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A Unified Analytical Framework for Ship Domains

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Most of the existing typical ship domains have been comprehensively reviewed and classified. Most of these ship domains are described in a geometrical manner that is difficult to apply to practices and simulations in marine traffic engineering. According to different types of geometrical ship domains, we have proposed mathematical models, based on which a unified analytical framework has been established. It is feasible and practical for the analytical models to be applied to the assessment of navigational safety, collision avoidance and trajectory planning, *etc*. Finally, some computer simulations and comparative studies of the proposed domain model have been presented and the simulation results show that the uniform analytical framework for ship domains is effective and identical to the original geometrical ones. It should be noted that the analytical domain models could be directly applied in any collision risk, collision avoidance or VTS system while the geometrical ones would be more illustrative but less practical or analytical.

KEY WORDS

1. Ship domain. 2. Collision avoidance. 3. Collision risk. 4. Analytical model

1. INTRODUCTION. The concept of ship domain, which was first defined by Fujii (Fujii, 1971) and Goodwin (Goodwin, 1975), has been widely used in marine traffic engineering since the 1970s. Following this concept over the past 30 years, some researchers (Davis, 1980, 1982; Coldwell, 1983; Zhao, 1993; Zhu, 2001; Smierzchalski, 2001; Kijima, 2001, 2003; Pietrzykowski, 2004, 2006, 2008) have also presented various ship domains with different shapes and sizes taking into account different factors affecting the domain parameters. Ship domains play a very important role in risk assessments (Pietrzykowski, 2008; Szlapczynski, 2006), collision avoidances (Hwang, 2002; Kao, 2007; Wilson, 2003), marine traffic simulations (Lisowski, 2000) and optimal trajectory planning (Smierzchalski, 2000), *etc*. Statistics show that human errors have caused 80% of marine accidents for which the main reason is inappropriate assessments of the navigational situation and the consequent wrong decisions. However, most of the existing ship domain models are described in a geometrical manner which is easy to understand but not conducive for application to practices or simulations. Therefore there is a need for a uniform analytical framework to describe ship domain models in order that these models could play a powerful role in marine traffic engineering. In this paper, the existing typical ship domains will be classified and described in a mathematical manner,

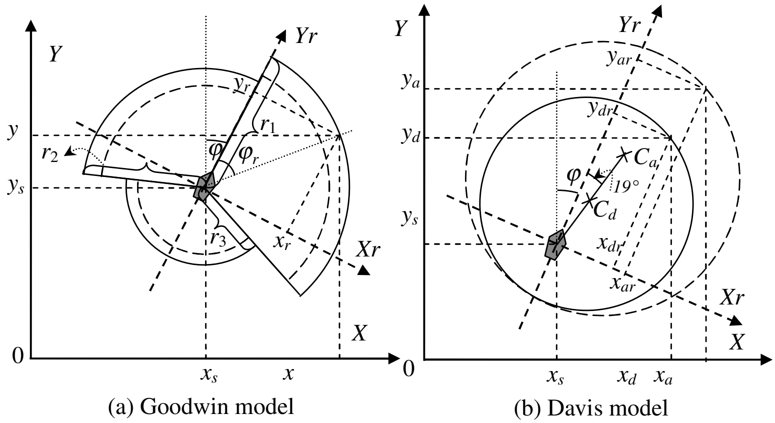

Figure 1. Circular ship domains, where *Xo-Yo* is the Earth-fixed coordinates system and *Xr-Yr* is the rotated ship-fixed coordinates system.

based on which a uniform analytical framework will be established. Some comparative simulation studies and analyses will also be presented.

2. THE EXISTING TYPICAL SHIP DOMAINS.

In the past thirty years or more, many researchers have presented various ship domains with different shapes and sizes having taken into account different factors that affect the parameters of ship domains. It should be noted that the determination of ship domains presented by statistical or intelligent methods strongly depend on the statistical data and navigators' experience. Looking deeply at most of the existing typical ship domains, we find they were apt to be described by geometrical figures including circle, ellipse, polygon and other complex figures rather than in an analytical manner since it is difficult to analytically describe the ship domains derived from statistical data or navigators' experience. Ship domain boundaries could not only be crisp but also fuzzy for assessing navigational safety and collision avoidance. In addition, for a resultant shape type of ship domain, the model could be represented as stationary or dynamic corresponding to the variables affecting ship domains. So, the existing typical ship domains could be roughly distinguished as circular, elliptical and polygonal ship domains according to the resulting domain shape regardless of what method had been used.

