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ASTIN BULLETIN

Volume **36** (1), 2005

ANGERS, J.-F., DESJARDINS, D., DIONNE, G. & GUERTIN, F. *Vehicle and fleet random effects in a model of insurance rating for fleets of vehicles.* 25-77. We are proposing a parametric model to rate insurance for vehicles belonging to a fleet. The tables of premiums presented take into account past vehicle accidents, observable characteristics of the vehicles and fleets, and violations of the road-safety code committed by drivers and carriers. The premiums are also adjusted according to accidents accumulated by the fleets over time. The proposed model accounts directly for explicit changes in the various components of the probability of accidents. It represents an extension of bonus malus-type automobile insurance models for individual premiums (Lemaire, 1985; Dionne and Vanasse, 1989 and 1992; Pinquet, 1997 and 1998; Frangos and Vrontos, 2001; Purcaru and Denuit, 2003). The extension adds a fleet effect to the vehicle effect so as to account for the impact that the unobservable characteristics or actions of carriers can have on truck accident rates. This form of rating makes it possible to visualize what impact the behaviors of owners and drivers can have on the predicted rate of accidents and, consequently, on premiums. The results are compared to those of the semiparametric approach.

BOUCHER, J.-P. & DENUIT, M. *Fixed versus random effects in Poisson regression models for claim counts: a case study with motor insurance.* 285-301. This paper examines the validity of some stylized statements that can be found in the actuarial literature about random effects models. Specifically, the actual meaning of the estimated parameters and the nature of the residual heterogeneity are discussed. A numerical illustration performed on a Belgian motor third party liability portfolio supports this discussion.

CAIRNS, A. J. G., BLAKE, D. & DOWD, K. *Pricing death: frameworks for the valuation and securitization of mortality risk.* 79-120. It is now widely accepted that stochastic mortality — the risk that aggregate mortality might differ from that anticipated — is an important risk factor in both life insurance and pensions. As such it affects how fair values, premium rates, and risk reserves are calculated.
This paper makes use of the similarities between the force of mortality and interest rates to examine how we might model mortality risks and price mortality-related instruments using adaptations of the arbitrage-free pricing frameworks that have been developed for interest-rate derivatives. In so doing, the paper pulls together a range of arbitrage-free (or risk-neutral) frameworks for pricing and hedging mortality risk that allow for both interest and mortality factors to be stochastic. The different frameworks that we describe — short-rate models, forward-mortality models, positive-mortality models and mortality market models — are all based on positive-interest-rate modelling frameworks since the force of mortality can be treated in a similar way to the short-term risk-free rate of interest. While much of this paper is a review of the possible frameworks, the key new development is the introduction of mortality market models equivalent to the LIBOR and swap market models in the interest-rate literature.
These frameworks can be applied to a great variety of mortality-related instruments, from vanilla longevity bonds to exotic mortality derivatives.

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GERBER, H. U., SHIU, E. S. W. & SMITH, N. *Maximizing dividends without bankruptcy.* 5-23. Consider the classical compound Poisson model of risk theory, in which dividends are paid to the shareholders according to a barrier strategy. Let b^* be the level of the barrier that maximizes the expectation of the discounted dividends until ruin. This paper is inspired by Dickson and Waters (2004). They point out that the shareholders should be liable to cover the deficit at ruin. Thus, they consider b^o, the level of the barrier that maximizes the expectation of the difference between the discounted dividends until ruin and the discounted deficit at ruin. In this paper, b^* and b^o are compared, when the claim amount distribution is exponential or a combination of exponentials.

HAMADA, M., SHERRIS, M. & VAN DER HOEK, J. *Dynamic portfolio allocation, the dual theory of choice and probability distortion functions.* 187-217. Standard optimal portfolio choice models assume that investors maximise the expected utility of their future outcomes. However, behaviour which is inconsistent with the expected utility theory has often been observed.

In a discrete time setting, we provide a formal treatment of risk measures based on distortion functions that are consistent with Yaari's dual (non-expected utility) theory of choice (1987), and set out a general layout for portfolio optimisation in this non-expected utility framework using the risk neutral computational approach.

As an application, we consider two particular risk measures. The first one is based on the PH-transform and treats the upsided and downside of the risk differently. The second one, introduced by Wang (2000) uses a probability distortion operator based on the cumulative normal distribution function. Both risk measures rank-order prospects and apply a distortion function to the entire vector of probabilities.

HEROLD, U. & MAURER, R. *Portfolio choice and estimation risk. A comparison of Bayesian to heuristic approaches.* 135-160. Estimation risk is known to have a huge impact on mean/ variance optimized portfolios, which is one of the primary reasons to make standard Markowitz optimization unfeasible in practice. This issue has attracted new interest in the last years, and several approaches to incorporate estimation risk into portfolio selection have been developed only recently. In this article, we review these approaches as well as some older ones and compare them in an empirical out-of-sample study. The approaches can be classified along two criteria. First, we can differentiate heuristic approaches (restricting portfolio weights and employing simulation techniques) and those based on Bayesian statistics (shrinking the portfolios towards a pre-determined target). Second, the assumptions about the return-generating process differ, either assuming returns to be IID distributed or to be partly predictable. The central result of our empirical study is that all of the IID approaches, whether they account for estimation risk or not, are not superior to simple investment strategies like holding the market portfolio. A risk-adjusted outperfomrance is possible only if sample means are substituted with conditional expected return estimates. Furthermore, the Bayesian approaches reduce turnover and stabilize portfolio weights.

KIJIMA, M. *A multivariate extension of equilibrium pricing transforms: the multivariate Esscher and Wang transforms for pricing financial and insurance risks.* 269-283. This paper proposes a multivariate extension of the equilibrium pricing transforms for pricing general financial and insurance risks. The multivariate Esscher and Wang transforms are derived from Bühlmann's equilibrium pricing model (1980) under some assumptions on the aggregate risk. It is shown that the Esscher and Wang transforms coincide with each other when the underlying risks are normally distributed.

OHLSSON, E. & JOHANSSON, B. *Exact credibility and Tweedie models.* 121-133. Kaas, Dannenburg & Goovaerts (1997) generalized Jewell's theorem on exact credibility, from the classical Bühlmann-Straub model. We extend this result further to the "Bühlmann-Straub model with a priori differences" (Bühlmann & Gisler, 2005). It turns out that exact credibility

holds for a class of Tweedie models, including the Poisson, gamma and compound Poisson distribution — the most important distributions for insurance applications of generalized linear models (GLMs). Our results can also be viewed as an alternative to the HGLM approach for combining credibility and GLMs, see Nelder and Verrall (1997).

STEFFENSEN, M. *Quadratic optimization of life and pension insurance payments.* 245-267. Quadratic optimization is the classical approach to optimal control of pension funds. Usually the payment stream is approximated by a diffusion process. Here we obtain semiexplicit solutions for quadratic optimization in the case where the payment process is driven by a finite state Markov chain model commonly used in life insurance mathematics. The optimal payments are affine in the surplus with state dependent coefficients. Also constraints on payments and surplus are studied.

TSANAKAS, A. & CHRISTOFIDES, N. *Risk exchange with distorted probabilities.* 219-243. An exchange economy is considered, where agents (insurers/banks) trade risks. Decision making takes place under distorted probabilities, which are used to represent either rank-dependence of preferences or ambiguity with respect to real-world probabilities. Pricing formulas and risk allocations, generalising the results of Bühlmann (1980, 1984) are obtained via the construction of aggregate preferences from heterogeneous agents' utility and distortion functions. This involves the introduction of a novel 'collective ambiguity aversion' coefficient. It is shown that probability distortion changes insurers' behaviour, who trade not only to share the aggregate market risk, but are also found to bet against each other. Moreover, probability distortion tends to increase the price of insurance (increase asset returns). While the cases of rank-dependence and ambiguity are formally similar, an important distinction emerges as for rank-dependent preferences equilibria are determinate, while for ambiguity they are generally indeterminate.

ZAKS, Y., FROSTIG, E. & LEVIKSON, B. *Optimal pricing of a heterogeneous portfolio for a given risk level.* 161-185. Consider a portfolio containing heterogeneous risks, where the policyholders' premiums to the insurance company might not cover the claim payments. This risk has to be taken into consideration in the premium pricing. On the other hand, the premium that the insureds pay has to be fair. This fairness is measured by the distance between the risk and the premium paid. We apply a non-linear programming formulation to find the optimal premium for each class so that the risk is below a given level and the weighted distance between the risk and the premium is minimized. We consider also the dual problem: minimizing the risk level for a given weighted distance between risks and premium.

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BARBE, P., FOUGÈRES, A.-L. & GENEST, C. *On the tail behaviour of sums of dependent risks.* 361-373. The tail behavior of sums of dependent risks was considered by Wüthrich (2003) and by Alink *et al.* (2004, 2005) in the case where the variables are exchangeable and connected through an Archimedean copula model. It is shown here how their result can be extended to a broader class of dependence structures using multivariate extreme-value theory. An explicit form is given for the asymptotic probability of extremal events, and the behavior of the latter is studied as a function of the indices of regular variation of both the copula and the common distribution of the risks.

BUCHWALDER, M., BÜHLMANN, H., MERZ, M. & WÜTHRICH, M. V. *The mean square error of prediction in the chain ladder reserving method (Mack and Murphy revisited).* 521-542. We

revisit the famous Mack formula [2], which gives an estimate for the mean square error of prediction MSEP of the chain ladder claims reserving method: We define a time series model for the chain ladder method. In this time series framework we give an approach for the estimation of the conditional MSEP. It turns out that our approach leads to results that differ from the Mack formula. But we also see that our derivation leads to the same formulas for the MSEP estimate as the ones given in Murphy [4]. We discuss the differences and similarities of these derivations.

BUCHWALDER, M., BÜHLMANN, H., MERZ, M. & WÜTHRICH, M. V. *The mean square error of prediction in the chain ladder reserving method. Final remark.* 553. Final remark on the comments in the article by Thomas Mack, Gerhard Quarg and Christian Braun (ASTIN **36** (2), 543-552).

CENCI, M., CORRADINI, M. & GHENO, A. *Dynamic portfolio selection in a dual expected utility theory framework.* 505-520. In this paper the dynamic portfolio selection problem is studied for the first time in a dual utility theory framework. The Wang transform is used as distortion function and well diversified optimal portfolios result both with and without short sales allowed.

DE JONG, P. & FERRIS, S. *Adverse selection spirals.* 589-628. This article discusses risk classification and develops and discusses a framework for estimating the effects of restrictions on risk classification. It is shown that expected losses due to adverse selection depend only on means, variances and covariances of insurance factors and rates of uptake of insurance. Percentage loadings required to avoid losses are displayed. Correlated information, such as family history, is also incorporated and it is seen how such information limits losses and decreases required loadings. Although the evidence suggests that adverse selection is not, at present, a severe problem for insurers, this might change if the authorities impose restrictions on risk classification and/or customers gain an informational advantage (such as better knowledge of their own risk levels). Application is made to unisex annuity pricing in the UK insurance market.

DICKSON, D. C. M. & WATERS, H. R. *Optimal dynamic reinsurance.* 415-432. We consider a classical surplus process where the insurer can choose a different level of reinsurance at the start of each year. We assume the insurer's objective is to minimise the probability of ruin up to some given time horizon, either in discrete or continuous time. We develop formulae for ruin probabilities under the optimal reinsurance strategy, i.e. the optimal retention each year as the surplus changes and the period until the time horizon shortens. For our compound Poisson process, it is not feasible to evaluate these formulae, and hence determine the optimal strategies, in any but the simplest cases. We show how we can determine the optimal strategies by approximating the (compound Poisson) aggregate claims distributions by translated gamma distributions, and, alternatively, by approximating the compound Poisson process by a translated gamma process.

FURMAN, E. & LANDSMAN, Z. M. *Tail variance premium with applications for elliptical portfolio of risks.* 433-462. In this paper we consider the important circumstances involved when risk managers are concerned with risks that exceed a certain threshold. Such conditions are well-known to insurance professionals, for instance in the context of policies involving deductibles and reinsurance contracts. We propose a new premium called tail variance premium (TVP) which answers the demands of these circumstances. In addition, we suggest a number of risk measures associated with TVP. While the well-known tail conditional expectation risk measure provides a risk manager with information about the average of the tail of the loss distribution, tail variance risk measure (TV) estimates the variability along such a tail. Furthermore, given a multivariate setup, we offer a number of allocation techniques which

preserve different desirable properties (sub-additivity and fulladditivity, for instance). We are able to derive explicit expressions for TV and TVP, and risk capital decomposition rules based on them, in the general framework of multivariate elliptical distributions. This class is very popular among actuaries and risk managers because it contains distributions with marginals whose tails are heavier than those of normal distributions. This distinctive feature is desirable when modeling financial datasets. Moreover, according to our results, in some cases there exists an optimal threshold, such that by choosing it, an insurance company minimizes its risk.

GERBER, H. U., LIN, X. S. & YANG, H. *A note on the dividends-penalty identity and the optimal dividend barrier.* 489-503. For a general class of risk models, the dividends-penalty identity is derived by probabilistic reasoning. This identity is the key for understanding and determining the optimal dividend barrier, which maximizes the difference between the expected present value of all dividends until ruin and the expected discounted value of a penalty at ruin (which is typically a function of the deficit at ruin). As an illustration, the optimal barrier is calculated in two classical models, for different penalty functions and a variety of parameter values.

GISLER, A. *The estimation error in the chain-ladder reserving method. A Bayesian approach.* 554-565. We derive the estimation error in a Bayesian framework and discuss the estimates of Mack [2] and of Buchwalder, Bühlmann, Merz and Wüthrich (BBMW) [1] from a Bayesian point of view.

HAINAUT, D. & DEVOLDER, P. *Life annuitization. Why and how much?* 629-654. This paper addresses some of the problems a majority of retired individuals face: Why and in what proportion should they invest in a life annuity to maximize the utility of their future consumption or a bequest? The market considered in this work is made up of three assets: a life annuity, a risky asset and a cash account. As this problem doesn't accept any suitable explicit solution, it is numerically solved by the Markov Chain approximation developed by Kushner and Dupuis. Without a bequest motive, we observe that the optimal planning of consumption is divided into two periods and that optimal asset allocation should include the risky asset. Next, the influence of a bequest on consumption and investment pattern is developed. We demonstrate that even with a bequest motive, pensioners should allocate a part of their wealth to the purchase of life annuities.

LAU, J. W., SIU, T. K. & YANG, H. *On Bayesian mixture credibility.* 573-588. We introduce a class of Bayesian infinite mixture models first introduced by Lo (1984) to determine the credibility premium for a non-homogeneous insurance portfolio. The Bayesian infinite mixture models provide us with much flexibility in the specification of the claim distribution. We employ the sampling scheme based on a weighted Chinese restaurant process introduced in Lo *et al.* (1996) to estimate a Bayesian infinite mixture model from the claim data. The Bayesian sampling scheme also provides a systematic way to cluster the claim data. This can provide some insights into the risk characteristics of the policyholders. The estimated credibility premium from the Bayesian infinite mixture model can be written as a linear combination of the prior estimate and the sample mean of the claim data. Estimation results for the Bayesian mixture credibility premiums will be presented.

