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Continuous wavelet transform analysis for self-similarity properties of turbulence in magnetized DC glow discharge plasma

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Abstract. Characterization of self-similarity properties of turbulence in magnetized plasma is being carried out in DC glow discharge plasma. The time series floating potential fluctuation experimental data are acquired from the plasma by Langmuir probe. Continuous wavelet transform (CWT) analysis considering db4 mother wavelet has been applied to the experimental data and self-similarity properties are detected by evaluating the Hurst exponent from the wavelet variance plotting. From the CWT spectrum, effort is made to extract a highly correlated frequency by locating the brightest spot. Accordingly, those signals are treated for finding out correlation dimension and the Liapunov exponent so that the exact frequency responsible for the chaotic behavior could be found out.

1. Introduction

Different methods arising from scientific investigation have been introduced so far to analyze the chaotic time series signals. Fourier analysis is a well-established and suitable tool for analyzing stationary time series whose statistical properties do not vary with time. The Fourier technique decomposes a signal into harmonic components, where the basic functions are trigonometric functions. Another tool for analyzing time series is the wavelet transform (WT; Daubechies 1992; Mallat 1999). The WT has been introduced and developed to study a large class of phenomena such as image processing, data compression, chaos, fractals, etc. The basic functions of the WT have the key property of localization in time (or space) and in frequency, contrary to what happens with trigonometric functions. In fact, the WT works as a mathematical microscope on a specific part of a signal to extract local structures and singularities (Daubechies 1992; Mallat 1999). This makes the wavelets ideal for handling non-stationary and transient signals as well as fractal-type structures. Since WT is a useful tool, many general aspects of nonlinear time series analysis are reviewed in Chua et al. (1992), Wornell et al. (1992), Carroll (1995), Abarbanel (1996), Staszewski et al. (1999), and Krommes (2002). The identification of the physical processes underlying low-frequency turbulence in magnetized plasmas is a challenging topic in contemporary plasma science (Horton 1999). By low frequency it is meant that the characteristic frequency of the fluctuating quantities, ω, is smaller than the ion cyclotron frequency Ωi. Significant interest in this topic arises from magnetic fusion research because turbulent fluctuations can enhance the transport of mass and energy (Chen 1965) and thus degrade the performance of confinement devices that aim to achieve fusion conditions. The topic is also of interest in space plasma investigations (Kamataki et al. 2007) in which enhanced transport across naturally existing sharp boundaries (in temperature, density, and magnetic field) can lead to major effects observable by spacecraft and ground-based instruments.

Significant experimental and theoretical effort has been devoted to the identification of universal trends in the spatial and temporal spectra of turbulent fluctuations in laboratory (Tchen 1973; Kuo 2001; Labit, et al. 2007; Škoric et al. 2008) and space plasmas (Jordan et al. 1997; Milano et al. 2004; Bale et al. 2005; Zimbardo 2006; Scipioni et al. 2008). Continuous WT analysis is one of the promising techniques to locate turbulent fluctuations in plasma system and it also helps to verify the persistence long-range correlation in the time series fluctuations (Pace et al. 2008). The present paper intends to highlight the turbulent fluctuations observed in an experimental magnetized DC glow discharge plasma device. With the help of CWT analysis, it has been also observed that self-similarity property prevails in the turbulent plasma behavior. The experimental setup and diagnostic tools are described in Sec. 2. Section 3 gives the experimental results and discussion.

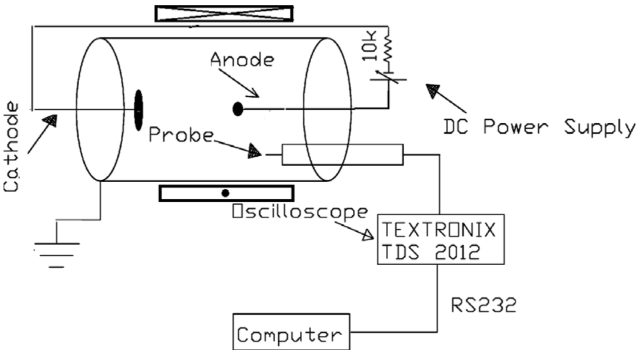

Figure 1. Schematic of the experimental setup.

2. Experimental setup

The experimental observation of nonlinear plasma oscillations is carried out in a DC glow discharge plasma chamber. Dimensions of the plasma device are 50 cm in length and 20 cm in diameter. The chamber is evacuated by rotary pump to attain a base pressure of 10^{-3} mb. Argon gas is then inserted into the chamber keeping the desired working pressure of around 0.2 to 0.6 mb. Plasma is produced by the DC glow discharge method by applying DC voltage across circular electrodes. To carry out the observations in presence of magnetic field, an external magnetic field of strength 10 to 60 G is applied to the plasma by passing DC current through the Helmholtz coils wound over the cylindrical chamber. The experimental setup is shown in Fig. 1. Plasma oscillations are extracted from the device as floating potential by Langmuir probe, which is inserted between cathode and anode. The characteristic plasma density and temperature are found to be 10^{12} cm^{-3} and 2 eV respectively. The time series floating potential data are collected in the oscilloscope, and further various nonlinear analyses are carried out with the help of MATLAB software.

