one of their great selling points, it would seem remiss not to include at least some of them'. The applications discussed in this chapter relate to geometric group theory, in particular to the concept of growth in (finitely generated) groups, which measures the asymptotic rate of growth of the cardinality of the n-fold product of certain kinds of subsets S of the group. A famous result in this area is Gromov's theorem, which can be proved and refined by using approximate groups.

The use of the word 'introduction' in the title of this book is not misleading: although the text discusses the results of recent research, it really is intended as a genuine introduction. As befits an entry in the London Mathematical Society Student Texts series, it reads as a text, not a research monograph, and even contains exercises. (The author recently taught a Part III course in this material at Cambridge using parts of the book.) Tointon writes clearly, but succinctly, and takes pains to motivate topics. At the same time, the book clearly contemplates that its readers may wish to pursue the subject further into currently unchartered territory, and provides excellent preparation for such beginning researchers. To this end, there is a detailed 75-entry bibliography, most of the entries of which are research papers, a handful of them not in English.

To summarize: any book that offers an introductory account of a hot new area of mathematics is, for that reason alone, a useful addition to the literature. The usefulness of such a book is, naturally, significantly enhanced when, as is the case here, the book is very nicely written, and there are few if any other such introductions to the subject in the textbook literature. Researchers and fledgling researchers in this area will want to own this book.

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Linear algebra for the young mathematician by Steven Weintraub, pp, 389, \$89.00 (hard), ISBN 978-1-47045-084-7, American Mathematical Society (AMS) (2019)

About 45 years ago, as a graduate student studying for my qualifying exams, I read Hoffman and Kunze's classic text on linear algebra (hereafter referred to as HK) from cover to cover. It was a memorable experience, and provided me with a base of knowledge that served me well not only on my exams but also in my subsequent doctoral research in Lie algebras. It has continued to be useful in my post-student years: whenever I have a linear algebra question, that book is usually the one that I go to first, and in many cases do not need to look elsewhere.

I mention this because the book now under review reminds me in some respects of HK. They share, for example, a basic philosophy of starting at the beginning with matrices and linear equations, but progressing quite far, all the while focusing on a conceptual understanding of the material, focusing on ideas and proof rather than just mechanical computations. In addition, the topic coverage of the two books is also fairly similar: both cover (in addition to the aforementioned matrices and linear equations) determinants, abstract vector spaces (over arbitrary fields), linear transformations, eigenvalues and eigenvectors (including diagonalisation and triangularisation), the Jordan canonical form, inner product spaces (and operators defined on them, up to and including variations of the spectral theorem), and bilinear and sesquilinear forms.

Both texts treat the subject of dimensionality in essentially the same way: the Axiom of Choice is mentioned but not dwelled on, so, for example, the existence of a basis is proved only for finite-dimensional spaces, but stated to be true for all. In

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addition, infinite-dimensional spaces are discussed throughout as examples, and specific mention is made when results require finite-dimensionality. This seems to strike just the right balance.

Both texts also do a good job of showing how linear algebra connects with other branches of mathematics, Weintraub perhaps doing a bit more with this than HK. For example, both texts prove (modulo some assumed facts about differential equations) that a linear ODE with constant coefficients of order n has an n-dimensional solution space, but Weintraub goes further and talks about linear equations with non-constant coefficients, as well as things like Jacobians and Wronskians.

There are some other differences between the books. The topic coverage, though similar, is not identical: HK covers the rational canonical form, which Weintraub does not; on the other hand, Weintraub covers singular values, which HK does not. Also, HK talks a bit about matrix groups, but Weintraub does not; his book, unlike HK, does, however, give (in an appendix) a brief peek at Banach and Hilbert spaces.

The bibliography in Weintraub's book is also far superior to the one in HK, which lists only six items, none of them with a publication date later than 1968 and none of them the kind of book an undergraduate could get much out of. Weintraub's section on further reading is more extensive, and helpfully arranged by subject matter (e.g. books dealing with applications of linear algebra to statistics or quantum theory).