LEITNER, J. *A note on credit insurance.* 347-360. In a simple stationary setting with constant interest rate, we derive pricing formulas for defaultable bonds with stochastic recovery rate using a replication argument. Replication is done by using an insurance contract (i.e. a kind of credit default swap), the price of which is determined by a dynamic premium calculation principle. We consider two cases, a linear one, where pricing amounts to solving an inhomogeneous linear ODE, and a super-linear case where a Riccati ODE has to be solved.

MACDONALD, A. S., PRITCHARD, D. J. & TAPADAR, P. *The impact of multifactorial genetic disorders on critical illness insurance: A simulation study based on UK Biobank.* 311-346. The UK Biobank project is a proposed large-scale investigation of the combined effects of genotype and environmental exposures on the risk of common diseases. It is intended to recruit 500,000 subjects aged 40-69, to obtain medical histories and blood samples at outset, and to follow them up for at least 10 years. This will have a major impact on our knowledge of multifactorial genetic disorders, rather than the rare but severe single-gene disorders that have been studied to date. What use may insurance companies make of this knowledge, particularly if genetic tests can identify persons at different risk? We describe here a simulation study of the UK Biobank project. We specify a simple hypothetical model of genetic and environmental influences on the risk of heart attack. A single simulation of UK Biobank consists of 500,000 life histories over 10 years; we suppose that case-control studies are carried out to estimate age specific odds ratios, and that an actuary uses these odds ratios to parameterise a model of critical illness insurance. From a large number of such simulations we obtain sampling distributions of premium rates in different strata defined by genotype and environmental exposure. We conclude that the ability of such a study reliably to discriminate between different underwriting classes is limited, and depends on large numbers of cases being analysed.

MACK, T., QUARG, G. & BRAUN, C. *The mean square error of prediction in the chain ladder reserving method. A comment.* 543-552. We discuss some questionable points of the approach taken in the paper by Buchwalder, Bühlmann, Merz and Wüthrich and come to the conclusion that this approach does not yield an improvement of Mack's original formula. The main reason is that the new approach disregards the negative correlation of the squares of the development factors. The same applies to the formula by Murphy (PCAS 1994).

SCOLLNIK, D. P. M. *A damaged generalised Poisson model and its application to reported and unreported accident counts.* 463-487. This paper investigates some models in which non-negative observations from a Poisson or generalised Poisson distribution are possibly damaged according to a binomial or quasi-binomial law. The latter case is appropriate when the observations are over-dispersed. Although the extent of the damage is not known, it is assumed that the event of whether or not damage occurred is discernible. The models are particularly suited for certain applications involving accident counts when evidence of certain accidents may be observed even though the accidents themselves may go unreported. Given the number of observed accidents and knowledge as to whether or not some additional accidents have gone unreported, these models may be used to make inferences concerning the actual number of unreported and total number of accidents in the current period, and the numbers of reported, unreported, and/or total accidents in a future period. The models are applied to a real data set giving reported and unreported patient accidents in a large hospital. Both maximum likelihood and Bayesian estimation methods are presented and discussed.

VENTER, G. G. *Discussion of 'The mean square error of prediction in the chain ladder reserving method'.* 566-571. For a dozen or so years the chain ladder models of Murphy and Mack have been known to give the same reserve estimates but differ slightly in their assumptions and in their calculations of the reserve variance. Up until now no one had provided a detailed analysis of the source of the difference in variance. The current (BBMW) paper shows that this arises from a slight difference in the estimates of the parameter variance piece, and this in turn comes from the fact that for Murphy the errors in the estimates of the development factors are independent, while under Mack's assumptions they are merely uncorrelated. This would suggest that any given data could be tested for the difference in assumptions by computing correlations of functions of the estimated factors. For instance a finding that some functions of the factors are correlated but the factors themselves are uncorrelated would support Mack's model over Murphy's.

VENTER, G. G., MAJOR, J. A. & KREPS, R. E. *Marginal decomposition of risk measures.* 375-413. The marginal approach to risk and return analysis compares the marginal return from a business decision to the marginal risk imposed. Allocation distributes the total company risk to business units and compares the profit/risk ratio of the units. These approaches coincide when the allocation actually assigns the marginal risk to each business unit, i.e., when the marginal impacts add up to the total risk measure. This is possible for one class of risk measures (scalable measures) under the assumption of homogeneous growth and by a subclass (transformed probability measures) otherwise. For homogeneous growth, the allocation of scalable measures can be accomplished by the directional derivative. The first well known additive marginal allocations were the Myers-Read method from Myers and Read (2001) and co-Tail Value at Risk, discussed in Tasche (2000). Now we see that there are many others, which allows the choice of risk measure to be based on economic meaning rather than the availability of an allocation method. We prefer the term "decomposition" to "allocation" here because of the use of the method of co-measures, which quantifies the component composition of a risk measure rather than allocating it proportionally to something. Risk adjusted profitability calculations that do not rely on capital allocation still may involve decomposition of risk measures. Such a case is discussed. Calculation issues for directional derivatives are also explored.

DUNSFORD, G. A. *A new superannuation model?* 181-319. The superannuation needs of most people should be capable of being satisfied by use of a simple vehicle which provides for the accumulation of funds while working and their drawdown when not working in retirement of semi retirement. A simple superannuation model could co-exist with the existing one as an alternative. With a ten page Product Disclosure Statement, simple investment choices, a simplified fee structure, bank account style periodic statements, the simple model could be favoured by many. The features of the simple model are not necessarily 'new' — hence the "?" in the title. The considerations for the development of the simple model include the purpose(s) of superannuation, the possible vehicles for its provision, a review of current problems and issues, member information, protection and decision making employer involvement and supervisory arrangements. The general taxation treatment of superannuation arrangements is not considered an integral part of the model. However, it is noted that taxation is a major issue and is the cause of many of the complications in the current system. Alternative tax models are considered. In order for a tax model to be able to stand the test of time without fundamental alteration, it is necessary to accommodate a government's need to alter tax revenue generated from time to time and ideally to provide incentives for voluntary savings — without the need for grandfathering complexities to maintain fairness. A tax model is put forward which is believed could meet these requirements over time. Key Words

simple superannuation product, retirement benefits only, seamless transition to retirement, superannuation imputation credits.

INSTITUTE OF ACTUARIES OF AUSTRALIA. *APRA Discussion Paper on Prudential Supervision of General Insurance.* 415-24.

LIFE INSURANCE PRACTICE COMMITTEE. *LIASB Discussion Drafts of Revised Actuarial Standards.* 407-14.

LIFE RISK INSURANCE COMMITTEE. *Report on the Lump Sum Experience Investigation 1998-99.* 321-402. This is the first report to be released by the Institute's new Life Risk Insurance Committee and covers the investigation of Lump Sum business, including Death, TPD and Trauma experience and Immediate Annuities. It covers the two year period from 1/1/1998 to 31/12/1999. Whilst data volumes and quality has made the drawing of credible conclusions about trends and rates difficult there are indications that there is a need to review the standard tables used for solvency calculations. A new format of investigation is in the development to allow better reporting and analysis of experience including the interaction between Death, TPD and Trauma cover to allow for better calculations of actual vs expected experience. The report also highlights the significant degree of variance in results between companies and the need to have full participation in the investigation if it is to produce credible and reliable results. Key Words experience, Mortality, Morbidity, Improvement, Insured Lives, Selection, Lump Sum, TPD, Trauma, Traditional Insurance, Term Insurance, Whole of Life, Endowment, Annuity.

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BRUYNEL, R. *Optimal investment strategies.* 425-488. This paper presents a simple stochastic model for examining the performance of proposed investment strategies on investment projects with cashflow over multiple periods. An optimal investment strategy is one having the lowest possible variance of return for its level of expected return. Algebraic expressions for optimal investment strategies are derived. Optimal investment strategies are identified for regular savers, lump sum investors, and retirees requiring regular drawings from an accumulated fund. The shape of the assumed efficient frontier is found to not significantly affect the optimal investment strategy. The efficiency of the common "Constant Portfolio", "Reducing Risk" and "Dollar Cost Averaging" investment strategies is measured under various scenarios. In some cases they produce a variance up to 40% higher than the variance achieved by the optimal investment strategy. Key Words Investment Strategy, Financial Planning, Dollar Cost Averaging, Efficient Frontier, Modern Portfolio Theory.

GAY, R. *Setting premiums for the largest heavy-tailed claims.* 489-529. For thin-tailed insurance claims governed by central limit theorems, insurance premiums are determined largely by the mean and variance of the claims distribution. Ft-tailed claims having limited moments are not necessarily subject to central limit theorems: they are characterised by the distance between the largest extremes. A new premium principle is described and its properties are discussed. Premiums determined by it in respect of fat-tailed claims are shown to depend directly the distance between extreme claims via their ratios. New results about the distribution of these ratios are established. The results have implications for estimation of the tail-index of the distribution, one of the outstanding statistical challenges facing the actuary, indeed in extreme value theory. Applications to premium determination for the largest claims of a data set in the face of tail-fatness uncertainty are provided. Key Words Heavy-tailed claims, premium principle, tail-index estimation.

INSTITUTE OF ACTUARIES OF AUSTRALIA. *Submission: ASIC Consultation Paper on online calculators: 23 September 2005.* 537-548.

INSTITUTE OF ACTUARIES OF AUSTRALIA. *Submission to Motor Accident Authority: Lifetime care and support: 21 September 2005.* 533-535.

AUSTRALIAN ACTUARIAL JOURNAL

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INSTITUTE OF ACTUARIES OF AUSTRALIA. *2006 Pre-Budget Submission to the Commonwealth Government, 20 November 2005.* 591-603.

INSTITUTE OF ACTUARIES OF AUSTRALIA. *LIASB exposure drafts of revised actuarial standards.* 621-634.

INSTITUTE OF ACTUARIES OF AUSTRALIA. *Submission to Emissions Trading Working Group.* 629-634.

INSTITUTE OF ACTUARIES OF AUSTRALIA. *Submission to IASB on ED proposed amendments to IAS 37.* 605-621.

NEUHAUS, W. *On equalisation by risk based capitation reinsurance.* 549-587. The paper points out that the Risk Based Capitation (RBC), as proposed by the Department of Health to equalise hospital benefits, has several shortcomings that could be eliminated. A theory of optimal equalisation is used to derive an equalisation scheme that properly recognises the form of partial community rating that is mandatory for private health insurers in Australia today. The optimal equalisation scheme is also of the RBC form, the only difference to the current proposal being that the Single Equivalent Units (SEU) needs to be calculated in a different way. Keywords: Health Insurance, Community Rating, Equalisation, Risk Based Capitation, Reinsurance.

GENEVA RISK AND INSURANCE REVIEW

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BRUNNER, J. K. & PECH, S. *Adverse selection in the annuity market with sequential and simultaneous insurance demand.* 111-146. This paper investigates the effect of adverse selection on the private annuity market in a model with two periods of retirement and two types of individuals, who differ in their life expectancy. In order to introduce the existence of time-limited pension insurance, we consider a model where for each period of retirement separate contracts can be purchased. Demand for the two periods can be decided sequentially or simultaneously. We show that only a situation where all risk types choose sequential contracts is an equilibrium and that this outcome is favourable for the long-living, but is unfavourable for the short-living individuals.

EMBRECHTS, P. & PUCCETTI, G. *Aggregating risk capital, with an application to operational risk.*

71-90. We describe a numerical procedure to obtain bounds on the distribution function of a sum of n dependent risks having fixed marginals. With respect to the existing literature, our method provides improved bounds and can be applied also to large non-homogeneous portfolios of risks. As an application, we compute the VaR-based minimum capital requirement for a portfolio of operational risk losses.

HUANG, H.-H. *Optimal insurance contract under a value-at-risk constraint.* 91-110. This study develops an optimal insurance contract endogenously under a value-at-risk (VaR) constraint. Although Wang et al [2005] had examined this problem, their assumption implied that the insured is risk neutral. Consequently, this study extends Wang et al [2005] and further considers a more realistic situation where the insured is risk averse. The study derives the optimal insurance contract as a single deductible insurance when the VaR constraint is redundant or as a double deductible insurance when the VaR constraint is binding. Finally, this study discusses the optimal coverage level from common forms of insurances, including deductible insurance, upper-limit insurance, and proportional coinsurance.

BAUER, D., KIESEL, R., KLING, A. & RUß, J. *Risk-neutral valuation of participating life insurance contracts.* 171-183. The valuation of life insurance contracts using concepts from financial mathematics has recently attracted considerable interest in academia as well as among practitioners. In this paper, we will investigate the valuation of participating contracts, which are characterized by embedded interest rate guarantees and some bonus distribution rules. We will model these under the specific regulatory framework in Germany; however, our analysis can be applied to any insurance market with cliquet-style guarantees. We will present a framework, in which different kinds of guarantees or options can be analyzed separately. Also, the practical implementation of such models is discussed. We use two different numerical approaches to derive fair parameter settings of such contracts and price the embedded options. The sensitivity of the contract value with respect to multiple parameters is studied. In particular, we find that life insurers offer interest rate guarantees below their risk-neutral value. Furthermore, the financial strength of an insurance company considerably affects the value of a contract.

DAHL, M. & MØLLER, T. *Valuation and hedging of life insurance liabilities with systematic mortality risk.* 193-217. This paper considers the problem of valuating and hedging life insurance contracts that are subject to systematic mortality risk in the sense that the mortality intensity of all policy-holders is affected by some underlying stochastic processes. In particular, this implies that the insurance risk cannot be eliminated by increasing the size of the portfolio and appealing to the law of large numbers. We propose to apply techniques from incomplete markets in order to hedge and valuate these contracts. We consider a special case of the affine mortality structures considered by Dahl [Dahl, M., 2004. Stochastic mortality in life insurance: market reserves and mortality-linked insurance contracts. IME 35], where the underlying mortality process is driven by a time-inhomogeneous Cox–Ingersoll–Ross (CIR) model. Within this model, we study a general set of equivalent martingale measures, and determine market reserves by applying these measures. In addition, we derive risk-minimizing strategies and mean-variance indifference prices and hedging strategies for the life insurance

liabilities considered. Numerical examples are included, and the use of the stochastic mortality model is compared with deterministic models.

HESS, K. T., SCHMIDT, K. D. & ZOCHER, M. *Multivariate loss prediction in the multivariate additive model.* 185-191. In the present paper we propose a multivariate version of the additive model of loss reserving. The multivariate additive model is a linear model with a particular design matrix and a particular variance structure and is suitable for certain portfolios consisting of several correlated subportfolios. Under the assumptions of the multivariate additive model, we derive a formula for the Gauss–Markov predictor for a non-observable incremental claim. We also show that the Gauss–Markov predictors for the reserve of a particular accident year and for the total reserve are obtained by summation over the Gauss–Markov predictors for the corresponding non-observable incremental claims, and that this is also true for the Gauss–Markov predictors for the corresponding quantities of the aggregate portfolio.

