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Optimal force distribution in multilegged vehicles

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SUMMARY

The force distribution problem in multilegged vehicle is a constrained, optimization problem. The solution to the problem is the setpoints of the leg contact forces for a particular system task. In this paper, the efficient Compact QP method which takes into account both the linear and quadratic objective functions is adopted to resolve this constrained, optimization problem. Various objective functions such as minimum force, load balance, safety margin on friction constraints can be considered by the Compact QP method. This method can also be applied to smooth discontinuities in commanded forces by manipulating the homogeneous solution and including smoothing periods when the leg phase alternates between support and transfer. This smoothing scheme does not require force sensors. Multiple goals which consider several alternative objective functions can also be achieved by the Compact QP method.

KEYWORDS: Multilegged vehicles; Optimal force; Compact QP method.

1. INTRODUCTION

It is interesting to notice the difference that animals use independent limbs while man has made almost exclusive use of wheeled vehicles to achieve locomotion. Each method has definite advantages. Legged configurations typically move slower but do not require hard, smooth surfaces. On the other hand, wheeled vehicles together with hard surfaces allow fast and efficient movement of heavy loads. Multilegged vehicles with their potential for terrain adaptability are now being developed with hopes that someday they can provide a valuable service to mankind. Among the ways in which a legged vehicle might be used are locating and disarming bombs, extinguishing fires, underground mining to recover natural resources, exploration of the ocean floors, or exploration of distant planets. Ultimately, such a vehicle could be utilized in almost any situation where a human cannot exist or would be in danger.

A multilegged vehicle is a robotic linkage system containing multiple chains and forming simple closed-kinematic loops.[1] The basic problem of controlling this system is that of coordination. In addition to the usually intra-chain, local coordination problem which involves control of the individual joints of a chain to achieve the desired tip control, there is an inter-chain, global coordination problem which involves coordination among the several chains. One of the major problem of inter-chain coordination is that of force distribution.[2,3]

Because multiple chains share the load, this global coordination problem is usually underspecified involving redundancy. Moreover, since friction constraints for preventing slippage, and the physical limits of the joint actuators often need to be considered, this problem is also constrained.

A multilegged vehicle is an example of a large-scale system.[4] With the legs of the vehicle forming closed-kinematic loops, the responses of individual legs are tightly coupled with one another through the body. The dynamic equations for each leg of a multilegged vehicle are:

$$\tau_k = (\mathbf{H}(\theta)\cdot\ddot{\theta} + \mathbf{E}(\theta, \dot{\theta}) + \mathbf{K}(\theta) + \mathbf{V}(\dot{\theta}))_k + (\mathbf{J}^T\cdot\mathbf{h}_C)_k \quad k = 1, \ldots, m \tag{1}$$

where

τ	= input joint torque/force vector $[N\times 1]$,
$\theta, \dot{\theta}, \ddot{\theta}$	= joint displacement, velocity, acceleration vectors, each $(N\times 1]$,
$\mathbf{H}(\theta)$	= inertia matrix $[N\times N]$,
$\mathbf{E}(\theta, \dot{\theta})$	= vector defining centrifugal and Coriolis effects $[N\times 1]$,
$\mathbf{K}(\theta)$	= vector defining the gravity terms $[N\times 1]$,
$\mathbf{V}(\dot{\theta})$	= vector defining the viscous friction terms $[N\times 1]$,
$\mathbf{J}^T$	= transpose of Jacobian matrix $[N\times 6]$,
$\mathbf{h}_C$	= chain tip contact force/moment vector $[6\times 1]$,
N	= number of degrees of freedom for each chain, and
m	= number of chains.

These m dynamic equations are tightly coupled through the terms, $(\mathbf{J}^T\cdot\mathbf{h}_C)_k$, where all of the $(\mathbf{h}_C)_k$ terms constitute the required wrench[5] (3 force and 3 moment components) to give the desired motion specified on the body. The equations which relate $(\mathbf{h}_C)_k$ to the required wrench are termed the force balance equations.

As for the cases of multifingered hands and multiple manipulators, a number of investigators[6–8] have aggregated all of the individual chain dynamic equations together with

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the force balance equations to solve this coupled, large-scale control problem. However, systematic and efficient methods to treat the coupling terms, $(\mathbf{J}^T\cdot\mathbf{h}_C)_k$, which must satisfy the force balance equations, friction constraints, and joint torque constraints, are yet needed. Essentially, this is the problem of force distribution.

A computationally efficient algorithm for solving the force distribution problem called the Compact-Dual LP (Linear Programming) method has been developed by Cheng and Orin.[2] The approach taken is based on efficient use of linear programming techniques,[9] and it makes real-time solution of the linear programming problem feasible.

Nahon and Angeles later pointed out several advantages of quadratic programming over linear programming in their study of grasping.[10] Quadratic programming produces continuous solutions under smooth constraint changes, does not require splitting design variables into positive and negative parts, is more efficient for large problems, and permits a quadratic optimization function rather than a linear one. They applied quadratic programming to minimize either the internal force or the norm of joint torques. Neither may be minimized using linear programming due to the quadratic optimization function.

Cheng, *et al*. have combined the merits of the Compact formulation (for reducing the optimization problem size) and the quadratic programming (for continuous solutions under smooth constraint changes and considering inequality constraints) to develop the Compact QP method[11] for resolving manipulator redundancy under inequality constraints.

In this paper, the Compact QP method[11] is applied to resolve the force distribution problem. The Compact QP method is a general optimal aglorithm which can accept most linear and quadratic objective functions. Therefore, several alternative objective functions are formulated and implemented in this paper. They are minimum force,[10] load balance,[3,12] and safety margins on friction constraints.[3]

Also, as indicated in reference 13, using the minimum force objective function, the force set-point solutions for all supporting legs show major discontinuities whenever the leg phase alternates between support and transfer. These discontinuities in the past caused large control impulses to the system. Klein and Chung[13] have applied the so-called minimum-perturbation solution method to minimize discontinuities in commanded forces by utilizing the actual contact forces (via force sensors) to form the proper homogeneous solution. However, force sensors are expensive, fragile, and may have slow reaction time. In this paper, a gradient projection method,[14] together with adding smoothing periods when the leg phase alternates between support and transfer, is proposed to smooth these discontinuities without the help of force sensors.

The Compact QP method can also be applied for multiple goals.[15] Three objective functions for minimum force, load balance, and smoothing discontinuities in commanded forces are considered simultaneously in a simulation and the solution sequences show that all of the three goals are accomplished.

The organization of the paper is described as follows. Section 2 describes the general formulation for the force distribution problem. Section 3 summarizes the Compact QP method. Section 4 presents the formulations to include the optimization criteria for load balance and safety margin on friction constraints into the general formulation. Section 5 describes the example mechanism, the TIT Quadruped, which will be used for all the simulations. Section 6 explains the general gait planning for the TIT Quadruped. Section 7 derives alternative objective functions for the force distribution problem. Finally, Section 8 presents the summary and conclusions.

2. GENERAL FORMULATION FOR THE FORCE DISTRIBUTION PROBLEM

The general formulation for the force distribution problem of a simple closed-chain mechanism, considering the chain dynamics and a general contact model of the chains with the load or support surface, has been derived in references 1 and 2.

The case of a multilegged vehicle with hard point contacts is presented in this section. This formulation includes equality constraints (force balance equations) and inequality constraints (friction constraints and maximum joint torque constraints) which will be derived in the following paragraphs.

2.1 Force balance equations

For hard point contact with friction, only forces are transmitted from the tips of the legs through the legs to the body, hereinafter the body is called the reference member (Fig. 1). The force balance equations on the reference member may then be written as:

$$ {}^{o}\mathbf{F}_{o} = \sum_{k=1}^{m} [-({}^{o}\mathbf{D}_{C}\cdot{}^{C}\mathbf{g}_{C})_k - ({}^{o}\mathbf{h}_{B})_k] + {}^{o}\mathbf{h}_{o} \tag{2} $$

where

${}^{o}\mathbf{F}_{o}$ = resultant force/moment vector applied to reference member expressed in reference member coordinate frame (O) $[6\times 1]$,

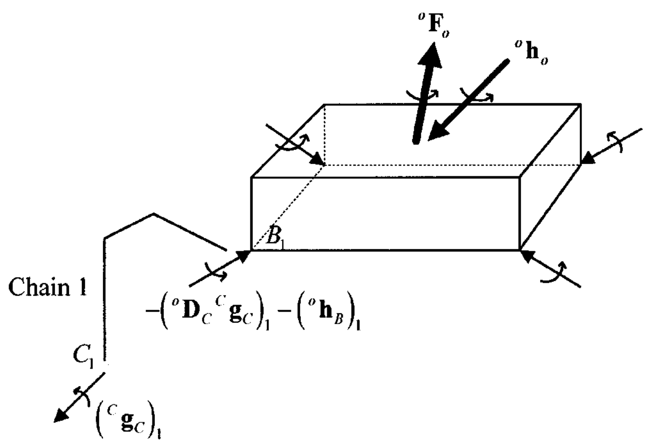

Fig. 1. Force balance on the body (reference member).