Weintraub's book is also written in a somewhat more reader-friendly style than is HK. The latter book originated as a text for an undergraduate course (at MIT), but there are relatively few undergraduates today who are likely to find this book accessible. It is succinct and rather dry, and does not engage in much or any hand-holding. The book under review, however, is very definitely intended as an undergraduate text, as its placement in the AMS Pure and Applied Undergraduate Texts series indicates. It is more conversational and chatty than is HK, with more motivation of the underlying ideas. Indeed, HK is not, according to the MIT linear algebra course webpage, even being used there anymore; Axler's *Linear Algebra Done Right* seems to be the standard text there now. In fact, HK no longer seems to appear on Pearson's website, so it may well be currently out of print.

Weintraub's book is not an easy one; the author has made every effort to make it comprehensible to good undergraduates, but has not shied away from doing things honestly and precisely, and the result is a text whose study requires some diligence. This effort will be rewarded at the end with a good understanding of linear algebra, but the book itself is still on the more difficult end of the spectrum and is likely best suited for honours introductory courses or courses at more elite universities.

Of course, reviewers are supposed to find nits to pick, and I have found a few. For one thing, the index could be improved. For example, what other linear algebra book has an index that omits both the words 'orthogonal' and 'orthonormal'? Also, while Cauchy sequences are discussed in the book, there is no reference to them in the Index, either under 'C' or 'S', Likewise, you won't find 'characteristic polynomial' under either 'C' or 'P', and a similar comment holds for 'minimal polynomial'.

Second, I would have preferred, both for pedagogical and practical reasons, to have the last two chapters (on forms and inner products, respectively) interchanged. I think it pedagogically advisable to do inner products first and then generalise to arbitrary forms. More than this, however, I think that there is more material in this text than can be covered in one semester, and since inner products are typically a part of an introductory linear algebra course and arbitrary forms are not, it would make sense to get to inner products first.

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Third, the author states in the Preface that '*Linear algebra is about vector spaces and linear transformations, not about matrices.*' (Italics in the original.) He made the same rather didactic statement in his earlier book *A Guide to Advanced Linear Algebra*, and, in my review of that book for the Mathematical Association of America online reviews column, I took exception to it, pointing out that quite a lot of current research in linear algebra is done in the area of matrix theory. The author is entitled to his opinion, of course, but, just as in the earlier book, this statement is presented here as one of established fact rather than personal opinion, and I know a number of linear algebraists who would disagree. (In the interest of full disclosure, I should point out that my wife Leslie Hogben is one of them.)

Finally, the author also states in the Preface, in describing this book, 'Of course, we think it is exceptionally well written.' Even if this statement was intended to be tongue-in-cheek, I can't help but wish the author had resisted the urge to put it in; statements like this seem more meaningful when made by somebody else.

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Linear algebra I by Frederick P. Greenleaf and Sophie Marques, pp. 261, $51 (paper), ISBN 978-1-4704-4871-4, American Mathematical Society (2019)

This is a text for a sophisticated, proof-based introduction to linear algebra at the advanced undergraduate/early graduate level. The book is, we are told, based on lecture notes from a course for Masters' degree students at the Courant Institute of Mathematics of New York University. It starts from scratch with the definition of a vector space, but is not intended as a text for the typical sophomore/junior level introductory course; the exposition is concise and efficient, results are done in more generality than is often the case in such courses (vector spaces, for example, are defined over an arbitrary field), and students are expected to understand and create rigorous proofs. The term 'field', though used at the outset, is not defined in the text, so presumably the intended audience is presumed to have at least a nodding acquaintance with the basic concepts of abstract algebra.

Topics covered in this book include vector spaces, linear transformations, duality, determinants, diagonalisation and inner product spaces (including some spectral theory). The theory of linear equations is discussed as needed in the chapter of vector spaces. Canonical forms are deferred to volume II (reviewed below).

As is hardly surprising, the level of difficulty of the material, and the demands placed on the reader, increase as the book proceeds. The 'difficulty gradient' is fairly steep, but not, given the intended audience, unreasonable. The first two chapters, on vector spaces and linear transformations, are pretty straightforward. Transfinite chicanery is avoided, so that the existence of a basis for a vector space, and the fact that two bases have the same cardinality, are proved only for finite-dimensional spaces. On the other hand, infinite-dimensional spaces are discussed in the text, especially as interesting examples, and the authors mention in the statement of a theorem when finite-dimensionality is required, instead of just assuming it throughout.

In the third chapter, on duality, we already start to see interesting things, such as a discussion of duality and the Fourier transform. The fourth chapter, on determinants, covers standard topics in this area, but the proofs (making use of the symmetric group) are rigorous and precise.