basis, triangularisation, inner products, orthonormal bases, orthogonal projection, optimisation, normed spaces, isometries, singular value decomposition, adjoint maps, the spectral theorem, determinants and characteristic polynomials.

So my 'falling out of love with the treatment' could be unfair. Perhaps I just have to acknowledge that linear algebra is tough and there is a lot of it. Failing to see the wood for the trees is probably the main problem, and these authors try very hard to avoid that by various methods: they provide plenty of interconnections between matrices, linear maps, geometry and some applications; they give a helping hand over the more difficult steps; extensive end-of-section exercises are provided (solutions to half of them), and quick exercises scattered throughout the text, having an 'are you with me so far?' function, with answers at the foot of the page.

It is a book well worth considering both for learning and teaching this important area of mathematics.

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Introduction to approximate groups by Matthew C. H. Tointon, pp. 205, £26.99 (paper), ISBN 978-1-10845-644-9, Cambridge University Press (2019)

The theory of approximate groups is a rapidly growing one; it is the focus of considerable current research and has surprising applications to many areas of mathematics, including graph theory, number theory, geometric group theory, and differential geometry. The book now under reviews offers an excellent introduction to this field, written at the level of a first or second year graduate student who has taken a good prior course on group theory and also has some acquaintance with basic notions of analysis such as open sets and measurable functions.

Approximate groups are not hard to define. If A is a subset of a group G and K is a positive integer, we say that A is a K-approximate group if A contains the identity element 1 of G, is symmetric (i.e. closed under the taking of group inverses), and satisfies an approximate closure condition. To define this condition more precisely, recall that, for subsets A and B of G, the set AB denotes the set of all elements ab ($a \in A, b \in B$); using this terminology, the requirement that A is closed under the group operation can be rephrased as $A^2 (= AA) \subseteq A$. In this notation, the approximate closure condition referred to earlier is that there exists a set X of size at most K such that $A^2 \subseteq XA$. An actual subgroup is obviously an approximate group with $K = 1$ and $X = \{1\}$. Of course, there are lots of approximate groups that fail to be actual groups: any finite subset of G of size K is a K-approximate group.

A slightly weaker notion is that of a *small doubling set*, namely a finite subset A of a group G with the property that the quotient of the cardinalities of A^2 and A is bounded by a constant $K \geqslant 1$. (Unfortunately there is a typo in the definition of 'small doubling' found in Definition 1.1.2, but it is fairly obvious and easily corrected.) While there are small doubling sets that are not approximate groups, the two notions are very closely related, and both objects are discussed in the book.

Chapter 1 of the text is introductory: it introduces the notion of an approximate group, gives a few examples, provides a useful two-page survey of the history of the subject, and discusses some background material and notation. Chapters 2 and 3 discuss basic results and examples. Chapters 4–10 discuss small doubling sets and approximate groups in various kinds of groups, including abelian and nilpotent groups and the general linear group. Chapter 11 is devoted to applications; as the author points out, there are too many examples to discuss in detail, but 'since the applicability of approximate groups is

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one of their great selling points, it would seem remiss not to include at least some of them'. The applications discussed in this chapter relate to geometric group theory, in particular to the concept of growth in (finitely generated) groups, which measures the asymptotic rate of growth of the cardinality of the n-fold product of certain kinds of subsets S of the group. A famous result in this area is Gromov's theorem, which can be proved and refined by using approximate groups.

The use of the word 'introduction' in the title of this book is not misleading: although the text discusses the results of recent research, it really is intended as a genuine introduction. As befits an entry in the London Mathematical Society Student Texts series, it reads as a text, not a research monograph, and even contains exercises. (The author recently taught a Part III course in this material at Cambridge using parts of the book.) Tointon writes clearly, but succinctly, and takes pains to motivate topics. At the same time, the book clearly contemplates that its readers may wish to pursue the subject further into currently unchartered territory, and provides excellent preparation for such beginning researchers. To this end, there is a detailed 75-entry bibliography, most of the entries of which are research papers, a handful of them not in English.

To summarize: any book that offers an introductory account of a hot new area of mathematics is, for that reason alone, a useful addition to the literature. The usefulness of such a book is, naturally, significantly enhanced when, as is the case here, the book is very nicely written, and there are few if any other such introductions to the subject in the textbook literature. Researchers and fledgling researchers in this area will want to own this book.

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Linear algebra for the young mathematician by Steven Weintraub, pp, 389, \$89.00 (hard), ISBN 978-1-47045-084-7, American Mathematical Society (AMS) (2019)

About 45 years ago, as a graduate student studying for my qualifying exams, I read Hoffman and Kunze's classic text on linear algebra (hereafter referred to as HK) from cover to cover. It was a memorable experience, and provided me with a base of knowledge that served me well not only on my exams but also in my subsequent doctoral research in Lie algebras. It has continued to be useful in my post-student years: whenever I have a linear algebra question, that book is usually the one that I go to first, and in many cases do not need to look elsewhere.

I mention this because the book now under review reminds me in some respects of HK. They share, for example, a basic philosophy of starting at the beginning with matrices and linear equations, but progressing quite far, all the while focusing on a conceptual understanding of the material, focusing on ideas and proof rather than just mechanical computations. In addition, the topic coverage of the two books is also fairly similar: both cover (in addition to the aforementioned matrices and linear equations) determinants, abstract vector spaces (over arbitrary fields), linear transformations, eigenvalues and eigenvectors (including diagonalisation and triangularisation), the Jordan canonical form, inner product spaces (and operators defined on them, up to and including variations of the spectral theorem), and bilinear and sesquilinear forms.

Both texts treat the subject of dimensionality in essentially the same way: the Axiom of Choice is mentioned but not dwelled on, so, for example, the existence of a basis is proved only for finite-dimensional spaces, but stated to be true for all. In