KHALAF-ALLAH, M., HABERMAN, S. & VERRALL, R. J. *Measuring the effect of mortality improvements on the cost of annuities.* 231-249. This paper uses the mortality projection model of Sithole, Haberman and Verrall [Sithole, T. Z., Haberman, S., Verrall, R. J., 2000. An investigation into parametric models for mortality projections, with applications to immediate annuitants' and life office pensioners' data. IME, 27] in order to investigate the effect of mortality improvements on the cost of annuities. Analytical results are derived using an approach which extends that of Pollard [Pollard, J. H., 1982. The expectation of life and its relationship to mortality. *JIA*, **109**, 225-240]. The results are illustrated using UK data from the Continuous Mortality Investigation Bureau. Simulation methods are used to consider the distribution of the annuity cost, as well as the more often quoted point estimates. The effects of the age of the pensioner at inception, the rate of interest and the gender of the pensioner are considered. Finally, a Bayesian model is developed which incorporates the estimation of the parameters of the mortality projection model with the simulation of the annuity cost. This allows us to consider the effect of parameter uncertainty on the projected distribution of the annuity cost.

MUERMANN, A., MITCHELL, O. S. & VOLKMAN, J. M. *Regret, portfolio choice, and guarantees in defined contribution schemes.* 219-229. We model how asset allocation decisions in a defined contribution (DC) pension plan might vary with participants' attitudes about risk and regret. We show that anticipated disutility from regret can have a potent effect on investment choices. Compared with a risk-averse investor, the investor who takes regret into account will hold more stock when the equity premium is low but less stock when the equity premium is high. We also assess how regret can influence a DC plan participant's view of rate-of-return guarantees, as measured by his willingness-to-pay. We find that regret increases the regret-averse investor's willingness-to-pay for a guarantee when the portfolio is relatively risky, but decreases it when the portfolio is relatively safe.

VALDEZ, E. A., PIGGOTT, J. & WANG, L. *Demand and adverse selection in a pooled annuity fund.* 251-266. In this paper, we construct a model for examining the demand for annuities together with the possible implications of adverse selection when an individual consumer has access to both a private annuity market and a market with a pooled annuity fund. An earlier paper by Piggott *et al.* [Piggott, J., Valdez, E. A., Detzel, B., 2005. The simple analytics of a pooled annuity fund. *J. Risk Insur.*, **72**, 497-520] provides a formal analysis of the payout adjustments from a longevity risk-pooling fund, an arrangement referred to in the paper as Group Self Annuitization (GSA). In such a pooled arrangement, the annuitants will bear their own cohorts' systematic risk, but the cohort will share the idiosyncratic risk. The resulting return on the pooled annuity fund can be expressed as the product of a return from an ordinary annuity multiplied by a random variable that accounts for the adjustment that is due

to deviations from expectation of mortality and investments. As demonstrated in this paper, a simple analysis of economic choice provides that it is possible to reduce the implications of adverse selection in a pooled annuity fund. It is well-documented that empirically, individuals do not find private annuity funds an attractive form of investment despite the potential welfare benefits that can be drawn from annuitization. A pooled annuity fund is an alternative to the conventional private annuity fund that may be considered more cost-effective.

YANG, J., CHENG, S. & ZHANG, L. *Bivariate copula decomposition in terms of comonotonicity, countermonotonicity and independence.* 267-284. Copulas are statistical tools for modelling the multivariate dependence structure among variables in a distribution free way. This paper investigates bivariate copula structure; the existence and uniqueness of a bivariate copula decomposition into a comonotonic, an independent, a countermonotonic, and an indecomposable part are proved, while the coefficients are determined from partial derivatives of the corresponding copula. Moreover, for the indecomposable part, an optimal convex approximation is provided and analyzed on the basis of the usual criterion. Some applications of the decomposition in finance and insurance are mentioned.

BALLOTTA, L., ESPOSITO, G. & HABERMAN, S. *The IASB Insurance Project for life insurance contracts: Impact on reserving methods and solvency requirements.* 356-375. In this communication, we review the fair value-based accounting framework promoted by the IASB Insurance Project for the case of a life insurance company. In particular, for the case of a simple participating contract with minimum guarantee, we show that the fair valuation process allows for the identification of a suitable safety loading to hedge against default risk; furthermore, we show that, when compared with the "traditional" accounting system based on the construction of mathematical reserves, the fair value approach offers a sounder reporting framework in terms of covering of the liability, implementation costs, volatility of assets and liabilities and solvency capital requirements.

CHUI, M. C. & LI, D. *Asset and liability management under a continuous-time mean–variance optimization framework.* 330-355. Asset and liability (AL) management under the mean–variance criteria refers to an optimization problem that maximizes the expected final surplus subject to a given variance of the final surplus or, equivalently, minimizes the variance of the final surplus subject to a given expected final surplus. We employ stochastic optimal control theory to analytically solve the AL management problem in a continuous-time setting. More specifically, we derive both the optimal policy and the mean–variance efficient frontier by a stochastic linear quadratic control framework. Then, the quality of the derived optimal AL management policy is examined by comparing it with those in the literature. We further discuss consequences of a discrepancy in objectives between equity holders and investors of a mutual fund. Finally, the optimal funding ratio, i.e., the wealth-to-liability ratio, is determined.

KAISHEV, V. K. & DIMITROVA, D. S. *Excess of loss reinsurance under joint survival optimality.* 376-389. Explicit expressions for the probability of joint survival up to time x of the cedent and the reinsurer, under an excess of loss reinsurance contract with a limiting and a retention level are obtained, under the reasonably general assumptions of any non-decreasing premium income function, Poisson claim arrivals and continuous claim amounts, modelled by any joint distribution. By stating appropriate optimality problems, we show that these results can be used to set the limiting and the retention levels in an optimal way with respect to the

probability of joint survival. Alternatively, for fixed retention and limiting levels, the results yield an optimal split of the total premium income between the two parties in the excess of loss contract. This methodology is illustrated numerically on several examples of independent and dependent claim severities. The latter are modelled by a copula function. The effect of varying its dependence parameter and the marginals, on the solutions of the optimality problems and the joint survival probability, has also been explored.

KOISSI, M.-C. & SHAPIRO, A. F. *Fuzzy formulation of the Lee–Carter model for mortality forecasting.* 287-309. In this paper, a fuzzy formulation of the Lee–Carter [JASA 87 (419) (1992), 659-675] model is proposed. The standard Lee–Carter (LC) model, which uses singular value decomposition, assumes that the errors have a constant variance over all ages. This statement, however, does not often hold. The advantage of such a fuzzy approach is that the errors are viewed as fuzziness of the model structure; hence the homoscedasticity is not an issue.

MELNIKOV, A. & ROMANIUK, Y. *Evaluating the performance of Gompertz, Makeham and Lee–Carter mortality models for risk management with unit-linked contracts.* 310-329. The paper compares the performance of three mortality models in the context of optimal pricing and hedging of unit-linked life insurance contracts. Two of the models are the classical parametric results of Gompertz and Makeham, the third is the recently developed method of Lee and Carter [JASA 87 (419) (1992) 659-675] for fitting mortality and forecasting it as a stochastic process. First, quantile hedging techniques of Föllmer and Leukert [Finance Stoch. 3 (1999) 251-273] are applied to price a unit-linked contract with payoff conditioned on the client's survival to the contract's maturity. Next, the paper analyzes the implications of the three mortality models on risk management possibilities for the insurance firm based on numerical illustrations with the Toronto Stock Exchange/Standard and Poor financial index and mortality data for the USA, Sweden and Japan. The strongest differences between the models are observed in Japan, where the lowest mortality for the next two decades is expected. The general mortality decline patterns, rectangularization of the survival curve and deceleration of mortality at older ages, are well pronounced in the results for all three countries.

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ANTONIO, K. & BEIRLANT, J. *Actuarial statistics with generalized linear mixed models.* 58-76. Over the last decade the use of generalized linear models (GLMs) in actuarial statistics has received a lot of attention, starting from the actuarial illustrations in the standard text by McCullagh and Nelder. Traditional GLMs however model a sample of independent random variables. Since actuaries very often have repeated measurements or longitudinal data (i.e. repeated measurements over time) at their disposal, this article considers statistical techniques for modelling such data within the framework of GLMs. Use is made of generalized linear mixed models (GLMMs) which model a transformation of the mean as a linear function of both fixed and random effects. The likelihood and Bayesian approaches to GLMMs are explained. The models are illustrated by considering classical credibility models and more general regression models for non-life ratemaking in the context of GLMMs. Details on computation and implementation (in SAS and WinBugs) are provided.

BELZUNCE, F., ORTEGA, E. M. & RUIZ, J. M. *On non-monotonic ageing properties from the Laplace transform, with actuarial applications.* 1-14. Non-monotonic ageing has been considered in the literature to model situations where initially the residual lifetime tends to decrease (increase) and after a certain period, it starts to increase (decrease) with increasing

age. On the other hand, the Laplace transform has been used to represent actuarial amounts, such as indemnities associated with risks, incomes associated with financial transactions or life premiums in life insurance. In this paper we introduce a new concept to describe non-monotonic ageing in actuarial sciences which is defined in terms of the Laplace transform of the residual life, and we derive its relationship with the net prospective premium reserve. We obtain some properties and applications in insurance related to this ageing class, the relationships among some non-monotonic ageing classes in the literature and closure under shock models, as well as several comparisons among the change points. Some illustrative examples are provided.

DE ANDRÉS SÁNCHEZ, J. *Claim reserving with fuzzy regression and Taylor's geometric separation method.* 145-163. Claim provisions are crucial for the financial stability of insurance companies. This is why actuarial literature has proposed several claim reserving methods, which are usually based on statistical concepts. However, the mutant and uncertain behaviour of insurance environments does not make it advisable to use a wide database when calculating claim reserves, and so it in fact makes the use of Fuzzy Set Theory very attractive. This paper develops a claim reserving method that combines Ishibuchi and Nii's extension [Fuzzy Sets and Systems 119, 273-290] to the fuzzy regression methods described by Tanaka *et al.* [IEEE Trans. Man Cybernetics 41, 389-396], Tanaka [*Fuzzy Sets and Systems*, **24**, 363-375], Savic and Pedrycz [in: Fuzzy Regression Analysis. Physica-Verlag, Heidelberg, pp91-100] and Tanaka and Ishibuchi (1992) [in: Fuzzy Regression Analysis. Physica-Verlag, Heidelberg, pp47-60] with the scheme for claim reserving proposed by Taylor (1978) [in: Proceedings of the First Meeting of the Contact Group Actuarial Sciences, pp37-64].

EMMS, P. & HABERMAN, S. *Asymptotic and numerical analysis of the optimal investment strategy for an insurer.* 113-134. The asymptotic behaviour of the optimal investment strategy for an insurer is analysed for a number of cash flow processes. The insurer's portfolio consists of a risky stock and a bond and the cash flow is assumed to be either a normal or a compound Poisson process. For a normally distributed cash flow, the asymptotic limits are found in the cases where the stock is very risky or very safe. For a compound Poisson risk process, a composite asymptotic expansion is found for the optimal investment strategy in the case where stock is very risky and the claim size distribution is of an exponential type.

In general for a compound Poisson cash flow, the outer asymptotic limit reduces the integro-differential equation describing the optimal stock allocation to an integral equation, which determines the classical survival probability in ruin theory. The leading order optimal asset allocation is derived from this survival probability through a feedback law. Calculation of the optimal asset allocation leads to a difficult numerical problem because of the boundary layer structure of the solution and the tail properties of the claim size distribution. A second order numerical method is successfully developed to calculate the optimal allocation for light and heavy-tailed claim size distributions.

EMMS, P., HABERMAN, S. & SAVOULLI, I. *Optimal strategies for pricing general insurance.* 15-34. Optimal premium pricing policies in a competitive insurance environment are investigated using approximation methods and simulation of sample paths. The market average premium is modelled as a diffusion process, with the premium as the control function and the maximisation of the expected total utility of wealth, over a finite time horizon, as the objective. In order to simplify the optimisation problem, a linear utility function is considered and two particular premium strategies are adopted. The first premium strategy is a linear function of the market average premium, while the second is a linear combination of the break-even premium and the market average premium. The optimal strategy is determined over the free parameters of each functional form.

It is found that for both forms the optimal strategy is either to set a premium close to the break-even or not to sell insurance depending on the model parameters. If conditions are

suitable for selling insurance then for the first premium strategy, in the case of no market average premium drift, the optimal premium rate is approximately $\bar{p}(0)/aT$ above break-even where $\bar{p}(0)$ is the initial market average premium, a is a constant related to the elasticity of demand and T is the time horizon. The optimal strategy for the second form of premium depends on the volatility of the market average premium. This leads to optimal strategies which generate substantial wealth since then the market average premium can be much larger than break-even leading to significant market exposure whilst simultaneously making a profit. Monte-Carlo simulation is used in order to study the parameter space in this case.

FISCHER, T. *A law of large numbers approach to valuation in life insurance.* 35-57. The classical Principle of Equivalence ensures that a life insurance company can accomplish that the mean balance per policy converges to zero almost surely for an increasing number of independent policyholders. By certain assumptions, this idea is adapted to the general case with stochastic financial markets. The implied minimum fair price of general life insurance policies is then uniquely determined by the product of the assumed unique equivalent martingale measure of the financial market with the physical measure for the biometric risks. The approach is compared with existing related results. Numeric examples are given.

GAO, F., SONG, F. M. & ZHANG, L. *Coherent risk measure, equilibrium and equilibrium pricing.* 85-94. In this paper, we use a coherent risk measure to define Pareto equilibrium which is consistent with the one in microeconomic theory, and present necessary and sufficient conditions for this equilibrium in both a complete market and an incomplete market. These results are generalizations of those of Heath and Ku. Moreover, we also study Arrow–Debreu equilibrium and give the equilibrium price in terms of risk measures. Some examples are given to illustrate the results intuitively.

KLING, A., RICHTER, A. & RUß, J. *The interaction of guarantees, surplus distribution, and asset allocation in with-profit life insurance policies.* 164-178. Traditional life insurance policies in many markets are sold with minimum interest rate guarantees. This paper concentrates on the risk cliquet-style guarantees impose on the insurer, measured by shortfall probabilities under the so-called "real-world probability measure P". We develop a general model and analyze the impact of interest rate guarantees on the risk of an insurance company. Furthermore the paper is concerned with how default risk depends on characteristics of the contract, on the insurer's reserve situation and asset allocation, and on management decisions as well as on regulatory parameters. In particular, the interaction of the parameters is analyzed yielding results that should be of interest for insurers as well as regulators.

LIU, G. & ZHAO, J. *Joint distributions of some actuarial random vectors in the compound binomial model.* 95-103. The compound binomial model is first proposed by Gerber [Gerber, H. U., 1988. Mathematical fun with compound binomial process, *Astin Bull.*, **18**, 161-168]. In this paper, we introduce a renewal mass function of a defective renewal sequence constituted by the up-crossing zero points of the model and get its explicit expression. By the mass function together with the strong Markov property of the surplus process $\{X(n)\}$, we obtain the explicit expressions of the ruin probability, the joint distribution of T, $X(T-1)$ and $|X(T)|$ (i.e., the time of ruin, the surplus immediately before ruin and the deficit at ruin) and distributions of some actuarial random vectors containing more than three variables.