2.1. *Circular Ship Domains.* In 1975, Goodwin (Goodwin, 1975) proposed a ship domain of which the boundary is divided into three sectors, as shown in Figure 1(a), according to the arcs of a ship's sidelights and stern light. The Goodwin model is also derived by statistical methods from a large number of records and simulator data, and contains further results of different sizes for different sea areas. Usually, we adopted the parameters that $r_1 = 0{\cdot}85$n.m., $r_2 = 0{\cdot}70$n.m. and $r_3 = 0{\cdot}45$n.m. Subsequently, a modified circular ship domain shown in Figure 1(b), which made its modelling easier, was proposed by Davis (Davis, 1980, 1982) in the 1980s. The ship domain is a circle of which the area is equal to the total of segments for the Goodwin model, but is obtained by off-centring the position of the ship within this circle so that the weighting of the differing areas for the various sectors is retained. The Davis model consists of two circles of which the second one with the ship off-centre was introduced and called the ship arena. This is used for navigators to be aware of other ships and decide what actions, if any, are needed to keep his own ship domain unviolated. On the basis of statistical data analysis, the popular parameters have been obtained as arena radius $r_a = 2{\cdot}7$n.m., domain radius $r_d = 1{\cdot}7$n.m., off-centring distance from arena circle centre $d_a = 1{\cdot}7$n.m., off-centring distance from domain circle centre $d_d = 0{\cdot}7$n.m. Later, Zhao *et al.* (Zhao, 1993) proposed a definition of fuzzy ship domain shown as broken lines in Figure 1(a) based on the Goodwin model using fuzzy sets theory, which determines a ship domain boundary and a fuzzy ship domain boundary of which the membership function value to the set "safe distance" is 0·5. It was assumed that only if the area defined by the fuzzy ship domain boundary were to be interrupted, would the navigator's action be necessary. A concept of subjective ship domains based on neural networks has been presented by Zhu (Zhu, 2001). Unlike the objective ones described in geometrical manners, the subjective domain based on neural networks is a nonlinear mapping from inputs to output and therefore it could express the effect of visibility and ship manoeuvrability and react quickly to a variety of situations. However, the model can only be applied to limited types of ships since it needs plenty of learning samples to train the network.

2.2. *Elliptical Ship Domains.* Originally, the first elliptical ship domain was derived by Fujii (Fujii, 1971) from a mass of recorded data registering ships' positions and movement trajectories in Japanese waters by using statistical methods. As depicted in Figure 2(a), it is an ellipse, of which the geometrical centre is identical to the position of the ship; the semi-major b and semi-minor a are four times and 1·6 times the ship length L respectively. In the 1980s, Coldwell (Coldwell, 1983) established another elliptical ship domain by similar statistical methods for head-on and overtaking encounter situations in restricted waters. As shown in Figure 2(b), it is a half ellipse of which the geometrical centre is no longer identical to the position of the ship for the head-on model, where the parameters are that $b = 6{\cdot}1L$, $a_1 = 1{\cdot}75L$, $a_2 = 3{\cdot}25L$. As shown in Figure 2(c), it is still an ellipse with the ship being on the geometrical centre except for the changed semi-major and minor for the overtaking model, where the parameters are $b = 6{\cdot}0L$ and $a = 1{\cdot}75L$. Both the Fujii and Coldwell models just adequately take into consideration international sea regulations. Recently, Kijima (Kijima, 2001, 2003) proposed a new ship domain modelled by "Blocking area" and "Watching area" which are defined as combinations of two ellipses of which the parameters are R_{bf}, R_{ba} and S_b, as well as R_{wf}, R_{wa} and S_w, as shown in Figure 2(d). R_{bf} and R_{ba} indicate longitudinal radius of the blocking area in fore and aft domains respectively and S_b is common transverse radius in both domains. Additionally, the parameters of ship length L, breadth B, relative speed ΔU and relative angle α between courses have been fully considered and introduced to define the ship domain using estimation formulae for these abovementioned area parameters. Obviously, this is a dynamic ship domain model which accounts for ship dimensions, manoeuvrability, encounter situations and target ship states.

2.3. *Polygonal Ship Domains.* Recently, some literature presented polygonal ship domains allowing the determination of dynamic dimensions of domains, which are mostly functions of ship dimensions and ship's speed in relation to other navigational objects. A version of the above approach features a relative domain for a target ship, as shown in Figure 3(a), proposed by Smierzchalski (Smierzchalski, 2001, 2003),

