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Design of an adaptive control law using trigonometric functions for robot manipulators

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SUMMARY
In this study, a new approach of adaptive control law for controlling robot manipulators using the Lyapunov based theory is derived, thus the stability of an uncertain system is guaranteed. The control law includes a PD feed forward part and a full dynamics feed forward compensation part with the unknown manipulator and payload parameters. The novelty of the obtained result is that an adaptive control algorithm is developed using trigonometric functions depending on manipulator kinematics, inertia parameters and tracking error, and both system parameters and adaptation gain matrix are updated in time.

KEYWORDS: Adaptive control; Robot manipulators; Parameter estimation; System stability; Parametric uncertainty.

1. INTRODUCTION

Adaptive control methods offer an attractive solution to the robot control problem, when neither an exact model of a manipulator nor accurate values of dynamic parameters may exist. Some of the control laws introduced by Craig et al.,[1] Middleton and Goodwin,[2] and Spong and Ortega[3] require acceleration measurements and/or computation of the inverse of the moments inertia matrix containing estimated parameters. Slotine and Li[4] derived an adaptive control algorithm without using the joint accelerations and the inverse of inertia matrix. It consists of a PD feedback part and a full dynamics feed forward compensation part with the unknown manipulator and payload parameters. In another study of Slotine and Li,[5] it is shown that position and velocity errors converge to zero but the Lyapunov stability was not established. In the reference study,[6] the adaptive robot controller[5] was proved in the Lyapunov sense, but in the proof the feedback matrix was assumed to be uniformly positive definite. Egeland and Godhavn[7] assumed that the feedback gain matrix is to be uniformly positive defined, possible time varying and proved stability in the sense of the Lyapunov. Burdet and Codourey[8] compared nine different adaptive control algorithms and, as a result, it is shown that the adaptive feed forward controllers are convenient for learning the parameters of the dynamic equation in the presence of friction and noise. In references [8,10], other comparative studies of adaptive control laws are given.

In this paper, a new adaptive control law is derived for n-link robot manipulators based on the Lyapunov-based theory. A parameter adaptation law is derived considering the Slotine and Li[4] and Sciavicco and Siciliano[11] approaches. Apart from similar studies, system parameters are estimated using trigonometric functions depending on manipulator kinematics, inertia parameters and tracking errors, and the adaptive gain matrix is also updated in time.

2. DERIVATION OF THE ADAPTATIVE CONTROL LAW

In the absence of friction or other disturbances, the dynamic model of an n-link manipulator can be written as[12]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where q denotes the n-dimensional vector of generalised coordinates, τ is the n-dimensional vector of applied torques (or forces), M(q) is the $n \times n$ symmetric positive definite inertia matrix, $C(q,\dot{q})\dot{q}$ is the n-dimensional vector of centripetal and Coriolis terms and G(q) is the n-dimensional vector of gravitational terms. Equation (1) can also be expressed in the following form.

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\pi \tag{2}$$

where π is a constant p-dimensional vector of inertia parameters and Y is an $n \times p$ matrix of known function of joint position, velocity and acceleration. Consider the control law

$$\tau = M(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + G(q) + K\sigma \tag{3}$$

where K is a positive definite matrix. The other quantities are given by

$$\tilde{q} = q_d - q \quad \dot{q}_r = \dot{q}_d + \Lambda\tilde{q} \quad \ddot{q}_r = \ddot{q}_d + \Lambda\dot{\tilde{q}} \tag{4}$$

where $\tilde{q}$ is the error between the desired and the actual position, Λ is a positive definite matrix that describes the nonlinear compensation and decoupling terms as a function of the desired velocity and acceleration, corrected by the current state (q and $\dot{q}$) of the manipulator. The term $K\sigma$ shows PD action on the error. σ is taken as

$$\sigma = \dot{q}_r - \dot{q} = \dot{\tilde{q}} + \Lambda\tilde{q} \tag{5}$$

Suppose that the computational model has the same structure as that of the manipulator dynamic model, but its parameters are not known exactly. The control law (3) is then modified into

$$\begin{aligned}\tau &= \hat{M}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G} + K\sigma \\ &= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\pi} + K\sigma\end{aligned} \tag{6}$$

where $\hat{\pi}$ represents the available estimate on the parameters and, accordingly, $\hat{M}$, $\hat{C}$, $\hat{G}$ denote the estimated terms in the dynamic model. Substituting (6) into (2) gives

$$\begin{aligned}M(q)\dot{\sigma} + C(q,\dot{q})\sigma + K\sigma &= -\tilde{M}(q)\ddot{q}_r - \tilde{C}(q,\dot{q})\dot{q}_r - \tilde{G} \\ &= -Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\pi}\end{aligned} \tag{7}$$

where the parameter error vector is

$$\tilde{\pi} = \hat{\pi} - \pi \tag{8}$$

Error quantities concerning the system parameters are characterised by

$$\tilde{M} = \hat{M} - M, \quad \tilde{C} = \hat{C} - C, \quad \tilde{G} = \hat{G} - G \tag{9}$$