MILEVSKY, M. A. & YOUNG, V. R. *The timing of annuitization: Investment dominance and mortality risk.* 135-144. We use preference-free dominance arguments to develop a framework for locating the optimal age (time) at which a retiree should purchase an irreversible life annuity, as a function of current annuity prices and mortality tables. Then, using the institutional characteristics of annuity markets in the US, we show that annuitization prior to age 65-70 is dominated by self-annuitization even in the absence of any bequest motives.

And, for retirees who are willing to accept some financial risk in exchange for retaining the benefits of liquidity and bequest, the optimal age can be even later. In addition to the normative implications, these results should help shed light on the so-called annuity puzzle which has been much debated by economists, by focusing attention on the specific ages for which a puzzle can actually be said to exist.

WANG, N. *Optimal investment for an insurer with exponential utility preference.* 77-84. This paper considers the optimal investment choice for a general insurer in the sense of maximizing the exponential utility of his or her reserve at a future time. The claim process is supposed to be a pure jump process (not necessarily compound Poisson) and the insurer has the option of investing in multiple risky assets whose price processes are described by the Black–Scholes market model. It is shown in this paper that the optimal strategy is to put a fixed amount of money in each risky asset if there is no risk-free asset. If there is a risk-free asset, the discounted amount held in each risky asset is fixed. In the case where the claim process is compound Poisson, the optimal strategy with respect to a properly selected utility function can result in a reserve process which is safer than that without risky investment.

YUEN, K. C., WANG, G. & LI, W. K. *The Gerber–Shiu expected discounted penalty function for risk processes with interest and a constant dividend barrier.* 104-112. In this paper, we consider the classical surplus process with interest and a constant dividend barrier. Under constant interest, we derive an integro-differential equation for the Gerber–Shiu expected discounted penalty function. Following an idea of Lin, Willmot and Drekic [*IME*, **33**, 551-566], we obtain the solution to the integro-differential equation which is in the form of an infinite series. In some special cases with exponential claims, we are able to find closed-form expressions for the Gerber–Shiu expected discounted penalty function. Finally, we extend the integro-differential equation to the case where the surplus is invested in an investment portfolio with stochastic return on investments.

BOYLE, P. P. & DRAVIAM, T. *Pricing exotic options under regime switching.* 267-282. This paper studies the pricing of options when the volatility of the underlying asset depends upon a hidden Markov process which takes discrete values. It is assumed that the regime switching process is generated by time-independent rate parameters and is independent of the Brownian motion. We derive the coupled Black–Scholes-type partial differential equations that govern the dynamics of several exotic options. These include European, Asian and lookback options. The difference in option prices with and without regime switching is substantial for lookback options and more moderate for European and Asian options.

CHEN, A. & SUCHANECKI, M. *Default risk, bankruptcy procedures and the market value of life insurance liabilities.* 231-255. The topic of insolvency risk in connection with life insurance companies has recently attracted a great deal of attention. In this paper, the question is investigated of how the values of the equity and of the liability of a life insurance company are affected by the default risk and the choice of the relevant bankruptcy procedure. As an example, the US Bankruptcy Code with Chapter 7 and Chapter 11 bankruptcy procedures is used. Grosen and Jørgensen's [*J. Risk Insur.*, **69**, 63-91] contingent claim model, implying only a Chapter 7 bankruptcy procedure, is extended to allow for more general bankruptcy procedures such as Chapter 11. Thus, more realistically, default and liquidation are modelled as distinguishable events. This is realized by using so-called standard and cumulative Parisian barrier option frameworks. It is shown that these options have appealing interpretations in

terms of the bankruptcy mechanism. Furthermore, a number of representative numerical analyses and comparative statics are performed in order to investigate the effects of different parameter changes on the values of the insurance company's equity and liability, and hence on the value of the life insurance contract. To complete the analysis, the shortfall probabilities of the insurance company implied by the proposed models are computed and compared.

DE SCHEPPER, A. & HEIJNEN, B. *Distribution-free option pricing.* 179-199. Nobody doubts the power of the Black and Scholes option pricing method, yet there are situations in which the hypothesis of a lognormal model is too restrictive. A natural way to deal with this problem consists of weakening the hypothesis, by fixing only successive moments and possibly the mode of the price process of a risky asset, and not the complete distribution. As a consequence of this generalization, the option price is no longer a unique value, but rather a range of possible values. In the present paper, we show how to find upper and lower bounds for this range, a range which turns out to be quite narrow in a lot of cases.

EMMS, P. *Pricing general insurance with constraints.* 335-355. Deterministic control theory is used to find the optimal premium strategy for an insurer in order to maximise a given objective. The optimal strategy can be loss-leading depending on the model parameters, which may result in negative premium values. In such circumstances, it is optimal to capture as much of the market as possible before making a profit towards the end of the time horizon. In reality, the amount by which an insurer can lower premiums is constrained by borrowing restrictions and the risk inherent in building up a large exposure. Consequently, the effect of constraining the pricing problem is analysed with two forms of constraint: a bounded premium and a solvency requirement. If a lower bound is placed on the premium then an analytical solution can be found, which is not necessarily a smooth function of time. The optimal premium strategy is described in qualitative terms, without recourse to specifying particular parameter values, by considering the value of the terminal optimal premium. Solvency constraints lead to an optimisation problem which is coupled to the state equations and so there is no analytical solution. Numerical results are presented for a subset of the parameter space using control parameterisation, which turns the optimisation problem into a nonlinear programming problem.

HASHORVA, E. *On the asymptotic distribution of certain bivariate reinsurance treaties.* 200-208. Let $\{(X_n, X_n)\}, n \geq 1$ be bivariate random claim sizes with common distribution function F and let $\{N(t), t \geq 0\}$ be a stochastic process which counts the number of claims that occur in the time interval $[0, t], t \geq 0$. In this paper we derive the joint asymptotic distribution of randomly indexed order statistics of the random sample $(X_1, Y_1), (X_2, Y_2), \ldots, (X_{N(t)}, Y_{N(t)})$ which is then used to obtain asymptotic representations for the joint distribution of two generalised largest claims reinsurance treaties available under specific insurance settings. As a by-product we obtain a stochastic representation of a m-dimensional Λ-extremal variate in terms of iid unit exponential random variables.

MORALES, M. *On the expected discounted penalty function for a perturbed risk process driven by a subordinator.* 293-301. The Expected Discounted Penalty Function (EDPF) was introduced in a series of now classical papers [*IME*, **21**, 129-137; *N. Am. Actuar. J.*, **2**(1), 48-78; *N. Am. Actuar. J.*, **2**(3), 101-112]. Later, Gerber and Landry [*IME*, **22**, 263-276] extended the concept to the perturbed case. Recent papers have extended these results in more general settings [for instance Tsai and Willmot [*IME*, **30**, 51-66], Li and Lu [*IME*, **36**, 179-193] and Li and Garrido [*Scand. Actuar. J.*, 2005, 161-186]. In this note we present yet another generalization that has not been considered before in the literature. We present a perturbed risk process with a subordinator as the model for the aggregate claims. We generalize existing results [*IME*, **30**, 51-66] on the EDPF for the subordinator case.

NGWIRA, B. C. & GERRARD, R. *Stochastic pension fund control in the presence of Poisson*

jumps. 283-292. We consider the problem of optimal funding and asset allocation for a defined benefit pension scheme by assuming that the pension fund can be invested in a risk-free asset and a risky asset whose return follows a jump diffusion process. We extend existing literature which mainly assumes that the risky asset's return follows a pure diffusion process. In a stochastic analysis of the optimal policies we show that the optimal contribution and asset allocation policies have similar forms as in the pure diffusion approaches, but with a modification for the effect of jumps. These results hold under both constant pension scheme benefit outgo and stochastic pension scheme benefit outgo. Using a sensitivity analysis of the effect of the mean jump magnitude on the asset allocation policy, we show that increasing (in absolute terms) the mean jump magnitude reduces the allocation in the risky asset and increases the allocation in the risk-free asset.

ROORDA, B. & SCHUMACHER, J. M. *Time consistency conditions for acceptability measures, with an application to Tail Value at Risk.* 209-230. An acceptability measure is a number that summarizes information on monetary outcomes of a given position in various scenarios, and that, depending on context, may be interpreted as a capital requirement or as a price. In a multiperiod setting, it is reasonable to require that an acceptability measure should satisfy certain conditions of time consistency. Various notions of time consistency may be considered. Within the framework of coherent risk measures as proposed by Artzner *et al.* [*Math. Fin.*, **9**, 203-228], we establish implication relations between a number of different notions, and we determine how each notion of time consistency is expressed through properties of a representing set of test measures. We propose modifications of the standard Tail-Value-at-Risk measure that have stronger consistency properties than the original.

TAKSAR, M. I. & HUNDERUP, C. L. *The influence of bankruptcy value on optimal risk control for diffusion models with proportional reinsurance.* 311-321. This paper considers the model of a financial entity such as an insurance company whose surplus is governed by a Brownian motion with constant drift and diffusion coefficient. A proportional reinsurance available to the company allows it to reduce its risk by simultaneously reducing the diffusion coefficient and the drift. The uncontrolled dividends are accumulated at the rate proportional to the current value of the surplus. It is assumed that at the time of bankruptcy the company liquidation (*bankruptcy* or *terminal*) value is P. The objective is to find the policy which maximizes the total discounted value of dividends and the terminal value of the company. We find the optimal policy and analyze its dependence on P.

WANG, Y., YANG, Y., WANG, K. & CHENG, D. *Some new equivalent conditions on asymptotics and local asymptotics for random sums and their applications.* 256-266. This paper uses a new method to achieve some new equivalent conditions on asymptotics and local asymptotics for random sums, modifies some results based on an incorrect lemma, and cancels some technical conditions on the existing corresponding results. The newly obtained equivalent conditions are applied to risk theory and infinite divisibility theory, and some new results are derived.

WANG, Z., XIA, J. & ZHANG, L. *Optimal investment for an insurer: the martingale approach.* 322-334. In this paper we apply the martingale approach, which has been widely used in mathematical finance, to investigate the optimal investment problem for an insurer. When the insurer's risk process is modeled by a Lévy process and the capital can be invested in a security market described by the standard Black–Scholes model, closed-form solutions to the problems of mean-variance efficient investment and expected CARA utility maximization are obtained. The effect of the claim process on the mean-variance efficient strategies and frontier is also analyzed.

XIAO, J., HONG, Z. & QIN, C. *The constant elasticity of variance (CEV) model and the Legendre transform–dual solution for annuity contracts.* 302-310. The paper focuses on the

constant elasticity of variance (CEV) model for studying a defined-contribution pension plan where benefits are paid by annuity. It also presents the process by which the Legendre transform and dual theory can be applied to find an optimal investment policy for a participant's whole life in the pension plan. Finally, it reveals two explicit solutions for the logarithm utility function in two different periods (before and after retirement). Hence, the optimal investment strategies in the two periods are obtained.

BAYRAKTAR, E. & YOUNG, V. R. *Hedging life insurance with pure endowments.* 435-444. We extend the work of Milevsky *et al.* [Milevsky, M. A., Promislow, S. D. & Young, V. R., 2005. Financial valuation of mortality risk via the instantaneous Sharpe ratio (preprint)], and Young [Young, V. R., 2006. Pricing life insurance under stochastic mortality via the instantaneous Sharpe ratio (preprint)], by pricing life insurance and pure endowments together. We assume that the company issuing the life insurance and pure endowment contracts requires compensation for their mortality risk in the form of a pre-specified instantaneous Sharpe ratio. We show that the price $P^{m,n}$ for m life insurances and n pure endowments is less than the sum of the price $P^{m,0}$ for m life insurances and the price $P^{0,n}$ for n pure endowments. Thereby, pure endowment contracts serve as a hedge against the (stochastic) mortality risk inherent in life insurance, and vice versa.

CHARPENTIER, A. & SEGERS, J. *Lower tail dependence for Archimedean copulas: characterizations and pitfalls.* 525-532. Tail dependence copulas provide a natural perspective from which one can study the dependence in the tail of a multivariate distribution. For Archimedean copulas with continuously differentiable generators, regular variation of the generator near the origin is known to be closely connected to convergence of the lower tail dependence copulas to the Clayton copula. In this paper, these characterizations are refined and extended to the case of generators which are not necessarily continuously differentiable. Moreover, a counterexample is constructed showing that even if the generator of a strict Archimedean copula is continuously differentiable and slowly varying at the origin, then the lower tail dependence copulas still do not need to converge to the independent copula.

CHEN, Y. Q. & NG, K. W. *The ruin probability of the renewal model with constant interest force and negatively dependent heavy-tailed claims.* 415-423. Recently, Tang [*SAJ*, 2005, 1-5] obtained a simple asymptotic formula for the ruin probability of the renewal risk model with constant interest force and regularly varying tailed claims. In this paper, we use a completely different approach to extend Tang's result to the case in which the claims are pairwise negatively dependent and extended regularly varying tailed.

DA ROCHA NEEVES, C. & MIGON, H. S. *Bayesian graduation of mortality rates: an application to reserve evaluation.* 424-434. This paper presents Bayesian graduation models of mortality rates, using Markov chain Monte Carlo (MCMC) techniques. Graduated annual death probabilities are estimated through the predictive distribution of the number of deaths, which is assumed to follow a Poisson process, considering that all individuals in the same age class die independently and with the same probability. The resulting mortality tables are formulated through dynamic Bayesian models. Calculation of adequate reserve levels is exemplified, via MCMC, making use of the value at risk concept, demonstrating the importance of using "true" observed mortality figures for the population exposed to risk in determining the survival coverage rate.

FROSTIG, E., ZAKS, Y. & LEVIKSON, B. *Optimal pricing for a heterogeneous portfolio for a given risk factor and convex distance measure.* 459-467. Consider a portfolio containing heterogeneous risks. The premiums of the policyholders might not cover the amount of the payments which an insurance company pays the policyholders. When setting the premium, this risk has to be taken into consideration. On the other hand the premium that the insured pays has to be fair. This fairness is measured by a function of the difference between the risk and the premium paid — we call this function a distance function. For a given small probability of insolvency, we find the premium for each class, such that the distance function is minimized. Next we formulate and solve the dual problem, which is minimizing the insolvency probability under the constraint that the distance function does not exceed a given level. This paper generalizes a previous paper [Zaks *et al.*, *Astin Bull.*, **36**(1)] where only a square distance function was considered.

KIM, B. & KIM, H.-S. *Moments of claims in a Markovian environment.* 485-497. This paper considers discounted aggregate claims when the claim rates and sizes fluctuate according to the state of the risk business. We provide a system of differential equations for the Laplace–Stieltjes transform of the distribution of discounted aggregate claims under this assumption. Using the differential equations, we present the first two moments of discounted aggregate claims in a Markovian environment. We also derive simple expressions for the moments of discounted aggregate claims when the Markovian environment has two states. Numerical examples are illustrated when the claim sizes are specified.

