THE α-REGULAR CLASSES OF THE GENERALIZED SYMMETRIC GROUP

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Introduction. The α-regular classes of any finite group G are important since they are those classes on which the projective characters of G with factor set α take non-zero value, and thus a knowledge of the α-regular classes gives the number of irreducible projective representations of G with factor set α (see [**4**]). Here we look at the particular case of the generalized symmetric group C_m wr S_l. The analogous problem of constructing the irreducible projective representations of C_m wr S_l has been dealt with in [**6**] by generalizing Clifford's theory of inducing from normal subgroups, but unfortunately, it is not in general possible to determine the irreducible projective characters (and hence the α-regular classes) by this method.

Necessary definitions of factor sets and properties of α-regular elements are given in §1, but a knowledge of the theory of projective representations of finite groups is assumed (see [**4**], [**2**]). In §2, we give a brief description of the group C_m wr S_l as well as the most important results of [**1**]. The α-regular classes of C_m wr S_l with respect to $(-1, 1, 1)$, $(-1, -1, -1)$, $(1, -1, 1)$ are determined in §3 and §4, and we tabulate these results, together with the α-regular classes of the remaining factor sets in Theorem 5.2.

In all that follows, G is a finite group, **C** the complex field, and $\mathbf{C}^*$ the group of non-zero elements of C.

1. Factor sets.

Definition 1.1. A mapping $\alpha: G \times G \to \mathbf{C}^*$ is called a *factor set* of G if

$$\alpha(x, y)\alpha(xy, z) = \alpha(x, yz)\alpha(y, z) \quad \text{for all } x, y, z \in G, \text{ and}$$

$$\alpha(1_G, 1_G) = 1,$$

where 1_G is the identity of G.

Definition 1.2. Let α be a factor set of G. We define $\alpha': G \times G \to \mathbf{C}^*$ by

$$\alpha'(x, y) = \alpha(x, y)\alpha(y, x)^{-1} \quad \text{for all } x, y \in G.$$

Definition 1.3. An element $a \in G$ is *α-regular* if $\alpha'(a, x) = 1$ for all $x \in C_G(a)$, the centralizer of a in G.

Lemma 1.4. *If $a \in G$ is α-regular, so is every conjugate of a in G. Thus the property of being α-regular is a class function on G.*

Proof. See [**4**].

Lemma 1.5. *Let T be a projective representation of G with factor set α, and let χ_T be the character of T. If $\chi_T(a) \neq 0$, then a is α-regular.*

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Proof. Let $x \in C_G(a)$. Then $T(x)T(a) = \alpha'(a, x)T(a)T(x)$, and so $\chi_T(a)(1-\alpha'(a, x)) = 0$, which gives the result.

LEMMA 1.6. *Let $a, b, c \in G$ be such that $b, c \in C_G(a)$. Then $\alpha'(a, bc) = \alpha'(a, b)\alpha'(a, c)$.*

Proof. By repeated applications of 1.1 we have

$$\alpha'(a, bc) = \alpha(a, bc)\alpha(bc, a)^{-1} = \alpha(a, bc)\alpha(b, c)\alpha(b, c)^{-1}\alpha(bc, a)^{-1}$$
$$= \alpha(a, b)\alpha(ab, c)\alpha(b, ca)^{-1}\alpha(c, a)^{-1} = \alpha(a, b)\alpha(a, c)\alpha(b, a)^{-1}\alpha(c, a)^{-1}$$

DEFINITION 1.7. Let α be a factor set of G, and assume there exists some integer n such that $\alpha(x, y)^n = 1$ for all $x, y \in G$. The smallest value of n such that this holds is called the *order* of α. If no such n exists, α is said to be of infinite order.

LEMMA 1.8. *Let α be a factor set of G of finite order n and let s be any integer such that $(n, s) = 1$. If $a \in G$ is such that a^s is α-regular, then a is also α-regular.*

Proof. Let $x \in C_G(a)$. Then $x \in C_G(a^s)$ and hence $\alpha'(a, x)^s = \alpha'(a^s, x) = 1$ by Lemma 1.6. However, as α is of order n, we must have $\alpha'(a, x)^n = 1$, and hence $\alpha'(a, x) = 1$.