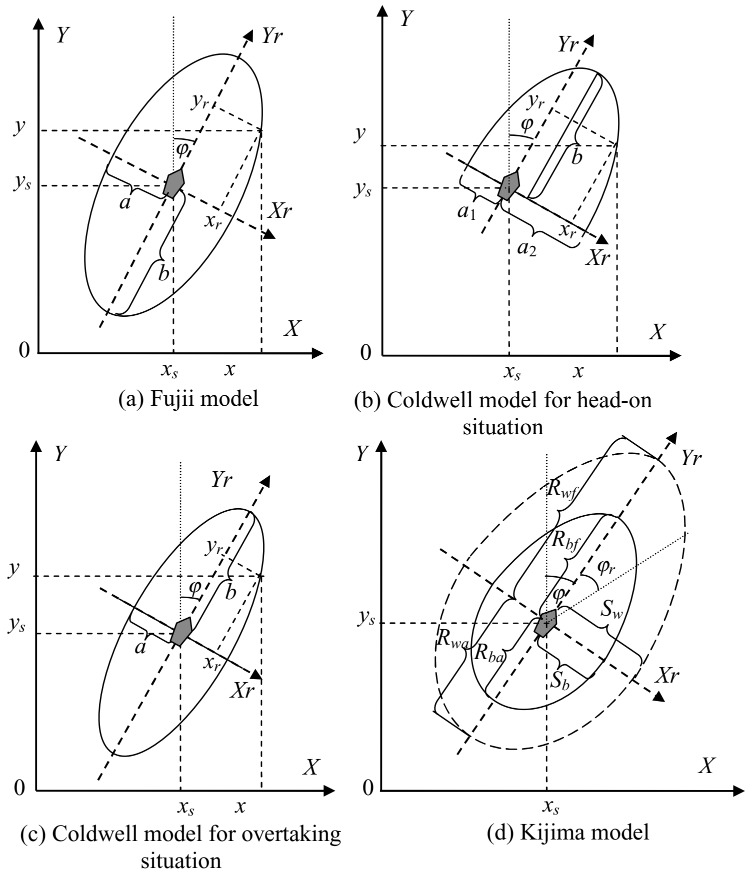

Figure 2. Elliptical ship domains, where *Xo-Yo* is the Earth-fixed coordinates system and *Xr-Yr* is the rotated ship-fixed coordinates system.

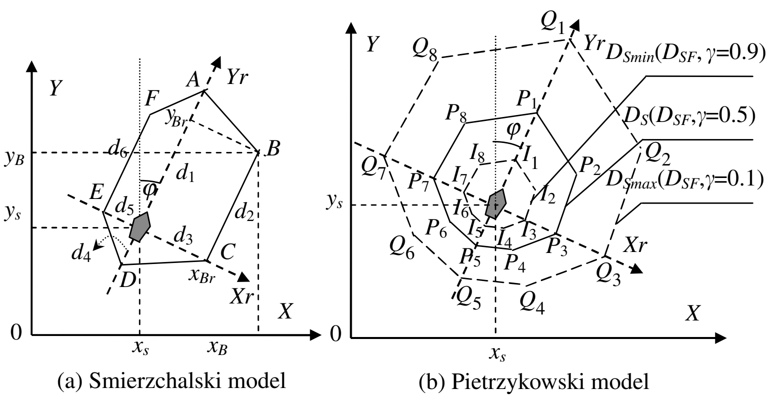

Figure 3. Polygonal ship domains, where *Xo-Yo* is the Earth-fixed coordinates system and *Xr-Yr* is the rotated ship-fixed coordinates system.

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of which the figure is a hexagon defined on the basis of dynamic parameters of own and target ship. This analytical method makes it possible to define a ship domain precisely; however, the human factor has not been accounted for. Another version, shown in Figure 3(b), proposed by Pietrzykowski (Pietrzykowski, 2004, 2006), defined ship domains as polygons of which the shapes depend on the discretization step of target ship course and usually is octagonal since the discretization steps adopt 45°. The expert research and questionnaires dealing with various ship encounter situations in an open sea in good visibility had determined the dynamically changing shapes and sizes of the ship domains D_S and ship fuzzy domains D_{SF} according to various situations. However, the dynamic domains would make the assessment of a navigation situation difficult. It follows that the mean ship domain D_S, the maximum ship domain D_{Smax} and the minimum ship domain D_{Smin} obtained by statistical methods, are more powerful and effective. It should be noted that D_{Smax} and D_{SF} with navigational safety $\gamma=0{\cdot}1$, D_S and D_{SF} with navigational safety $\gamma=0{\cdot}5$, as well as D_{Smin} and D_{SF} with navigational safety $\gamma=0{\cdot}9$, are comparable, respectively.

3. MATHEMATICAL DESCRIPTIONS OF SHIP DOMAINS DERIVED FROM GEOMETRIES.

Due to different determinations and representations of various ship domains, the resultant domain models vary from geometrical ones to intelligent ones even for the same shape type, saying nothing of the different types of domain shape. For comparative study of computer simulations, therefore, the first step is that all the abovementioned ship domain models should be mathematically described by putting them onto the Earth reference coordinate system and ship reference coordinate system, shown in Figures 1–3. Furthermore, it will be more feasible if the ship domains classified into the same type could be modelled mathematically in the same manner. In other words, there will be three mathematical frameworks describing the existing ship domains.