KLEINOW, T. & WILLDER, M. *The effect of management discretion on hedging and fair valuation of participating policies with maturity guarantees.* 445-458. In this paper we consider how an insurer should invest in order to hedge the maturity guarantees inherent in participating policies. Many papers have considered the case where the guarantee is increased each year according to the performance of an exogenously given reference portfolio subject to some guaranteed rate. However, in this paper we will consider the more realistic case whereby the reference portfolio is replaced by the insurer's own investments which are controlled completely at the discretion of the insurer's management. Hence in our case any change in the insurer's investment strategy leads to a change in the underlying value process of the participating contract. We use a binomial tree model to show how this risk can be hedged, and hence calculate the fair value of the contract at the outset.

KOCH, I. & DE SCHEPPER, A. *An application of comonotonicity and convex ordering to present values with truncated stochastic interest rates.* 386-402. A subject often recurring in recent financial and actuarial research is the investigation of present value functions with stochastic interest rates. Only in the case of uncomplicated payment streams and rather basic interest rate models is an exact analytical result for the distribution function available. In the present contribution, we introduce the concept of truncated stochastic interest rates, useful to adapt general stochastic models to specific financial requirements, and we show how to obtain analytical results for bounds for the present value. We elaborate our method in extension for the Hull and White model and related models. We illustrate the accuracy of the approximations graphically, and we use the bounds to estimate the Value-at-Risk.

LEIPUS, R. & ŠIAULYS, J. *Asymptotic behaviour of the finite-time ruin probability under subexponential claim sizes.* 498-508. The paper deals with the Sparre Andersen risk model. We study the tail behaviour of the finite-time ruin probability, $\Psi(x, t)$, in the case of subexponential claim sizes as initial risk reserve x tends to infinity. The asymptotic formula holds uniformly for t in a corresponding region and reestablishes a formula of Tang [Tang, Q., 2004a. Asymptotics for the finite time ruin probability in the renewal model with consistent variation. *Stochastic Models*, **20**, 281-297] obtained for the class of claim distributions having consistent variation.

Roos, B. *On variational bounds in the compound Poisson approximation of the individual risk model.* 403-414. We present new upper bounds for the total variation distance between the aggregate claims distribution in the individual risk model and a suitable compound Poisson distribution. It turns out that the bounds are generally valid and contain so-called magic factors. Higher-order approximations, including the signed Kornya–Presman measures, are also investigated. In contrast to results of a previous paper by the author, the results do not depend on a joint decomposition of the individual claim amount distributions. Further, we do not need to assume the finiteness of moments.

Wan, N. *Dividend payments with a threshold strategy in the compound Poisson risk model perturbed by diffusion.* 509-523. In the absence of dividends, the surplus of an insurance company is modelled by a compound Poisson process perturbed by diffusion. Dividends are paid at a constant rate whenever the modified surplus is above the threshold, otherwise no dividends are paid. Two integro-differential equations for the expected discounted dividend payments prior to ruin are derived and closed-form solutions are given. Accordingly, the Gerber–Shiu expected discounted penalty function and some ruin related functionals, the probability of ultimate ruin, the time of ruin and the surplus before ruin and the deficit at ruin, are considered and their analytic expressions are given by general solution formulas. Finally the moment-generating function of the total discounted dividends until ruin is discussed.

Willemse, W. J. & Kaas, R. *Rational reconstruction of frailty-based mortality models by a generalisation of Gompertz' law of mortality.* 468-484. A generalisation of Gompertz' distribution is proposed, and it is shown that continuous heterogeneous mortality models with Gamma distributed frailty have lifetime random variables distributed as the difference of two such generalised Gompertz random variables. With this result, limitations of existing frailty-based mortality models are identified. The approach taken in this paper allows the frailty distribution to be interpreted as a lifetime reduction distribution and enables application of heterogeneous survival models with a stronger relation to empirically identifiable concepts.

Wong, H. Y. & Chan, C. M. *Lookback options and dynamic fund protection under multiscale stochastic volatility.* 357-385. This paper investigates the valuation of lookback options and dynamic fund protection under a multiscale stochastic volatility model. The underlying asset price is assumed to follow a geometric Brownian motion with a volatility that is driven by two stochastic processes with one persistent factor and one fast mean-reverting factor. Semi-analytical pricing formulas for lookback options are derived by means of a multiscale asymptotic technique. The effects of stochastic volatility on options with lookback payoffs are examined. By calibrating effective parameters from the volatility smile of vanilla options, our model improves the valuation of options with the lookback feature. We also develop a model-independent parity relation between the price functions of dynamic fund protection and quanto lookback options. This enables us to assess the effect of multiscale stochastic volatility on the price of dynamic fund protection.

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Brockett, P. L., Golden, L. L., Jang, J. & Yang, C. *A comparison of neural network, statistical methods, and variable choice for life insurers' financial distress prediction.* 397-419.

This study examines the effect of the statistical/mathematical model selected and the variable set considered on the ability to identify financially troubled life insurers. Models considered are two artificial neural network methods (back-propagation and learning vector quantization (LVQ)) and two more standard statistical methods (multiple discriminant analysis and logistic regression analysis). The variable sets considered are the insurance regulatory information system (IRIS) variables, the financial analysis solvency tracking (FAST) variables, and Texas early warning information system (EWIS) variables, and a data set consisting of twenty-two variables selected by us in conjunction with the research staff at TDI and a review of the insolvency prediction literature. The results show that the back-propagation (BP) and LVQ outperform the traditional statistical approaches for all four variable sets with a consistent superiority across the two different evaluation criteria (total misclassification cost and resubstitution risk criteria), and that the twenty-two variables and the Texas EWIS variable sets are more efficient than the IRIS and the FAST variable sets for identification of financially troubled life insurers in most comparisons.

CUMMINS, J. D. & DOHERTY, N. A. *The economics of insurance intermediaries.* 359-396. This article analyzes the economic functions of independent insurance intermediaries (brokers and independent agents), focusing on the commercial property–casualty insurance market. The article investigates the functions performed by intermediaries, the competitiveness of the market, the compensation arrangements for intermediaries, and the process by which policies are placed with insurers. Insurance intermediaries are essentially market makers who match the insurance needs of policyholders with insurers who have the capability of meeting those needs. Intermediary compensation comprises premium-based commissions, expressed as a percentage of the premium paid, and contingent commissions based on the profitability, persistency, and/or volume of the business placed with the insurer. Empirical evidence is provided that premium-based and contingent commissions are passed on to policyholders in the premium. However, contingent commissions can enhance competitive bidding by aligning the insurer's and the intermediary's interests. This alignment of interests gives insurers more confidence in the selection of risks and thus helps to break the "winner's curse" and encourages insurers to bid more aggressively. Independent intermediaries also help markets operate more efficiently by reducing the information asymmetries between insurers and buyers that can cause adverse selection.

GOLUBIN, A. Y. *Pareto-optimal insurance policies in the models with a premium based on the actuarial value.* 469-487. This article analyzes the problem of designing Pareto-optimal insurance policies when both the insurer and the insured are risk averse and the premium is calculated as a function of the actuarial value of the insurer's risk. Two models are considered: in the first, the set of admissible policies is constrained by a given size of the premium; in the second, the premium size is not constrained so that it varies with the actuarial value of a policy chosen by the agents. For both cases a characterization of the Pareto-optimal policies is derived. The corresponding optimality equations for the Pareto-optimal policies are obtained and compared with the results on the classical risk exchange model.

PITREBOIS, S., DENUIT, M. & WALHIN, J. F. *Multi-event bonus-malus scales.* 517-528. This article is devoted to the design of bonus-malus scales involving different types of claims. Typically, claims with or without bodily injuries, or claims with full or partial liability of the insured driver, are distinguished and entail different penalties. Under mild assumptions, claim severities can also be taken into account in this way. Numerical illustrations enhance the interest of the approach.

SCHILLER, J. *The impact of insurance fraud detection systems.* 421-438. This article analyzes the impact of detection systems in an insurance fraud context. In a noncommitment Costly State Verification setting insurers can only detect fraudulent claims by performing costly audits, and

policyholders are overcompensated by the optimal insurance contract. We show that auditing becomes more effective and overcompensation can be reduced, when insurers are able to condition their audits on the information provided by detection systems.

Van Kleef, R. C., Van de Ven, W. P. M. M. & Van Vliet, R. C. J. A. *A voluntary deductible in social health insurance with risk equalization: "community-rated or risk-rated premium rebate?"* 529-550. On January 1, 2006 a new mandatory basic health insurance will be introduced in the Netherlands. One aspect of the new scheme is that the insured can choose to have a deductible. This option should increase the individual responsibility and reduce moral hazard. In the new scheme, a risk equalization system is aimed at avoiding preferred risk selection and insolvency of insurance companies with a relatively high-risk pool. A crucial issue with respect to a voluntary deductible in this type of social health insurance is whether the premium rebate should be community rated or risk rated. The Dutch government has chosen the former, which means that the premium rebate will be independent of health status and risk. Our analysis shows that, in a situation with "accurate" risk equalization, a community-rated premium rebate could lead to an adverse selection spiral. Over time, this spiral results in none of the insured taking a deductible and thus no reduction in moral hazard.

Vidal-Meliá, C., Domínguez-Fabián, I. & Devesa-Carpio, J. E. *Subjective economic risk to beneficiaries in notional defined contribution accounts.* 489-515. This article aims to quantify the aggregate subjective economic risk to which beneficiaries would be exposed if a retirement pension system based on notional account philosophy were introduced. We use scenario generation techniques to make projections of the factors that determine the real expected internal rate of return (IRR) and the expected replacement rate (RR) for the beneficiary according to six retirement formulae based on the most widely accepted rates or indices. We then apply the model to the case of Spain. Our projections are based on Herce and Alonso's macroeconomic scenario 2000-2050 (2000) and include information about the past performance of the indices and the time period the forecast is to cover. The results of the IRR calculation — average value, standard deviation, and value-at-risk (VaR) — are analyzed both in objective terms and for different degrees of participants' risk aversion.

Viswanathan, K. S. *The pricing of insurer demutualization initial public offerings.* 439-468. This article examines the pricing of the initial public offerings (IPOs) that follow insurance company demutualizations. Insurers that convert from mutual to stock form typically cite the need for capital as a key motivation. Given that capital adequacy is a primary regulatory objective for insurers, one would expect that for a given number of shares to be sold, these firms would price their offerings to maximize proceeds. However, the vast literature on IPO pricing suggests various theories as to why it may be in the issuing firm's best interest to underprice its offering. By examining the initial and long-run stock returns for these conversion IPOs, the existence and degree of underpricing, as characterized by large initial returns, can be determined. It is observed that on average demutualization insurer IPOs post significantly higher first-day returns than nondemutualization insurer IPOs. These gains would accrue to the initial investors and to those policyholders who receive compensation in the form of shares in the newly created stock insurer. Attractive returns are sustained for both groups of insurers during the first few years after IPO.

Blake, D., Cairns, A. J. G., Dowd, K. & MacMinn, R. D. *Longevity bonds: financial engineering, valuation, and hedging.* 647-672. This article examines the main characteristics of

longevity bonds (LBs) and shows that they can take a large variety of forms which can vary enormously in their sensitivities to longevity shocks. We examine different ways of financially engineering LBs and consider problems arising from the dearth of ultra-long government bonds and the choice of the reference population index. The article also looks at valuation issues in an incomplete markets context and finishes with an examination of how LBs can be used as a risk management tool for hedging longevity risks.

BOARDMAN, T. *Annuitization lessons from the UK: money-back annuities and other developments.* 633-646.

BROWN, J. R. & ORSZAG, P. R. *The political economy of government-issued longevity bonds.* 611-631. This article explores the trade-offs associated with government issuance of longevity bonds as a way of stimulating private annuity supply in the presence of aggregate mortality risk. We provide new calculations suggesting a 5 per cent chance that aggregate mortality risk could ex post raise annuity costs for private insurers by as much as 5-10 percentage points, with the most likely effect based on historical patterns toward the lower end of that range. While we suspect that aggregate mortality risk does exert some upward pressure on annuity prices, evidence from private market pricing suggests that, to the extent that private insurers are accurately pricing this risk, the effect is less than 5 percentage points. We discuss ways that the private market can spread this risk, while emphasizing that the government has the unique ability to spread aggregate risk across generations. We note factors that might hamper such an efficient allocation of risk, including potential political incentives for the government to shift more than the optimal amount of risk onto future generations, and the possibility that government fiscal policy might allocate risk less efficiently within each generation than would private markets. We also discuss how large-scale longevity bond issuance might affect government borrowing costs, as well as political economy aspects of how the proceeds from such a bond issuance might be used.

CAIRNS, A. J. G., BLAKE, D. & DOWD, K. *A two-factor model for stochastic mortality with parameter uncertainty: theory and calibration.* 687-718. In this article, we consider the evolution of the post-age-60 mortality curve in the United Kingdom and its impact on the pricing of the risk associated with aggregate mortality improvements over time: so-called longevity risk. We introduce a two-factor stochastic model for the development of this curve through time. The first factor affects mortality-rate dynamics at all ages in the same way, whereas the second factor affects mortality-rate dynamics at higher ages much more than at lower ages. The article then examines the pricing of longevity bonds with different terms to maturity referenced to different cohorts. We find that longevity risk over relatively short time horizons is very low, but at horizons in excess of ten years it begins to pick up very rapidly. A key component of the article is the proposal and development of a method for calculating the market risk-adjusted price of a longevity bond. The proposed adjustment includes not just an allowance for the underlying stochastic mortality, but also makes an allowance for parameter risk. We utilize the pricing information contained in the November 2004 European Investment Bank longevity bond to make inferences about the likely market prices of the risks in the model. Based on these, we investigate how future issues might be priced to ensure an absence of arbitrage between bonds with different characteristics.

COX, S. H., LIN, Y. & WANG, S. *Multivariate exponential tilting and pricing implications for mortality securitization.* 719-736. Normalized exponential tilting is an extension of classical theories, including the Capital Asset Pricing Model (CAPM) and the Black–Merton–Scholes model, to price risks with general-shaped distributions. The need for changing multivariate probability measures arises in pricing contingent claims on multiple underlying assets or liabilities. In this article, we apply it to valuation of mortality-based securities written on mortality indices of several countries. We show how to use multivariate exponential tilting to

price the first pure mortality security, the Swiss Re bond. The same technique can be applied in other mortality securitization pricing.

MILEVSKY, M. A., PROMISLOW, S. D. & YOUNG, V. R. *Killing the law of large numbers: mortality risk premiums and the Sharpe ratio.* 673-686. We provide an overview of how the law of large numbers breaks down when pricing life-contingent claims under stochastic as opposed to deterministic mortality (probability, hazard) rates. In a stylized situation, we derive the limiting per-policy risk and show that it goes to a non-zero constant. This is in contrast to the classical situation when the underlying mortality decrements are known with certainty, per policy risk goes to zero. We decompose the standard deviation per policy into systematic and non-systematic components, akin to the analysis of individual stock (equity) risk in a Markowitz portfolio framework. Finally, we draw upon the financial analogy of the Sharpe Ratio to develop a premium pricing methodology under aggregate mortality risk.