In order to derive a new adaptive control law, the following Lyapunov function candidate is defined as

$$V(\sigma,\tilde{q},\tilde{\pi}) = \frac{1}{2}\sigma^T M(q)\sigma + \frac{1}{2}\tilde{q}^T B\tilde{q} + \frac{1}{2}\tilde{\pi}^T\Omega(t)^2\tilde{\pi} \tag{10}$$

where B is a positive definite matrix. As the novelty of this study, $\Omega(t)$ is chosen as a $p \times p$ dimensional diagonal matrix and changes in time. The time derivative of Equation (10) is written as

$$\begin{aligned}\dot{V}(\sigma,\tilde{q},\tilde{\pi}) &= \sigma^T M(q)\dot{\sigma} + \frac{1}{2}\sigma^T\dot{M}(q)\sigma + \tilde{q}^T B\dot{\tilde{q}} \\ &\quad + \tilde{\pi}^T\dot{\Omega}(t)\Omega(t)\tilde{\pi} + \tilde{\pi}^T\Omega(t)^2\dot{\tilde{\pi}}\end{aligned} \tag{11}$$

Taking $B = 2\Lambda K$, using the property $\sigma^T[\dot{M}(q) - 2C(q,\dot{q})]\sigma = 0\ \forall\sigma \in R^n$, the time derivative of $V(\sigma,\tilde{q},\tilde{\pi})$ along the trajectory of system (7) is

$$\begin{aligned}\dot{V}(\sigma,\tilde{q},\tilde{\pi}) &= -\dot{\tilde{q}}^T K\dot{\tilde{q}} - \tilde{q}^T\Lambda K\Lambda\tilde{q} + \tilde{\pi}^T(\Omega(t)\Omega(t)\dot{\tilde{\pi}} \\ &\quad + \dot{\Omega}(t)\Omega(t)\tilde{\pi} - Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)\sigma)\end{aligned} \tag{12}$$

Since $K > 0$, and $\Lambda > 0$ the first terms of Equation (12) are less or equal to zero that is:

$$\dot{V}(\sigma,\tilde{q},\tilde{\pi}) = -\dot{\tilde{q}}^T K\dot{\tilde{q}} - \tilde{q}^T\Lambda K\Lambda\tilde{q} \le 0 \tag{13}$$

Hence, we look for the conditions for which the equation $\Omega(t)\Omega(t)\dot{\tilde{\pi}} + \dot{\Omega}(t)\Omega(t)\tilde{\pi} - Y^T\sigma = 0$ is satisfied. The remaining terms in Equation (13) are

$$\Omega(t)\Omega(t)\dot{\tilde{\pi}} + \dot{\Omega}(t)\Omega(t)\tilde{\pi} - Y^T\sigma = 0 \tag{14}$$

Equation (14) is arranged as

$$\Omega(t)\dot{\tilde{\pi}} + \dot{\Omega}(t)\tilde{\pi} = \frac{Y^T\sigma}{\Omega(t)} \tag{15}$$

Then

$$\Omega(t)\dot{\hat{\pi}} + \dot{\Omega}(t)\hat{\pi} = \frac{Y^T\sigma}{\Omega(t)} + \dot{\Omega}(t)\pi \tag{16}$$

since $\dot{\tilde{\pi}} = \dot{\hat{\pi}}$ (π is a constant). Equation (16) is arranged as

$$\frac{d}{dt}(\Omega(t)\hat{\pi}) = \frac{Y^T\sigma}{\Omega(t)} + \dot{\Omega}(t)\pi \tag{17}$$

Integration of both side of Equation (17) yields

$$\Omega(t)\hat{\pi} = \int\frac{Y^T\sigma}{\Omega(t)}dt + \int\dot{\Omega}(t)\pi\,dt + C \tag{18}$$

Then

$$\Omega(t)\hat{\pi} = \int\frac{Y^T\sigma}{\Omega(t)}dt + \Omega(t)\pi + C \tag{19}$$

