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DYNAMIC CONTROLLABILITY WITH OVERLAPPING TARGETS: OR WHY TARGET INDEPENDENCE MAY NOT BE GOOD FOR YOU

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We generalize some recent results developed in static policy games with multiple players, to a dynamic context. We find that the classical theory of economic policy, static or dynamic, can be usefully applied to a strategic context of difference games: if one player satisfies the *Golden Rule*, then either all other players' policies are ineffective with respect to the dynamic target variables shared with that player. Or no Nash Feedback Equilibrium can exist, unless they all share target values for those variables. We extend those results to the case in which there are also nondynamic targets, to show that policy effectiveness (a Nash equilibrium) can continue to exist if some players satisfy the *Golden Rule* but target values differ between players in their nondynamic targets. We demonstrate the practical importance of these results by showing how policy effectiveness (a policy equilibrium) can appear or disappear with small variations in the expectations process or policy rule in a widely used model of monetary policy with the possibility of target independence.

Keywords: Policy Games, Policy Ineffectiveness, Static Controllability, Existence of Equilibria, Nash Feedback Equilibrium

1. INTRODUCTION

The issue of the effectiveness of public policy is central to economic analysis. The initial contributions by Tinbergen, Theil, and others stated the conditions for policy effectiveness, both static and dynamic, in a *parametric* context.[1] In the last

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two decades, a new approach to economic policy problems has developed, immune from the Lucas (1976) critique, in which the strategic interactions between the government, the central bank, and other agents are modeled explicitly.[2] However, abstract conditions for policy effectiveness have not been studied in that context until recently. Acocella and Di Bartolomeo (2004, 2006) provide general conditions for policy ineffectiveness and equilibrium existence in static LQ-games of the kind stated by the classical theory of economic policy, and show how this can be profitably used to define some general properties of policy games.

This paper extends the idea of controllability to *dynamic* difference games, and in that context we consider the importance of target independence (as opposed to instrument independence) which has been a point of particular controversy in monetary policy design. Our approach is to consider the Nash Feedback Equilibrium for LQ-difference games,[3] and derive conditions for policy ineffectiveness and the equilibrium existence for that case. We then demonstrate the usefulness of our results by showing how easily policy effectiveness, or a policy equilibrium, can appear or disappear with small variations in the expectations process or in the policy rule using a standard model of monetary theory—illustrating, as we do so, how certain variations in the problem can permit or take away the opportunity for policy makers to operate with differing target values for their policy objectives. To do this, we make use of some properties of sparse matrices since nearly all economic models display sparseness.

The rest of the paper is organized as follows. Section 2 defines basic concepts and introduces a formal framework to describe LQ-difference games. Section 3 derives two theorems stating a sufficient condition for policy ineffectiveness and a necessary condition for the equilibrium existence in the traditional Tinbergen framework. Section 4 provides a formal relaxation of the two theorems for the case of sparse economic systems. Section 5 illustrates the application of our results to one of the most widely used models in monetary theory. The paper ends with some conclusions and some ideas for further research.

2. THE BASIC SETUP

We consider the problem where n players try to minimize their individual quadratic criterion. Each player controls a different set of inputs to a single system, which is described by the following difference equation:

$$x(t+1) = Ax(t) + \sum_{i \in N} B_i u_i(t), \tag{1}$$

where N is the set of the players, $x \in \mathbf{R}^M$, is the vector of the states of the system; $u_i \in \mathbf{R}^{m(i)}$ is the (control variable) vector that player i can manipulate; and $A \in \mathbf{R}^{M \times M}$ and $B_i \in \mathbf{R}^{M \times m(i)}$ are full-rank matrices describing the system parameters which (for simplicity) are constant.