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$({}^{o}\mathbf{D}_{C})_{k}$ = transform to resolve contact forces from contact coordinate frame for chain $k(C_k)$ to reference member coordinate frame (O) $[6\times 3]$,

$({}^{C}\mathbf{g}_{C})_{k}$ = unknown contact force vector onto support surface from chain k expressed in contact coordinate frame (C_k) $[3\times 1]$,

$({}^{o}\mathbf{h}_{B})_{k}$ = force/moment vector applied to chain k at its base when unconstrained (open chain) expressed in reference member coordinate frame (O) $[6\times 1]$, and

${}^{o}\mathbf{h}_{o}$ = external force/moment vector on reference member (including gravity) expressed in reference member coordinate frame (O) $[6\times 1]$.

Moreover, the components of $({}^{C}\mathbf{g}_{C})_{k}$ are:

$$({}^{C}\mathbf{g}_{C})_{k}=\begin{bmatrix} {}^{C}f^{x} \\ {}^{C}f^{y} \\ {}^{C}f^{z} \end{bmatrix}_{k} \tag{3}$$

where, for example, ${}^{C}f^{x}$ is the x component of the contact force expressed in the contact coordinate frame. Also, note that $(\mathbf{g}_{C})_{k}$ is just the unknown part of $(\mathbf{h}_{C})_{k}$ (Eq. (1)).

2.2 *Friction constraints*

With the contact normal along the z direction and directed outward and a coefficient of friction of μ, the friction force constraints may be expressed as:

$$-{}^{C}f_{k}^{x}+\frac{\mu}{\sqrt{2}}\cdot{}^{C}f_{k}^{z}\leq 0, \tag{4}$$

$${}^{C}f_{k}^{x}+\frac{\mu}{\sqrt{2}}\cdot{}^{C}f_{k}^{z}\leq 0, \tag{5}$$

$$-{}^{C}f_{k}^{y}+\frac{\mu}{\sqrt{2}}\cdot{}^{C}f_{k}^{z}\leq 0, \tag{6}$$

$${}^{C}f_{k}^{y}+\frac{\mu}{\sqrt{2}}\cdot{}^{C}f_{k}^{z}\leq 0, \tag{7}$$

which give a conservative but linear set of constraints describing a friction pyramid inscribed within the desired friction cone [1]. It is noted that Eqs. (4)–(7) imply ${}^{C}f_{k}^{z}\leq 0$. The friction force constraints may be rewritten as:

$$\begin{bmatrix} -1 & 0 & \mu/\sqrt{2} \\ 1 & 0 & \mu/\sqrt{2} \\ 0 & -1 & \mu/\sqrt{2} \\ 0 & 1 & \mu/\sqrt{2} \end{bmatrix}\begin{bmatrix} {}^{C}f^{x} \\ {}^{C}f^{y} \\ {}^{C}f^{z} \end{bmatrix}_{k}\leq\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{8}$$

In short, Eq. (8) yields:

$$(\mathbf{S}\cdot{}^{C}\mathbf{g}_{C})_{k}\leq\mathbf{0} \tag{9}$$

where $\mathbf{S}$ is the matrix coefficients of the friction constraints.

2.3 *Maximum joint torque constraints*

To keep the joint torques within the physical limits of the actuators, maximum joint torque constraints may be written[1] as

$$\tau_{k\ \min}\leq\tau_{k}\leq\tau_{k\ \max} \tag{10}$$

where

$$\tau_{k}=({}^{C}\hat{\mathbf{J}}_{C}^{T}\cdot{}^{C}\mathbf{g}_{C})_{k}+\mathbf{t}_{k} \tag{11}$$

and

$\tau_{k\ \max}$ = maximum actuator torque vector for chain k $[N\times 1]$,

$\tau_{k\ \min}$ = minimum actuator torque vector for chain k $(N\times 1]$,

$({}^{C}\hat{\mathbf{J}}_{C}^{T})_{k}$ = transpose of Jacobian for linear velocity of chain k using the origin of contact coordinate frame as velocity reference point, expressed in contact coordinate frame $[N\times 3]$, and

$\mathbf{t}_{k}$ = joint torque/force vector for chain k when unconstrained (open chain) $[N\times 1]$.

Combining Eqs. (10) and (11) results in:

$$({}^{C}\hat{\mathbf{J}}_{C}^{T}\cdot{}^{C}\mathbf{g}_{C})_{k}\leq(\tau_{max}-\mathbf{t})_{k}, \tag{12}$$

$$-({}^{C}\hat{\mathbf{J}}_{C}^{T}\cdot{}^{C}\mathbf{g}_{C})_{k}\leq(-\tau_{min}+\mathbf{t})_{k}, \tag{13}$$

If all of the $({}^{C}\mathbf{g}_{C})_{k}$, $k=1,\ldots,m$, are organized into a composite contact force vector, $\mathbf{G}$, of size $[3m\times 1]$, and the legs' dynamics $({}^{o}\mathbf{h}_{B})_{k}$ are not considered, the force balance equations Eq. (2) may be expressed as:

$$\mathbf{W}\cdot\mathbf{G}=\mathbf{F} \tag{14}$$

where $\mathbf{F}={}^{o}\mathbf{F}_{o}-{}^{o}\mathbf{h}_{o}$ and $\mathbf{W}$ is the composite vector space of $\mathbf{G}$ with size $[6\times 3m]$. And, the inequality constraints which include the friction constraints (Eq. (9)) and the maximum joint torque constraints (Eqs. (12) and (13)) may be combined and written as:

$$\mathbf{A}\cdot\mathbf{G}\leq\mathbf{B} \tag{15}$$

$\mathbf{A}$ = coefficient matrix of inequality constraints $[l\times 3m]$,

$\mathbf{B}$ = boundary-value vector of inequality constraints $[l\times 1]$, and

l = number of inequality constraints $(4m+2Nm)$.

Equations (14) and (15) define an underspecified problem which has multiple solutions. The general form of objective functions for a QP problem may be formulated as:

$$\text{Minimize }\ \frac{1}{2}\cdot(\mathbf{G}-\mathbf{Z})^{T}\cdot\mathbf{H}\cdot(\mathbf{G}-\mathbf{Z}) \tag{16}$$

where $\mathbf{H}$ is a $3m\times 3m$ positive semidefinite cost matrix and $\mathbf{Z}$ is an arbitrary vector which has the same dimension as $\mathbf{G}$. This objective function will search for an optimal solution, $\mathbf{G}^{*}$, which is nearest to a given vector $\mathbf{Z}$.

The objective function in Eq. (16) can be rewritten in the standard QP form as:

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$$\text{Minimize } \mathbf{C}\cdot\mathbf{G}+\frac{1}{2}\cdot\mathbf{G}^T\cdot\mathbf{H}\cdot\mathbf{G} \tag{17}$$

with

$$\mathbf{C}=-\mathbf{Z}^T\cdot\mathbf{H} \tag{18}$$

Combining Eqs. (14), (15) and (17), a constrained, optimization problem, the force distribution problem, has been formulated as:

$$\text{Minimize } \mathbf{C}\cdot\mathbf{G}+\frac{1}{2}\cdot\mathbf{G}^T\cdot\mathbf{H}\cdot\mathbf{G} \tag{19}$$

$$\text{Subject to } \mathbf{W}\cdot\mathbf{G}=\mathbf{F}, \tag{20}$$

$$\mathbf{A}\cdot\mathbf{G}\leq\mathbf{B}, \tag{21}$$

which is in the Original QP form. If a proper objective function is selected (with **C** and **H** specified), then a QP routine can be applied to obtain the optimal solution. However, the Original QP form may be computationally intensive. In order to improve the computational efficiency, the Compact QP method is introduced in the following section. Note that, the Compact QP method was first proposed by Cheng *et al*. to resolve manipulator redundancy under inequality constraints.[11]

3. THE COMPACT QP METHOD

The problem size of QP is mainly determined by the number of variables and constraints. Therefore, in order to maintain a smaller problem size, the numbers of variables and constraints should be as few as possible. In the area of numerical analysis, the elimination of the equality constraints by using them to reduce the number of independent variables is a standard approach.[16,17] Also, from the robotics literature, Cheng and Orin[2] applied the Gaussian elimination, Abdel-Rahman[18] used the Gram-Schmidt orthogonalization, and Nahon and Angeles[10,19] adopted the Householder reflections to eliminate the equality constraints. In this paper, the so-called Compact formulation as derived in reference 2 is adopted since the technique of Gaussian elimination with partial pivoting can ensure adequately numerical stability with better computational efficiency.[20,21] Furthermore, through the process of Gaussian elimination with partial pivoting, the decisions on which variables to be eliminated (i.e. basic variables) and which to be retained (i.e. free variables) are also made. In the following section, the Compact formulation for obtaining the general solution of the force balance equations is briefly described.