STALLARD, E. *Demographic issues in longevity risk analysis.* 575-609. Fundamental to the modeling of longevity risk is the specification of the assumptions used in demographic forecasting models that are designed to project past experience into future years, with or without modifications based on expert opinion about influential factors not represented in the historical data. Stochastic forecasts are required to explicitly quantify the uncertainty of forecasted cohort survival functions, including uncertainty due to process variance, parameter errors, and model misspecification errors. Current applications typically ignore the latter two sources although the potential impact of model misspecification errors is substantial. Such errors arise from a lack of understanding of the nature and causes of historical changes in longevity and the implications of these factors for the future. This article reviews the literature on the nature and causes of historical changes in longevity and recent efforts at deterministic and stochastic forecasting based on these data. The review reveals that plausible alternative sets of forecasting assumptions have been derived from the same sets of historical data, implying that further methodological development will be needed to integrate the various assumptions into a single coherent forecasting model. Illustrative calculations based on existing forecasts indicate that the ranges of uncertainty for older cohorts' survival functions will be at a manageable level. Uncertainty ranges for younger cohorts will be larger and the need for greater precision will likely motivate further model development.

TURNER, A. *Pension, risks, and capital markets.* 559-574.

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NORTH AMERICAN ACTUARIAL JOURNAL

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ANTONIO, K., BEIRLANT, J., HOEDEMAKERS, T. & VERLAAK, R. *Lognormal mixed models for reported claims reserves.* 30-48. Traditional claims-reserving techniques are based on so-called run-off triangles containing aggregate claim figures. Such a triangle provides a summary of an underlying data set with individual claim figures. This contribution explores the interpretation of the available individual data in the framework of longitudinal data analysis. Making use of the theory of linear mixed models, a flexible model for loss reserving is built. Whereas traditional claims-reserving techniques don't lead directly to predictions for individual claims, the mixed model enables such predictions on a sound statistical basis with, for example, confidence regions. Both a likelihood-based as well as a Bayesian approach are considered. In

the frequentist approach, expressions for the mean squared error of prediction of an individual claim reserve, origin year reserves, and the total reserve are derived. Using MCMC techniques, the Bayesian approach allows simulation from the complete predictive distribution of the reserves and the calculation of various risk measures. The paper ends with an illustration of the suggested techniques on a data set from practice, consisting of Belgian automotive third-party liability claims. The results for the mixed-model analysis are compared with those obtained from traditional claims-reserving techniques for run-off triangles. For the data under consideration, the lognormal mixed model fits the observed individual data well. It leads to individual predictions comparable to those obtained by applying chain-ladder development factors to individual data. Concerning the predictive power on the aggregate level, the mixed model leads to reasonable predictions and performs comparable to and often better than the stochastic chain ladder for aggregate data.

FINKELSTEIN, M., TUCKER, H. G. & VEEH, J. A. *Pareto tail index estimation revisited.* 1-10. An estimator of the tail index of a Pareto distribution is given that is based on the use of the probability integral transform. This new estimator provides performance that is comparable to the best robust estimators, while retaining conceptual and computational simplicity. A tuning parameter in the new estimator can be adjusted to control the tradeoff between robustness and efficiency. The method used to compute the estimator also can be used to find a confidence interval for the tail index that is guaranteed to have the nominal confidence level for any given sample size. Guidelines for the use of the new estimator are provided.

GERRARD, R., HABERMAN, S. & VIGNA, E. *The management of decumulation risks in a defined contribution pension plan.* 84-110. The aim of the paper is to lay the theoretical foundations for the construction of a flexible tool that can be used by pensioners to find optimal investment and consumption choices in the distribution phase of a defined contribution pension plan. The investment/consumption plan is adopted until the time of compulsory annuitization, taking into account the possibility of earlier death. The effect of the bequest motive and the desire to buy a higher annuity than the one purchasable at retirement are included in the objective function. The mathematical tools provided by dynamic programming techniques are applied to find closed-form solutions: numerical examples are also presented. In the model, the tradeoff between the different desires of the individual regarding consumption and final annuity can be dealt with by choosing appropriate weights for these factors in the setting of the problem. Conclusions are twofold. First, we find that there is a natural time-varying target for the size of the fund, which acts as a sort of safety level for the needs of the pensioner. Second, the personal preferences of the pensioner can be translated into optimal choices, which in turn affect the distribution of the consumption path and of the final annuity.

GOULET, V., FORGUES, A. & LU, J. *Credibility for severity revisited.* 49-62. It is basic actuarial knowledge that the pure premium of an insurance contract can be written as the product of the expected claim number and the expected claim amount. Actuaries use credibility theory to incorporate the contract's individual experience into this calculation in a statistically optimal way. For many years, however, the use of credibility was limited to the frequency component. Starting with the paper by Hewitt (1971), there have been various suggestions as to how credibility theory also can be applied to the severity component of the pure premium. The latest such suggestion, Frees (2003), revived the interest in the problem. In this paper, we review four different formulas incorporating frequency and severity into credibility calculations. We then compare by simulation which one is most accurate at predicting a contract's next-year outcome. It is found that the classical formula of Bühlmann (1967) is as good as the other ones in many cases. Alternatives, however, may offer easier analysis of the separate effects of frequency and severity on the premium. We also show that all the formulas reviewed in this paper stem from the same minimization problem, and we present a general, integrated, solution. At the same time, we complete Gerber (1972) by providing a proof to the main result of this paper and by stating required additional assumptions.

KLUGMAN, S. A. & RIOUX, J. *Toward a unified approach to fitting loss models.* 63-83. There are four components to fitting models: selecting a set of candidate distributions, estimating parameters, evaluating the appropriateness of a model, and determining which member fits best. It is important to have the candidate set be small to avoid overfitting. Finite mixture models using a small number of base distributions provide an ideal set. Because actuaries fit models for a variety of situations, particularly with regard to data modifications, it is useful to have a single approach. Although not optimal or exact for a particular model or data structure, the method should be reasonable for most all settings. Such a method is proposed in this article.

WALLACE, M. *The problem with current accounting: A critique of SFAS 115 and SFAS 133 using an equity-indexed annuity example.* 11-29. In recent decades, as the use of derivatives by financial institutions has expanded, the shortcomings of historical cost accounting approaches have become increasingly apparent. Since derivatives can create large exposures to risk that go unnoticed under historical standards, the accounting industry has focused on how to change the standards so that these risks are reflected appropriately in a company's accounting statements. New standards such as SFAS 115 and SFAS 133 have been adopted in part to achieve this goal. However, both of these standards use a piecemeal approach to risk measurement that may be adding to the problem rather than creating a solution. This paper will use a simple equity-indexed annuity to illustrate the problem with historical cost accounting and with the standards that have been adopted to correct it. The paper then argues that the only legitimate means of reflecting risk properly on a company's accounting statements is to adopt full fair value accounting for all assets and liabilities on the company's books.

CAI, J. & XU, C. *On the decomposition of the ruin probability for a jump-diffusion surplus process compounded by a geometric Brownian motion.* 120-132. If one assumes that the surplus of an insurer follows a jump-diffusion process and the insurer would invest its surplus in a risky asset, whose prices are modeled by a geometric Brownian motion, the resulting surplus for the insurer is called a jump-diffusion surplus process compounded by a geometric Brownian motion. In this resulting surplus process, ruin may be caused by a claim or oscillation. We decompose the ruin probability in the resulting surplus process into the sum of two ruin probabilities: the probability that ruin is caused by a claim, and the probability that ruin is caused by oscillation. Integro-differential equations for these ruin probabilities are derived. When claim sizes are exponentially distributed, asymptotical formulas of the ruin probabilities are derived from the integro-differential equations, and it is shown that all three ruin probabilities are asymptotical power functions with the same orders and that the orders of the power functions are determined by the drift and volatility parameters of the geometric Brownian motion. It is known that the ruin probability for a jump-diffusion surplus process is an asymptotical exponential function when claim sizes are exponentially distributed. The results of this paper further confirm that risky investments for an insurer are dangerous in the sense that either ruin is certain or the ruin probabilities are asymptotical power functions, not asymptotical exponential functions, when claim sizes are exponentially distributed.

CAI, J., GERBER, H. U. & YANG, H. *Optimal dividends in an Ornstein-Uhlenbeck type model with credit and debit interest.* 94-119. In the absence of investment and dividend payments, the surplus is modeled by a Brownian motion. But now assume that the surplus earns investment income at a constant rate of credit interest. Dividends are paid to the shareholders according

to a barrier strategy. It is shown how the expected discounted value of the dividends and the optimal dividend barrier can be calculated; Kummer's confluent hypergeometric differential equation plays a key role in this context. An alternative assumption is that business can go on after ruin, as long as it is profitable. When the surplus is negative, a higher rate of debit interest is applied. Several numerical examples document the influence of the parameters on the optimal dividend strategy.

DE JONG, P. *Forecasting runoff triangles.* 28-38. This paper deals with the methodology of liability forecasting using the runoff triangle data. Techniques are based on time series models and methods that facilitate the calculation of forecast distributions and the assessment of model fit. The models deal with correlation within triangles. Correlations are critical to proper reserving. The output of the methodology is the complete shape of the liability distribution. Methods are applied to a well-known runoff triangle and results compared to those from previous studies.

GERBER, H. U. & SHIU, E. S. W. *On optimal dividend strategies in the compound Poisson model.* 76-93. The optimal dividend problem goes back to a paper that Bruno De Finetti presented to the International Congress of Actuaries in New York (1975). For a stock company that pays dividends to its shareholders, what is the strategy that maximizes the expectation of the discounted dividends (until possible ruin)? Jeanblanc-Picque and Shiryaev (1995) and Asmussen and Taksar (1997) solved the problem in the Brownian motion model, when a ceiling is imposed for the dividend rate. Here we study the problem with the Brownian motion generalized to a compound Poisson process. In particular, we derive a rule for deciding plowback and dividend payout, which is the key issue in corporate finance.

SCHIRMACHER, E. & FELDBLUM, S. *Financial pricing models for property-casualty insurance products: retrospective analysis.* 1-27. Insurance companies sell products without knowing what the ultimate costs will be. Moreover, in many cases the time between receipt of premium and the payment of claims could span many years. To manage the risk that companies have assumed, they need special tools to monitor the ongoing profitability of their products. In this paper we introduce one tool — retrospective analysis — to monitor profitability. Retrospective analysis is mainly concerned with incorporating all available information about a block of business to determine the current estimate of profitability and to attribute any differences to the appropriate sources. To illustrate the methods used in retrospective analysis we present a simplified but complete model of an insurance policy. We also introduce two economic accounting systems — the net present value system and the internal rate of return system — and analyze how emerging experience alters the profitability of our insurance policy. We pay special attention to quantifying the changes under both accounting systems.

SHERRIS, M. & VAN DER HOEK, J. *Capital allocation in insurance: Economic capital and the allocation of the default option value.* 39-61. The determination and allocation of economic capital is important for pricing, risk management, and related insurer financial decision making. This paper considers the allocation of economic capital to lines of business in insurance. We show how to derive closed-form results for the complete markets, arbitrage - free allocation of the insurer default option value, or insolvency exchange option, to lines of business for insurer balance sheet. We assume that individual lines of business and the surplus ratio are joint log-normal although the method we adopt allows other assumptions. The allocation of the default option value is required for fair pricing in the multiline insurer. We discuss and illustrate other methods of capital allocation, including Myers-Read, and give numerical examples for the capital allocation of the default option value based on explicit payoffs by line.

SIU, T. K. *Option pricing under autoregressive random variable models.* 62-75. The

autoregressive random variance (ARV) model introduced by Taylor (1980, 1982, 1986) is a popular version of stochastic volatility (SV) models and a discrete-time simplification of the continuous-time diffusion SV models. This paper introduces a valuation model for options under a discrete-time ARV model with general stock and volatility innovations. It employs the discrete time version of the Esscher transform to determine an equivalent martingale measure under an incomplete market. Various parametric cases of the ARV models, are considered, namely, the lognormal ARV models, the jump-type Poisson ARV models, and the gamma ARV models, and more explicit pricing formulas of a European call option under these parametric cases are provided. A Monte Carlo experiment for some parametric cases is also conducted.

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Colquitt, L. L., Hoyt, R. E. & McCullough, K. A. *The impact of asbestos and environmental reserves increases on shareholder wealth.* 17-31. Between 1992 and 2001 significant reserves increase announcements were made by several major property/liability insurers. These reserves increases were for the purpose of recognizing expected asbestos and environmental (A&E) liability. Although most analysts agree that U.S. insurers are underreserved for asbestos and environmental liability, how the market reacts to an insurer's announcement of an increase in these reserves has not been analyzed. An insurer that is significantly underreserved is likely to be viewed by the market as lacking financial stability for the long term. However, when a company increases its reserves, there is a charge to income and a reduction in capital. If surplus is diminished sufficiently as a result of the increased reserving, regulatory attention and eroding shareholder and market confidence could result as well. By calculating the sample insurers' cumulative abnormal returns surrounding the largest asbestos and environmental reserves increase announcements made between 1992 and 2001, the study estimates and documents the market's reaction to these reserves increase announcements. We further explore the potential impact of additional asbestos and environmental liability exposure reporting requirements. Starting with 1995 statutory annual accounting statements, Footnote 24 required additional reporting by insurers of their asbestos and environmental liability exposure (1995 statements were publicly available by the end of the first quarter of 1996). When looking at reserves increase announcements prior to this additional reporting requirement, we find that most insurers announcing large increases in asbestos and environmental reserves prior to 1996 experience a significant reduction in stock price in the days surrounding their announcement. However, consistent with the notion that the additional accounting disclosure requirements after 1995 (Footnote 24) provide valuable information on insurers' exposure, we find that the announcement of A&E reserves increases after 1995 had no statistically significant effect on the market value of announcing insurers.

de Alba, E. *Claims reserving when there are negative values in the runoff triangle: Bayesian analysis using the three-parameter log-normal distribution.* 45-59. This paper is concerned with the situation that occurs in claims reserving when there are negative values in the development triangle of incremental claim amounts. Typically these negative values will be the result of salvage recoveries, payments from third parties, total or partial cancellation of outstanding claims due to initial overestimation of the loss or to a possible favorable jury decision in favor of the insurer, rejection by the insurer, or just plain errors. Some of the traditional methods of claims reserving, such as the chain-ladder technique, may produce estimates of the reserves even when there are negative values. However, many methods can break down in the presence of enough (in number and/or size) negative incremental claims if certain constraints are not met. Historically the chain-ladder method has been used as a gold standard

(benchmark) because of its generalized use and ease of application. A method that improves on the gold standard is one that can handle situations where there are many negative incremental claims and/ or some of these are large. This paper presents a Bayesian model to consider negative incremental values, based on a three-parameter log-normal distribution. The model presented here allows the actuary to provide point estimates and measures of dispersion, as well as the complete distribution for outstanding claims from which the reserves can be derived. It is concluded that the method has a clear advantage over other existing methods. A Markov chain Monte Carlo simulation is applied using the package WinBUGS.