The criterion player $i \in N$ aims to minimize is:

$$J_i(u_1, u_2, \ldots, u_n) = \sum_{t=0}^{+\infty} (x_i(t) - \bar{x}_i)' Q_i (x_i(t) - \bar{x}_i), \tag{2}$$

where $\bar{x}_i \in \mathbf{R}^{M(i)}$ is a vector of target values. For player i, the relevant subsystem of (1) is:

$$x_i(t+1) = A_i x_i(t) + \sum_{j \in N} B_{ij} u_j(t), \tag{3}$$

where $A_i \in \mathbf{R}^{M(i) \times M(i)}$ and $B_{ij} \in \mathbf{R}^{M(i) \times m(i)}$ are appropriate submatrices of A and B_i. We assume that all matrices are of full rank, and that $M(i) \geq m(i)$. The economic interpretation of these assumptions is straightforward.

The Nash Feedback Equilibrium can now be defined as follows.

DEFINITION (Nash Feedback Equilibrium). *The vector* $u^*(t) = (u_1^*(t), u_2^*(t), \ldots, u_i^*(t) \ldots, u_n^*(t))$ *defines a Nash Feedback Equilibrium if* $J_i u^*(t)) \geq J_i(u_1^*(t), u_2^*(t), \ldots, u_i(t) \ldots, u_n^*(t))$ *for any* $u_i(t)$ *and for any player i, where* $u_i(t)$ *is a feedback strategy given the information available at period t.*

Operationally, a feedback strategy means that a contingent rule (dependent on the system's state vector) is provided for each player, and that the rules themselves can be obtained from the backward recursions of dynamic programming (Holly and Hughes Hallett, 1989: 176–179).

3. THE GOLDEN RULE AND THE EQUILIBRIUM PROPERTIES

In order to apply the traditional theory of economic policy to study the properties of Nash Feedback Equilibrium, we first recall the traditional Tinbergen idea of *static controllability*:

DEFINITION (Golden Rule). *A policy maker satisfies the Golden Rule of economic policy if the number of its independent instruments* (*at least*) *equals the number of its independent targets.*

Second, we need to redefine policy ineffectiveness because its classical definition[4] cannot be maintained in the realm of multiplayer policy games in which policies become endogenous variables. Instead, the following definition of ineffectiveness can be applied:[5]

DEFINITION (ineffectiveness). *A policy is ineffective if the equilibrium values of the targets are never affected by changes in the parameters of its criterion function.*

Controllability, in the terms of the *Golden Rule* of economic policy, ineffectiveness and the Nash Feedback Equilibrium existence, are related through the following two theorems:

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THEOREM 1 (ineffectiveness). *Provided that an equilibrium exists, if one player satisfies the Golden Rule, all the other players' policies are ineffective with respect to the target variables shared with that player.*

Proof. We start by *assuming* that the policy makers' value functions are quadratic,[6] $V_i(x) = (x_i(t) - \bar{x}_i)' P_i (x_i(t) - \bar{x}_i)$, where P_i are negative definite symmetric matrices so that there are no redundant targets (and for the sake of simplicity, time indexes are omitted). By using the transition law to eliminate the next period state, the n Bellman equations become:

$$(x_i - \bar{x}_i)' P_i (x_i - \bar{x}_i)$$
$$= \max_{u_i} \left\{ (x_i - \bar{x}_i)' Q_i (x_i - \bar{x}_i) + \left(A_i x + \sum_{j \in N} B_{ij} u_j \right)' P_i \left(A_i x + \sum_{j \in N} B_{ij} u_j \right) \right\}. \quad \textbf{(4)}$$

A Nash Feedback Equilibrium must satisfy the first-order conditions:

$$(B'_{ii} P_i B_{ii}) u_i = -B'_{ii} P_i \left(A_i (x_i - \bar{x}_i) + \sum_{j \neq i \in N} B_{ij} u_j \right), \quad \textbf{(5)}$$

which yields the following policy rule:

$$u_i = -(B'_{ii} P_i B_{ii})^{-1} B'_{ii} P_i A_i (x_i - \bar{x}_i) - (B'_{ii} P_i B_{ii})^{-1} B'_{ii} P \sum_{j \neq i \in N} B_{ij} u_j \quad \textbf{(6)}$$

