT}}{\partial q_t}\right] - q_t (1-q_t) A_t^{MT}\left[N_t^T + N_t^M + (1-q_t) \frac{\partial N_t^T}{\partial q_t} + q_t \frac{\partial N_t^M}{\partial q_t}\right]}{\left[q_t N_t^M + (1-q_t) N_t^T\right]^2}, \quad \textbf{(A.1)}$$

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Where $A_t^{MT} = (\tau_t^M)^{\frac{1}{2}} N_t^M - (\tau_t^T)^{\frac{1}{2}} N_t^T$. A solution to (5) will be a stable steady state if and only if, at this point, $\frac{\partial[q_{t+1}-q_t]}{\partial q_t} \leq 0$. It follows from (A.1) that $\frac{\partial[q_{t+1}-q_t]}{\partial q_t}|_{q_t=0} = \frac{A_t^{MT}|_{q_t=0}}{N_t^T(0)}$ and $\frac{\partial[q_{t+1}-q_t]}{\partial q_t}|_{q_t=1} = -\frac{A_t^{MT}|_{q_t=1}}{N_t^M(1)}$. If $\tau_t^M(1) = 0$, $\tau_t^T(1) > 0$, $N_t^M(1) \geq 0$, $N_t^T(1) > 0$ and $\tau_t^M(0) > 0$, $\tau_t^T(0) = 0$, $N_t^M(0) > 0$, $N_t^T(0) \geq 0$, then $\frac{\partial[q_{t+1}-q_t]}{\partial q_t}|_{q_t=0} > 0$, and $\frac{\partial[q_{t+1}-q_t]}{\partial q_t}|_{q_t=1} > 0$. This finally implies that $q_t = \{0, 1\}$ are unstable steady states. So, if $q_{t+1} - q_t$ is continuous in q_t, there exists, at least one interior stable steady. ■

LEMMA A.2. *If $\Delta V_t^M \leq 0$, then $q_t = \{0, 1\}$ are the only steady states of (A.1). Furthermore, $q_t = 0$ is globally stable and $q_t = 1$ is unstable. If $\Delta V_t^T \leq 0$, then $q_t = \{0, 1\}$ are the only steady states of (A.1). Furthermore, $q_t = 0$ is unstable and $q_t = 1$ is globally stable.*

Proof. From (8), if $\Delta V_t^M \leq 0$, $\tau_t^M = 0\ \forall q_t \in [0, 1]$. It follows that $\forall q_t \in [0, 1]$;

$$q_{t+1} - q_t = -\frac{q_t (1 - q_t) \left(\tau_t^T\right)^{\frac{1}{2}} N_t^T}{q_t N_t^M + (1 - q_t) N_t^T} < 0. \tag{A.2}$$

By (7), (9), and (10), it is obvious that (A.2) is continuous in q_t. (A.2) implies that there does not exist any interior steady state, $\frac{\partial[q_{t+1}-q_t]}{\partial q_t}|_{q_t=0} < 0$, and $\frac{\partial[q_{t+1}-q_t]}{\partial q_t}|_{q_t=1} > 0$. It follows that $q_t = 0$ is unstable and $q_t = 1$ is globally stable.

With the same method, from (10), $\forall q_t \in [0, 1]$, if $\Delta V_t^T \leq 0$, $\tau_t^T = 0$ and

$$q_{t+1} - q_t = \frac{q_t (1 - q_t) \left(\tau_t^M\right)^{\frac{1}{2}} N_t^M}{q_t N_t^M + (1 - q_t) N_t^T} > 0. \tag{A.3}$$

By (7)–(9), it is obvious that (A.3) is continuous in q_t. (A.3) implies that there does not exist any interior steady state, $\frac{\partial[q_{t+1}-q_t]}{\partial q_t}|_{q_t=0} > 0$, and $\frac{\partial[q_{t+1}-q_t]}{\partial q_t}|_{q_t=1} < 0$. It follows that $q_t = 0$ is unstable and $q_t = 1$ is globally stable. ■

LEMMA A.3. *If A_t^{MT}, N_t^M, N_t^T are continuous in q_t and $\forall q_t \in [0, 1]$, $(N_t^M, N_t^T) > (0, 0)$, then $q_{t+1} - q_t$ is continuous in q_t at the equilibrium.*