In order to solve the above equation, $\Omega(t)$ is chosen as a time varying function such that

$$1/\Omega(t) = \left(\gamma\cos\left(\int Y^T\sigma\,dt\right) + \beta\right)I \tag{20}$$

where γ and β are positive real numbers, I is a $p \times p$ dimensional identity matrix. Substitution Equation (20) into Equation (19) yields

$$\begin{aligned}&\frac{\hat{\pi}}{\gamma\cos\left(\int Y^T\sigma\,dt\right) + \beta} \\ &\quad = \int(Y^T\sigma)\left(\gamma\cos\left(\int Y^T\sigma\,dt\right) + \beta\right)dt \\ &\quad + \frac{\pi}{\gamma\cos\left(\int Y^T\sigma\,dt\right) + \beta} + C\end{aligned} \tag{21}$$

After integration, the result is

$$\begin{aligned}\frac{\hat{\pi}}{\gamma\cos\left(\int Y^T\sigma\,dt\right) + \beta} &= \gamma\sin\left(\int Y^T\sigma\,dt\right) + \beta\int Y^T\sigma\,dt \\ &\quad + \frac{\pi}{\gamma\cos\left(\int Y^T\sigma\,dt\right) + \beta} + C\end{aligned} \tag{22}$$

If the condition of $\hat{\pi}(0) = \pi$ is taken as an initial condition, the constant C is equivalent to zero. Hence, the parameter adaptation law is derived as

$$\begin{aligned}\hat{\pi} &= \left(\gamma\cos\left(\int Y^T\sigma\,dt\right) + \beta\right)\left[\gamma\sin\left(\int Y^T\sigma\,dt\right)\right. \\ &\quad \left. + \beta\int Y^T\sigma\,dt\right] + \pi\end{aligned} \tag{23}$$

The resulting block diagram of adaptive control is illustrated in Fig. 1.

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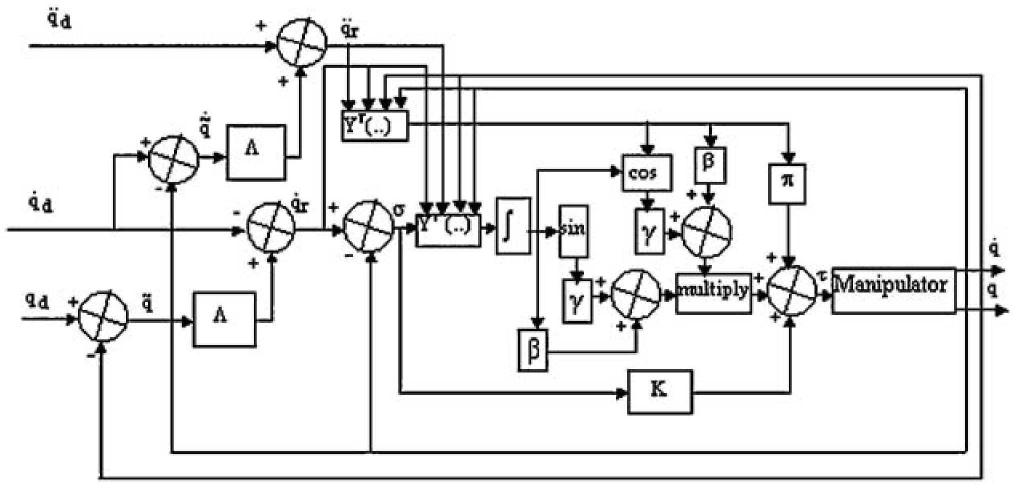

Fig. 1. Block diagram of the proposed adaptive control (23).

3. EXTENSIONS

In order to derive the other parameter estimation law, Ω(t) is chosen as

$$1/\Omega(\mathrm{t}) = \left(\cos\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)\right)\mathrm{I} \tag{24}$$

where α is a positive real number. Substituting Equation (24) into Equation (19) yields

$$\frac{\hat{\pi}}{\cos\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)} = \int\left[(\mathrm{Y}^{\mathrm{T}}\sigma)\cos\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)\right]\mathrm{dt} + \frac{\pi}{\cos\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)} + \mathrm{C} \tag{25}$$

After integration, the result is

$$\frac{\hat{\pi}}{\cos\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)} = (1/\alpha)\sin\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) + \frac{\pi}{\cos\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)} + \mathrm{C} \tag{26}$$

Then

$$\hat{\pi} = \cos\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)(1/\alpha)\sin\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) + \pi + \mathrm{C}\cos\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) \tag{27}$$

If the condition of $\hat{\pi}(0) = \pi$ is taken as an initial condition, the constant C is equivalent to zero. Hence, the parameter adaptation law is derived as

$$\hat{\pi} = \cos\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)\left[(1/\alpha)\sin\left(\alpha\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)\right] + \pi \tag{28}$$

The resulting block diagram is shown in Fig. 2.