Now, to demonstrate Theorem 1, we focus (without loss of generality) on player 1. If player 1 satisfies the *Golden Rule*, then $m(1) = M(1)$ and $B_{11} \in \mathbf{R}^{M(1) \times M(1)}$ is square and nonsingular. Equation (6) then becomes:

$$u_1 = -B_{11}^{-1} A_1 (x_1 - \bar{x}_1) - B_{11}^{-1} \sum_{j=2}^{N} B_{1j} u_j, \quad \textbf{(7)}$$

since P_i is also nonsingular. That implies:

$$x_1(t+1) = \bar{x}_1 \quad \text{for all } t \in [0, +\infty]. \quad \textbf{(8)}$$

Thus, if a Nash Feedback Equilibrium exists, the value of the target vector x_1 is time invariant and only depends on the preferences of player 1 since, in that case, condition (7) will hold for all periods $t \in [0, +\infty]$. This completes the proof of Theorem 1. ■

THEOREM 2 (nonexistence). *The Nash Feedback Equilibrium of the policy game described does not exist if two or more players satisfy the Golden Rule and at least two of them share one or more target variables.*

Proof. To prove Theorem 2, we only need to show that if also another player (e.g., player 2) satisfies his/her *Golden Rule*, the equilibrium does not exist.

Assume a solution exists and that this solution implies the following optimal policy vector $u^* = (u_1^*, u_2^*, \dots, u_n^*)$ at time t. Then, given $u_3^*(t), \dots, u_n^*(t)$, $u_1^*(t)$ and $u_2^*(t)$ must satisfy the following system [obtained from (5)]:

$$\begin{bmatrix} B_{11}' P_1 B_{11} & B_{22}' P_2 B_{12} \\ B_{11}' P_1 B_{21} & B_{22}' P_2 B_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$= - \begin{bmatrix} B_{11}' P_1 & \emptyset \\ \emptyset & B_{22}' P_2 \end{bmatrix} \begin{bmatrix} A_1(x_1 - \bar{x}_1) + \sum_{j \neq 1} B_{1j} u_j^* \\ A_2(x_2 - \bar{x}_2) + \sum_{j \neq 2} B_{2j} u_j^* \end{bmatrix}. \tag{9}$$

Notice that the first partitioned matrix in (9) is always square; and that if both players satisfy their *Golden Rule*, then all the matrices therein are also square. Now *assume that both players share the same target variables*, that is, $x_1 = x_2$. In this case, we have $A_1 = A_2$ and $B_{ij} = B_{ij}$ for $i \in \{1, 2\}$ and $j \in N$. The first partitioned matrix of (9) therefore has a zero determinant ($B_{11} = B_{21}$ and $B_{12} = B_{22}$) and cannot be inverted. Hence, a couple (u_1^*, u_2^*) satisfying (9) does not exist and u^* cannot be the solution, as claimed by the theorem.

Conversely, consider now target space instead of instrument space. If the first two players both satisfy the *Golden Rule*, it is easy to show that by substituting the first-order condition for u_2 from (5) into (7) for u_1, the first-order conditions for both players cannot both be satisfied unless they both share the same target values, that is, unless the following holds:

$$A(\bar{x}_1 - \bar{x}_2) = 0 \quad \text{or } \bar{x}_1 = \bar{x}_2. \tag{10}$$ [7]

Next, consider the case in which *the first two players do not share all their targets*. When the system can be controlled, this case can be solved by decomposing the problem of each player into two mutually interdependent problems: (A) to minimize the quadratic deviations of the shared targets from their shared target values using an equal number of (arbitrary selected) instruments from u_1, assuming that nonshared target values can be reached; (B) to minimize the quadratic deviations of the nonshared targets from their target values with respect to the remaining instruments, assuming that the shared targets are satisfied (and equal to their target values because of the *Golden Rule*). Given (10), the impossibility of a solution now emerges from the first-order conditions for the first of the two problems (A).[8] Hence, as claimed, if at least two players can control their subsystems and share at least one target variable, the Nash Feedback Equilibrium cannot exist. ■

Comment 1: Theorem 1 gives a sufficient condition for policy effectiveness. But this does not assure the existence of an equilibrium, which may fail to occur. By contrast, Theorem 2 gives a necessary condition for an equilibrium to exist because it states a sufficient condition for the opposite. However, it may not be sufficient for existence.[9] Note also that if Theorem 1 is satisfied, Theorem 2 is not (and vice versa).