3.1. *Circle Domains*. The Goodwin, Davis, Zhao and Zhu models have been distinguished as circular ship domains, which can be described as:

$$\begin{cases} f_{circle}(x,y)>0, \text{ while } (x,y) \text{ is out of domain} \\ f_{circle}(x,y)=0, \text{ while } (x,y) \text{ is on the boundary} \\ f_{circle}(x,y)<0, \text{ while } (x,y) \text{ is in the domain} \end{cases} \tag{1}$$

$$f_{circle}(x,y)=\begin{cases} (x-x_t)^2+(y-y_t)^2-r_{t,1}^2, & \text{if } 0^\circ \leqslant \varphi_t \leqslant 112{\cdot}5^\circ \\ (x-x_t)^2+(y-y_t)^2-r_{t,2}^2, & \text{if } 247{\cdot}5^\circ \leqslant \varphi_t < 360^\circ, \\ (x-x_t)^2+(y-y_t)^2-r_{t,3}^2, & \text{if } 112{\cdot}5^\circ < \varphi_t < 247{\cdot}5^\circ \end{cases} \quad t \in \{a,d\} \tag{2}$$

$$\varphi_t=\begin{cases} \arccos\dfrac{y_{t,r}}{\sqrt{x_{t,r}^2+y_{t,r}^2}}, & x_{t,r} \geqslant 0 \\ 360^\circ-\arccos\dfrac{y_{t,r}}{\sqrt{x_{t,r}^2+y_{t,r}^2}}, & x_{t,r}<0 \end{cases} \tag{3}$$

$$\begin{cases} x_{t,r}=(x-x_t)\cos\varphi-(y-y_t)\sin\varphi \\ y_{t,r}=(x-x_t)\sin\varphi+(y-y_t)\cos\varphi \end{cases} \tag{4}$$

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Table 1. The parameters of the circle ship domains (n.m.)

Ship Domains	Goodwin	Davis	Zhao	Zhu
$r_{a,1}$, $r_{d,1}$	0·85, 0·85	2·7, 1·7	0·85, 0·68	3·2f(X), f(X)
$r_{a,1}$, $r_{d,2}$	0·70, 0·70	2·7, 1·7	0·70, 0·56	1·2f(X), f(X)
$r_{a,1}$, $r_{d,3}$	0·45, 0·45	2·7, 1·7	0·45, 0·36	1·2f(X), f(X)
d_a, d_d	0, 0	1·7, 0·7	0, 0	0, 0

$$\begin{cases} x_t = x_s + d_t \sin(\varphi + 19^\circ) \\ y_t = y_s + d_t \cos(\varphi + 19^\circ) \end{cases} \tag{5}$$

where, $r_{a,i}$, $r_{d,i}$, $i=1,2,3$ are the radii of arena and domain respectively, d_a, d_d, are the distances from own ship to centres of arena and domain circles respectively, φ is the course of own ship, (x_s, y_s) are coordinates of own ship in the Earth reference coordinates system, and other numerical parameters in the above-mentioned equations according to individual domain model have been listed in Table 1.

Where, $f(X)$ is the dynamic radius of ship domain, which is the response of the input X based on the neural network, where $X=(x_0, x_1, x_2, x_3, x_4)^T=(D/D_{\max}, B/L, T/B, C_b, \Phi/180^\circ)^T$, D is the visible distance, $D_{\max}$ is valued to be 5n.m., B/L is the ratio of breadth to length, T/B is the ratio of draft to breadth, C_b is the block coefficient and Φ is the bearing of the CPA (Closest Point of Approach).

3.2. *Elliptical Domains.* Fujii, Coldwell and Kijima models have been distinguished as elliptical ship domains, which can be described as:

$$\begin{cases} f_{ellipse}(x,y) > 0, \textit{while } (x,y) \textit{ is out of domain} \\ f_{ellipse}(x,y) = 0, \textit{while } (x,y) \textit{ is on the boundary} \\ f_{ellipse}(x,y) < 0, \textit{while } (x,y) \textit{ is in the domain} \end{cases} \tag{6}$$

$$f_{ellipse}(x,y) = \begin{cases} \left(\dfrac{x-x_0}{S_t}\right)^2 + \left(\dfrac{y-y_0}{R_{t,f}}\right)^2 - 1, \\ \text{if } \ 0^\circ \leqslant \varphi_r \leqslant 90^\circ \text{ or } 270^\circ \leqslant \varphi_r < 360^\circ, \quad t \in \{b, w\} \\ \left(\dfrac{x-x_0}{S_t}\right)^2 + \left(\dfrac{y-y_0}{R_{t,a}}\right)^2 - 1, \\ \text{if } \ 90^\circ < \varphi_r < 270^\circ \end{cases} \tag{7}$$

$$\varphi_r = \begin{cases} \arccos \dfrac{y_r}{\sqrt{x_r^2 + y_r^2}}, & x_r \geq 0 \\ 360^\circ - \arccos \dfrac{y_r}{\sqrt{x_r^2 + y_r^2}}, & x_r < 0 \end{cases} \tag{8}$$

$$\begin{cases} x_r = (x-x_0)\cos\varphi - (y-y_0)\sin\varphi \\ y_r = (x-x_0)\sin\varphi + (y-y_0)\cos\varphi \end{cases} \tag{9}$$

$$\begin{cases} x_0 = x_s + d_c \sin(\varphi + 90^\circ) \\ y_0 = y_s + d_c \cos(\varphi + 90^\circ) \end{cases} \tag{10}$$

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Table 2. The parameters of the elliptical ship domains.