For the third derivation, Ω(t) is chosen as

$$1/\Omega(t) = \cos\left(\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) + \delta\cos\left(\kappa\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) \tag{29}$$

Substituting Equation (29) into Equation (19) yields

$$\begin{aligned}\frac{\hat{\pi}}{\cos\left(\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) + \delta\cos\left(\kappa\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)} &= \int\left[(\mathrm{Y}^{\mathrm{T}}\sigma)\cos\left(\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)\right.\\ &\quad \left. + (\mathrm{Y}^{\mathrm{T}}\sigma)\delta\cos\left(\kappa\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)\right]\mathrm{dt}\\ &\quad + \frac{\pi}{\cos\left(\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) + \delta\cos\left(\kappa\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)} + \mathrm{C}\end{aligned} \tag{30}$$

The result after integration is

$$\begin{aligned}\frac{\hat{\pi}}{\cos\left(\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) + \delta\cos\left(\kappa\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)} &= \sin\left(\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) + (\delta/\kappa)\sin\left(\kappa\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)\\ &\quad + \frac{\pi}{\cos\left(\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) + \delta\cos\mathrm{s}\left(\kappa\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)} + \mathrm{C}\end{aligned} \tag{31}$$

If the condition of $\hat{\pi}(0) = \pi$ is taken as an initial condition, the constant C is equivalent to zero. Hence, the parameter adaptation law is derived as

$$\begin{aligned}\hat{\pi} &= \left[\cos\left(\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) + \delta\cos\left(\kappa\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)\right]\\ &\quad \times\left[\sin\left(\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right) + (\delta/\kappa)\sin\left(\kappa\int \mathrm{Y}^{\mathrm{T}}\sigma\,\mathrm{dt}\right)\right] + \pi\end{aligned} \tag{32}$$

The resulting block diagram is shown in Fig. 3.

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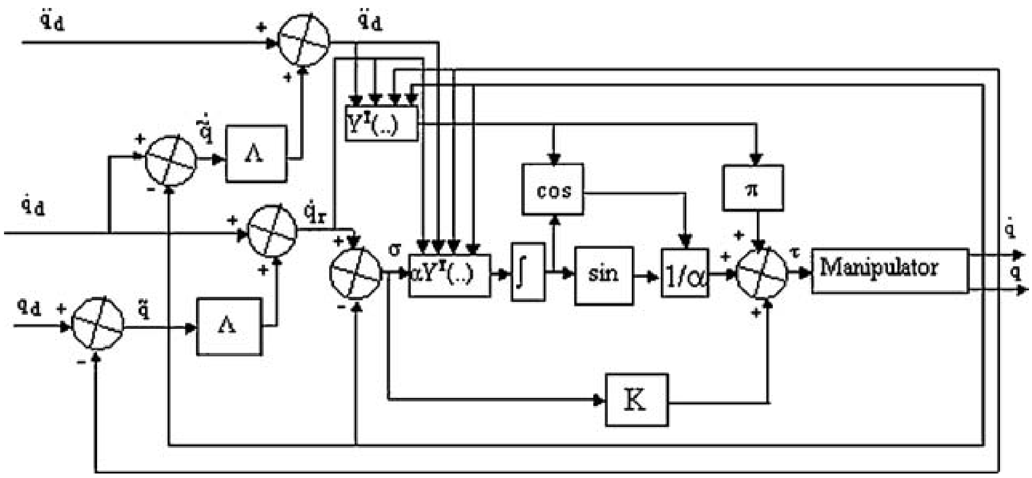

Fig. 2. Block diagram of the proposed adaptive control law (28).

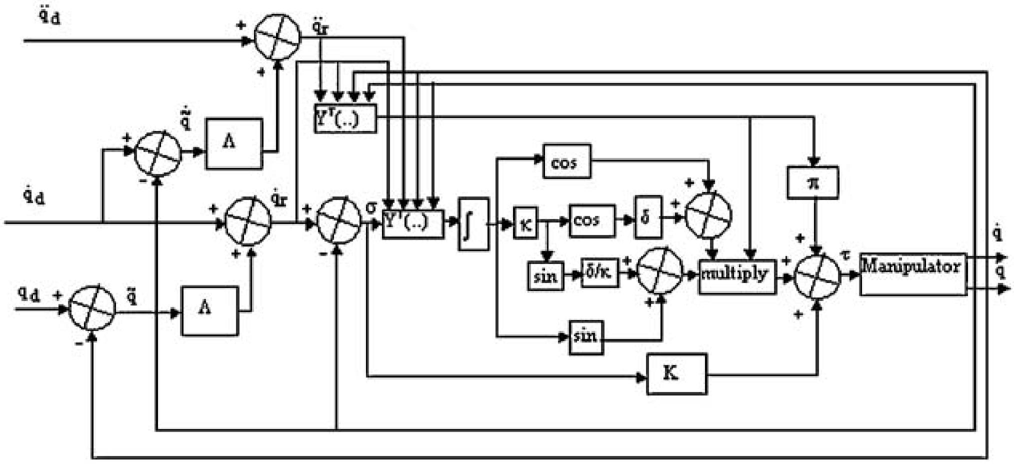

Fig. 3. Block diagram of the proposed adaptive control law (32).