Proof. This is straightforward because, by (3), (5) can be written as follows:

$$q_{t+1} - q_t = \frac{q_t (1 - q_t) \left[\left(\tau_t^M\right)^{\frac{1}{2}} N_t^M - \left(\tau_t^T\right)^{\frac{1}{2}} N_t^T\right]}{q_t N_t^M + (1 - q_t) N_t^T}. \tag{A.4}$$

■

LEMMA A.4. *If (a) A_t^{MT} is quadratic in q_t, (b) $\tau_t^M(1) = 0$, $\tau_t^T(1) > 0$, $N_t^M(1) \geq 0$, $N_t^T(1) > 0$ and $\tau_t^M(0) > 0$, $\tau_t^T(0) = 0$, $N_t^M(0) > 0$, $N_t^T(0) \geq 0$, (c) A_t^{MT}, N_t^M, N_t^T are continuous in q_t and $\forall q_t \in [0, 1]$, $(N_t^M, N_t^T) > (0, 0)$, and (d) $(\Delta V_t^M, \Delta V_t^T) > (0, 0)$, there exists only one interior steady state $\overline{q} \in]0, 1[$ that is globally stable.*

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Proof. By Lemmas A.1–A.3, it is obvious that there exists an odd-numbered set of steady states between $q_t = 0$ and $q_t = 1$ when $(\Delta V_t^M, \Delta V_t^T) > (0, 0)$. From (A.4), $\forall q_t \in]0, 1[$, $q_{t+1} - q_t = 0$ if and only if $A_t^{MT} = 0$. If A_t^{MT} is quadratic in q_t, $A_t^{MT} = 0$ has at most two real solutions. Then, there exists only one interior steady state $\overline{q} \in]0, 1[$. By Lemmas A.1 and A.3, it is straightforward that $\frac{\partial[q_{t+1} - q_t]}{\partial q_t}|_{q_t=\overline{q}} < 0$. Thus $\overline{q}$ is globally stable. ■

From (7)–(10), it appears that, whatever the values of $\widehat{\Omega}$ and $\widetilde{\Omega}$, when $\Omega_t^M \in]\Omega^T - \theta^M, \Omega^T + \theta^T[$, A_t^{MT} is a quadratic of q_t. From Lemmas A.1, A.3 and A.4, there exists a unique interior cultural steady state $\overline{q}$ that is globally stable. From Lemma A.2, when $\Omega_t^M \leq \Omega_t^T - \theta^M$, $q_t = \{0, 1\}$ are the only existing steady states, and $q_t = 0$ is globally stable while $q_t = 1$ is unstable. Also from Lemma A.2, when $\Omega_t^M \geq \Omega_t^T + \theta^T$, $q_t = \{0, 1\}$ are also the only existing steady states; however, $q_t = 0$ is unstable while $q_t = 1$ is globally stable.

APPENDIX C

From Appendix B, $\overline{q}$ is the unique interior solution of $A_t^{MT} = 0$. Because A_t^{MT} depends on both q_t and Ω_t^M, it directly follows that

$$\left.\frac{dq_t}{d\Omega_t^M}\right|_{q_t=\overline{q}} = -\left.\frac{\frac{\partial A_t^{MT}}{\partial \Omega_t^M}}{\frac{\partial A_t^{MT}}{\partial q_t}}\right|_{q_t=\overline{q}}. \tag{A.5}$$

From Proposition 1, $\frac{\partial A_t^{MT}}{\partial q_t}|_{q_t=\overline{q}} < 0$ when $\Omega_t^M \in]\Omega^T - \theta^M, \Omega^T + \theta^T[$. Thus, $\frac{dq_t}{\partial \Omega_t^M}|_{q_t=\overline{q}} > 0 \Leftrightarrow \frac{\partial A_t^{MT}}{\partial \Omega_t^M}|_{q_t=\overline{q}} > 0$. Differentiating A_t^{MT} with respect to Ω_t^M leads to the following condition:

$$\left[\frac{1}{2}\left(\tau_t^M\right)^{-\frac{1}{2}} N_t^M \frac{\partial \tau_t^M}{\partial \Omega_t^M} - \frac{1}{2}\left(\tau_t^T\right)^{-\frac{1}{2}} N_t^T \frac{\partial \tau_t^T}{\partial \Omega_t^M}\right] + \left[\left(\tau_t^M\right)^{\frac{1}{2}} \frac{\partial N_t^M}{\partial \Omega_t^M} - \left(\tau_t^T\right)^{\frac{1}{2}} \frac{\partial N_t^T}{\partial \Omega_t^M}\right] > 0. \tag{A.6}$$

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