Ship Domains	Fujii	Coldwell Head-on	Coldwell Overtaking	Kijima
S_b, $R_{b,f}$, $R_{b,a}$	$1{\cdot}6L$,$4{\cdot}0L$,$4{\cdot}0L$	$2{\cdot}5L$,$6{\cdot}1L$, -	$1{\cdot}75L$,$6{\cdot}0L$,$6{\cdot}0L$	S_b, R_{bf}, R_{ba}
S_w, $R_{w,f}$, $R_{w,a}$	$1{\cdot}6L$,$4{\cdot}0L$,$4{\cdot}0L$	$2{\cdot}5L$, $6{\cdot}1L$, -	$1{\cdot}75L$,$6{\cdot}0L$,$6{\cdot}0L$	S_w, R_{wf}, R_{wa}
d_c	0	$0{\cdot}75L$	0	0

where, S_w, S_b are the latitudinal radii of arena (watching area) and domain (blocking area) respectively, $R_{b,f}$, $R_{b,a}$ are the longitudinal radii of the area in fore and aft domains respectively, $R_{w,f}$, $R_{w,a}$ are the longitudinal radii of the area in fore and aft arenas respectively, d_c is the distance from own ship to centre of the domain, and some numerical parameters have been listed in Table 2.

While, for the Kijima model,

$$\begin{cases} R_{wf}=L+2(R_{bf}-L) \\ R_{wa}=L+2(R_{ba}-L), \\ S_w=B+2(S_b-B) \end{cases} \quad \begin{cases} R_{bf}=L+(1+s)T_{90}U \\ R_{ba}=L+T_{90}U \\ S_b=B+(1+t)D_T \end{cases} \tag{11}$$

$$T_{90}=0{\cdot}67\sqrt{A_D^2+(D_T/2)^2}/U \tag{12}$$

$$\begin{cases} A_D=L\exp(0{\cdot}3591\log U+0{\cdot}0952) \\ D_T=L\exp(0{\cdot}5441\log U-0{\cdot}0795) \end{cases} \tag{13}$$

$$\begin{cases} s=2-(U-U_T)/U, t=1, & \textit{while head}-\textit{on situation} \\ s=2-\alpha/\pi, t=\alpha/\pi, & \textit{while crossing situation} \\ s=1, t=1, & \textit{while overtaking situation} \end{cases} \tag{14}$$

Where, U and U_T are the speeds represented in knots of own and target ship respectively, s and t are coefficients to consider influence of encounter situation, α is relative angle between courses of two ships.

3.3. *Polygon Domains*. Mainly, the Smierzchalski and Pietrzykowski models have been distinguished as polygonal ship domains, of which the mathematical description would be more different and difficult because of its inherent characteristics. We can calculate the bearing angles of the target with respect to the vertices of the polygonal ship domain and compare the angles with the bearings of the segment vectors of the boundaries. It is noted that the target will be inside the ship domain if it lies on the right side of all the segment vectors of the boundaries. The formulas can therefore be described as:

$$f_{polygon}(x,y)=\max_i\{mark_i, i=1,2,\ldots,n\} \tag{15}$$

$$mark_i=\begin{cases} 1, & \textit{if } 0\leqslant\varphi_i<\phi_i \textit{ or } \phi_i+180<\varphi_i<360,\ \textit{while } \phi_i\in[0,180) \\ & \textit{if } \phi_i-180<\varphi_i<\phi_i, \textit{while } \phi_i\in[180,360) \\ 0, & \textit{if } |\varphi_i-\phi_i|=0 \textit{ or } 180 \\ -1, & \textit{if } \phi_i<\varphi_i<\phi_i+180, \textit{while } \phi_i\in[0,180) \\ & \textit{if } 0\leqslant\varphi_i<\phi_i-180 \textit{ or } \phi_i<\varphi_i<360, \textit{while } \phi_i\in[180,360) \end{cases} \tag{16}$$

Table 3. The parameters of the polygon ship domains (n.m.)