3.1 The compact formulation

The Compact formulation was originally derived by the Cheng and Orin [2]. If **W** has full rank, then, using Gaussian elimination with partial pivoting, Eq. (20) may be transformed into a desired row-reduced echelon form[22] which has an identity matrix of order 6, $\mathbf{I}_6$ in the first 6 columns of the resulting matrix:

$$[\mathbf{I}_6\ \mathbf{W}_r]\begin{bmatrix}\mathbf{G}_b\\ \mathbf{G}_f\end{bmatrix}=[\mathbf{F}_r] \tag{22}$$

where

$\mathbf{I}_6$ = identity matrix of order $6[6\times6]$,

$\mathbf{W}_r$ = remaining columns of the matrix **W** after transformation $[6\times(3m-6)]$,

$\mathbf{G}_b$ = partial vector for basic variables of **G** $[6\times1]$,

$\mathbf{G}_f$ = partial vector for free variables of **G** $[(3m-6)\times1]$, and

$\mathbf{F}_r$ = the resulting vector of **F** after transformation $[6\times1]$.

Equation (22) may be rewritten as:

$$\mathbf{G}=\begin{bmatrix}\mathbf{G}_b\\ \mathbf{G}_f\end{bmatrix}=\begin{bmatrix}\mathbf{F}_r\\ 0\end{bmatrix}+\begin{bmatrix}-\mathbf{W}_r\\ \mathbf{I}\end{bmatrix}[\mathbf{G}_f]$$

$$=\mathbf{G}_p+\mathbf{N}\cdot\mathbf{G}_f \tag{23}$$

where

$$\mathbf{G}_p=\begin{bmatrix}\mathbf{F}_r\\ 0\end{bmatrix}, \tag{24}$$

$$\mathbf{N}=\begin{bmatrix}-\mathbf{W}_r\\ \mathbf{I}\end{bmatrix}, \tag{25}$$

and

$\mathbf{G}_p$ = particular solution of **G** $[3m\times1]$,

$\mathbf{N}\cdot\mathbf{G}_f$ = homogeneous solution of **G** $[3m\times1]$,

$\mathbf{N}$ = matrix which maps $\mathbf{G}_f$ into the nullspace of **W** $[3m\times(3m-6)]$,

$\mathbf{0}$ = zero column vector $[(3m-6)\times1]$, and

$\mathbf{I}$ = identity matrix $[(3m-6)\times(3m-6)]$.

In addition, Eq. (20) may be decomposed such that the computation time for obtaining the general solution may be further reduced.[23]

The purpose for obtaining the general solution is to eliminate the linear equality constraints (formed by the force balance equations) of the optimization problem such that the problem size is reduced.[2] After reducing the problem size, a suitable optimization method may then be applied to solve this underspecified, optimization problem efficiently.

3.2 The compact QP method

The force distribution problem formulated in the Original QP form was expressed in Eqs. (19)–(21). Substituting Eq.

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(23) (the general solution of Eq. (20)) into Eqs. (19) and (21) result in the Compact QP form:

$$\text{Minimize } \hat{\mathbf{C}}\cdot\mathbf{G}_f + \frac{1}{2}\cdot\mathbf{G}_f^T\cdot\hat{\mathbf{H}}\cdot\mathbf{G}_f \tag{26}$$

$$\text{Subject to } \hat{\mathbf{A}}\cdot\mathbf{G}_f \le \hat{\mathbf{B}}, \tag{27}$$

with

$$\hat{\mathbf{C}} = \mathbf{C}\cdot\mathbf{N} + \mathbf{G}_p^T\cdot\mathbf{H}\cdot\mathbf{N} \quad [1\times(3m-6)], \tag{28}$$

$$\hat{\mathbf{H}} = \mathbf{N}^T\cdot\mathbf{H}\cdot\mathbf{N} \quad [(3m-6)\times(3m-6)], \tag{29}$$

$$\hat{\mathbf{A}} = \mathbf{A}\cdot\mathbf{N} \quad [l\times(3m-6)], \tag{30}$$

$$\hat{\mathbf{B}} = \mathbf{B} - \mathbf{A}\cdot\mathbf{G}_p \quad [l\times 1]. \tag{31}$$

Finally, a computationally efficient QP algorithm[24] is applied to obtain the constrained, optimal solution for the force distribution problem.

4. OPTIMIZATION CRITERIA FOR LOAD BALANCE AND SAFETY MARGIN ON FRICTION CONSTRAINTS

The choice of an objective function is critical. In this research, the optimization criteria for minimum force,[19] load balance,[3,12] slippage avoidance,[3] and minimizing discontinuities in commanded forces[13] have been considered. While friction constraints were included in the formulation in the previous section to avoid slippage, an additional margin of safety may be introduced and maximized. This will involve introduction of an additional variable into the formulation, and details of this will be given later in this section. Also, to include the load balance criterion into the objective function, it is often necessary to introduce additional variables and constraints, and this will be discussed in the following paragraphs. The criteria for minimum force and minimizing discontinuities in commanded forces do not involve any major changes in the formulation, and they will be considered further in a later section of the paper.

4.1. Load balance

The optimization criterion for load balance was first developed by Orin and Oh.[12] The approach they used was to minimize the maximum normal component of contact force among the chains, ${}^{C}f^{z}_{\max}$. This resulted in a criterion which was linear and relatively straightforward to include in the formulation. This new positive variable, ${}^{C}f^{z}_{\max}$, is defined as:[3,12]

$$-{}^{C}f^{z}_{k} \le {}^{C}f^{z}_{\max} \quad \mathrm{k}=1,\ldots,\mathrm{m} \tag{32}$$

where ${}^{C}f^{z}_{k}$ is the normal component of the contact force at chain k with the support surface and as expressed in a contact coordinate frame. Consequently, Eq. (32) shall be included into the force distribution formulation so that with a suitable objective function, the load may be evenly distributed among the chains. This will be explained later.

From Eq. (3), ${}^{C}f^{z}_{k}$ can be expressed in terms of $({}^{C}\mathbf{g}_C)_k$ as:

$${}^{C}f^{z}_{k} = [0\ 0\ 1]\cdot({}^{C}\mathbf{g}_C)_k. \tag{33}$$

Then

$${}^{C}f^{z}_{k} = [0\ 0\ 1]\cdot({}^{C}\mathbf{R}_o)_k\cdot({}^{o}\mathbf{g}_C)_k \tag{34}$$

where

$({}^{C}\mathbf{R}_o)_k$ = rotation matrix which transforms the components expressed in reference member coordinates into those expressed in kth contact coordinates $(C_k)[3\times 3]$, and

$({}^{o}\mathbf{g}_C)_k$ = unknown contact force vector onto reference member expressed in reference member coordinate frame (O) $[3\times 1]$.

Or

$${}^{C}f^{z}_{k} = \mathbf{a}_k\cdot({}^{o}\mathbf{g}_C)_k \tag{35}$$

with

$$\mathbf{a}_k = [0\ 0\ 1]\cdot({}^{C}\mathbf{R}_o)_k. \tag{36}$$

Combining these m equations yields:

$$-\begin{bmatrix} {}^{C}f^{z}_{1} \\ {}^{C}f^{z}_{2} \\ \vdots \\ {}^{C}f^{z}_{m} \end{bmatrix} - \begin{bmatrix} {}^{C}f^{z}_{\max} \\ {}^{C}f^{z}_{\max} \\ \vdots \\ {}^{C}f^{z}_{\max} \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{37}$$

or

$$-\begin{bmatrix} \mathbf{a}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_2 & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{a}_m \end{bmatrix}\begin{bmatrix} ({}^{o}\mathbf{g}_C)_1 \\ ({}^{o}\mathbf{g}_C)_2 \\ \vdots \\ ({}^{o}\mathbf{g}_C)_m \end{bmatrix} - \begin{bmatrix} {}^{C}f^{z}_{\max} \\ {}^{C}f^{z}_{\max} \\ \vdots \\ {}^{C}fz_{\max} \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{38}$$

In short, they are:

$$-{}^{C}\mathbf{f}^{z} - {}^{C}\mathbf{f}^{z}_{\max} \le \mathbf{0} \tag{39}$$

or

$$-\mathbf{L}\cdot\mathbf{G} - {}^{C}\mathbf{f}^{z}_{\max} \le \mathbf{0} \tag{40}$$

where ${}^{C}\mathbf{f}^{z}$ is the aggregate vector of ${}^{C}f^{z}_{k}$, $k=1,\ldots,m$, ${}^{C}\mathbf{f}^{z}_{max} = [1\ 1\ \ldots\ 1]^T\cdot{}^{C}f^{z}_{max}$, and $\mathbf{L}$ is the associated coefficient matrix with dimensions $[m\times 3m]$. Define

$$\underline{\mathbf{G}} = \begin{bmatrix} \mathbf{G} \\ \hline {}^{C}f^{z}_{\max} \end{bmatrix}_{(3m+1)\times 1} = \begin{bmatrix} ({}^{o}\mathbf{g}_C)_1 \\ \vdots \\ ({}^{o}\mathbf{g}_C)_m \\ {}^{C}f^{z}_{\max} \end{bmatrix}. \tag{41}$$

Then Eq. (40) can be included into the Original QP formulation as follows:

Original QP Form

$$\text{Minimize } \underline{\mathbf{C}}\cdot\underline{\mathbf{G}} + \frac{1}{2}\cdot\underline{G}^T\cdot\underline{H}\cdot\underline{G} \tag{42}$$

$$\text{Subject to } \underline{\mathbf{W}}\cdot\underline{\mathbf{G}} = \mathbf{F}, \tag{43}$$

$$\underline{\mathbf{A}}\cdot\underline{\mathbf{G}} \le \underline{\mathbf{B}}, \tag{44}$$

where

$$\underline{\mathbf{C}} = [\mathbf{C}|\rho]_{1\times(3m+1)}, \tag{45}$$

$$\underline{\mathbf{H}} = \left[\begin{array}{c|c} \mathbf{H} & \mathbf{0}_{3m\times 1} \\ \hline \mathbf{0}_{1\times 3m} & h_b \end{array}\right]_{(3m+1)\times(3m+1)}, \tag{46}$$

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$$\underline{\mathbf{W}} = [\mathbf{W}|\mathbf{0}_{6\times 1}]_{6\times(3m+1)}, \tag{47}$$

$$\underline{\mathbf{A}} = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{0}_{l\times 1} \\ \hline -\mathbf{L} & -\mathbf{1}_{m\times 1} \end{array}\right]_{(l+m)\times(3m+1)}, \tag{48}$$

$$\underline{\mathbf{B}} = \left[\begin{array}{c} \mathbf{B} \\ \hline \mathbf{0}_{m\times 1} \end{array}\right]_{(l+m)\times 1}, \tag{49}$$

with

$\mathbf{1}$ = $[1\ 1\ \ldots\ 1]^T$,

ρ = the weight of ${}^C f^z_{\max}$ in the linear objective function, and

h_b = the weight of ${}^C f^z_{\max}$ in the quadratic objective function.

Note that the range on $\mathbf{G}$ is unrestricted, while ${}^C f^z_{\max} > 0$.

Similarly, Eq. (40) can also be included into the Compact QP formulation:

Compact QP Form

$$\text{minimize } \underline{\hat{\mathbf{C}}} \cdot \underline{\mathbf{G}}_f + \frac{1}{2} \cdot \underline{\mathbf{G}}_f^T \cdot \underline{\hat{\mathbf{H}}} \cdot \underline{\mathbf{G}}_f \tag{50}$$

$$\text{Subject to } \underline{\hat{\mathbf{A}}} \cdot \mathbf{G}_f \le \underline{\hat{\mathbf{B}}}, \tag{51}$$

where

$$\underline{\hat{\mathbf{C}}} = [\hat{\mathbf{C}}|\rho]_{1\times(3m-5)}, \tag{52}$$

$$\underline{\hat{\mathbf{H}}} = \left[\begin{array}{c|c} \hat{\mathbf{H}} & \mathbf{0}_{(3m-6)\times 1} \\ \hline \mathbf{0}_{1\times(3m-6)} & h_b \end{array}\right]_{(3m-5)\times(3m-5)}, \tag{53}$$

$$\underline{\hat{\mathbf{A}}} = \left[\begin{array}{c|c} \hat{\mathbf{A}} & \mathbf{O}_{(l\times 1)} \\ \hline -\mathbf{L}\cdot\mathbf{N} & -\mathbf{1}_{m\times 1} \end{array}\right]_{(l+m)\times(3m-5)}, \tag{54}$$

$$\underline{\hat{\mathbf{B}}} = \left[\begin{array}{c} \hat{\mathbf{B}} \\ \hline \mathbf{L}\cdot\mathbf{G}_p \end{array}\right]_{(l+m)\times 1}, \tag{55}$$

$$\underline{\mathbf{G}}_f = \left[\begin{array}{c} \mathbf{G}_f \\ \hline {}^C f^z_{\max} \end{array}\right]_{(3m-5)\times 1}, \tag{56}$$

with $\mathbf{G}_f$ unrestricted and ${}^C f^z_{\max} > 0$.

4.2 Safety margin on friction constraints

A friction pyramid (Eqs. (4)–(7)) is used in the formulation to give linear friction force constraints. The physical interpretation for these equations is that the contact force should be constrained to be inside the friction pyramid so as to avoid slippage. A nonnegative variable, s, termed the safety margin with units of force, may be added to the friction force constraints as given in Eq. (4):

$$-{}^C f^x_k + \frac{\mu}{\sqrt{2}} \cdot {}^C f^z_k + s \le 0. \tag{57}$$

Then a positive value for s provides an additional margin within the friction pyramid for safety. Hence, slippage avoidance may be further enhanced[3] by maximizing s.

Using the aggregate vector $\mathbf{G}$, the friction force constraints (Eq. (9)) become:

$$\mathbf{Q}\cdot\mathbf{G} \le \mathbf{0}, \tag{58}$$

where $\mathbf{Q}$ is the matrix coefficients for the friction constraints and has diagonal elements of $\mathbf{S}_k$. The safety margin, s, may be added to the friction inequalities by modifying Eq. (58) as:

$$\mathbf{Q}\cdot\mathbf{G} + \mathrm{s} \le \mathbf{0}, \tag{59}$$

where $\mathrm{s} = [s\ s\ \ldots\ s]^T$ with dimensions $[4m \times 1]$.

By the same token, Eq. (59) can be included into the Original QP, and Compact QP formulations. After including the safety-margin variable, s, into the formulations, the primary variables become:

$$\underline{\underline{\mathbf{G}}} = \left[\begin{array}{c} \mathbf{G} \\ \hline {}^C f^z_{max} \\ \hline s \end{array}\right]_{(3m+2)\times 1} \tag{60}$$

and the free variables are:

$$\underline{\underline{\mathbf{G}}}_f = \left[\begin{array}{c} \mathbf{G}_f \\ \hline {}^C f^z_{max} \\ \hline s \end{array}\right]_{(3m-4)\times 1} \tag{61}$$

Then, the objective function yields

$$\underline{\underline{\mathbf{C}}} \cdot \underline{\underline{\mathbf{G}}} + \frac{1}{2} \cdot \underline{\underline{\mathbf{G}}}^T \cdot \underline{\underline{\mathbf{H}}} \cdot \underline{\underline{\mathbf{G}}} \tag{62}$$

where

$$\underline{\underline{\mathbf{C}}} = [\mathbf{C}|\rho|\eta]_{1\times(3m+2)} \tag{63}$$

$$\underline{\underline{\mathbf{H}}} = \left[\begin{array}{c|cc} \mathbf{H} & \multicolumn{2}{c}{0_{3m\times 2}} \\ \hline O_{2\times 3m} & h_b & 0 \\ & 0 & h_s \end{array}\right]_{(3m+2)\times(3m+2)} \tag{64}$$

with η representing the weight for s in the linear objective function and h_s representing the weight for s in the quadratic objective function.

With completion of the above formulation for the force distribution problem, a specific case will be considered to illustrate the approach. The force distribution problem of the TIT Quadruped provides such an example and will be presented in the next section.

5. EXAMPLE MECHANISM

The schematic diagram and graphical simulation model for the TIT Quadruped are shown in Fig. 2. See Leg 1 of Fig. 2, the weights for Links 1–4 of Leg 1 are 20.4, 11, 3.92, and 1.8 kg, respectively. The body weight is 120 kg. Therefore, the total weight of the TIT Quadruped is

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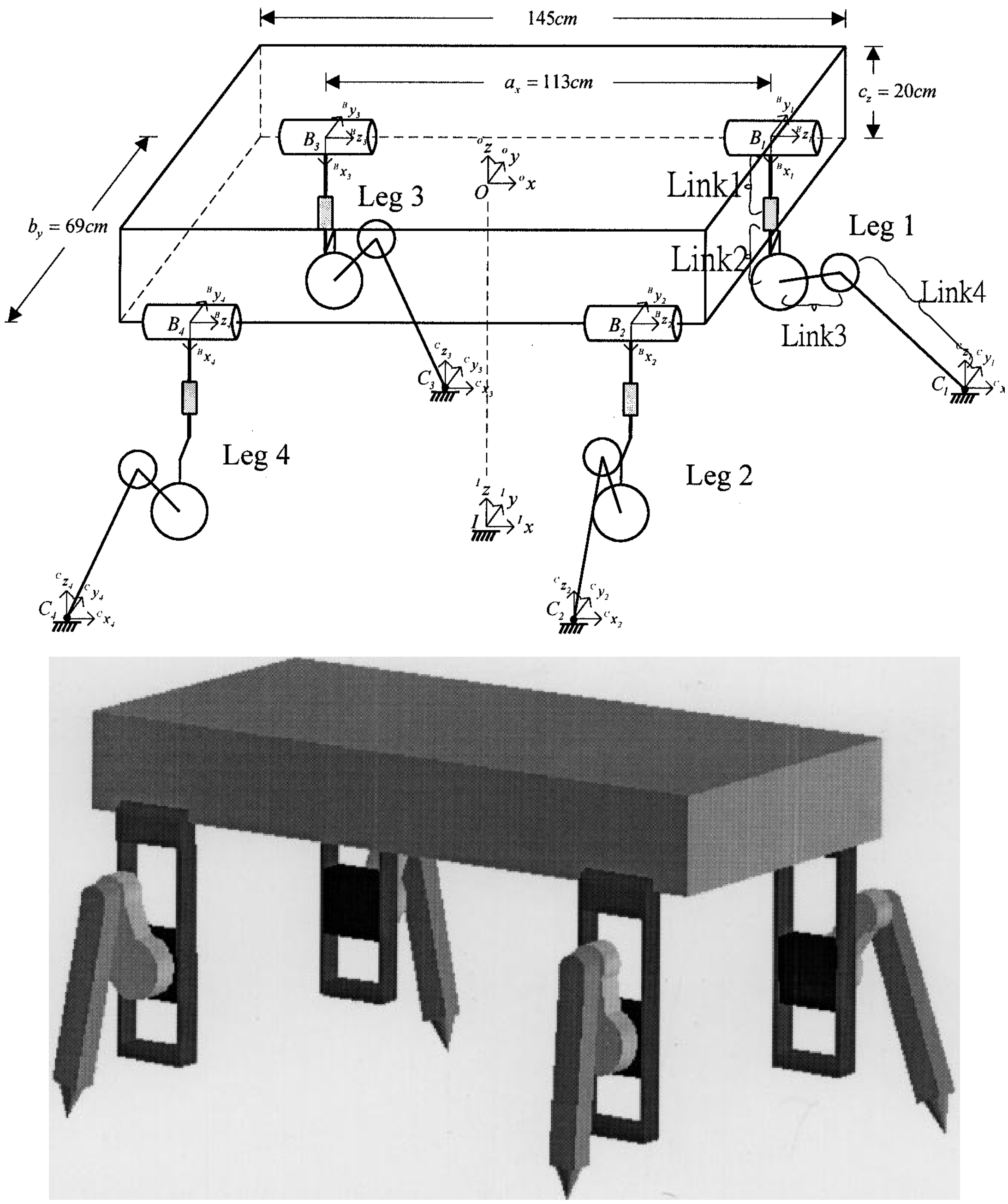