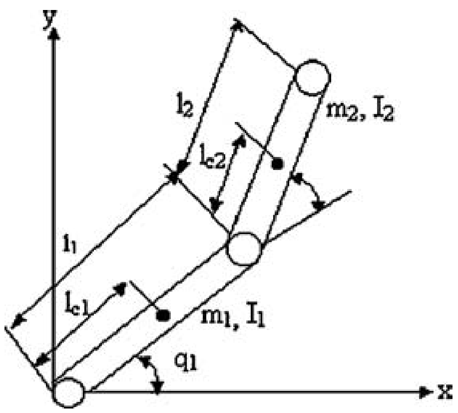

Fig. 4. Two-link planar robot.[13]

4. SIMULATION EXAMPLE

The control algorithms have been applied to a two-link robot manipulator, as shown in Fig. 4, to illustrate the proposed controllers performance. Computer simulations have been carried out with the same Spong's model[13] and one parameterization of this robot is given by

$$\pi_1 = m_1 l_{c1}^2 + m_2 l_1^2 + I_1, \quad \pi_2 = m_2 l_{c2}^2 + I_2, \quad \pi_3 = m_2 l_1 l_{c2},$$
$$\pi_4 = m_1 l_{c1}, \quad \pi_5 = m_2 l_1, \quad \pi_6 = m_2 l_{c2}, \qquad (33)$$

With this parameterisation, the dynamic model in Equation 1 can be written as

$$Y(q, \dot{q}, \ddot{q})\pi = \tau \qquad (34)$$

The component y_{ij} of $Y(q, \dot{q}, \ddot{q})$ are given as

$$y_{11} = \ddot{q}_1; \quad y_{12} = \ddot{q}_1 + \ddot{q}_2;$$
$$y_{13} = \cos(q_2)(2\ddot{q}_1 + \ddot{q}_2) - \sin(q_2)\left(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2\right);$$
$$y_{14} = g_c \cos(q_1); \quad y_{15} = g_c \cos(q_1); \quad y_{16} = g_c \cos(q_1 + q_2);$$
$$y_{21} = 0; \quad y_{22} = \ddot{q}_1 + \ddot{q}_2; \quad y_{23} = \cos(q_2)\ddot{q}_1 + \sin(q_2)\left(\dot{q}_1^2\right);$$
$$y_{24} = 0; \quad y_{25} = 0; \quad y_{26} = g_c \cos(q_1 + q_2). \qquad (35)$$

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Table I. Parameters of the unloaded arm[13].

m_1	m_2	l_1	l_2	l_{c1}	l_{c2}	I_1	I_2
10	5	1	1	0.5	0.5	10/12	5/12

Table II. π_i for the unloaded arm[13].

π_1	π_2	π_3	π_4	π_5	π_6
8.33	1.67	2.5	5	5	2.5

Table III. π_{0i} for the loaded arm[13].

π_{01}	π_{02}	π_{03}	π_{04}	π_{05}	π_{06}
13.33	8.96	8.75	5	10	8.75

$Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$in Equation (6) has the component

$$y_{11} = \ddot{q}_{r1}; \quad y_{12} = \ddot{q}_{r1} + \ddot{q}_{r2};$$
$$y_{13} = \cos(q_2)(2\ddot{q}_{r1} + \ddot{q}_{r2}) - \sin(q_2)(\dot{q}_1\dot{q}_{r2} + \dot{q}_1\dot{q}_{r2} + \dot{q}_2\dot{q}_{r2});$$
$$y_{14} = g_c\cos(q_1); \quad y_{15} = g_c\cos(q_1); \quad y_{16} = g_c\cos(q_1 + q_2)$$
$$y_{21} = 0; \quad y_{22} = \ddot{q}_{r1} + \ddot{q}_{r2}; \quad y_{23} = \cos(q_2)\ddot{q}_{r1} + \sin(q_2)(\dot{q}_1\dot{q}_{r1});$$
$$y_{24} = 0; \quad y_{25} = 0; \quad y_{26} = g_c\cos(q_1 + q_2). \tag{36}$$

For illustrative purposes let us assume that the parameters of the unloaded manipulator are known and Table I gives the relevant values. Using the values in Table I, Equation (33) gives the *ith* component of π as shown in Table II; Table II also shows the lower bounds of parameters.