2. The generalized symmetric group. C_m wr S_l is the wreath product of the cyclic group C_m of order m with the symmetric group S_l on l symbols (see e.g. [**3**]). Here, however, it is more convenient to think of the group in other terms. It has a presentation

$$C_m \text{ wr } S_l = \langle r_i\,(i = 1, \dots, l-1),\ w_j\,(j = 1, \dots, l) \mid r_i^2 = (r_i r_{i+1})^3 = (r_i r_j)^2 = 1, \quad (|j-i| \geqq 2),$$
$$w_j^m = 1,\ w_i w_j = w_j w_i,\ r_i w_i = w_{i+1} r_i,\ r_i w_j = w_j r_i, \quad j \neq i, i+1\rangle$$

(see [**6**]). It is called the generalized symmetric group because we may think of r_i as the transposition $(i, i+1)$ and w_j as the mapping $j \to \xi j$, where ξ is some primitive mth root of 1. Thus C_m wr S_l permutes the letters $\{1, \dots, l\}$ as well as multiplying any number of them by some power of ξ.

Let

$$\sigma = \begin{pmatrix} 1 & 2 & & l \\ & & \cdots & \\ \xi^{k_1} b_1 & \xi^{k_2} b_2 & & \xi^{k_l} b_l \end{pmatrix} \in C_m \text{ wr } S_l,$$

where $\{b_1, \dots, b_l\} = \{1, \dots, l\}$, and the k_i are positive integers. We define $\Phi: C_m \text{ wr } S_l \to S_l$ by

$$\Phi(\sigma) = \begin{pmatrix} 1 & 2 & & l \\ & & \cdots & \\ b_1 & b_2 & & b_l \end{pmatrix}.$$

Then Φ is a homomorphism.

DEFINITION 2.1. $\sigma \in C_m$ wr S_l is *even* if $\Phi(\sigma)$ is an even element of S_l and *odd* otherwise. In terms of the generators $\{r_i, w_j\}$ given above, σ is even if and only if the number of r_i appearing in any expression for σ is even.

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Any element $\sigma \in C_m$ wr S_l may be written down uniquely (up to reordering) as a product of disjoint cycles $\sigma = \theta_1 \dots \theta_t$ for some t, where

$$\theta_i = \begin{pmatrix} b_{i_1} & b_{i_2} & \cdots & b_{i_{l_i}} \\ \xi^{k_{i_1}} b_{i_1} & \xi^{k_{i_2}} b_{i_2} & & \xi^{k_{i l_i}} b_{i_{l_i}} \end{pmatrix}, \; i = 1, \dots, t. \tag{1}$$

DEFINITION 2.2. Let θ_i be any cycle of the above form. We define

$$\operatorname{diag} \theta_i = \begin{pmatrix} b_{i_1} \\ \xi b_{i_1} \end{pmatrix} \begin{pmatrix} b_{i_2} \\ \xi b_{i_2} \end{pmatrix} \cdots \begin{pmatrix} b_{i_{l_i}} \\ \xi b_{i_{l_i}} \end{pmatrix} = w_{b_{i_1}} w_{b_{i_2}} \cdots w_{b_{i_{l_i}}}$$

We note that diag θ_i is always an even element of C_m wr S_l.

DEFINITION 2.3. Let $\sigma = \theta_1 \dots \theta_t \in C_m$ wr S_l, where $\{\theta_i \mid i = 1, \dots, t\}$ are the disjoint cycles of σ. For any such θ_i we define $f(\theta_i) = \sum_{j=1}^{l_i} k_{i_j}$ (as in (1)).

Let a_{pq} be the number of cycles θ_i of σ of length q such that $f(\theta_i) \equiv p \pmod{m}$, for $1 \leqq p \leqq m$, $1 \leqq q \leqq l$. The $m \times l$ matrix (a_{pq}) is called the *type* $\mathrm{Ty}(\sigma)$ of σ.

THEOREM 2.4. *$\sigma, \sigma_1 \in C_m$ wr S_l are conjugate if and only if* $\mathrm{Ty}(\sigma) = \mathrm{Ty}(\sigma_1)$.

Proof. See [**3**].

In [**1**], the following results are proved.