Fig. 2. Schematic diagram and graphical simulation model for the TIT Quadruped (I = initial, O = reference member, B = base, and C = contact coordinate frames.)

268.48 kg. The TIT Quadruped has four legs, each with 3 degrees of freedom. Joint 1 is a rotational joint for sidestepping movement; Joint 2 is a sliding joint for up/down motion; and Joint 3 is a rotational joint in forward/backward direction. The body is the reference member and the chains (legs) are in frictional contact with the ground. The case of 4 legs is considered (m = 4), and each leg has 3 degrees of freedom ($N = 3$). Also, for hard point contact with friction, 3 degrees of constraint are imposed at the contact point. For this example, a reference member coordinate frame (O) is defined with its origin at the geometric center of the object and with the x-axis parallel with the body central line forward direction. The z-axis of the reference member coordinate frame is pointing upward, while the y-axis is assigned by the right-hand rule.

The definition of the contact coordinate frame (C) is also shown in Fig. 2.

Before evaluating the effects of various objective functions, the motion planning for the TIT Quadruped shall be defined first. In the following section, wave gaits are described.

6. GAIT PLANNING

A gait is the sequence of lifting and placing of legs during the locomotion of a legged system. A specific type of gait called a wave gait is used in this paper. Wave gaits are common in nature and are characterized by a longitudinally symmetric legged system. In this section, a brief summary of McGhee's work in formalizing gait definitions and specifications is presented.[25]

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A periodic gait is a specific type of gait in which every limb has the same cycle time. Periodic gaits can greatly simplify motion planning. The period, T, is the time required for one locomotion cycle of a periodic gait. The stride length, λ, is the distance by which the center of gravity of a system is translated during a complete locomotion cycle. Stride length, λ, and period, T, together determine the velocity of the system.

Periodic gaits may be characterized by two parameters for each leg. The relative leg phase, ϕ_i, is the fraction of the locomotion cycle by which the contact of leg i with the supporting surface lags the contact of leg 1 (front left leg). The duty factor, β_i, of leg i is the fraction of a locomotion cycle during which leg i is in contact with the supporting surface. Gaits in which all legs share a common duty factor, β, are called regular gaits, and ones in which the motion of each left-right pair is exactly one-half cycle out of phase are known as symmetric gaits.

A parameter known as the stroke, R, may be related to the stride length and duty factor for a gait. Stroke is the distance by which the center of gravity of a system is translated during the support phase of a leg. Stroke is equal to stride length times duty factor.

An important parameter of gaits is the longitudinal stability margin. This is the shortest distance, over an entire cycle of locomotion, from the vertical projection of the center of gravity onto the supporting surface, to an edge of the support pattern as measured in the direction of travel. A statically stable gait is a gait in which the longitudinal stability margin is always positive. Note that this depends on kinematics as well as leg sequencing.

Wave gaits are the unique gaits which maximize longitudinal stability margin for any given duty factor in longitudinally symmetric walking vehicles.[26] They are periodic, regular, and symmetric, and are characterized by a progression of stepping events from back to front of a walking vehicle.

The quadruped gait diagram and leg placing sequence for wave gaits with $\beta = \frac{5}{6}$ is shown in Fig. 3, where hollow circles represent leg lifting and solid circles stand for leg placing.

Using the general formulation of Section 2, the Compact QP method of Section 3, together with various optimization criteria of Section 4, and the gait planning of this section, it is now possible to study various effects for alternative objective functions. Section 7 describes the resulting simulation.

7. ALTERNATIVE OBJECTIVE FUNCTIONS

The Compact QP method is a general optimization algorithm which can accept most linear and quadratic objective functions without reformulation. Therefore, we may apply this method to accomplish a variety of objectives.

In order to demonstrate the versatility of the Compact QP method, several objective functions have been considered to solve the force distribution problem of the example mechanism, the TIT Quadruped. The associated weights of $\mathbf{C}$, ρ, η, and $\mathbf{H}$, h_b, h_s in Eqs. (63) and (64) as well as the desired vector $\mathbf{Z}$ in Eq. (16) will be defined, followed by presenting the corresponding simulation results.

7.1 *Circular motion planning*

In order to evaluate the proper objective functions (such as minimum force, load balance, and safety margins on friction constraints) for the optimal force distribution, a suitable motion planning shall be defined. We have tried several body motion plannings such as circular motion, up-and-down simple harmonic motion, left-and-right simple harmonic motion, etc. We discovered that the circular motion is the most general one because it can generate suitable varying forces in all x, y, and z directions for evaluating those objective functions.

The circular motion planning is designed with all the 4 legs of the quadruped being on the ground and the center of gravity of the body performs a plannar circular trajectory. The center of the circle is at the geometric center, the radius of the circle is 20 cm, and the period is 5 sec. This motion planning is applied to evaluate the effectiveness of the objective functions for the minimum force, load balance, and safety margins on friction constraints.

7.2. *Minimum force*

The most common minimum effort criterion is the minimum norm of all the contact forces (for the case of hard

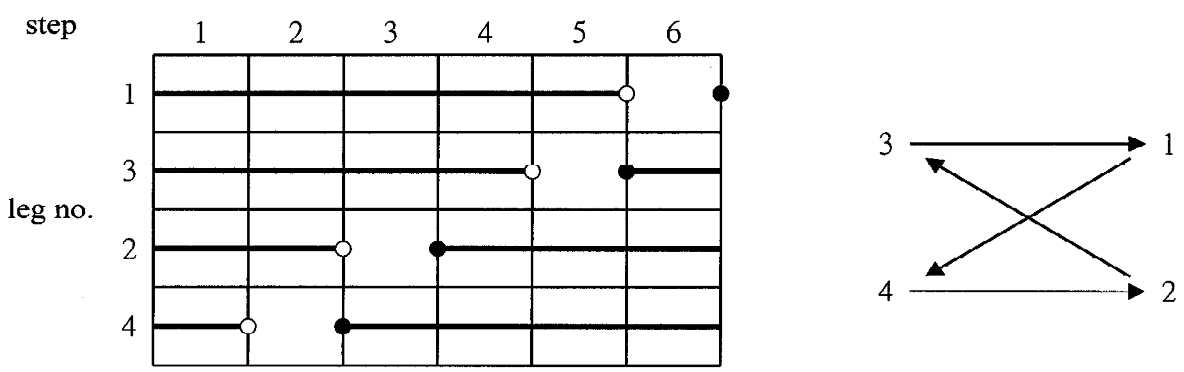

Fig. 3. Quadruped gait diagram and leg placing sequence for wave gaits with $\beta = \frac{5}{6}$.