If an unknown load carried by the robot is regarded as part of the second link then the parameters m_2, l_{c2}, and I_2 will change to $m_2 + \Delta m_2$, $l_{c2} + \Delta l_{c2}$ and $I_2 + \Delta I_2$ respectively. A controller will be designed so as to provide robustness in the following intervals.

$$0 \le \Delta m_2 \le 10; \quad 0 \le \Delta l_{c2} \le 0.5; \quad 0 \le I_2 \le \frac{15}{12} \tag{37}$$

With this choice of the range given by (37), π_0 is a vector chosen as a loaded arm parameters and their upper bound. The computed values for the *ith* component of π_0 are given in Table III.[13]

5. CONCLUSION AND DISCUSSION

In this paper, a new parameter estimation law for controlling robot manipulators based on the Lyapunov stability theory has been derived. In this scheme, the main structure of this control law is similar to (in fact, based on) Sciavicco and Siciliano,[11] and Slotine and Li's algorithm.[4] For illustrative purpose, the adaptive algorithm of Sciavicco and Siciliano[11] is given by

$$\dot{\hat{\pi}} = K_\pi^{-1} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\sigma \tag{38}$$

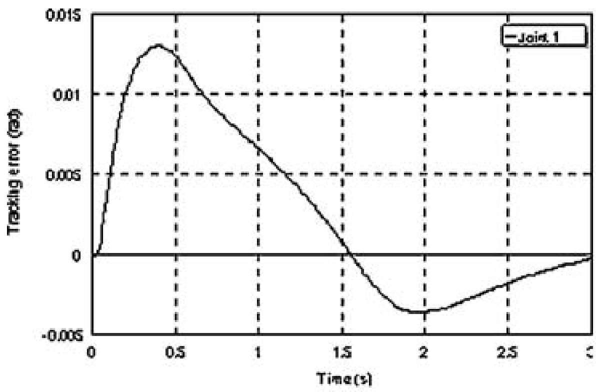
where the Equation (38) gives the parameter update law, and K_π is a $p \times p$ dimensional positive definite gain matrix. Then, the parameter estimation law (38) can be modified into

$$\hat{\pi} = \int K_\pi^{-1} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\sigma\, dt + \pi \tag{39}$$

where $\hat{\pi}(0) = \pi$.

In the reference study,[11] the adaptation gain matrix K_π is fixed and the parameters estimated depend on the values of $\int Y^T\sigma dt$. However, in the new scheme, the functions $\gamma\sin(\int Y^T\sigma\, dt) + \beta\int Y^T\sigma\, dt$, $(1/\alpha)\sin(\alpha\int Y^T\sigma\, dt)$ and $\sin(\int Y^T\sigma\, dt) + (\delta/\kappa)\sin(\kappa\int Y^T\sigma\, dt)$ are parameter estimation laws used instead of $\int Y^T\sigma dt$, and the functions $\gamma\cos(\int Y^T\sigma\, dt) + \beta$, $\cos(\alpha\int Y^T\sigma\, dt)$ and $\cos(\int Y^T\sigma\, dt) + \delta\cos(\kappa\int Y^T\sigma\, dt)$ can be considered adaptation gain matrices used instead of K_π^{-1}. Both system parameters and adaptation gain matrices are updated in time and converge to their asymptotic values during control process.

For computer simulation, a fifth order polynomial is considered as a reference trajectory for both joints. The joint angles change from 0 to 2.5 rad in 3 seconds and the sampling time is taken to be 0.01 s. For a comparison of the proposed control laws with the known controller (39), using the same parameters such as K and Λ, the developed control algorithms are applied to the model system for the same trajectory in order to analyse the performance of each control law. Consequently, the matrices K and Λ are chosen as $K = \mathrm{diag}(80\ 80)$, $\Lambda = \mathrm{diag}(80\ 80)$. The relevant results are plotted in Fig. 5 for the controller (39), and are given in Figs 6–8 for the proposed controllers, respectively.



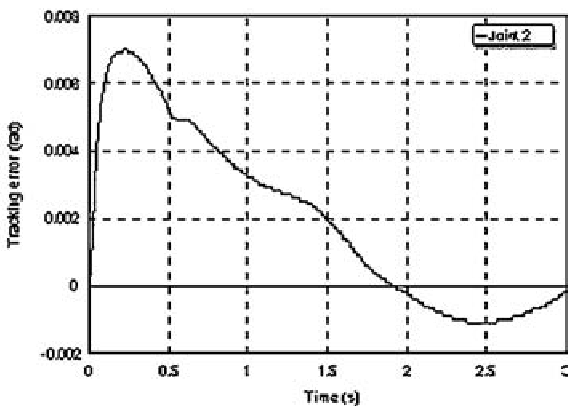


Fig. 5. Response using the adaptive control law[11] (39) when $K_\pi = \mathrm{diag}([1\ 1\ 1\ 1\ 1\ 1])$, $\Lambda = \mathrm{diag}([80\ 80])$ and $K = \mathrm{diag}([80\ 80])$.