3. Results and discussion

3.1. *Analysis of periodic and relaxation oscillations*

The plasma floating potential fluctuations are observed at a filling pressure of 0.4 mb in absence and presence of magnetic field. It is found that with increasing discharge voltage, the pattern of fluctuations gradually changes toward relaxation oscillation mode. Figure 2 shows change of pattern from periodic to relaxation oscillation with increasing discharge voltage and in absence of magnetic field. At $V_d = 596$ V, one periodic fluctuation is obtained, which becomes two periodic at 604 V, three periodic at 619 V, and finally at 629–630 V, relaxation mode of oscillation is observed. On the other hand, Fig. 3 implies that in presence of external magnetic field, viz. at 46 G, an opposite transition, i.e. from chaotic to quasi-periodic oscillation occurs with increased discharge voltage. It is attributed likely to the inclusion of external magnetic field.

A chaotic pattern of plasma oscillations is noted when the external magnetic field is gradually changed to 9, 28, and 46 G keeping the discharge voltage, $V_d = 596$ V (Fig. 4). The chaotic behavior is further verified by adopting the 'Largest Liapunov Exponent' (LLE) technique for the signal at $V_d = 596$ V and a magnetic field of 46 G (Fig. 5). At this condition, LLE is found to be 0.1324 whereas for a magnetic field of 28 G it is 0.0842. This implies that chaoticity of the system is enhanced with increasing magnetic field.

Moreover, estimation of correlation dimension of the plasma floating potential signals at various magnetic fields shows that the complexity of the system also increases gradually with the higher external magnetic field (Fig. 6). Hence, it can be inferred that with the imposed external magnetic field, the complexity as well as the chaoticity of the plasma system enhances.

3.2. *Continuous Wavelet Transform (CWT) analysis*

Characterization of self-similarity properties of chaotic signals in magnetized plasma is being carried out by CWT analysis. The time series floating potential fluctuation experimental data are acquired from the magnetized DC glow discharge plasma by Langmuir probe. CWT analysis considering db4 mother wavelet has been applied to the experimental data and self-similarity properties are detected. Continuous wavelet spectrums of

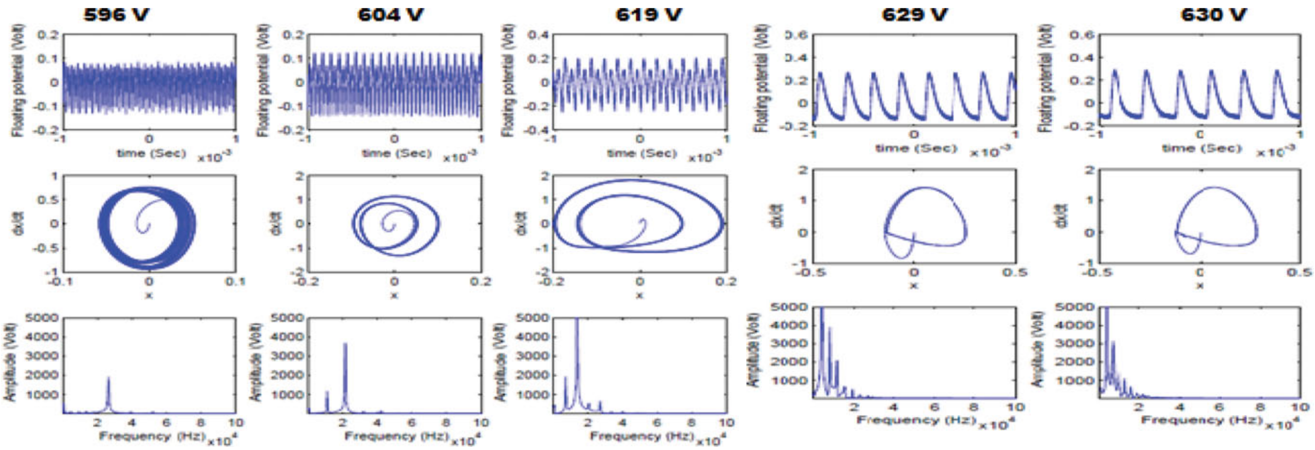

Figure 2. (Colour online) Periodic and relaxation oscillations of floating potential fluctuations at zero external magnetic field.