GERBER, H. U. & SHIU, E. S. W. *On the merger of two companies.* 60-67. This paper examines the merger of two stock companies under the premise, due to Bruno de Finetti, that the companies pay out dividends to their shareholders in such a way so as to maximize the expectation of the discounted dividends until (possible) ruin or insolvency. The aggregate net income streams of the two companies are modeled by a bivariate Wiener process. Explicit results are presented. In particular, it is shown that if for each company the product of the valuation force of interest and the square of the coefficient of variation of its aggregate net income process is less than 6.87%, the merger of the two companies would result in a gain.

KESTENBAUM, B. M. & FERGUSON, B. R. *The number of centenarians in the United States on January 1, 1990, 2000, and 2010 based on improved medicare data.* 1-6.

ORTH, B. J. *Managing longevity risk in U.S. retirement plans through mandatory annuitization.* 32-44. Over the past 20 years, the United States has experienced a profound shift in the way that employment-based retirement benefits are delivered to workers. The traditional life annuity from a defined benefit (DB) plan has been largely replaced by lump sums from defined contribution (DC) plans. Along with investment risk, American workers are bearing a larger share of the longevity risk inherent in all retirement systems. As Americans benefit from longer lives, they are facing a harsh reality: will their retirement assets last long enough? Workers have embraced the flexibility offered by the widely available, and very popular, 401(k) plan. Often described as a do-it-yourself retirement program, these plans have allowed workers to accumulate significant levels of retirement savings. Employers like them too because they are less costly and easier to administer than traditional DB plans. Will this enthusiasm wane as baby boomers retire and face the daunting task of managing this pool of assets over retirements that can span 30 or 40 years or longer? Retirees have been reluctant to annuitize their assets for many reasons, and the annuity market in the United States is relatively small. The shift from DB to DC plans has left a majority of workers with only one form of annuitized benefit: Social Security. Yet life annuities offer the best method of managing longevity risk, both for the individual and for society. This paper suggests a possible framework for the mandatory annuitization of U.S. retirement savings, considers the experience of other countries, and analyzes the advantages and disadvantages of mandatory annuitization. If properly structured, it is possible that the benefits of an annuity mandate would outweigh the drawbacks.

PINKHAM, C. A., CUMMING, M. E. & MINUK, H. *The metabolic syndrome and all-cause mortality in an insured lives population.* 7-16. Metabolic syndrome and its association with mortality have not been studied in insured lives populations. The Swiss Re Study evaluated metabolic syndrome prevalence and associated mortality from all causes and circulatory disease in a cohort of 35,470 predominantly healthy individuals, aged 18-83 years, who were issued life insurance policies between 1986 and 1997. Metabolic syndrome was defined using the National Cholesterol Education Program (NCEP) Expert Panel Adult Treatment Panel (ATP) III guidelines. The NCEP obesity criteria were modified with a prediction equation using body mass index, gender, and age substituted for waist circumference. Adjustments also were made for nonfasting triglyceride and blood glucose values. Risk ratios for policyholders

identified with metabolic syndrome were 1.16 ($P = .156$) for mortality from all causes and 1.45 ($P = .080$) for mortality from circulatory disease compared with individuals without the syndrome. Risk was proportional to the number of components, or score, of the metabolic syndrome present. Risk ratios for metabolic syndrome score were 1.14 ($P < .001$) for mortality from all causes and 1.38 ($P < .001$) for mortality from circulatory disease compared with individuals without metabolic syndrome factors. In both all-cause and circulatory death models, relative risk was highest for the blood pressure risk factor. Based on a modified NCEP definition, increased mortality risk is associated with metabolic syndrome in an insured lives cohort and has life insurance mortality pricing implications.

BACHLER, R., DUNCAN, I. & JUSTER, I. *A comparative analysis of chronic and nonchronic insured commercial member cost trends.* 76-89. Disease management (DM) is increasingly encountered in health plans and employer groups as a health care intervention targeted to individuals with chronic diseases ("Chronics"). To justify the investment by payers in DM, it is important to demonstrate beneficial clinical and financial outcomes. In the absence of randomized control studies, financial results are often estimated in a pre/post-study in which the cost of Chronics in the absence of DM can be predicted by their pre-DM year cost (on a per member per month basis) adjusted for the Nonchronic population's cost trend. The assumption made, not previously tested, is that absent DM, the Chronic and Nonchronic trends are identical. We calculated Chronic and Nonchronic trends between 1999 and 2002 and compared them under different assumptions regarding identification of chronic disease and medical services. Qualification for the Chronic group was defined as having coronary artery disease, heart failure, diabetes, asthma, or chronic obstructive lung disease. Our base case used an algorithm that identified a member as Chronic prospectively (that is, from the point of identification forward), with one or more of the chronic conditions. We used a data set of 1.5 million commercially insured members. When Chronic and Nonchronic members are identified and included in the population prospectively, the average three-year trend over the study period for chronic and nonchronic members adjusted for high cost outliers were 4.9% and 13.9%, respectively. Adjusting the population experience for differences in service mix had little effect on the divergence in trends. However, altering the Chronic selection algorithm to eliminate migration between groups (thus classifying a member as always Chronic if identified as Chronic at any point in the four years) caused the trends to converge (Chronics, 16.3%; Nonchronics 17.2%; total 16.0%). Using the original selection algorithm but risk-adjusting the populations annually also caused their trends to converge (Chronics, 12.5%; Nonchronics 11.9%). Finally, applying an annual "requalification" process (in which members who qualify as Chronic in one year but not the next are excluded in the year in which they fail to qualify), we see some, although not complete, convergence of trends. Estimating DM program financial outcomes based on the assumption that absent the program, the Chronic population would have had the same trend as the Nonchronic population can lead to erroneous conclusions. Identification of a Chronic member and the point at which that member is reclassified from one subpopulation to another can significantly affect the observed trends in both subpopulations, implying that great care must be taken over classification and interpretation of the resulting trends, and their use in DM savings calculations. Trends calculated using a prospective identification methodology introduce a bias into estimates of outcomes. We refer to this effect, which has not previously been described or discussed in the literature, as "migration bias." It is critical to understand how trends in a reference population can vary according to selection criteria for disease in the chronic population, service mix, and changes in risk over time.

BALLS, K. G. *Immediate annuity pricing in the presence of unobserved heterogeneity.* 103-116. One of the acknowledged difficulties with pricing immediate annuities is that underwriting the annuitant's life is the exception rather than the rule. In the absence of underwriting, the price paid for a life-contingent annuity is the same for all sales at a given age. This exposes the market (insurance company and potential policyholder alike) to antiselection. The insurance company worries that only the healthiest people choose a life-contingent annuity and therefore adjust mortality accordingly. The potential policyholders worry that they are not being compensated for their relatively poor health and choose not to purchase what would otherwise be a very beneficial product. This paper develops a model of underlying, unobserved health. Health is a state variable that follows a first-order Markov process. An individual reaches the state "death" either by accident from any health state or by progressively declining health state. Health state is one-dimensional, in the sense that health can either "improve" or "deteriorate" by moving farther from or close to the "death" state, respectively. The probability of death in a given year is a function of health state, not of age. Therefore, in this model a person is exactly as old as he or she feels. I first demonstrate that a multistate, ageless Markov model can match the mortality patterns in the common annuity mortality tables. The model is extended to consider several types of mortality improvements: permanent through decreasing probability of deteriorating health, temporary through improved distribution of initial health state, and plateau through the effects of past health improvements. I then construct an economic model of optimal policyholder behavior, assuming that the policyholder either knows his or her health state or has some limited information. The value of mortality risk transfer through purchasing a life-contingent annuity is estimated for each health state under various risk-aversion parameters. Given the economic model for optimal purchasing of annuities, the value of underwriting (limited information about policyholder health state) is demonstrated.

BERNARD, C., LE COURTOIS, O. & QUITTARD-PINON, F. *Development and pricing of a new participating contract.* 179-195. This article designs and prices a new type of participating life insurance contract. Participating contracts are popular in the United States and European countries. They present many different covenants and depend on national regulations. In the present article we design a new type of participating contract very similar to the one considered in other studies, but with the guaranteed rate matching the return of a government bond. We prove that this new type of contract can be valued in closed form when interest rates are stochastic and when the company can default.

CHAN, W.-S. & ZHANG, L. *Direct derivation of finite-time ruin probabilities in the discrete risk model with exponential or geometric claims.* 269-79. Growing research interest has been shown in finite-time ruin probabilities for discrete risk processes, even though the literature is not as extensive as for continuous-time models. The general approach is through the so-called Gerber-Shiu discounted penalty function, obtained for large families of claim severities and discrete risk models. This paper proposes another approach to deriving recursive and explicit formulas for finite-time ruin probabilities with exponential or geometric claim severities. The proposed method, as compared to the general Gerber-Shiu approach, is able to provide simpler derivation and straightforward expressions for these two special families of claims.

DUPUIS, D. J. & JONES, B. L. *Multivariate extreme value theory and its usefulness in understanding risk.* 1-27. This paper gathers recent results in the analysis of multivariate extreme values and illustrates their actuarial application. We review basic and essential background on univariate extreme value theory and stochastic dependence and then provide an introduction to multivariate extreme value theory. We present important concepts for the analysis of multivariate extreme values and collect research results in this area. We draw particular attention to issues related to extremal dependence and show the importance of model selection when fitting an upper tail copula to observed joint exceedances. These ideas are illustrated

on four data sets: loss amount and allocated loss adjustment expense under insurance company indemnity claims, lifetimes of pairs of joint and last survivor annuitants, hurricane losses in two states, and returns on two stocks. In each case the extremal dependence structure has an important financial impact.

GAILLARDETZ, P. & LIN, X. S. *Valuation of equity-linked insurance and annuity products with binomial models.* 117-144. In this paper we develop a valuation method for equity-linked insurance products. We assume that the premium information of term life insurances, pure endowment insurances, and endowment insurances at all maturities is obtainable within a company or from the insurance market. Using a method similar to that of Jarrow and Turnbull (1995), we derive three martingale probability measures associated with these basic insurance products. These measures are age dependent, include an adjustment for the mortality risk, and reproduce the premiums of the respective insurance products. We then extend the martingale measures to include the financial market information using copulas and use them to evaluate equity-linked insurance contracts and equity-indexed annuities in particular. This is different from the traditional approach under which diversification of mortality risk is assumed. A detailed numerical analysis is performed for various existing equity-indexed annuities in the North American market.

GARRIDO, J. & MORALES, M. *On the expected discounted penalty function for Levy risk processes.* 196-218. Dufresne *et al.* (1991) introduced a general risk model defined as the limit of compound Poisson processes. Such a model is either a compound Poisson process itself or a process with an infinite number of small jumps. Later, in a series of now classical papers, the joint distribution of the time of ruin, the surplus before ruin, and the deficit at ruin was studied (Gerber and Shiu 1997, 1998a, 1998b; Gerber and Landry 1998). These works use the classical and the perturbed risk models and hint that the results can be extended to gamma and inverse Gaussian risk processes. In this paper we work out this extension to a generalized risk model driven by a nondecreasing Levy process. Unlike the classical case that models the individual claim size distribution and obtains from it the aggregate claims distribution, here the aggregate claims distribution is known in closed form. It is simply the one-dimensional distribution of a subordinator. Embedded in this wide family of risk models we find the gamma, inverse Gaussian, and generalized inverse Gaussian processes. Expressions for the Gerber-Shiu function are given in some of these special cases, and numerical illustrations are provided.

HARDY, M. R., FREELAND, R. K. & TILL, M. C. *Validation of long-term equity return models for equity-linked guarantees.* 28-47. A number of models have been proposed for the equity return process for equity-linked guarantees, following the introduction of stochastic modeling requirements by the Canadian Institute of Actuaries and the American Academy of Actuaries. In this paper we present some of the models that have become well known and discuss the use of residuals to test the fit. After showing that the use of the static, "actuarial approach" to risk management can result in two models with very similar likelihood and residuals giving very different capital requirements, we propose an extension of the bootstrap to compare all the models and to determine whether the optimistic or pessimistic view of the long-term left tail risk is more consistent with the data. Our context is the determination of capital requirements, so we are concerned in this work with real-world rather than risk neutral processes.

KAISER, T. & BRAZAUSKAS, V. *Interval estimation of actuarial risk measures.* 249-268. This article investigates performance of interval estimators of various actuarial risk measures. We consider the following risk measures: proportional hazards transform (PHT), Wang transform (WT), value-at-risk (VaR), and conditional tail expectation (CTE). Confidence intervals for these measures are constructed by applying nonparametric approaches (empirical and

bootstrap), the strict parametric approach (based on the maximum likelihood estimators), and robust parametric procedures (based on trimmed means). Using Monte Carlo simulations, we compare the average lengths and proportions of coverage (of the true measure) of the intervals under two data-generating scenarios: "clean" data and "contaminated" data. In the "clean" case, data sets are generated by the following (similar shape) parametric families: exponential, Pareto, and lognormal. Parameters of these distributions are selected so that all three families are equally risky with respect to a fixed risk measure. In the "contaminated" case, the "clean" data sets from these distributions are mixed with a small fraction of unusual observations (outliers). It is found that approximate knowledge of the underlying distribution combined with a sufficiently robust estimator (designed for that distribution) yields intervals with satisfactory performance under both scenarios.

LU, Y. & GARRIDO, J. *Regime-switching periodic models for claim counts.* 235-248. We study a Cox risk model that accounts for both seasonal variations and random fluctuations in the claims intensity. This occurs with natural phenomena that evolve in a seasonal environment and affect insurance claims, such as hurricanes. More precisely, we define an intensity process governed by a periodic function with a random peak level. The periodic intensity function follows a deterministic pattern in each short-term period and is illustrated by a beta-type function. A Markov chain with m states, corresponding to different risk levels, is chosen for the level process, yielding a so-called regime-switching process. The properties of the corresponding claim-counting process are discussed in detail. By properly defining a Lundberg-type coefficient, we derive upper bounds for finite time ruin probabilities in a two-state case.

MOORE, K. S. & YOUNG, V. R. *Optimal and simple, nearly optimal rules for minimizing the probability of financial ruin in retirement.* 145-161. The increasing risk of poverty in retirement has been well documented; it is projected that current and future retirees' living expenses will significantly exceed their savings and income. In this paper, we consider a retiree who does not have sufficient wealth and income to fund her future expenses, and we seek the asset allocation that minimizes the probability of financial ruin during her lifetime. Building on the work of Young (2004) and Milevsky, Moore, and Young (2006), under general mortality assumptions, we derive a variational inequality that governs the ruin probability and optimal asset allocation. We explore the qualitative properties of the ruin probability and optimal strategy, present a numerical method for their estimation, and examine their sensitivity to changes in model parameters for specific examples. We then present an easy-to-implement allocation rule and demonstrate via simulation that it yields nearly optimal ruin probability, even under discrete portfolio rebalancing.