THEOREM 2.5. *Each projective representation T of C_m wr S_l may be generated (in the sense of* [**6**]*) by a set of matrices $\{R_1, \dots, R_{l-1}, V_1, \dots, V_l\}$, where $R_i = T(r_i)$, and $V_j = T(w_j)$, in which case the corresponding factor set α of T is of order 2 or 1. Furthermore the matrices $\{R_i, V_j\}$ satisfy the following relations: $R_i^2 = I$, $(R_i R_{i+1})^3 = I$, $(R_i R_j)^2 = \gamma I$, $|i-j| \geqq 2$, $V_i^m = I$, $V_i V_j = \mu V_j V_i$, $j \neq i$, $R_i V_i = V_{i+1} R_i$, $R_i V_j = \lambda V_j R_i$, $j \neq i, i+1$, where*

$$\gamma^2 = \lambda^{(2,m)} = \mu^{(2,m)} = 1.$$

The Schur Multiplier of C_m wr S_l is then given by

$$H^2(C_m \text{ wr } S_l, \mathbf{C}^*) = \begin{cases} C_2 = \{(\gamma)\} & \text{if } m \text{ is odd, } l \geqq 4, \\ \{1\} & \text{if } m \text{ is odd, } l < 4, \\ C_2^3 = \{(\gamma, \lambda, \mu)\}, & \text{if } m \text{ is even, } l \geqq 4, \\ C_2^2 = \{(\lambda, \mu)\} & \text{if } m \text{ is even, } l = 3, \\ C_2 = \{(\mu)\} & \text{if } m \text{ is even, } l = 2, \\ \{1\} & \text{if } m \text{ is even, } l = 1. \end{cases}$$

For simplicity of notation, we will always use $\{(\gamma, \lambda, \mu)\}$ to denote the multiplier, with the convention that γ, λ, or $\mu = 1$ for certain values of m, l (given by the above result).

The relations between the $\{R_i, V_j\}$ given in 2.5 imply the following result which is expressed in terms of the factor set α of the projective representation T of C_m wr S_l generated by $\{R_i, V_j\}$.

LEMMA 2.6. (i) $\alpha'(r_i, r_j) = \gamma, \quad |i-j| \geqq 2$
(ii) $\alpha'(r_i, w_j) = \lambda, \quad j \neq i, i+1$
(iii) $\alpha'(w_i, w_j) = \mu, \quad j \neq i.$

The proof is easy and is omitted.

Before proceeding to determine the α regular classes of C_m wr S_l for all (γ, λ, μ), we describe a class of matrices which will be used to construct matrices $\{R_i, V_j\}$ satisfying 2.5.

LEMMA 2.7. *Let k be any positive integer. There exist matrices $\{N_1, \ldots, N_{2k+1}\}$ of degree 2^k satisfying*

(i) $N_j^2 = I, \quad j = 1, \ldots, 2k+1,$

(ii) $N_j N_h = -N_h N_j \quad j \neq h,$

(iii) $N_1 N_2 \ldots N_{2k+1} = (i)^k I, \quad (i = \sqrt{-1}),$

(iv) *No other product of distinct matrices $N_{j_1} \ldots N_{j_t} = \zeta I$, for any $\zeta \in \mathbf{C}^*$ (apart from a reordering of* (iii)).

(v) *$N_{j_1} \ldots N_{j_t}$ has non-zero trace if and only if $N_{j_1} \ldots N_{j_t} = \zeta I$, for some $\zeta \in \mathbf{C}^*$.*

Proof. Let $M_1, \ldots, M_{2k+1}$ be defined as in [7, p. 198]. Put $N_j = M_{2k+2-j}$ if j is odd, and $N_j = (i)M_{2k+2-j}$ if j is even.

3. In the case $(\gamma, \lambda, \mu) = (1, 1, 1)$, the projective representations of C_m wr S_l are linear (ordinary) representations, and hence all classes are α-regular. Next we consider the α-regular classes of C_m wr S_l, when α is the factor set of the projective representation T generated by matrices $\{R_i, V_j\}$ for $(\gamma, \lambda, \mu) = (-1, 1, 1)$. (Henceforth, we will write $\alpha \in (-1, 1, 1)$). Let $\{N_1, \ldots, N_{2k+1}\}$ be the matrices defined in Lemma 2.7, where $k = [\frac{1}{2}l]$ (integer part). Putting $R_i = (1/\sqrt{2})(N_i - N_{i+1})$, $i = 1, \ldots, l-1$, $V_j = I$, $j = 1, \ldots, l$, we see that $\{R_i, V_j\}$ satisfy the conditions of 2.5 for $(-1, 1, 1)$.