Comment 2: The importance of these results for economic policy is exemplified by Theorem 2. It says that if two independent policy authorities—say, fiscal policy makers and the central bank—decide to pursue different inflation targets, then the Nash equilibrium may not exist and the economy may not be able to reach an equilibrium when both policy makers try to optimize their policies. The conditions for this to happen are not particularly stringent. In other words, except for certain sparse economies discussed below, target independence may be unhelpful—not because fiscal and monetary policies cannot be coordinated properly, but because the underlying equilibrium cannot be reached if both policy makers try to optimize their policy choices independently.

4. A GENERALIZATION: SPARSE ECONOMIC SYSTEMS

We now relax Theorems 1 and 2 in a way that may prove important in economic models, but which is less often observed in physical systems. Most economic models display sparseness. That is to say, when written in structural form, they typically relate each endogenous variable to just one or two other endogenous variables; and a small number of lagged endogenous variables, control variables, or predetermined variables. In that case, the structural model from which (1) is derived can be written as:

$$x(t+1) = Cx(t+1) + Dx(t) + \sum_{i \in N} F_i u_i(t), \tag{11}$$

where C, D and F_i are sparse matrices (predominantly zero matrices, with just a few nonzero elements per row). For the sake of simplicity, we assume that all the players share all the target variables (as discussed in the previous section, this assumption can be easily relaxed). In that case, the index on matrices A can be removed, together with the second index on the B matrices. In this situation, (1) has:

$$A = (I - C)^{-1} D \quad \text{and} \quad B_i = (I - C)^{-1} F_i, \tag{12}$$

where $(I - C)^{-1}$ exists by virtue of the normalization in (11), irrespective of the definitions of C, D and F_i. But A and B_i are now no longer of full rank. However, we can premultiply (11) by a permutation matrix T; and insert $T^{-1}T$ (where $T^{-1} = T'$, a property of permutation matrices) into the first two terms on the right of (11). This allows us to separate those target variables which are affected *directly* by dynamic adjustments over time from those which are not. We get the reordered system:

$$\tilde{x}(t+1) = \tilde{A}\tilde{x}(t) + \sum_{i \in N} \tilde{B}_i u_i(t), \tag{13}$$

where $\tilde{x}(t) = Tx(t)$, $\tilde{A} = (I - TCT')^{-1}\, TDT'$ and $\tilde{B}_i = (I - TCT')^{-1}\, TF_iT'$. But this formulation then implies $\tilde{A} = \begin{bmatrix} A_{11} & 0 \\ A_{21} & 0 \end{bmatrix}$, where A_{11} is a square full rank matrix of order ℓ, $A_{21} \in \mathbf{R}^{(M-\ell)\times\ell}$, and where ℓ is the number of target variables in the system that are *directly* subject to dynamic adjustments (i.e., the rank of C).

Hence, M–ℓ target variables are not directly subject to dynamic adjustments. They appear in the second subvector of $\tilde{x}(t)$.

Now we can rework Theorems 1 and 2. We get:

THEOREM 3 (ineffectiveness and nonneutrality in sparse economies). *If the targets of one (and only one) player which are directly subject to dynamic adjustments also satisfy the Golden Rule among themselves, then the policies of all other players will be ineffective with respect to their dynamic targets. Conversely, no Nash Feedback Equilibrium exists in this policy game if two or more players satisfy the Golden Rule for their dynamic targets—unless they happen to share target values for those variables. But the Nash equilibrium may still exist if the Golden Rule is satisfied and the target values for the nondynamic targets differ across players; and the policies of the other players will still be effective for those targets even if one (or some) player satisfies the Golden Rule.*