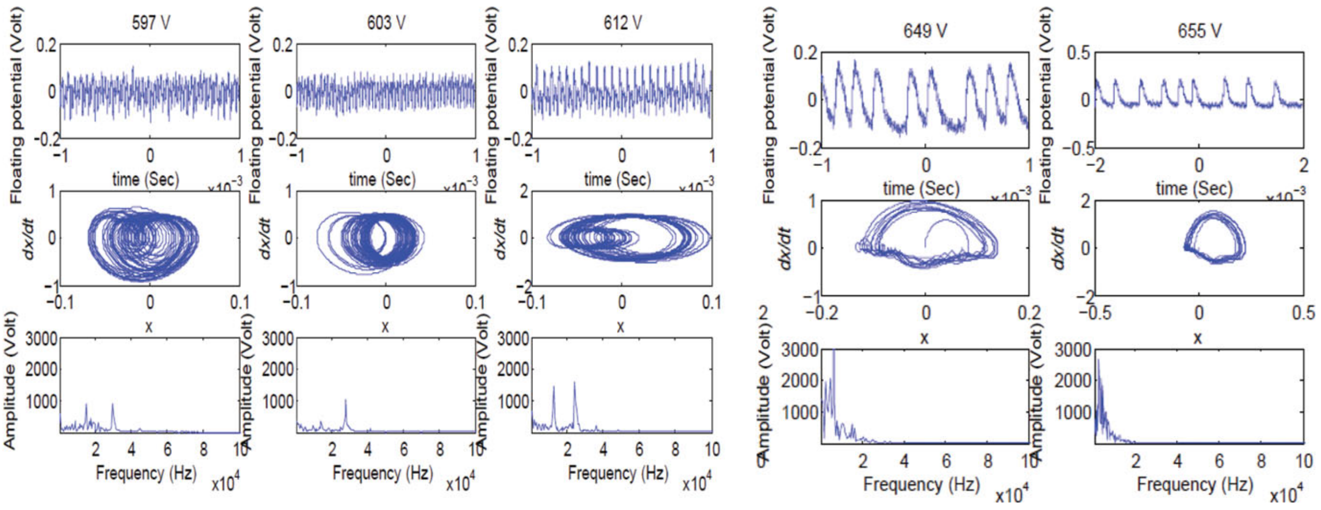

Figure 3. (Colour online) Chaotic and relaxation oscillations of floating potential fluctuations at 46 G.

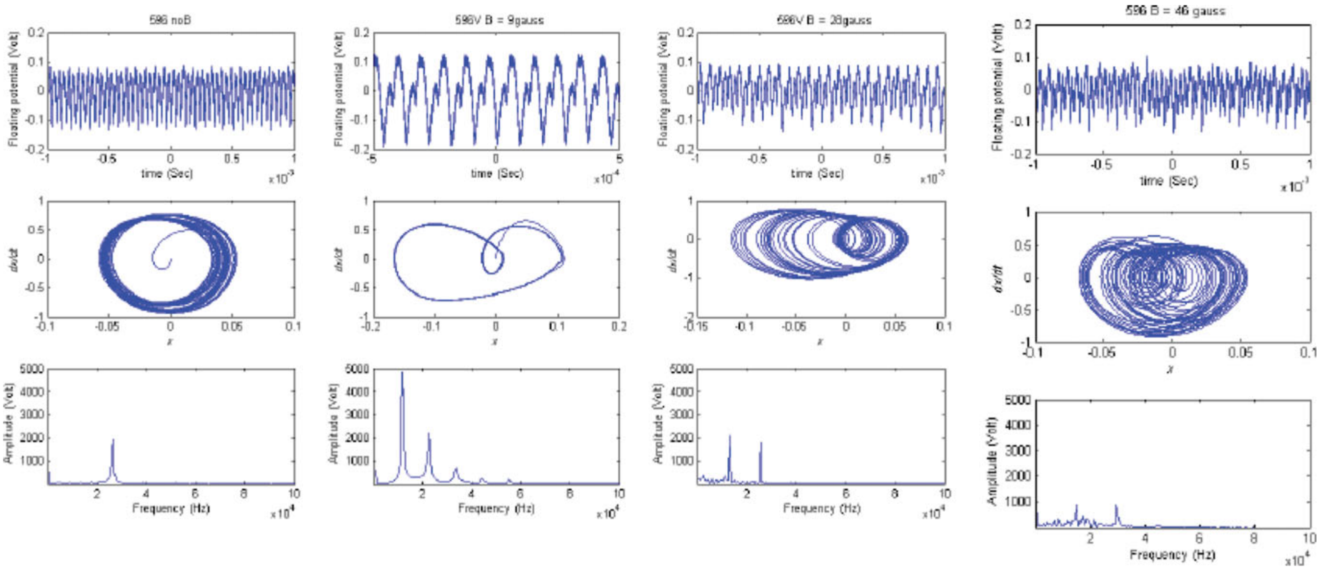

Figure 4. (Colour online) Chaoticity of the system increases for increasing magnetic field keeping the fixed discharge voltage, $V_d = 596$ V.