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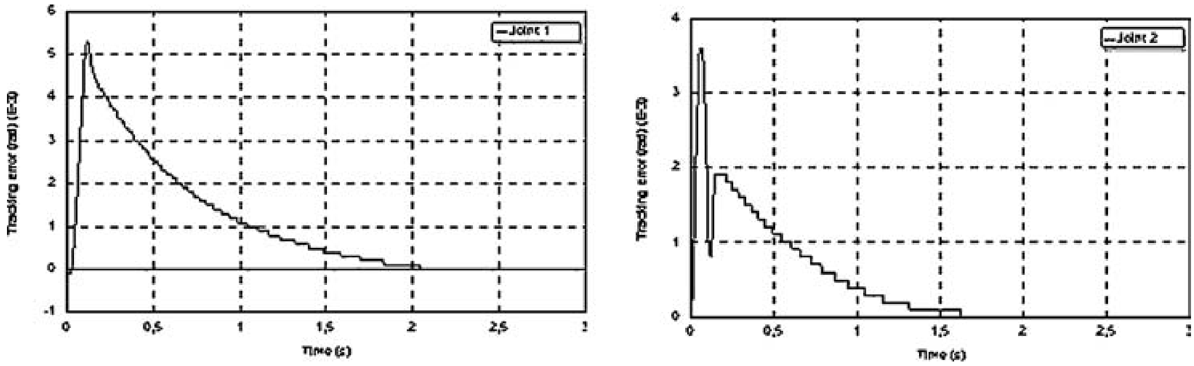

Fig. 6. Response of the adaptive control law (23) when $\Lambda = \mathrm{diag}([80\ 80])$, $\mathrm{K} = \mathrm{diag}([80\ 80])$, $\gamma = 1$ and $\beta = 1$.

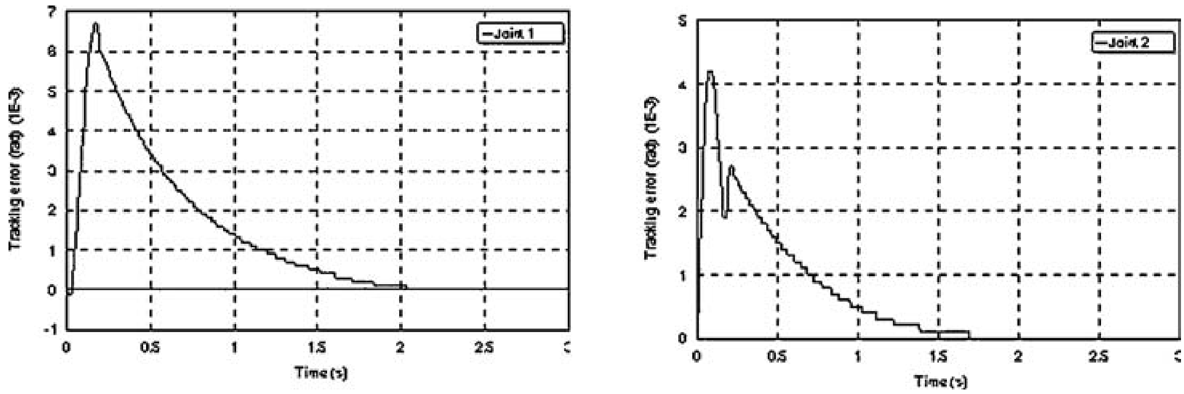

Fig. 7. Response of the adaptive control law (28) when $\Lambda = \mathrm{diag}([80\ 80])$, $\mathrm{K} = \mathrm{diag}([80\ 80])$ and $\alpha = 0.2$.