Ship Domains	Smierzchalski	Pietrzykowski
$(x_{1,r}, y_{1,r})$	$(0, d_1)$	(0, 1·7)
$(x_{2,r}, y_{2,r})$	(d_3, d_2)	(1·0, 1·1)
$(x_{3,r}, y_{3,r})$	$(d_3, 0)$	(1·2, 0)
$(x_{4,r}, y_{4,r})$	$(0, -d_4)$	(0·6, -0·6)
$(x_{5,r}, y_{5,r})$	$(-d_5, 0)$	(0, -0·8)
$(x_{6,r}, y_{6,r})$	$(-d_5, d_6)$	(−0·6, -0·6)
$(x_{7,r}, y_{7,r})$	—	(−1·2, 0)
$(x_{8,r}, y_{8,r})$	—	(−1·0, 1·1)

$$\varphi_i(x, y) = \begin{cases} \arccos \dfrac{y - y_i}{\sqrt{(x - x_i)^2 + (y - y_i)^2}}, & x \geqslant x_i \\ 360^\circ - \arccos \dfrac{y - y_i}{\sqrt{(x - x_i)^2 + (y - y_i)^2}}, & x < x_i \end{cases} \tag{17}$$

$$\phi_i = \begin{cases} \arccos \dfrac{y_{i+1} - y_i}{\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}}, & x_{i+1} \geqslant x_i \\ 360^\circ - \arccos \dfrac{y_{i+1} - y_i}{\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}}, & x_{i+1} < x_i \end{cases} \tag{18}$$

$$\begin{cases} x_i = x_s + x_{i,r} \cos \varphi + y_{i,r} \sin \varphi \\ y_i = y_s + y_{i,r} \cos \varphi - x_{i,r} \sin \varphi \end{cases} \tag{19}$$

and the parameters for the polygon domains are in Table 3.

While, for the Smierzchalski model,

$$\begin{cases} d_1 = LU_m^{1\cdot 26} + 30U_m \\ d_2 = T_{CPAo} U_m \\ d_3 = BU_m^{0\cdot 44}, d_3 > D_{CPAo} \\ d_4 = D_b/2 \ or \ d_4 = D_b E \\ d_5 = D_{CPAo}/2 \ or \ d_5 = D_{CPAo} E \\ d_6 = T_{CPAo} U_m \end{cases}, \tag{20}$$

$$E = U_{REL}/U \tag{21}$$

$$U_m = \max\{U, U_{REL}\} \tag{22}$$

Where, U_{REL} is relative speed of target ship, D_{CPAo} and T_{CPAo} are assumed values of $DCPA \leqslant D_b$ and $TCPA$ respectively, D_b is assumed here to be 2n.m., d_4 and d_5 should be no less than 0·5n.m.

4. SIMULATION AND ANALYSIS.

For a comparative study of these three large classes of ship domains, we produced some computer simulations for analysis in various encounter situations with other target ships. The principal

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Table 4. The principal dimensions of the own ship and target ship.

	Own Ship	Target Ship
L (m)	175·0	325·0
B (m)	25·4	53·0
T (m)	9·5	22·0
C_b	0·57	0·83

Table 5. The initial conditions for own ship and target ship.

	Head-on (n.m., knots, °)			Crossing (n.m., knots, °)			Overtaking (n.m., knots, °)		
Ships	Pos.	Vel.	Cour.	Pos.	Vel.	Cour.	Pos.	Vel.	Cour.
Own	(−2, 0)	13	10	(−2, 0)	13	10	(−2, 0)	16	10
Target	(−2, 6)	10	170	(1·5, 6)	10	235	(−1·4,4)	6	10

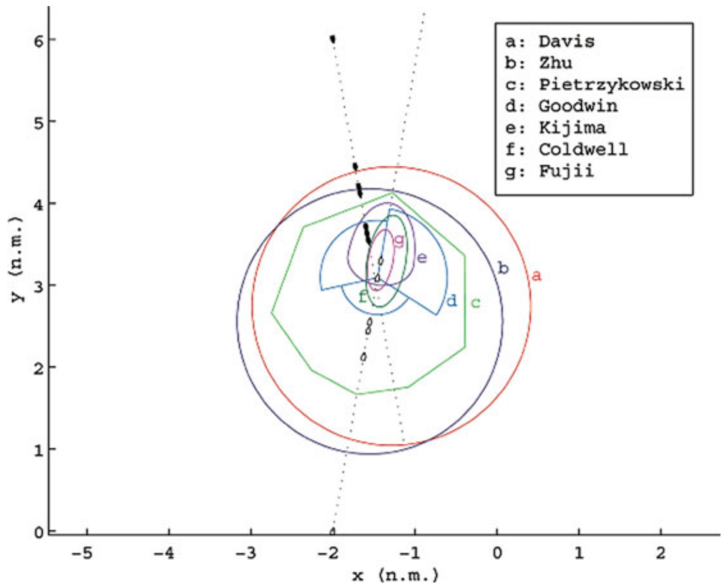

Figure 4. Ship domains evoked in head-on encounter situation.

dimensions of the own ship and target ship are listed in Table 4. Three encounter situations were considered, for which the initial conditions are listed in Table 5. Consider the movements of the two ships with respect to Earth-fixed coordinates in good ($D=5$n.m.) visibility.