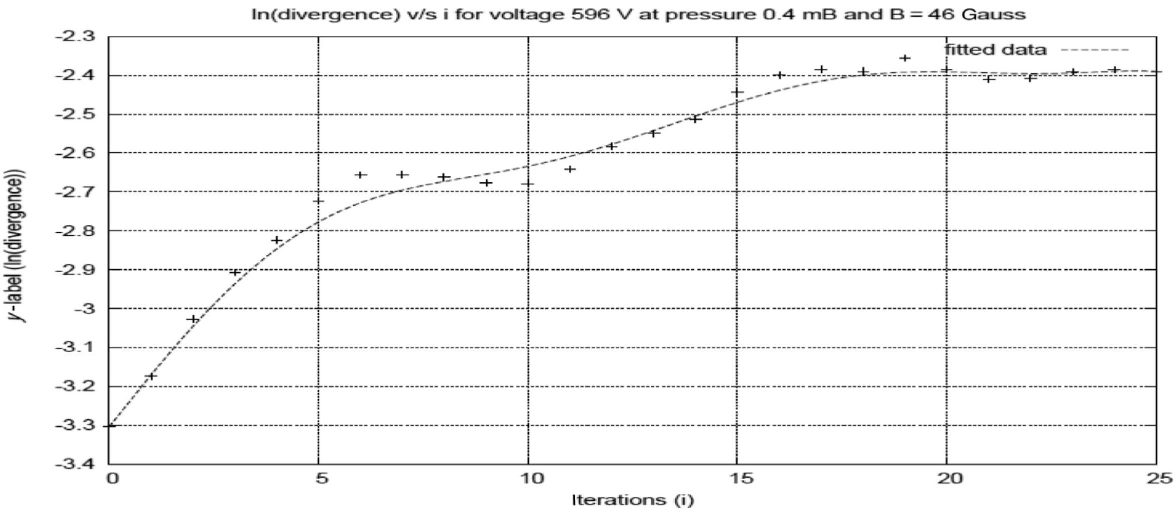

Figure 5. The largest Lyapunov exponent (LLE) using the Rosenstein algorithm is determined for chaotic signals at a discharge voltage, $V_d = 596$ V and a magnetic field of 46 G.

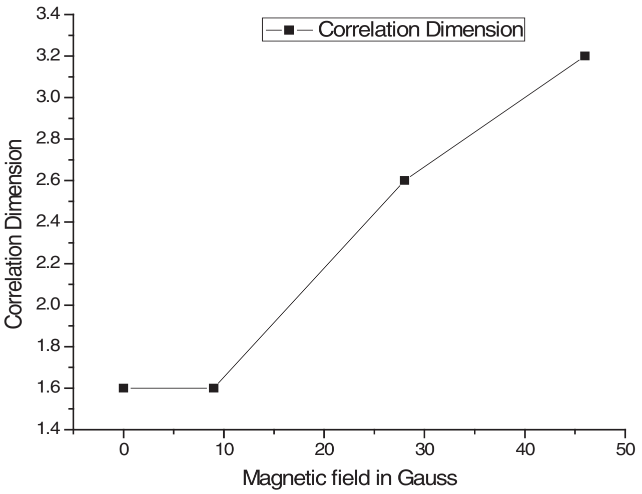

Figure 6. Correlation dimensions of floating potential fluctuations for increasing magnetic field at a discharge voltage, $V_d = 596$ V and a pressure of 0.4 mb.

various chaotic signals are depicted in Figs. 7(a)–(e). The repeating pattern of all the spectrums at different scales qualitatively tells about self-similarity property of the turbulent plasma potential measured in magnetized as well as unmagnetized DC discharge plasma.

Table 1. Estimation of Hurst exponent for floating potential fluctuations at various magnetic fields.

External magnetic field	Hurst exponent
9 G	0.37
28 G	0.41
46 G	0.77

This self-similar property is further verified by evaluating the Hurst exponent of plasma signals by the Wavelet Variance method (Table 1). It is evident from Table 1 that the Hurst exponent becomes more than 0.5 at a higher magnetic field of 46 G, which shows long-range persistent behavior of the signal.

Now effort has been made to extract a particular signal responsible for the chaotic nature of the plasma system at higher external magnetic field regime. Figure 8 shows a zoomed wavelet spectrum for the plasma signal at B = 46 G and $V_d = 596$ V. From the spectrum, the frequency corresponding to the highly superimposed