(i) Let $\sigma = \theta_1 \ldots \theta_t$ be even, where $\{\theta_i \,|\, i = 1, \ldots, t\}$ are the disjoint cycles of σ. Assume further that all θ_i are even cycles. Thus, there exists an odd integer p such that

$$\sigma^p = w_{i_1}^{a_{i_1}} \ldots w_{i_r}^{a_{i_r}}, \quad (a_{i_j} \in \mathbf{Z}_+)$$

If T is the projective representation of C_m wr S_l generated by the above matrices, (see (6)), then

$$T(\sigma^p) = I, \text{ and hence } \sigma^p \text{ is } \alpha\text{-regular by 1.5.}$$

Thus σ is α-regular by Lemma 1.8. If θ_1 is an odd cycle,

$$\alpha'(\theta_1, \sigma) = \alpha'(\theta_1, \theta_1)\alpha'(\theta_1, \theta_2, \ldots, \theta_t) \text{ (by Lemma 1.6)}$$

$$= -1 \text{ (by Lemma 2.6)}$$

and as $\theta_1 \in C_{C_m \text{ wr } S_l}(\sigma)$, σ is not α-regular.

(ii) Now assume $\sigma = \theta_1 \ldots \theta_t$ is odd, and let $\text{Ty}(\theta_1) = \text{Ty}(\theta_2)$. By Lemma 1.4, we may assume, without loss of generality that

$$\theta_1 = \begin{pmatrix} a_1 & \ldots & a_q \\ a_2 & \ldots & \xi^p a_1 \end{pmatrix} \text{ and } \theta_2 = \begin{pmatrix} b_1 & \ldots & b_q \\ b_2 & \ldots & \xi^p b_1 \end{pmatrix},$$

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where $\{a_i, b_j\} \subseteq \{1, \ldots, l\}$. Define

$$\theta_\dagger = \begin{pmatrix} a_1 & b_1 & a_2 & \ldots & a_q & b_q \\ b_1 & a_2 & b_2 & \ldots & b_q & \xi^p a_1 \end{pmatrix}$$

Then $\theta_\dagger^2 = \theta_1\theta_2$, and thus $\theta_\dagger \in C_{C_m \text{ wr } S_l}(\sigma)$. Further,

$$\begin{aligned} \alpha'(\theta_\dagger, \sigma) &= \alpha'(\theta_\dagger, \theta_1\theta_2)\alpha'(\theta_\dagger, \theta_3 \ldots, \theta_t) \text{ (by Lemma 1.6)} \\ &= -1 \text{ (by Lemma 2.6)} \end{aligned}$$

and hence σ is not α-regular. If all θ_i are of different type, $C_{C_m \text{ wr } S_l}(\sigma)$ consists of elements of the form

$$\prod_{i=1}^{t} (\theta_i)^{u_i}(\operatorname{diag}\theta_i)^{v_i}, \quad \text{where} \quad u_i, v_i \in \mathbf{Z}_+$$

However, diag θ_i is α-regular by (i), and thus $\alpha'(\operatorname{diag}\theta_i, \sigma) = 1$. Finally $\alpha'(\theta_i, \sigma) = 1$ by Lemmas 1.6 and 2.6 (henceforth these references will be omitted), and thus σ is α-regular.

4. We now consider the remaining factor sets. By Theorem 2.5, $m \equiv 0 \pmod 2$, and in this case, we can make the following definition.

DEFINITION 4.1. $\sigma \in C_m$ wr S_l of type (a_{pq}) is *positive* if $\sum_q \sum_{p \text{ odd}} a_{pq} \equiv 0 \pmod 2$, and *negative* otherwise.

In terms of the generators of C_m wr S_l given in §2, we see that σ is positive if and only if the number of w_j appearing in any expression for σ is even.

In the following, v_i will always denote a positive cycle, and τ_j a negative cycle.

4.2. $\alpha \in (-1, -1, -1)$. This case is only briefly sketched, since it is a straightforward generalization of the particular case $m = 2$ given in [5]. If $k = [\frac{1}{2}l]$, we define $\{N_1 \ldots, N_{2k+1}\}$ as in Lemma 2.7, and put

$$R_i = (1/\sqrt{2})(N_i - N_{i+1}) \ i = 1, \ldots, l-1, \quad \text{and} \quad V_j = (-1)^j N_j, \ j = 1, \ldots, l.$$

Then $\{R_i, V_j\}$ generate a projective representation T of C_m wr S_l, whose factor set $\alpha \in (-1, -1, -1)$. By using an argument similar to that used by Schur in [7], it is easy to show that $\chi_\tau(\sigma) \neq 0$ if and only if either

(i) $\sigma = v_1 \ldots v_r \tau_1 \ldots \tau_s$, where all v_i are even and all τ_j are odd, or (only when l is odd)
(ii) $\sigma = \tau_1 \ldots \tau_s$

(see [5]). All σ of the above form are α-regular by Lemma 1.5. We can however, by using the argument given in [5], prove that these are the only α-regular elements (details omitted).