4.1. *Head-on Situation*. In this encounter situation, the target ship comes towards own ship from the front. As these two ships approach closer and closer, the different ship domains will be violated sequentially. As shown in Figure 4, seven ship domains in three classes have been considered in the process of approaching. It can

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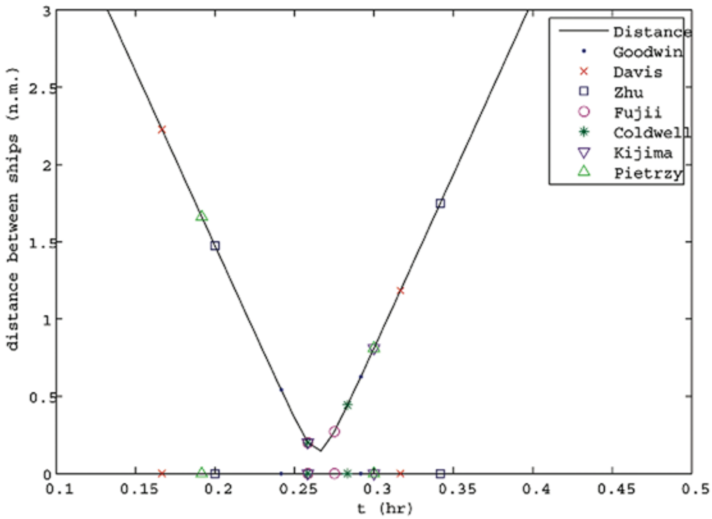

Figure 5. Distance between the two ships and the points where various ship domains will be violated during the head-on encounter situation.

be seen that the Davis and Fujii models are the most conservative and most risky ones, respectively. Moreover, Figure 5 shows in detail the points where the various ship domains would be violated when the distance between the two ships changes. It shows the Goodwin model to be moderate among the ship domains considered. According to these simulation results, the Davis, Pietrzykowski and Zhu models tend to be used as risk assessment while the remaining models are more suitable for collision avoidance. However, in practice, it should be noted that the Fujii and Coldwell models seem too risky for navigators to take action for collision avoidance.

4.2. *Crossing Situation.* In this encounter situation the simulation results, shown in Figure 6 and Figure 7, are very similar to the those in the head-on situation except that the distances where some of the ship domains would be violated increase because most of them have taken into consideration the international sea regulations which result in an asymmetrical model. The Zhu and Kijima models increase their extent most in this encounter situation since the relative course and speed have been accounted for, respectively. For symmetric ship domains, Fujii and Pietrzykowski models keep the same extents although the encounter situation has largely changed. And the Fujii ship domain would not be invaded until the distance between the two ships decreased nearly to DCPA, which is considered to be very dangerous.

4.3. *Overtaking Situation.* In this situation, the own ship will overtake the target ship from the stern of the target. It is expected that the precaution taken in the bow direction is as important as the head-on situation although the relative speed between the overtaking ship and target ship is ordinarily much smaller than that in a head-on situation. The simulation results, shown in Figures 8 and 9, indicate that the extents of ship domains are mostly similar to those in head-on situations except for the Kijima model which increases the extent since the relative speed and course have been taken into account and the fore and aft domains are considered differently.

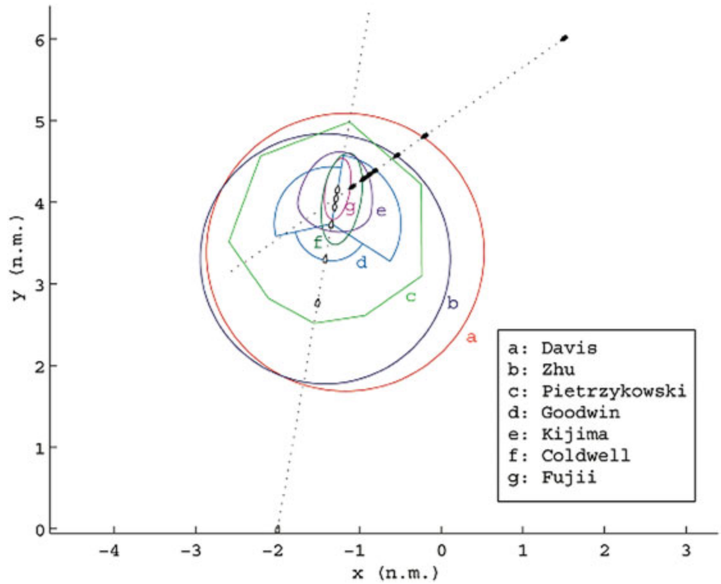

Figure 6. Ship domains evoked in the crossing encounter situation.