Proof. Recall that, until now, if players 1 and 2 satisfy the *Golden Rule*, their reaction functions imply $A(\bar{x}_1 - \bar{x}_2) = 0$. In a sparse economic system, the equivalent condition is $\tilde{A}\,(\bar{x}_1 - \bar{x}_2) = 0$ (note that $\tilde{B}_1^{-1}$ still exists if it is square, and the *Golden Rule* applies to player 1). We now write $\bar{x}_{11}$ as the first ℓ elements of $\bar{x}_1$ (corresponding to the first ℓ elements, or dynamic targets, in $\tilde{x}$) and $\bar{x}_{21}$ as the remaining M–ℓ elements of $\bar{x}_1$. Similarly, we define $\bar{x}_{12}$ and $\bar{x}_{22}$ to be the associated subvectors of $\bar{x}_2$. These partitions conform to that in $\tilde{A}$. Our theorem now follows from the fact that both $A_{11}(\bar{x}_{11} - \bar{x}_{12}) = 0$ and $A_{21}(\bar{x}_{11} - \bar{x}_{12}) = 0$, and hence $\bar{x}_{11} = \bar{x}_{12}$ (since A_{11} and A_{21} differ in dimension and A_{11} is of full rank), will be needed to satisfy the replacement for (10) in this case: namely, $\tilde{A}(\bar{x}_1 - \bar{x}_2) = 0$. However $\bar{x}_{21} - \bar{x}_{22} \neq 0$ is consistent with $\tilde{A}(\bar{x}_1 - \bar{x}_2) = 0$. That completes the proof. ■

5. AN EXAMPLE

We turn now to some simple examples to illustrate the usefulness of these results in practice. Consider an economy that can be described by the following well-known model:

$$y_t = \rho y_{t-1} + \alpha\left(\pi_t - \pi_t^e\right) - \beta\left(i_t - \pi_t^e\right) + \varepsilon_t \tag{14}$$

$$i_t = c_0 + c_1(\pi_t - \pi^*) + c_2 y_t \tag{15}$$

$$\pi_t^e - \pi_{t-1}^e = d\left(\pi_{t-1} - \pi_{t-1}^e\right) \quad \text{with } 0 < d < 1. \tag{16}$$

Equation (14) is an elaboration of the standard workhorse model that has been part of the theory of monetary policy since the Barro-Gordon model was introduced in 1983. It consists of a short-run Phillips curve with a persistence parameter ($\rho \neq 0$), set within a standard Lucas supply function (long-run Phillips curve) and elaborated to include the effects of interest rate changes on output. It could therefore be interpreted as either a dynamic open economy Phillips curve; or a new Keynesian IS curve with dynamics. In that context, y_t is the deviation of output from its natural rate (the output gap); π_t is the rate of inflation, and π_t^e the expected rate of inflation in the private sector; i_t is the nominal rate of interest ($i_t - \pi_t^e$, the corresponding real rate of interest); and ε_t a supply shock with mean zero and constant variance.

The chief policy instrument (control variable) in this example will be i_t. Equation (15) is therefore a Taylor rule: c_0 is a constant term, reflecting control errors or the equilibrium rate of interest; π^* is the target inflation rate, and determinacy (the Taylor principle) suggests $c_1 > 1$. Finally, (16) says that expectations are formed by the adaptive principle (we improve on that below); and all parameters, in all three equations, are defined to be positive. This model has lags in all three endogenous variables: y_t, π_t and π_t^e.