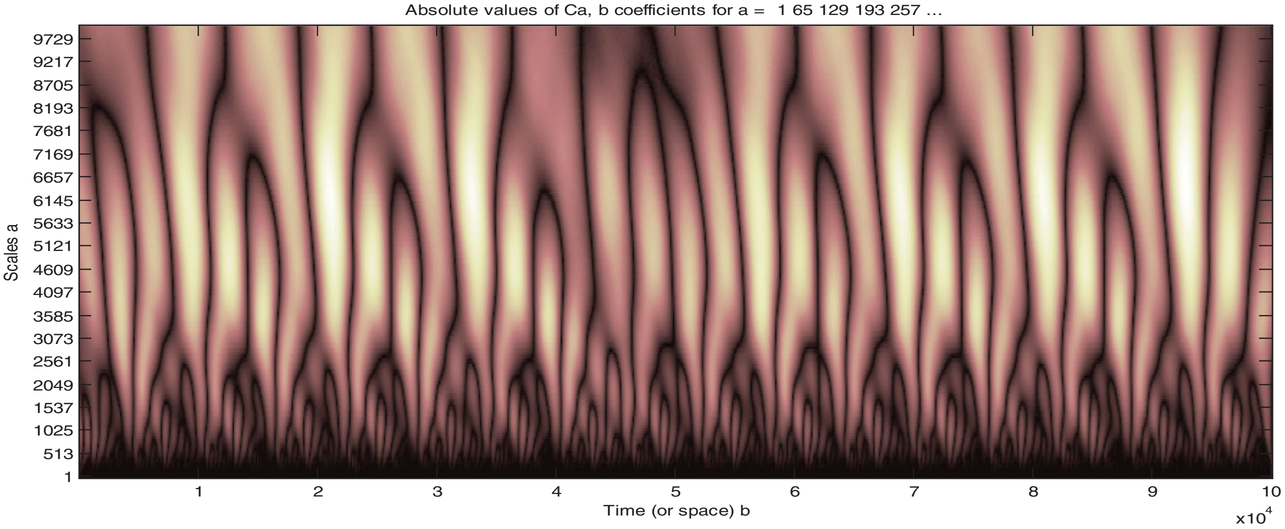

Figure 7a. (Colour online) Wavelet spectrum at B = 0 G, $V_d = 473$ V, 0.4 mb.

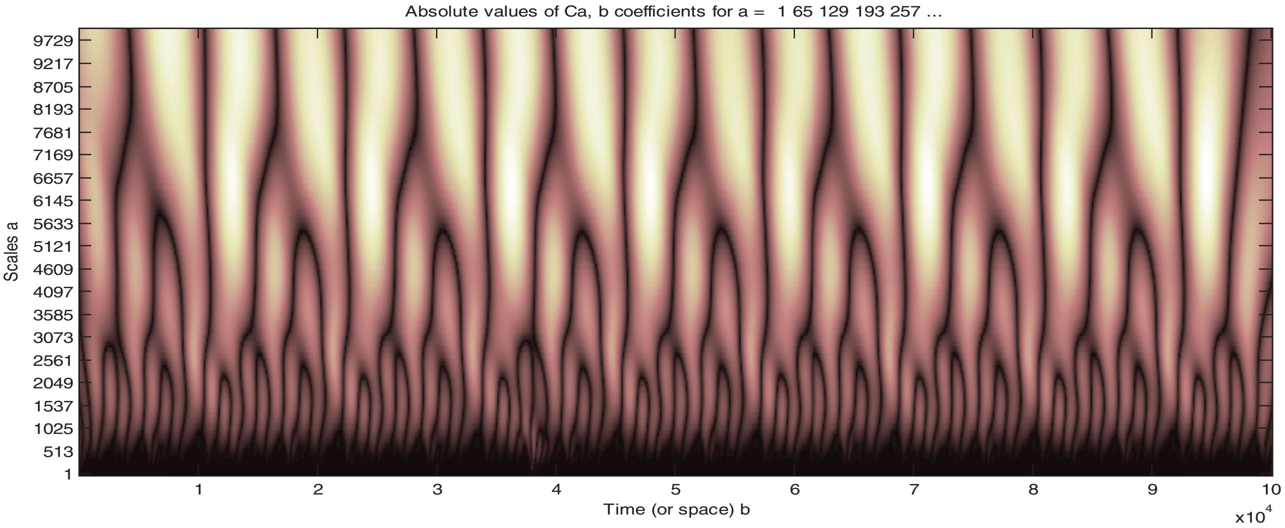

Figure 7b. (Colour online) Wavelet spectrum at B = 9 G, $V_d = 429$ V, 0.4 mb.