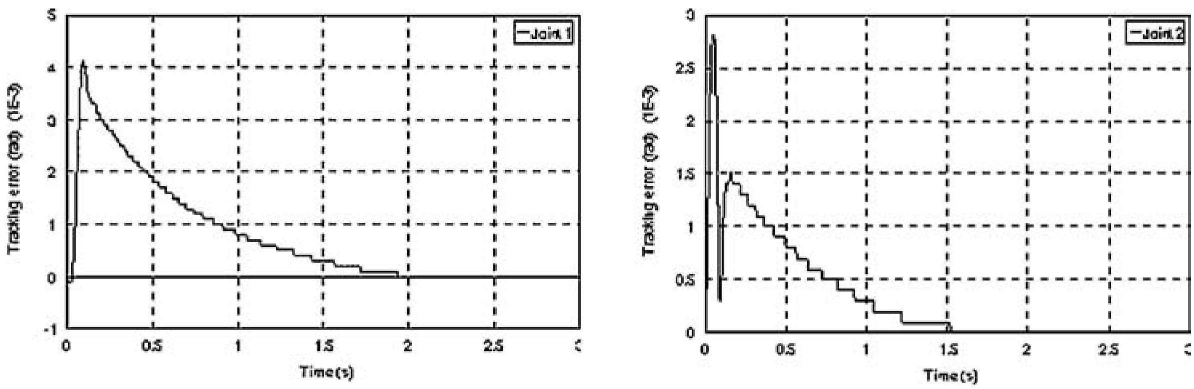

Fig. 8. Response of the adaptive control law (33) when $\Lambda = \mathrm{diag}([80\ 80])$, $\mathrm{K} = \mathrm{diag}([80\ 80])$, $\kappa = 2$ and $\delta = 0.2$.

As seen from Fig. 5, the maximum tracking error in transient state is about 0.013 rad for the first joint, 0.007 rad for the second joint with the control parameters $\mathrm{K} = \mathrm{diag}(80\ 80)$ and $\Lambda = \mathrm{diag}(80\ 80)$. As shown in Figs 6–8, the performance of the proposed control laws are better than the known control law (39), and its maximum tracking error is about 0.0051 rad for the first joint, 0.0036 rad for the second joint for control law (23); 0.0068 rad for the first joint, 0.0041 rad for the second joint for the control law (28); 0.0040 rad for the firsf joint, 0.0027 rad for the second joint for the control law (32) in transient state with the same control parameters of $\mathrm{K} = \mathrm{diag}(80\ 80)$ and $\Lambda = \mathrm{diag}(80\ 80)$.

In the control process, $\mathrm{Y}^{\mathrm{T}}\sigma$ converges to zero, and the parameter estimation speed and the convergence rate of estimation law (39) decreases. Therefore, the tracking performance in a transient and steady state response is not better as compared to the proposed control laws. However, in the new scheme, parameter estimation speed does not decreases as much as the control law (39) when $\mathrm{Y}^{\mathrm{T}}\sigma$ converges to zero because of the trigonometric function. Thus, the parameter estimation speed of the proposed controllers is high and accurate convergence of parameters to their asymptotic values can be achieved. As a result, the tracking performance in a transient and steady state has been improved.

References

1. J. J. Craig, P. Hsu and S. S. Sastry, "Adaptive control of robot manipulator", *Int. J. Robotics Research* **6**(3), 16–28 (1987).
2. R. H. Middleton and G. C. Goodwin, "Adaptive computed torque control for rigid link manipulators", *System Control Letters* **10**, 9–16 (1988).
3. M. W. Spong and R. Ortega, "On adaptive inverse dynamics control of rigid robots", *IEEE Transactions on Automatic Control* **35**(1), 92–95 (1990).
4. J. J. Slotine and W. Li, "On the adaptive control of robotic manipulator", *Int. J. Robotics Research* **6**(3), 49–59 (1987).
5. J. J. Slotine and W. Li, "Adaptive manipulator control: A case study", *IEEE Transactions on Automatic Control* **33**(11), 994–1003 (1988).
6. M. W. Spong, R. Ortega and R. Kelley, "Comment on adaptive manipulator control: A case study", *IEEE Transactions on Automatic Control* **35**(6), 761–762 (1990).
7. O. Egeland and J. M. Godhavn, "A note on Lyapunov stability for adaptive robot controller", *IEEE Transactions on Automatic Control* **39**(8), 1671–1673 (1994).
8. E. Burdet and A Codourey, "Evaluation of parametric and nonparametric nonlinear adaptive controllers", *Robotica* **16**, Part 1, 59–73 (1998).
9. R. Colbaugh, K. Glass and H. Seraji, "Adaptive tracking control of manipulators: Theory and experiments", *Robotics & Computer-Integrated Manufacturing* **12**(3), 209–216 (1996).
10. R. Ortega and M. W. Spong, "Adaptive motion control of rigid robots: A tutorial", *Automatica* **25**(6), 877–888 (1989).
11. L. Sciavicco and B. Siciliano, *Modeling and Control of Robot Manipulators* (The McGraw-Hill Companies, 1996.)
12. M. W. Spong and M. Vidyasagar, *Robot Dynamics and Control* (Wiley, New York, 1989.)
13. M. W. Spong, "On the robust control of robot manipulators", *IEEE Transactions on Automatic Control* **37**, 1782–1786 (1992).