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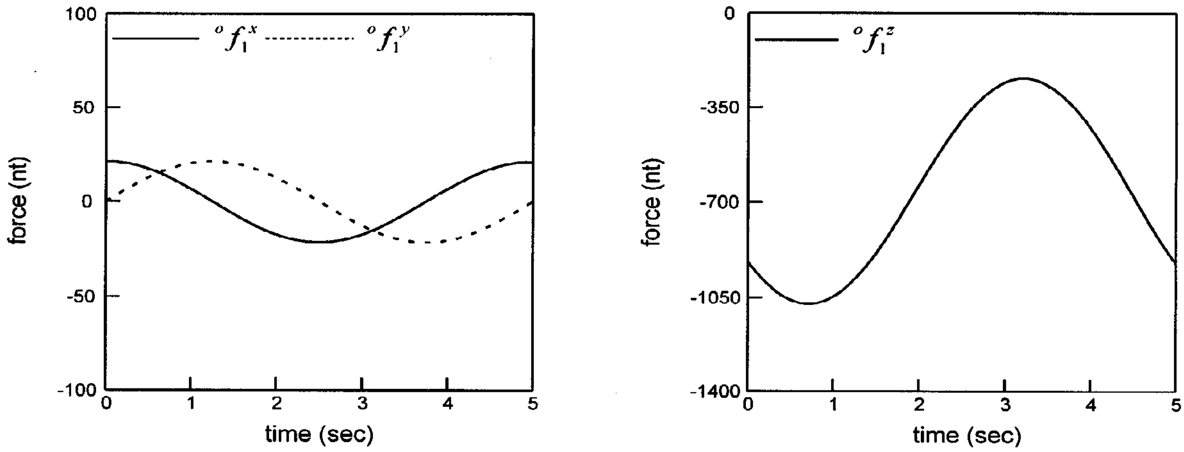

Fig. 4. The contact forces for foot 1 at C_1 expressed in the reference member coordinate frame (O) under circular motion and with $\mu = 0.5$. Objective function for MF used.

point contacts).[10] The objective function for minimum force (MF) is:

$$\text{Minimize } \frac{1}{2} \cdot \mathbf{G}^T \cdot \mathbf{H} \cdot \mathbf{G} \tag{65}$$

Therefore, the associated weights are:

$$\mathbf{C} = \mathbf{0}, \tag{66}$$

$$\rho = 0, \tag{67}$$

$$\eta = 0, \tag{68}$$

$$\mathbf{H} = \mathbf{I}, \tag{69}$$

$$h_b = 0, \tag{70}$$

$$h_s = 0. \tag{71}$$

Figure 4 shows the simulation results of the contact forces using this objective function.

7.3 *Load balance*

In order to take advantage of using all the feet on the ground, the load should be more equally shared by them. The strategy of load balance (LB) is to minimize the maximum normal component of the contact forces, $^{C}f^{z}_{\max}$. Therefore, the objective function yields:

$$\text{Minimize } \; ^{C}f^{z}_{\max} \tag{72}$$

or

$$\text{Minimize } \; (^{C}f^{z}_{\max})^2. \tag{73}$$

The corresponding weights for Eq. (72) are $\rho = 1$ and all the others being set to zeros. As for the associated weights for Eq. (73), they are $h_b = 1$ and all the others being set to zeros. Both objective functions will result in similar results. The simulation results of Eq. (73) are shown in Fig. 5. Comparing the magnitudes of the z-component contact forces between MF and LB, indeed those for LB objective are smaller. However, the x- and y-component contact forces display discontinuous results. It is due to the fact that in this motion planning, all four contact points are on the ground and in the same x–y plane, therefore the force balance equation (Eq. (20)) can be decomposed into 2 smaller linear equations with one possesses the variables of x and y components only and the other has the variables of z component only.[23] Also, because the objective function specified in Eq. (73) can merely affect the z-component contact forces, this optimization process cannot control the

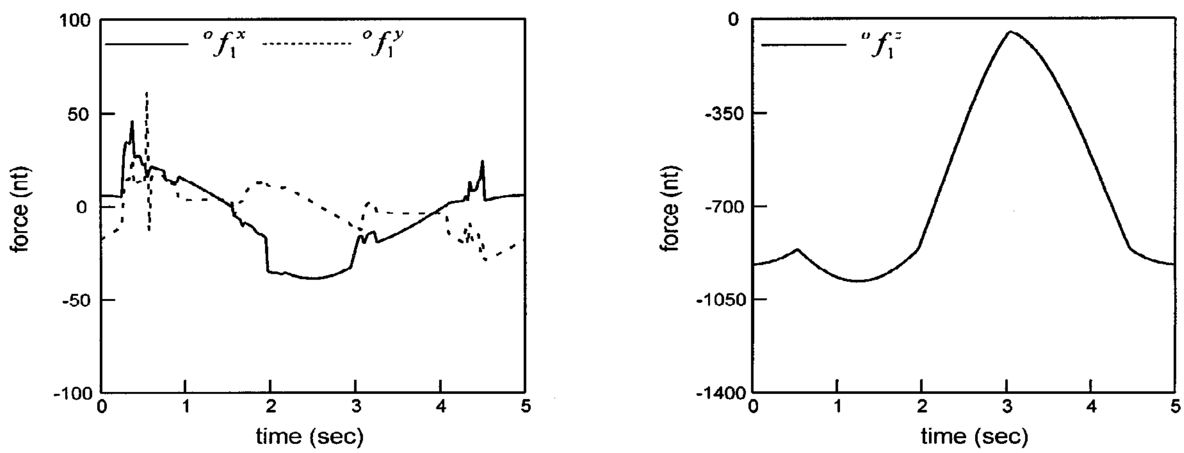

Fig. 5. The contact forces for foot 1 at C_1 expressed in the reference member coordinate frame (O) under circular motion and with $\mu = 0.5$. Objective function for LB used.

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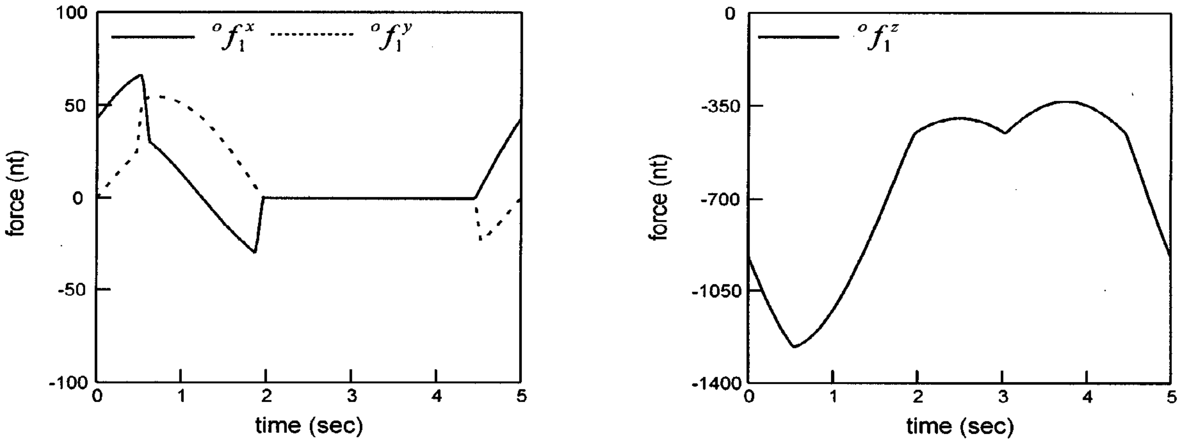

Fig. 6. The contact forces for foot 1 at C_1 expressed in the reference member coordinate frame (O) under circular motion and with $\mu = 0.5$. Objective function for SM used.

results of x- and y-component contact forces. Further, since the decomposed, smaller equations which possess the variables of x and y components only are redundant (8 variables in 3 equations), multiple solutions exist.[23] As such, any feasible solutions may result, it causes these discontinuous solution sequences in x and y directions as shown in Fig. 5.

7.4 Safety margins on friction constraints

For the purpose of avoiding slippage, the objective function of safety margin (SM) is specified as:

$$\text{Minimize} \quad -s \tag{74}$$

which results in $\eta = -1$ and all the other weights being set to zeros. This will bring the contact forces away from the edge of the friction force constraints. Figure 6 shows the simulation results for the contact forces over time, with $\mu = 0.5$.

The curves of ${}^C f^z_{\max}$ for MF, LB, and SM are depicted in Fig. 7. Figure 7 shows that LB produces the minimum-effort results among those three objective functions. Also shown in Fig. 7, to maximize slippage avoidance, the TIT Quadruped exerts more effort than the cases for MF and LB. Hence, it is recommended not to use the objective function of SM. If indeed safety margins on friction constraints need to be considered, we may assign a smaller value of μ such that the desired safety margins can be achieved in the inequality constraints.

7.5. Minimum force plus load balance

From the above analysis, it appears that we may combine the advantages obtained from both MF and LB such that smooth solution sequences and less normal contact forces may be achieved. The desired objective function becomes:

$$\text{Minimize} \quad \frac{1}{2} \cdot \mathbf{G}^T \cdot \mathbf{H} \cdot \mathbf{G} + ({}^C f^z_{\max})^2 \tag{75}$$

Figure 8 shows the simulation results of all the contact forces of feet 1–4 for applying MF + LB. From Fig. 8, the results of the optimal force distribution between the legs are clearly displayed. Comparing the contact forces for foot 1 among Figures 4, 5, and 8, we can conclude that the x- and y-component contact forces of MF + LB are the same as those of MF and the z-component contact forces of MF + LB are identical to those of LB.