4.3. $\alpha \in (1, -1, 1)$. Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$, and put $R_j = A$, $j = 1, \ldots, l-1$ $V_j = (-1)^j B$, $j = 1, \ldots, l$. The projective representation T generated by $\{R_i, V_j\}$ has factor set $\alpha \in (1, -1, 1)$ (see Theorem 2.5).

(i) Let $\sigma = v_1 \ldots v_r \tau_1 \ldots \tau_s$, where σ is even, and s is even. $T(\sigma) = \pm I$, and thus all elements of this form are α-regular by 1.5.

(ii) Assume $\sigma = \nu_1 \dots \nu_r \tau_1 \dots \tau_s$ (σ even, s odd) is α-regular. Then $T(\sigma) = \pm B$ (see [6]). If ν_1 is odd, $\alpha'(\nu_1, \sigma) = -1$, and similarly if τ_1 is odd, $\alpha'(\tau_1, \sigma) = -1$. Thus all cycles must be even. If $\mathrm{Ty}(\nu_1) = \mathrm{Ty}(\nu_2)$, we define $\nu_\dagger$ as in §3(ii), and $\alpha'(\nu_\dagger, \sigma) = -1$. Similarly if $\mathrm{Ty}(\tau_1) = \mathrm{Ty}(\tau_2)$, $\alpha'(\tau_\dagger, \sigma) = -1$, and thus all cycles must be of different type. However, in this case, $C_{C_m \mathrm{wr} S_t}(\sigma)$ consists of elements ϕ of the form

$$\phi = \prod_i^r (\mathrm{diag}\, \nu_i)^{a_i} (\nu_i)^{b_i} \prod_{j=1}^s (\mathrm{diag}\, \tau_j)^{c_j} (\tau_j)^{d_j}$$

(see [3]), and as all ν_i, τ_j are even cycles, $T(\phi) = \pm I$ or $\pm B$. Thus $\alpha'(\phi, \sigma) = 1$.

(iii) Assume $\sigma = \nu_1 \dots \nu_r \tau_1 \dots \tau_s$ (σ odd, s even) is α-regular. If ν_1 is even, ν_1 is α-regular by (i) and thus $\alpha'(\mathrm{diag}\, \nu_1, \nu_1) = 1$. $\alpha'(\mathrm{diag}\, \nu_1, \nu_2 \dots \nu_r \tau_1 \dots \tau_s) = -1$, and hence $\alpha'(\mathrm{diag}\, \nu_1, \sigma) = -1$. If τ_1 exists, even or odd, then $\alpha'(\tau_1, \sigma) = -1$, and thus we must have $\sigma = \nu_1 \dots \nu_r$, all ν_i odd. $C_{C_m \mathrm{wr} S_t}(\sigma)$ consists of elements ϕ of the form

$$\phi = \prod_{i=1}^r \nu_i^{a_i} (\mathrm{diag}\, \nu_i)^{b_i} \theta,$$

where θ is conjugate to an element permuting the sets of symbols in cycles with similar type as they stand (see [3]), and as each cycle is of even length, θ is itself an even, positive element of C_m wr S_t. Thus $T(\phi) = \pm A$ or $\pm I$, and as $T(\sigma) = \pm A$, $\alpha'(\sigma, \phi) = 1$.

(iv) Assume $\sigma = \nu_1 \dots \nu_r \tau_1 \dots \tau_s$ (σ odd, s odd) is α-regular. By a similar argument to (iii), we can show that σ must be of the form $\sigma = \tau_1 \dots \tau_s$, with all τ_i odd. As above, $C_{C_m \mathrm{wr} S_t}(\sigma)$ consists of elements ϕ of the form

$$\phi = \prod_{i=1}^s (\tau_i)^{a_i} (\mathrm{diag}\, \tau_i)^{b_i} \theta,$$

where θ is again even and positive. Thus $T(\phi) = \pm I$ or $\pm BA$, according to whether $\sum_{i=1}^t a_i$ is even or odd. However, $T(\sigma) = \pm BA$, and thus, $\alpha'(\phi, \sigma) = 1$.