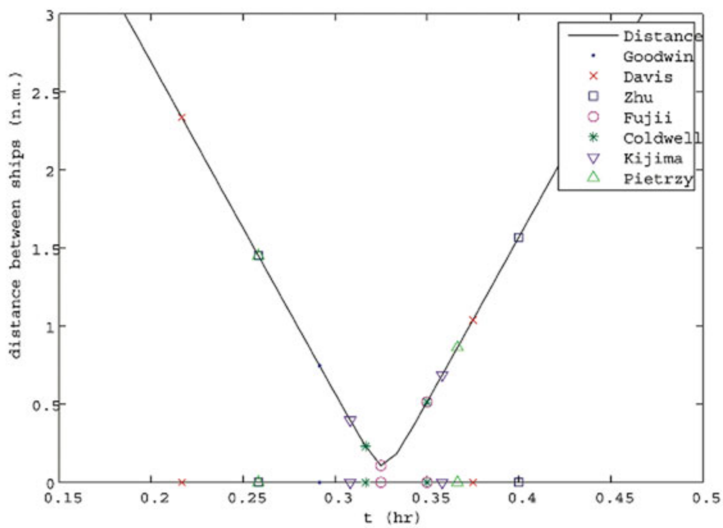

Figure 7. Distance between the two ships and the points where various ship domains will be violated during the crossing encounter situation.

5. CONCLUSIONS.

The typical ship domains have been reviewed and classified into three large classes, as circle, ellipse and polygon ship domains. It is known that most of the ship domains were illustrated in different geometrical manners that would be more descriptive and declarative. According to the classifications, we have proposed the mathematical descriptions for each type of ship domain. The uniform

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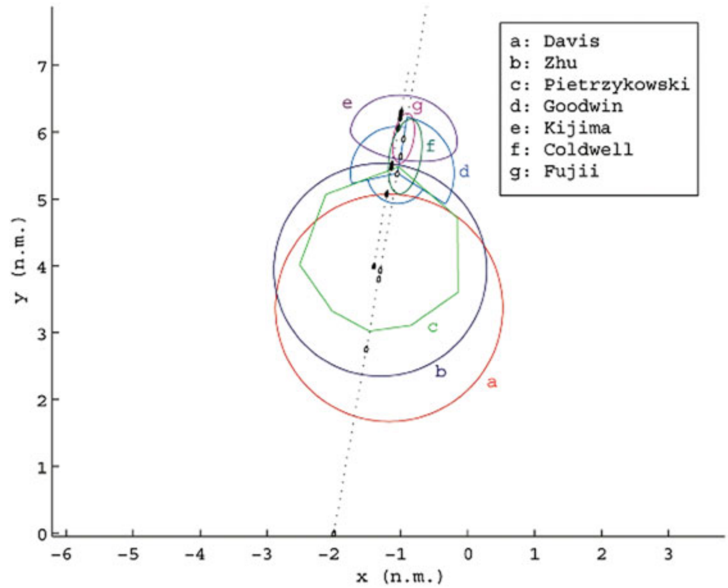

Figure 8. Ship domains evoked in the overtaking situation.

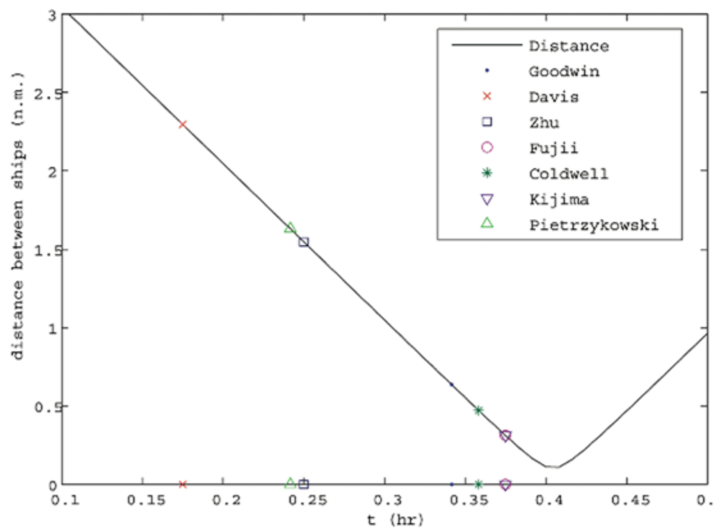

Figure 9. Distance between the two ships and the points where various ship domains will be violated during the overtaking encounter.

analytical framework has been established for various ship domain models, which would make it more feasible and practical to apply the models for risk assessment and collision avoidance, regardless of practices or simulations. Finally, several computer simulations on different encounter situations have been presented for validation of the analytical domain models and analysis of the comparative study for

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the different ship domains. The results show that the analytical ship domain models are effective and identical to the original geometrical ones. It should be noted that most of the existing ship domains have not adequately taken the human factors and environmental states into account. Future research on ship domains will focus on how these vital factors affect the shape and size of ship domains.

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