The objective function for MF + LB will also be applied to the TIT Quadruped for walking on an even terrain with wave gaits. The setup for the straight, even-terrain motion planning is $\lambda = 60$ cm, $T = 9.6$ sec, and $\beta = \frac{5}{6}$. Also, the gait diagram and leg placing sequence shown in Fig. 3 are adopted. As depicted in Fig. 9, with the objective function of MF + LB, the curve of the force setpoint is piecewise linear. Notice that the curve shows discontinuities whenever the set of supporting legs is changed. These discontinuities in the past caused large control impulses to the system.[13] In this research, we would like to modify the gait diagram and

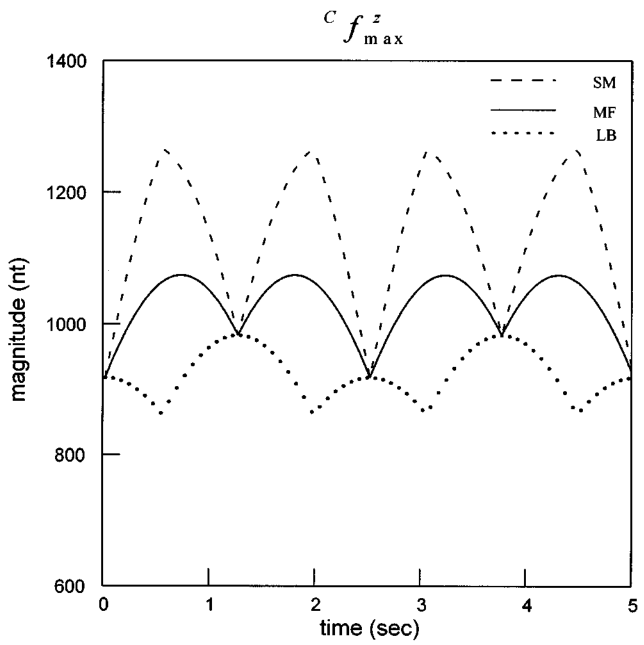

Fig. 7. Comparison of ${}^C f^z_{\max}$ among the objective functions of MF, LB and SM.

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derive a proper set of desired vector, $\mathbf{G}_d$, to manipulate the homogeneous solution for solving this problem. It will be presented next.

7.6. Smoothing discontinuities in commanded forces

Klein and Chung[13] have applied the so-called minimum-perturbation solution method to solve this problem. In their

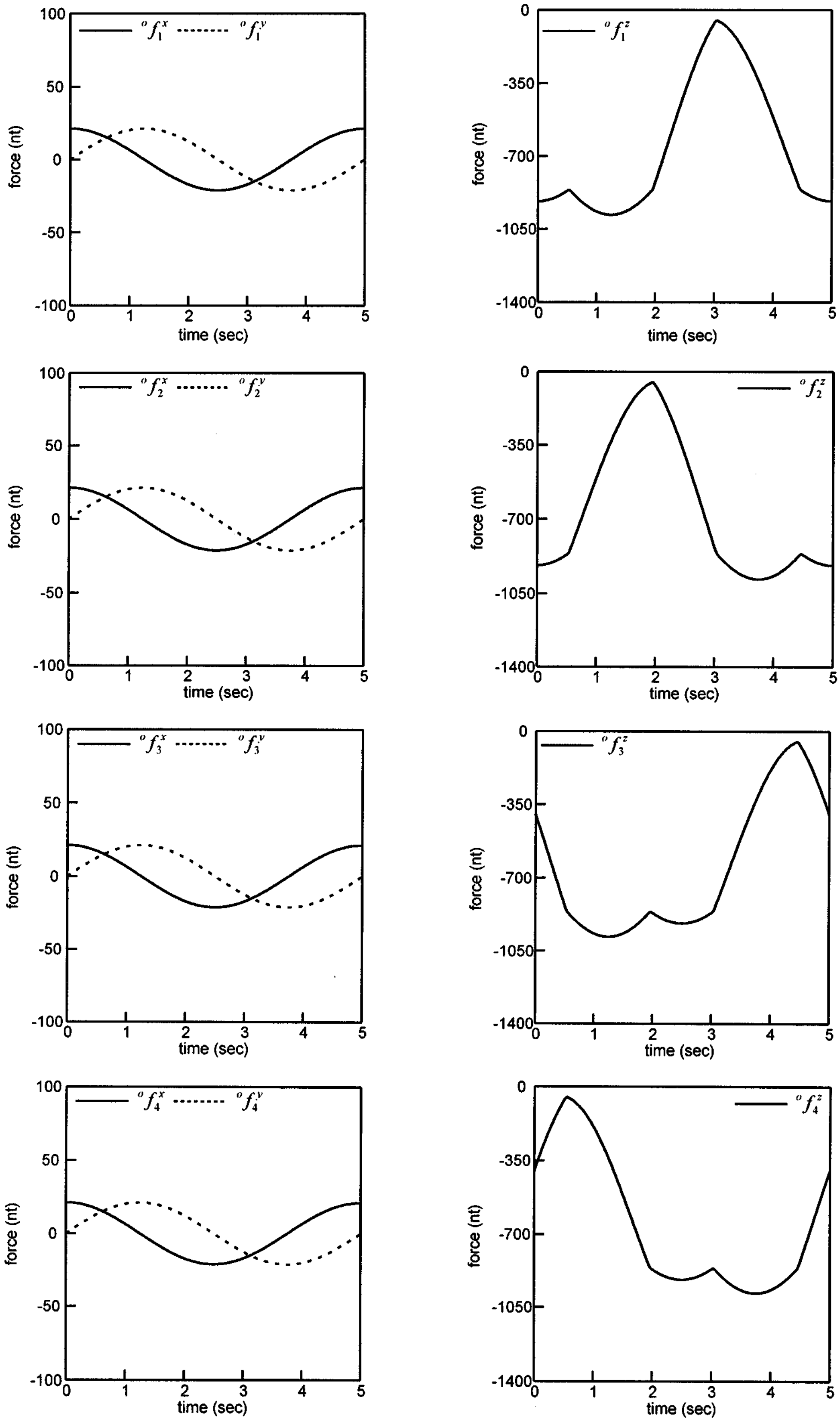

Fig. 8. The contact forces for feet 1–4 at C_1–C_4 expressed in the reference member coordinate frame (O) under circular motion and with $\mu=0.5$. Objective function for MF+LB used.

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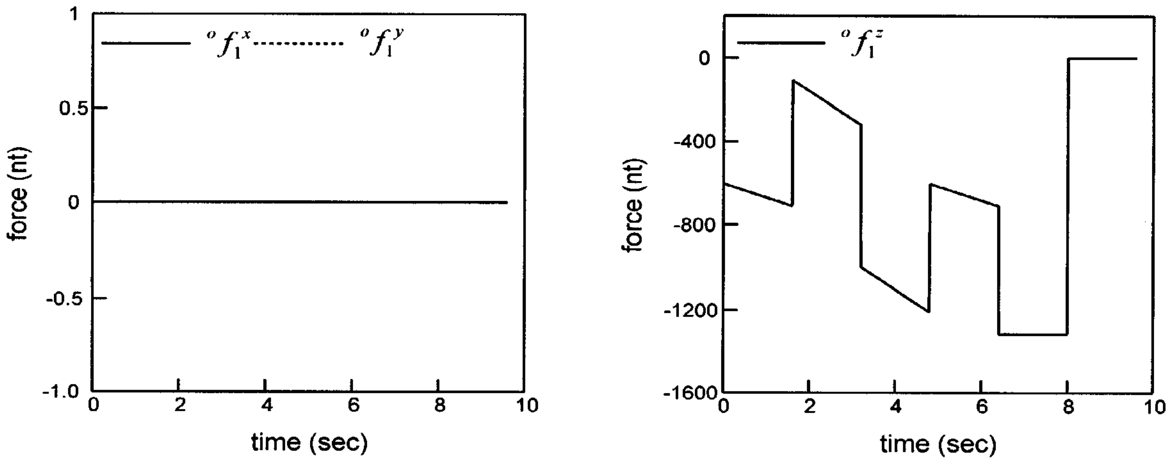

Fig. 9. The contact forces for foot 1 at C_1 expressed in the reference member coordinate frame (O) for walking on an even terrain using wave gaits with $T = 0.6$. sec and $\beta = \frac{5}{6}$. Objective function for MF + LB used.

approach, the actual contact forces shall be available (via force sensors) and assigned to be the desired vector, **Z**, such that the objective function as shown in Eq. (16) can be applied to minimize the differences between the actual contact forces, **Z**, and the optimal solution of **G**. However, force sensors are expensive, fragile, and may have slow reaction time. It is desirable to propose an alternative which does not require force sensors.

The approach proposed in this research is to add 20-sample smoothing periods (the number, 20, is simply a design that we chose to undertake) when the leg phase alternates between support and transfer. During the smoothing period, a desired contact-force sequence will be defined wbich will guide the optimal solution to gradually reduce the magnitude of the contact forces when lifting or gradually increase the magnitude of the contact forces when placing. The gait diagram of Fig. 3 is then modified as in Fig. 10.

As depicted in Fig. 10, a hollow triangle represents begin-lifting and a hollow circle stands for end-lifting. Also, a solid diamond represents begin-placing and a solid circle stands for end-placing. Two more 20-sample smoothing periods (for 0.2 sec each) are included during leg switching between Leg 2 and Leg 4 as well as between Leg 1 and Leg 3. Therefore, the corresponding duty factor and leg phases yield $\beta = 0.84$, $\phi_1 = 0$, $\phi_2 = 0.5$, $\phi_3 = 0.82$, and $\phi_4 = 0.32$.