To obtain the reduced form of (14)–(16), corresponding to (1), we renormalize (15) on π_t. This then yields, corresponding to (11),

$$\begin{bmatrix} 1 & -\alpha & \alpha-\beta \\ c_2 c_1^{-1} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} y_t \\ \pi_t \\ \pi_t^e \end{pmatrix} = \begin{bmatrix} \rho & 0 & 0 \\ 0 & 0 & 0 \\ 0 & d & 1-d \end{bmatrix} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ \pi_{t-1}^e \end{pmatrix} + \begin{pmatrix} -\beta \\ c_1^{-1} \\ 0 \end{pmatrix} i_t + \begin{pmatrix} \varepsilon_t \\ \pi^* - c_1^{-1} \\ 0 \end{pmatrix}. \tag{17}$$

From here we can determine the value of A for this model, using (12). It is:

$$A = \Delta^{-1} \begin{bmatrix} \rho & -d(\alpha-\beta) & -(1-d)(\alpha-\beta) \\ -\rho c_2 c_1^{-1} & d(\alpha-\beta)c_2 c_1^{-1} & (1-d)(\alpha-\beta)c_2 c_1^{-1} \\ 0 & d\left(1 + c_2 c_1^{-1}\alpha\right) & \left(1 + c_2 c_1^{-1}\alpha\right)(1-d) \end{bmatrix}, \tag{18}$$

where $\Delta = 1 + \alpha c_2 c_1^{-1}$, the determinant of the Jacobian matrix in (17), is nonzero as long as $\alpha c_2 + c_1 \neq 0$, a condition that always holds. But (18) cannot be reorganized to deliver zeros in the right-hand column (the condition that allows one target to be decoupled). Hence, if there are multiple policy makers in this model, they would have to set identical target values for the output gap, the inflation rate, *and* the inflation expectations that they want the markets to have, if there is to be an equilibrium for the policy game and if those targets are to be controllable. Moreover, there could be competing policy makers: for example, where the central bank uses nominal interest rates to control inflation, but another authority (the government) sets the long-term inflation target π^*; or where fiscal policy makers try to moderate the effects of monetary policy by means of tax breaks or suitable budgetary policies; or where policy makers try to influence inflation expectations by setting intermediate targets, or by talking the exchange rate up or down [this would require an extra "constant" term in (16) and hence the third equation of (17)]. These are all situations that are common in practice. The

Bank of England is an example of the first case; the United States, or Italy and France in the Euro, is an example of the second; and Turkey or many high-inflation countries an example of the third.

Next, we consider a variant on this example. Suppose, because of data revisions, policy makers recognize that it is difficult to measure the current output gap accurately, and use a more reliable past measure, y_{t-1}, in equation (15) instead. Suppose also that the private sector, perhaps for similar reasons, find that imperfect expectations introduce too much volatility into the system, and find it cheaper to use simple lagged expectations instead: $\pi_t^e = d\pi_{t-1}$. The model now has no lags in π_t^e. Solving through (11) and (12), we now get:

$$\tilde{A} = \Delta^{-1} \begin{bmatrix} \rho & -d(\alpha-\beta) & 0 \\ -c_2 c_1^{-1}\rho & d(\alpha-\beta)c_2 c_1^{-1} & 0 \\ 0 & d\left(1 + c_2 c_1^{-1}\alpha\right) & 0 \end{bmatrix}. \tag{19}$$

This allows our potential policy makers to disagree on the (intermediate) inflation targets they announce to the markets (π_t^e), but still have controllable target variables and a reachable Nash equilibrium. This happens because there is now a delay before some of the target variables are affected by the policy instruments. So they can set policies to reach some agreed targets first, allowing differences to persist elsewhere, and then use them again to reach the other target values later.

A stronger version of this result is obtained if the contemporaneous output gap is restored to the Taylor rule (15) but expectations are *rational*. That means (16) is replaced by:

$$\pi_t^e = \pi_t - v_t, \tag{20}$$

where v_t is a random expectations error with mean zero. This is the form of the model that most theorists would favor. It implies that we now have no lags in either π_t or π_t^e, and that

$$\tilde{A} = \Delta^{-1}\Gamma^{-1} \begin{bmatrix} 1 & 0 & 0 \\ -c_2 c_1^{-1} & 0 & 0 \\ c_2 c_1^{-1} & 0 & 0 \end{bmatrix}, \tag{21}$$

where $\Gamma = (\Delta + c_2 c_1^{-1}(\alpha - \beta))\rho^{-1}$. Evidently, in this model, the policy makers could have different target values for both π_t^* and π_t^e and still reach a Nash equilibrium outcome for their target variables.