5. The α-regular classes. We now tabulate the α-regular classes of C_m wr S_t for all factor sets (γ, λ, μ), the results in the cases $(-1, 1, -1)$, $(-1, -1, 1)$, $(1, 1, -1)$, $(1, -1, -1)$ being given without proof. The techniques used in these cases are, however, the same as those used in §3 and §4. In the actual computations, the following result was used repeatedly.

LEMMA 5.1. *If $\sigma \in C_m$ wr S_t is (γ, λ, μ)-regular then it is $(\gamma_1, \lambda_1, \mu_1)$-regular if and only if it is $(\gamma\gamma_1, \lambda\lambda_1, \mu\mu_1)$-regular.*

THEOREM 5.2. *The α-regular elements σ of C_m wr S_t are the following.*

(a) $\alpha \in (1, 1, 1)$. *All classes are α-regular.*

(b) $\alpha \in (-1, 1, 1)$. *$\sigma = \theta_1 \dots \theta_t$, where the θ_i are the disjoint cycles of σ, and either*
(i) *σ is even and all θ_i are even, or*
(ii) *σ is odd and all θ_i are of different type.*

Henceforth m is even, and $\sigma = \nu_1 \dots \nu_r \tau_1 \dots \tau_s$, where $\{\nu_i\}$ are the disjoint positive cycles, and $\{\tau_i\}$ are the disjoint negative cycles of σ.

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(c) $\alpha \in (-1, -1, -1)$. *Either*
(i) *all* v_i *are even and all* τ_j *are odd, or*
(ii) $\sigma = \tau_1 \dots \tau_s$ *(only when* l *is odd).*

(d) $\alpha \in (1, -1, 1)$. *Either*
(i) σ *is even and* s *is even, or*
(ii) σ *is even,* s *is odd and all cycles are even and of different type, or*
(iii) σ *is odd and all cycles are odd and positive, or*
(iv) σ *is odd and all cycles odd and negative.*

(e) $\alpha \in (-1, 1, -1)$. *Either*
(i) σ *is even,* s *is even, all* v_i *are even and all* τ_j *odd, or*
(ii) σ *is even,* s *is odd and all cycles are even, negative and of different type, or*
(iii) σ *is odd,* s *is odd and all cycles are negative.*

(f) $\alpha \in (-1, -1, 1)$. *Either*
(i) σ *is even,* s *is even and all cycles are even, or*
(ii) σ *is even,* s *is odd and all cycles of different type, or*
(iii) σ *is odd and all cycles are odd, positive and of different type, or*
(iv) σ *is odd,* s *is odd and all cycles are odd, negative and of different type.*

(g) $\alpha \in (1, 1, -1)$. *Either*
(i) $\sigma = \tau_1 \dots \tau_s$, s *is odd and all* τ_i *are of different type, or*
(ii) $\sigma = v_1 \dots v_r$ *and all* v_i *are even.*

(h) $\alpha \in (1, -1, -1)$. *Either*
(i) σ *is even and all cycles are even and positive, or*
(ii) σ *is even,* s *is odd and all cycles are even and negative, or*
(iii) σ *is odd,* s *is even and all cycles are negative and of different type, or*
(iv) σ *is odd,* s *is odd, all* v_i *are even, all* τ_j *are odd and all cycles are of different type.*

REFERENCES

1. J. W. Davies and A. O. Morris, The Schur multiplier of the generalized symmetric group, *J. London Math. Soc.* (2), **8** (1974), 615–620.

2. B. Huppert, *Endliche Gruppen* (Springer-Verlag, 1967).

3. A. Kerber, *Representations of permutation groups*, Lecture notes in Mathematics No. 240 (Springer-Verlag, 1971).

4. A. O. Morris, *Projective representations of finite groups*, Proceedings of the conference on Clifford Algebras, Matscience, Madras 1971 (1972), 43–86.

5. E. W. Read, On projective representations of the finite reflection groups of type B_l and D_l, *J. London Math. Soc.* (2) **10** (1975), 129–142.

6. E. W. Read, The projective representations of the generalized symmetric group; to appear.

7. I. Schur, Über die Darstellungen der symmetrischen und der alternierden Gruppe durch gebrochene lineare Substitutionen, *J. Reine Angew. Math.* **139** (1911), 155–250.

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