PAI, J. S., SHAND, K. J. & WANG, X. *Compound Poisson model with covariates: A case study on pet insurance.* 219-234. Pet insurance in North America continues to be a growing industry. Unlike in Europe, where some countries have as much as 50% of the pet population insured, very few pets in North America are insured. Pricing practices in the past have relied on market share objectives more so than on actual experience. Pricing still continues to be performed on this basis with little consideration for actuarial principles and techniques. Developments of mortality and morbidity models to be used in the pricing model and new product development are essential for pet insurance. This paper examines insurance claims as experienced in the Canadian market. The time-to-event data are investigated using the Cox's proportional hazards model. The claim number follows a nonhomogenous Poisson process with covariates. The claim size random variable is assumed to follow a lognormal distribution. These two models work well for aggregate claims with covariates. The first three central moments of the aggregate claims for one insured animal, as well as for a block of insured animals, are derived. We illustrate the models using data collected over an eight-year period.

PRITCHARD, D. J. *Modeling disability in long-term care insurance.* 48-75. Long-term care (LTC) costs and, in particular, those arising under an LTC insurance contract, are difficult to estimate. This is because of the complex effects of the processes of aging — disability and cognitive impairment. As disability is a gradual, as opposed to a discrete, process, and as the effects are sometimes reversible, a fairly complex model is necessary to capture its nature. This paper concentrates on modeling the disability process of aging only and, in particular, fully incorporates the recovery process as dictated by the data. With the recovery process modeled, the effect on the estimated model costs of disability of the common simplifying assumption that recoveries can be ignored is easily assessed. This paper has twin objectives: (1) to present novel methodology, the penalized likelihood, for using interval-censored longitudinal data, such as the National Long-Term Care Study, to parameterize Markov models; and (2) to estimate the costs arising under an LTC insurance contract in respect of disability. The model is also used to show that ignoring recovery from disability can lead to significant overestimation of LTC insurance costs — suggesting that claims underwriting in LTC insurance may be an important factor in managing claims costs.

STUCKI, B. R. *Using reverse mortgages to manage the financial risk of long-term care.* 90-102. A penny saved is a penny earned. Poor Richards Almanack (1737). This sage advice of Benjamin Franklin highlights the fact that the basic strategy for ensuring retirement security has changed little over the past 200 years. The traditional formula is simple — accumulate assets during one's working years and systematically draw down these assets after retirement. In recent years, however, more and more Americans are finding it difficult to save enough for retirement from earnings. The dramatic fall of the stock market between March 2000 and March 2001 exacerbated the problem, reducing personal wealth by an estimated $3.5 trillion in just one year (Weller 2003). These trends are troubling at a time when rising longevity places seniors at greater financial risk due to a chronic illness or disability. There is one bright spot, however — housing wealth has continued to rise. Average home equity in the United States increased more than 10 per cent between 2003 and 2004 (Joint Center for Housing Studies 2005). Many older families have substantial amounts of untapped housing wealth, including households whose other retirement resources may be very modest. With more than $2 trillion tied up in home equity, this financial asset has the potential to dramatically increase the ability of seniors to pay for the services and supports that can help them to stay at home. Unlocking illiquid assets such as housing wealth requires us to look more closely at asset decumulation in retirement. Typically, elders sell their home to access the equity they have built up over time. When they move to a more appropriate living situation, the sale of a house can be very beneficial. Those elders with a chronic health condition, who are forced to sell their home to pay for long-term care, however, could face serious problems. Relocating often entails the loss of familiar activities along with support from family and friends. This can reduce quality of life and accelerate cognitive decline (Bassuk 1999). For physically or mentally impaired elders, a better approach would be to use the equity in the home to purchase services and devices that could enable them to stay at home. A new type of financial tool — the reverse mortgage — can help older Americans achieve this goal. Little work has been done to examine the role of reverse mortgages in managing the financial risk of long-term care among older households. Here we address this issue by examining the use of reverse mortgages to help impaired elders continue to live at home (termed "aging in place"). The article also identifies the potential links between reverse mortgages and long-term care insurance. The research presented here is part of a study conducted by the National Council on the Aging, which was funded by grants from the Robert Wood Johnson Foundation and the Centers for Medicare and Medicaid Services (Stucki 2005). The analysis is based on the 2000 Health and Retirement Study and data from the housing and mortgage industries. This study focuses specifically on households where the youngest homeowner is at least age 62, since this is the minimum age to qualify for a reverse mortgage. Because home values have increased substantially since 2000, the numbers presented here will likely underrepresent the potential of this financing option for long-term care.

WATTS, K. A., DUPUIS, D. J. & JONES, B. L. *An extreme value analysis of advanced age mortality data.* 162-178. Extreme value theory describes the behavior of random variables at extremely high or low levels. The application of extreme value theory to statistics allows us to fit models to data from the upper tail of a distribution. This paper presents a statistical analysis of advanced age mortality data, using extreme value models to quantify the upper tail of the distribution of human life spans. Our analysis focuses on mortality data from two sources. Statistics Canada publishes the annual number of deaths in Canada, broken down by gender and age. We use the deaths data from 1949 to 1997 in our analysis. The Japanese Ministry of Health, Labor, and Welfare also publishes detailed annual mortality data, including the 10 oldest reported ages at death in each year. We analyze the Japanese data over the period from 1980 to 2000. Using the *r*-largest and peaks-over-threshold approaches to extreme value modeling, we fit generalized extreme value and generalized Pareto distributions to the life span data. Changes in distribution by birth cohort or over time are modeled through the use of covariates. We then evaluate the appropriateness of the fitted models and discuss reasons for their shortcomings. Finally, we use our findings to address the existence of a finite upper bound on the life span distribution and the behavior of the force of mortality at advanced ages.

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KOISSI, M.-C. *Longevity and adjustment in pension annuities, with application to Finland.* 226-242. The aim of this paper is twofold. First, the improvement in adult mortality in Finland is studied. Lee-Carter (LC) Poisson log-bilinear model is used for mortality forecasting. Secondly, the paper studies how the pension annuities are adjusted to unexpected mortality pattern. A formula for funded plan is proposed. Application is made with Finnish mortality rates predicted using the LC model.

LESCOURRET, L. & ROBERT, C. Y. *Extreme dependence of multivariate catastrophic losses.* 203-225. Natural catastrophes cause insurance losses in several different lines of business. An approach to modelling the dependence in loss severities is to assume that they are related to the intensity of the natural disaster. In this paper we introduce a factor model and investigate the extreme dependence. We derive a specific extreme dependence structure when considering an heavy-tailed intensity. Estimation procedures are presented and their moderate sample properties are compared in a simulation study. We also motivate our approach by an illustrative example from storm insurance.

LU, Y. *On the severity of ruin in a Markov-modulated risk model.* 183-202. We consider a Markov-modulated risk model in which the claim inter-arrivals, amounts and premiums are influenced by an external Markovian environment process. A system of Laplace transforms of the probabilities of the severity of ruin, given the initial environment state, is established from a system of integro-differential equations derived by Snoussi [The severity of ruin in Markov-modulated risk models Schweiz Aktuarver. Mitt., 2002, **1**, 31-43]. In the two-state model, explicit formulas for probabilities of the severity of ruin are derived, when the initial reserve is zero or when both claim amount distributions are from the rational family. Numerical illustrations are also given.

BOUDREAULT, M., COSSETTE, H., LANDRIAULT, D. & MARCEAU, É. *On a risk model with dependence between interclaim arrivals and claim sizes.* 265-285. We consider an extension to the classical compound Poisson risk model for which the increments of the aggregate claim amount process are independent. In Albrecher and Teugels (2006), an arbitrary dependence structure among the interclaim time and the subsequent claim size expressed through a copula is considered and they derived asymptotic results for both the finite and infinite-time ruin probabilities. In this paper, we consider a particular dependence structure among the interclaim time and the subsequent claim size and we derive the defective renewal equation satisfied by the expected discounted penalty function. Based on the compound geometric tail representation of the Laplace transform of the time to ruin, we also obtain an explicit expression for this Laplace transform for a large class of claim size distributions. The ruin probability being a special case of the Laplace transform of the time to ruin, explicit expressions are therefore obtained for this particular ruin related quantity. Finally, we measure the impact of the various dependence structures in the risk model on the ruin probability via the comparison of their Lundberg coefficients.

PITREBOIS, S., DENUIT, M. & WALHIN, J.-F. *An actuarial analysis of the French bonus-malus system.* 247-264. The bonus-malus system in force in France differs from most of those used in industrialized countries around the world. Policyholders do not move inside a scale but their premium is obtained with the help of multiplicative CRM coefficients (CRM stands for the acronym of the French coefficient de réduction-majoration). The French bonus-malus system has been the topic of very few scientific investigations in the actuarial literature. This paper purposes to analyze this bonus-malus system in details. Despite its apparent simplicity, it will be seen that it leads to nontrivial mathematical problems. The financial equilibrium of the bonus-malus system is also investigated thanks to the multivariate De Pril's algorithm for the convolution of independent and identically distributed random vectors.

SPREEUW, J. *Types of dependence and time-dependent association between two lifetimes in single parameter copula models.* 286-309. Most publications on modeling insurance contracts on two lives, assuming dependence of the two lifetimes involved, focus on the time of inception of the contract. The dependence between the lifetimes is usually modeled through a copula and the effect of this dependence on the pricing of a joint life policy is measured. This paper investigates the effect of association at the outset on the mortality in the future. The conditional law of mortality of an individual, given his survival and given the life status of the partner is derived. The conditional joint survival distribution of a couple at any duration, given that the two lives are then alive, is also derived. We analyze how the degree of dependence between the two members of a couple varies throughout the duration of a contract. We have done that for (mainly Archimedean) copula models, with one parameter for the degree of dependence. The conditional distributions hence derived provide the basis for the calculation of prospective provisions.

GUI, E. H., LU, B., MACDONALD, A. S., WATERS, H. R. & WEKWETE, C. T. *The genetics of breast and ovarian cancer III: a new model of family history with insurance applications.* 338-367. Insurers' access to genetic test results is often restricted and the only genetic information that might be collected during underwriting in some countries is family history. Previous

studies have included family history in a simple way but only for diseases which have no cause other than gene mutations, because then the event 'affected parent' contributes all possible information short of a genetic test result. We construct a model of breast cancer (BC) and ovarian cancer (OC) — common diseases with rare genetic variants — in which the development of a family history is represented explicitly as a transition between states, hence as part of the applicant's own life history. This allows the impact of a moratorium to be modelled. We then apply this family history model to life insurance in a semi-Markov framework and to critical illness (CI) insurance in a Markov framework to: (a) estimate premium ratings depending on genotype or family history; and (b) model the potential cost of adverse selection.

HERBERT, T. K. J. & SCOTT, W. F. *On de-seasonalising adjusted-average formulae.* 368-377. The techniques of Borgan (1979) are extended to cover data with seasonal variations. Examples are given, and it is suggested that the formulae presented here give smoother results than those traditionally employed to deal with economic time series subject to seasonal variations.

MA, J. & YU, Y. *Principle of equivalent utility and universal variable life insurance pricing.* 311-337. In this paper we study the pricing problem for a class of universal variable life (UVL) insurance products, using the idea of principle of equivalent utility. As the main features of UVL products we allow the (death) benefit to depend on certain indices or assets that are not necessarily tradable (e.g., pension plans), and we also consider the "multiple decrement" cases in which various status of the insured are allowed and the benefit varies in accordance with the status. Following the general theory of indifference pricing, we formulate the pricing problem as stochastic control problems, and derive the corresponding HJB equations for the value functions. In the case of exponential utilities, we show that the prices can be expressed explicitly in terms of the global, bounded solutions of a class of semilinear parabolic PDEs with exponential growth. In the case of general insurance models where multiple decrements and random time benefit payments are all allowed, we show that the price should be determined by the solutions to a system of HJB equations, each component corresponds to the value function of an optimization problem with the particular status of the insurer.

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ASHER, A. *The relative investment performance of the Community Growth Fund.* 1-26. This paper examines the impact of social criteria on the investment performance of the Community Growth Fund, a trade-union-controlled South African unit trust. It gives a brief history of the fund, discusses reasons for performance deviations, and shows that there may be reason for believing that some social criteria improve performance.

MCLEOD, H. D. *Mutuality and solidarity in health care in South Africa.* 135-167. This paper uses Wilkie's definition so of mutuality and solidarity to review the history of private health care in South Africa. The vision for a future unified national health care system is given and the phases of reform are outlined. The first phase of reforms has been completed and these are contextualised in terms so far return to solidarity principles. The elements of a planned social health- insurance system are described and it is shown how income cross-subsidies will further entrench the principles of solidarity. In the past, health care actuaries largely supported mutuality principles. The implications for the actuarial profession are suggested.

O'MALLEY, B. R., DORRINGTON, R. E., JURISICH, S. C., VALENTINI, J.-A., COHEN, T. M. & ROSS, B. J. *An investigation of the mortality of South African assured lives, 1995-1998.* 27-59. This paper outlines the finding so fan investigation of the mortality of South African assured lives, covering the period 1995 to 1998. The investigation was carried out by the Continuous Statistical Investigations (CSI) Committee of the Actuarial Society of South Africa (ASSA). Some of the major findings of the investigation were worsening mortality below age 50, general improvements in mortality above age 50, substantial differences in mortality between the different socio-economic classes and signs of anti-selection below age 50, especially in the better socio-economic classes.

RUSCONI, R. D. *Saving for retirement in South Africa: charges to the consumer.* 61-133. Whether in occupational plans or private supplementary arrangements, workers pay for the opportunity to save for retirement through a variety of charges that erode the prosperity of their retirement years. This paper provides an analysis of the administrative charges paid by South Africans saving for retirement in the areas of occupational retirement funds, individual-life products and unit trusts, using a model designed to evaluate the lifetime effect of these charges. The analysis suggests that retirement funds are cheapest, followed by unit trusts and then individual-life products. These results are consistent with the flexibility of the more expensive products. Overall, however, charges appear to be high. Comparison with international benchmarks appears to confirm these concerns. South Africa is in need of new pension-fund legislation and initial thinking on the frame work has begun. The paper ends with thoughts on the implications of this research for policy makers.

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LOWTHER, M. W. & MCMILLAN, W. *Planning lifelong professionalisation learning for actuaries.* 1-17. This paper presents a model for what is termed Lifelong Professionalisation Learning for actuaries. The model is grounded on the proposition that professions are dynamic, offering the public varying quantities and qualities of professional aspects over time. The overall curriculum for the model is devived by ordering these aspects into cognitive, normative and organisational strands. Different aspects of the curriculum need to be addressed at the different stages of an actuary's career. Delivery of the curriculum may take many forms, and should be planned by national actuarial associations under the guidance of an education expert.

THOMSON, R. J. *A typology of models use in actuarial science.* 19-36. This paper proposes a categorisation of the models used in actuarial science. It illustrates the application of that categorisation by using it to classify numerous such models. It is suggested that this categorisation, together with the illustrative classification, may be used as a typology for the classification of other such models, and that this typology may be found useful as a candidate exemplar, or as a basis for further refinement, in the discourse of actuarial science.

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