Once again, different policy makers (in government and the central bank) could have target independence (and, hence, different inflation targets) and still expect to reach an equilibrium position. But it could nevertheless prove to be a dream since, if expectations are not rational (because it is too expensive to gather the necessary information accurately), or if it is difficult to measure the current output gap reliably, then they will not be able to reach this idealized equilibrium—or, indeed, any other solution that allows both to optimize their policies.

6. CONCLUDING REMARKS

This paper represents an attempt to generalize some recent results developed in static policy games to a dynamic model. We find that the classical theory of economic policy can be usefully applied to a strategic context of difference games: namely, if one player satisfies the *Golden Rule*, either all the other players' policies are ineffective with respect to their dynamic target variables shared with that player or no Nash Feedback Equilibrium exists without exact agreement on all the dynamic target values. We illustrate the usefulness of our results with reference to a model incorporating a Taylor rule, a description of expectations formation and a relation that can be interpreted as either a dynamic open economy Phillips curve or a New-Keynesian IS curve with dynamics. Small variations in the model specification can bring, or take away, policy effectiveness—allowing the policy makers the latitude to disagree on none, one, or several of the exact target values in their common objectives. Likewise, our general results show how easily target independence, in a world where institutional and policy independence are considered important, can prove to be counterproductive if policy makers try to optimize their choices.

These results lead to three obvious topics for further research. First, our theorems are based on a specific concept of strong controllability, usually known as "static" controllability: that is, the target values are intended to be reached in successive time periods. It is well known, in fact, that in general fewer instruments than targets are needed to control a dynamic system when the targets are to be reached only after a given number of time periods have elapsed. Once the theorems are reformulated in terms of that form of dynamic controllability, it may be possible to define more general and less stringent conditions than those discussed here. Second, we have introduced contemporaneous and adaptive expectations based on the argument that backward-looking models fit the data better than forward-looking models (Gali and Gertler, 1999). But that may conflict with actual practice. For example, Central Banks react to inflation forecasts, and the private sector may be forward looking in their wage bargains or asset holdings. It would be useful to check if our examples continue to apply in such cases. Third, our results are designed to deal with cases of devolved decision making within a single economy, where the government, central bank, employers, and unions are concerned with output, employment, and inflation for that economy, and have a variety of fiscal, monetary, and labor market instruments to reach their targets. It would be interesting, therefore, to extend our analysis to a multicountry setting, where some targets (for example, exchange rates, bilateral trade balances, and inflation if in a currency union) are held in common, but the other targets are not.

NOTES

1. Tinbergen (1952, 1956); Theil (1964).
2. Hughes Hallett (1984, 1986), Levine and Brociner (1994), Aarle et al. (1997), Engwerda et al. (2002), Pappa (2004).

3. Note that we are concerned here with dynamic controllability in the sense of achieving certain target values in successive periods of time, but not in the alternative (classical) sense of reaching those target values only after a certain number of time periods has elapsed.

4. The classical definition of policy ineffectiveness implies that autonomous changes in the policy maker's instruments can have no influence on the targets (Hughes Hallett, 1989). However, that does not allow for the possible blocking moves by other policy players in the game. We therefore adopt a more general definition here.

5. See Gylfason and Lindbeck (1994).

6. Indeed, we know that the value function must be convex for a solution to exist (see, e.g., Başar and Olsder, 1995; Sargent, 1987: 42–48; Dockner et al., 2000). See also Engwerda (2000a, 2000b) for a more advanced exposition.

7. $\bar{x}_1 \neq \bar{x}_2$ is not possible here because A is of full rank. We consider the case in which r(A) < M in the next section.

8. Notice that, because the targets are controllable, this result is independent of the assignment of the instruments.

9. Existence is a rather complex matter in this context. For example, being in a dynamic system, stability is also required. See Engwerda (2000a, 2000b).

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