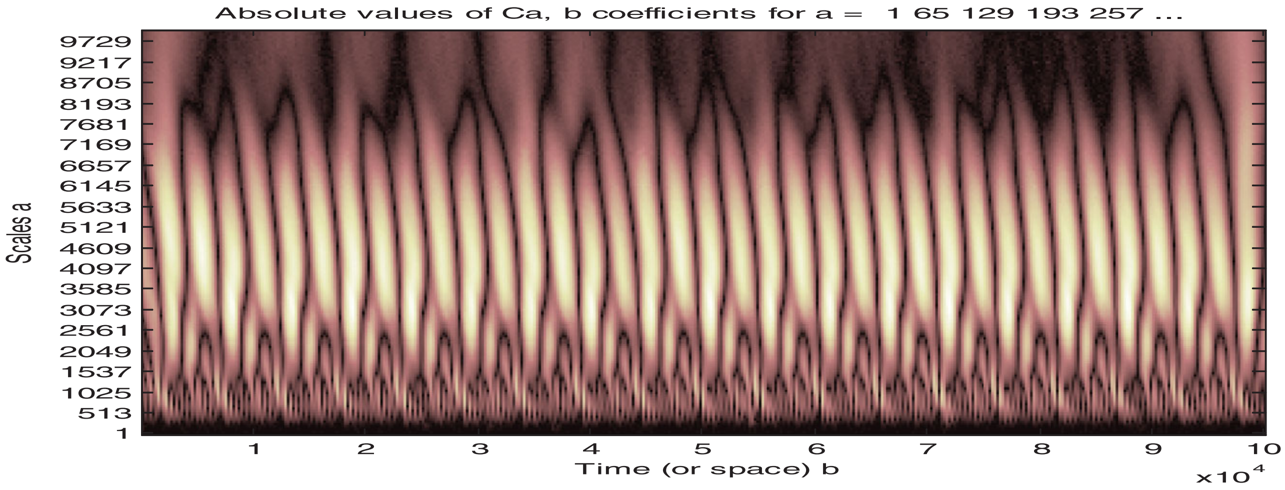

Figure 7c. (Colour online) Wavelet spectrum at B = 28 G, V_d = 432 V, 0.4 mb.

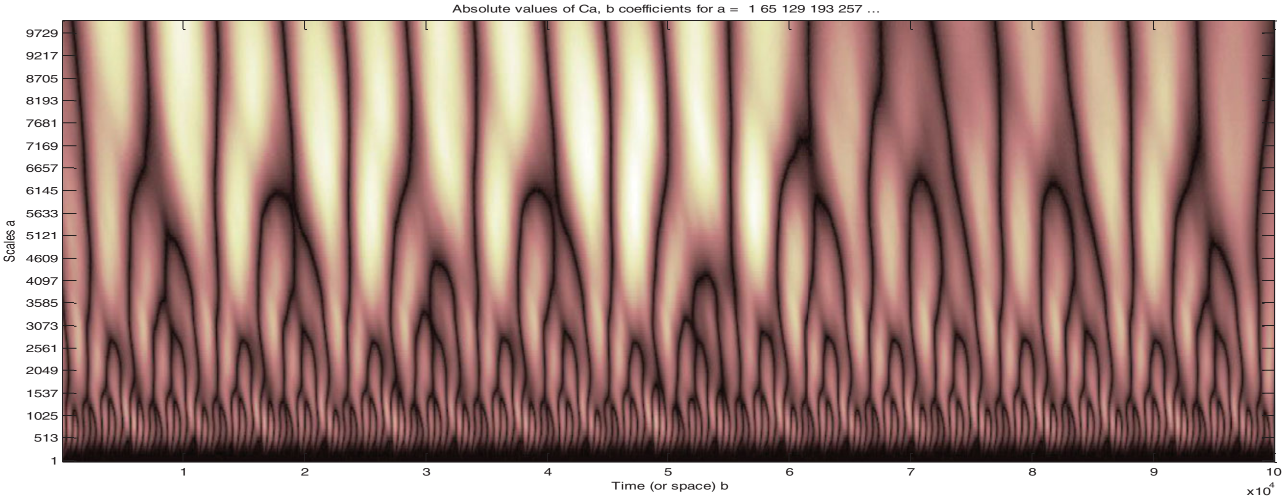

Figure 7d. (Colour online) Wavelet spectrum at B = 28 G, V_d = 435 V, 0.4 mb.