Three cases exist for determining the desired contact-force sequence, $\mathbf{G}_d$.

Case 1: Leg Lifting

Suppose that Leg 4 is lifting and only the normal contact forces are considered, then, we may assign the desired $[{}^{C}f_1^z, {}^{C}f_2^z, {}^{C}f_3^z, {}^{C}f_4^z]$ to change from

$$\left[\frac{M}{4}, \frac{M}{4}, \frac{M}{4}, \frac{M}{4}\right]$$

to

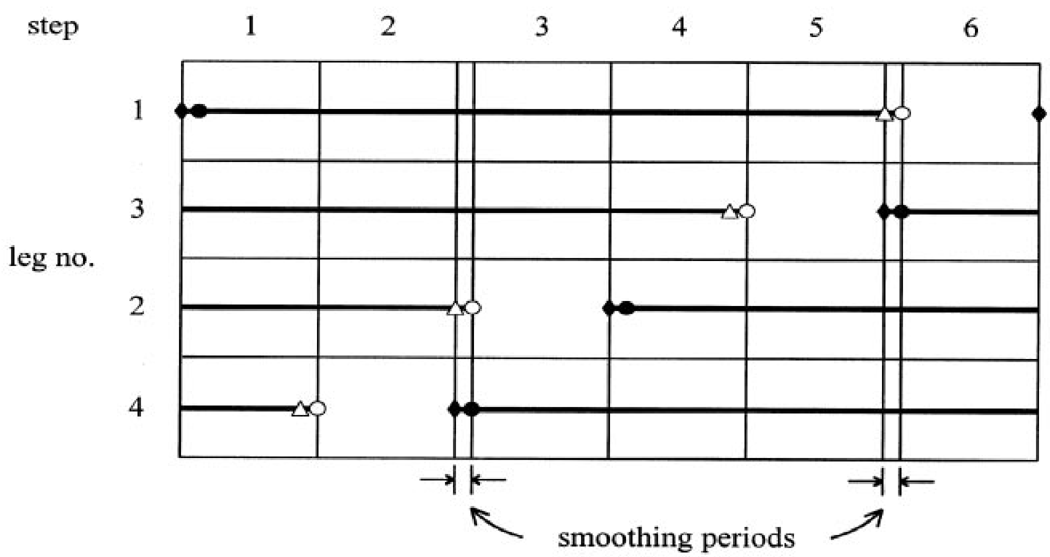

Fig. 10. Quadruped gait diagram for including two more 20-sample smoothing periods with $\beta = 0.84$, $\phi_1 = 0$, $\phi_2 = 0.5$, $\phi_3 = 0.82$, and $\phi_4 = 0.32$.

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$$\left[\frac{M}{3}, \frac{M}{3}, \frac{M}{3}, 0\right]$$

gradually, where M stands for the mass of the quadruped. A half-cosine curve is chosen to smooth the desired contact-force sequence:

i) from $\frac{M}{4}$ to 0:

$$\gamma_{10} = \frac{1}{2}\left[1 + \cos\left(\frac{\pi}{20} \cdot t_s\right)\right] \quad t_s = 0 - 20 \tag{76}$$

ii) from $\frac{M}{4}$ to $\frac{M}{3}$ $\left(\text{or from 0 to } \left(\frac{M}{3} - \frac{M}{4}\right)\right)$:

$$\gamma_{01} = 1 - \frac{1}{2}\left[1 + \cos\left(\frac{\pi}{20} \cdot t_s\right)\right] \quad t_s = 0 - 20 \tag{77}$$

Thus, $\mathbf{G}_d$ is defined as:

$$\mathbf{G}_d = \left[0\ 0\ \left(\frac{M}{4} - \frac{M}{3}\right) \times \gamma_{01} - \frac{M}{4}\ \middle|\ 0\ 0\ \left(\frac{M}{4} - \frac{M}{3}\right) \times \gamma_{01} - \frac{M}{4}\ \middle|\ 0\ 0\ \left(\frac{M}{4} - \frac{M}{3}\right) \times \gamma_{01} - \frac{M}{4}\ \middle|\ 0\ 0\ -\frac{M}{4} \times \gamma_{10}\right]^T \tag{78}$$

Case 2: Leg Placing

Suppose that Leg 1 is placing and only the normal contact forces are considered, then, we may assign the desired $[{}^C f_1^z, {}^C f_2^z, {}^C f_3^z, {}^C f_4^z]$ to change from

$$\left[0, \frac{M}{3}, \frac{M}{3}, \frac{M}{3}\right]$$

to

$$\left[\frac{M}{4}, \frac{M}{4}, \frac{M}{4}, \frac{M}{4}\right]$$

gradually. Therefore, $\mathbf{G}_d$ yields:

$$\mathbf{G}_d = \left[0\ 0\ -\frac{M}{4} \times \gamma_{01}\ \middle|\ 0\ 0\ \left(\frac{M}{4} - \frac{M}{3}\right) \times \gamma_{10} - \frac{M}{4}\ \middle|\ 0\ 0\ \left(\frac{M}{4} - \frac{M}{3}\right) \times \gamma_{10} - \frac{M}{4}\ \middle|\ 0\ 0\ \left(\frac{M}{4} - \frac{M}{3}\right) \times \gamma_{10} - \frac{M}{4}\right]^T \tag{79}$$

Case 3: Leg Switching on the Same Side

Suppose that Leg 4 is placing and Leg 2 is lifting simultaneously and only the normal contact forces are considered, then, we may assign the desired $[{}^C f_1^z, {}^C f_2^z, {}^C f_3^z, {}^C f_4^z]$ to change from

$$\left[\frac{M}{3}, \frac{M}{3}, \frac{M}{3}, 0\right]$$

to

$$\left[\frac{M}{3}, 0, \frac{M}{3}, \frac{M}{3}\right]$$

gradually. As such, $\mathbf{G}_d$ becomes:

$$\mathbf{G}_d = \left[0\ 0\ -\frac{M}{3}\ \middle|\ 0\ 0\ -\frac{M}{3} \times \gamma_{10}\ \middle|\ 0\ 0\ -\frac{M}{3}\ \middle|\ 0\ 0 - \frac{M}{3} \times \gamma_{01}\right]^T \tag{80}$$

Now, we may assign

$$\mathbf{Z} = \alpha \cdot \mathbf{G}_d \tag{81}$$

where α is a positive scaling factor. Note that, while this method provides the optimal direction for the contact force vector within the null space, its magnitude is not unique and is adjusted[27] by the scalar coefficient, α.

The simulation results by using the gait diagram shown in Fig. 10 are depicted in Fig. 11. Two cases are shown: one

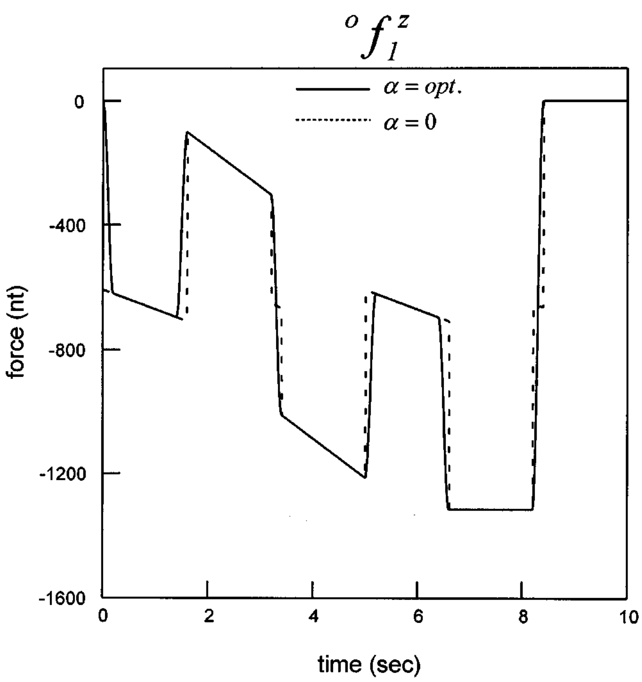

Fig. 11. Comparison of the smoothing effect between $\alpha = 0$ and $\alpha =$ optimal value.

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for $\alpha = 0$ (without smoothing) and the other for $\alpha =$ optimal value. The smoothing effect by using this approach is clearly displayed in Fig. 11.

8. CONCLUSIONS

Optimal force distribution in multilegged vehicles was studied in this paper. It began by summarizing the general formulation for the force distribution problem. Then, the Compact QP method was introduced and the optimization criteria for load balance and safety margin of friction constraints were also formulated. The TIT Quadruped was adopted as the example mechanism. Then, the gait planning was introduced followed by evaluating alternative objective functions. The combined objective function for MF + LB was proposed to be the basic objective function for optimal force distribution. Finally, a scheme for smoothing discontinuities in commanded forces was proposed. In conclusion, the Compact QP method is a general and efficient optimization scheme for resolving the constrained, optimal force distribution problem with multiple criteria. Also, the proposed smoothing scheme can effectively manipulate the null space solution to smooth the discontinuities in commanded forces without the help of force sensors.

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