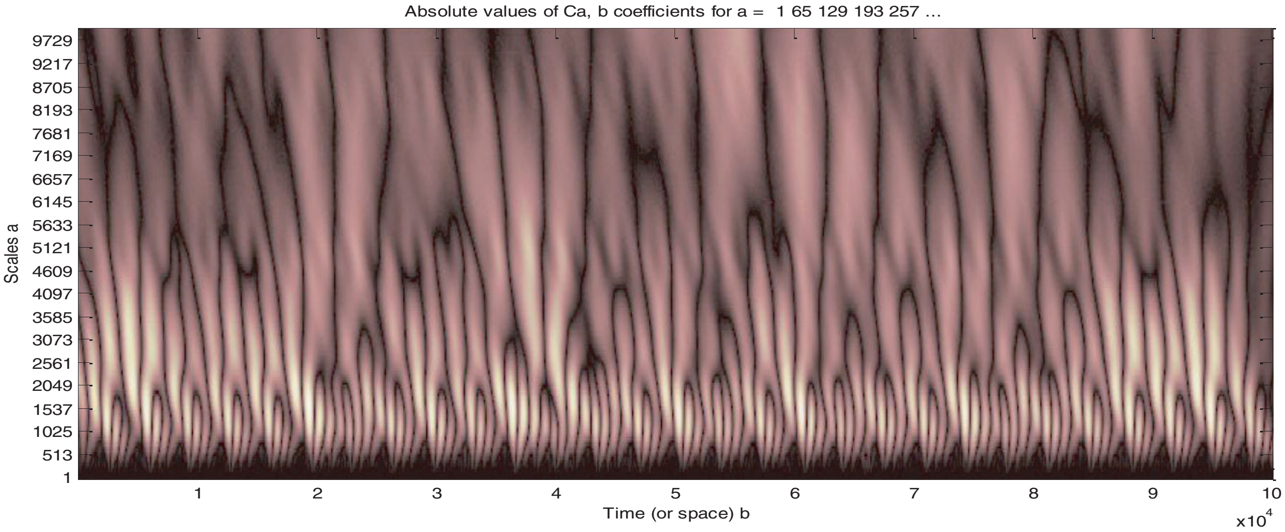

Figure 7e. (Colour online) Wavelet spectrum at B = 46 G, V_d = 596 V, 0.4 mb.

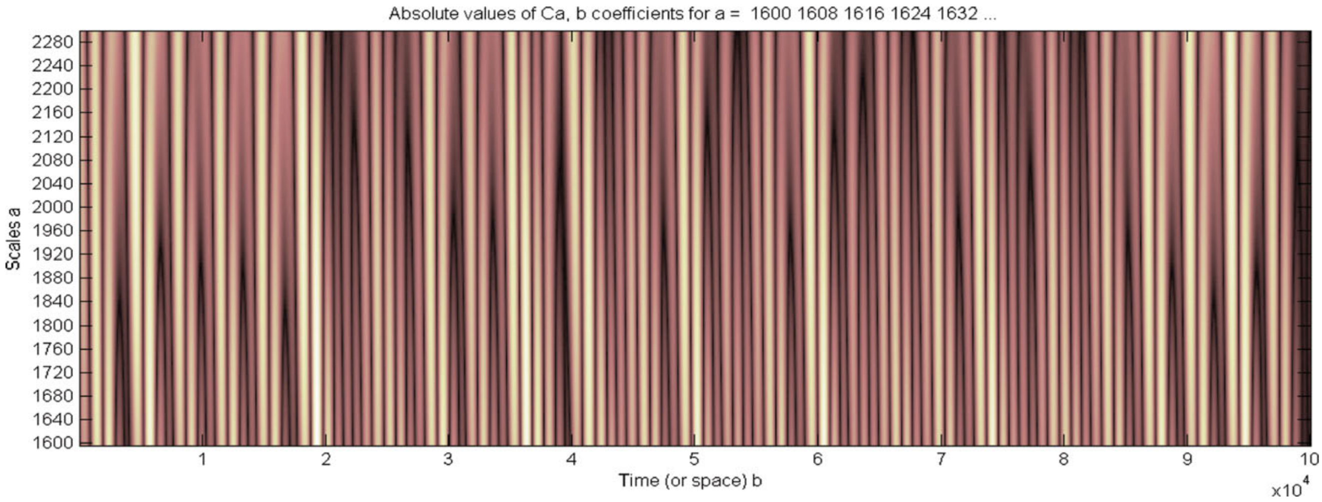

Figure 8. (Colour online) Wavelet spectrum at B = 46 G, V_d = 596 V, 0.4 mb (zoomed).

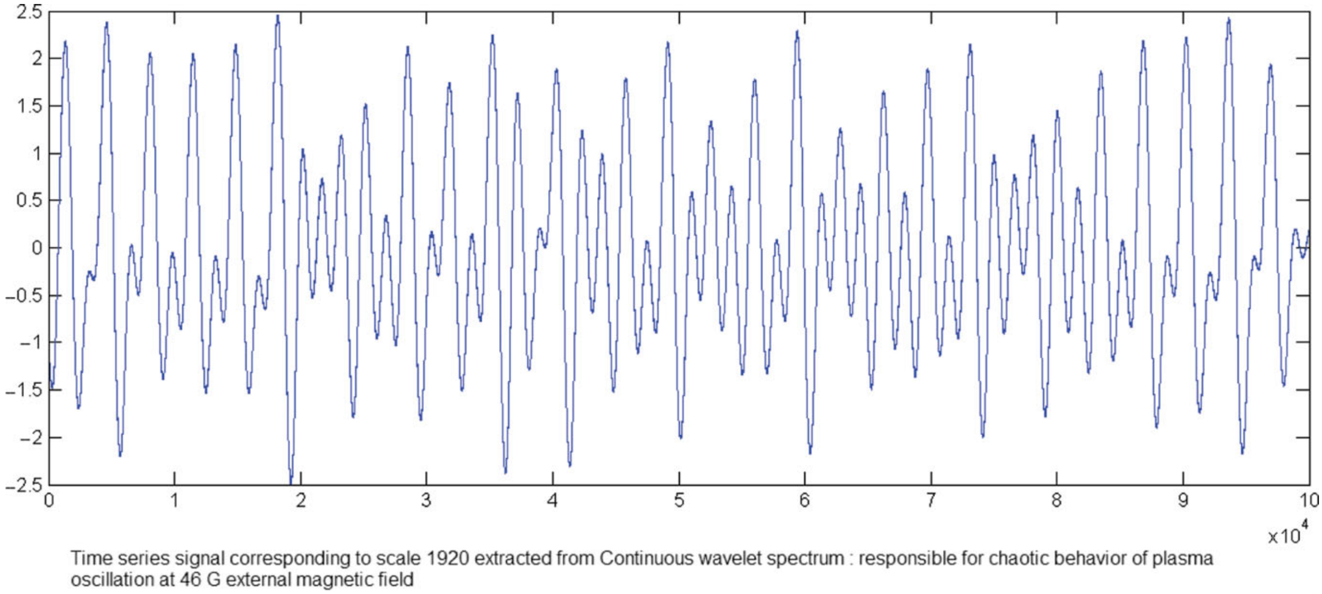

Figure 9. (Colour online) Time series plasma floating potential oscillation corresponding to scale 1920 at B = 46 G, V_d = 596 V, 0.4 mb.

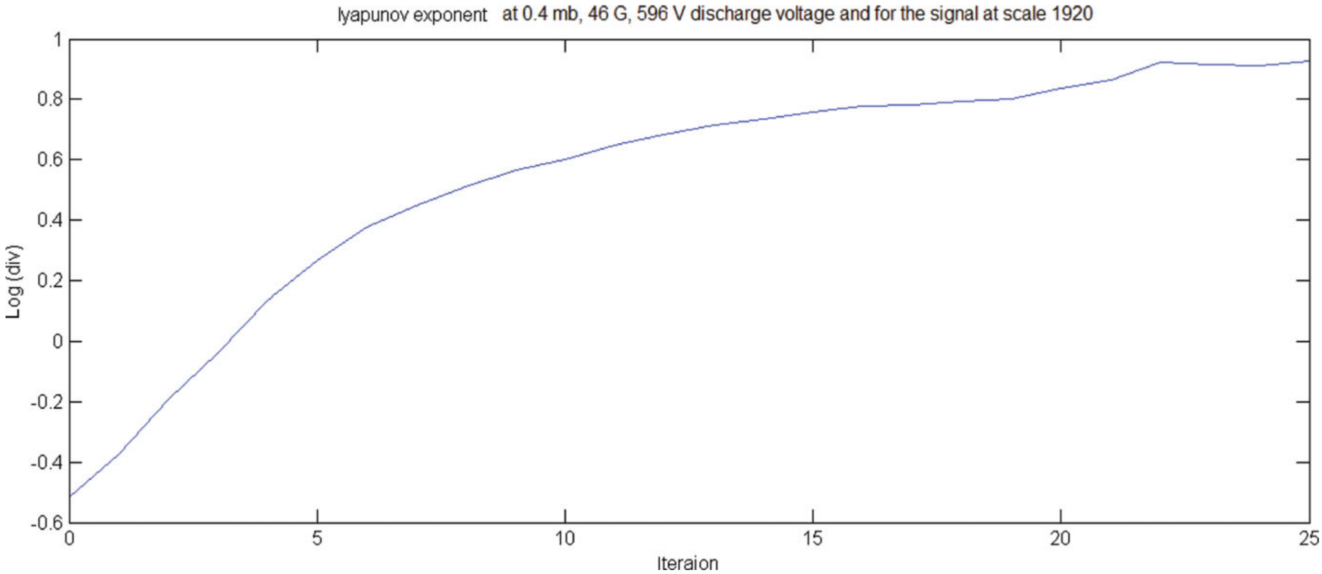

Figure 10. (Colour online) Lyapunov exponent of the signal corresponding to scale 1920 at B = 46 G, V_d = 596 V, 0.4 mb.

(brightest spot) scale (y-axis parameter) is extracted and is depicted in Fig. 9.

This particular signal is then treated for chaoticity by LLE (Fig. 10), which shows that Lyapunov exponent is positive for this signal with a value of 0.1630 corresponding to scale 1920. On the other hand, at B = 28 G, signal corresponding to scale 2116, Lyapunov exponent is also positive with a value of 0.0829, implying a less chaotic plasma system.

To conclude, it is observed that CWT analysis is extremely helpful to investigate the self-similar persistent long-range behavior of chaotic signals. A decisive advantage of the wavelet-based method is that it makes possible to obtain directly at all scales the self-similarity properties and the corresponding exponents.

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