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Book Review

Stochastic Processes, Richard F. Bass, Cambridge University Press, 2011, 390pp. (hardback), £45.00. ISBN: 9781107008007

Stochastic Processes written by Richard F. Bass aims to provide a complete overview of stochastic processes. This book is ambitious in the breadth of ground it covers but it is nevertheless a valuable resource for a variety of readerships, including graduate students and researchers who are interested in this subject matter. In my view, to fully appreciate the material developed, the readers require familiarity with the theory and techniques studied during an undergraduate or perhaps a graduate probability course, including basic measure theory. As such, the book is not suitable as an introduction for someone completely unfamiliar with the mathematical and probabilistic concepts that underpin the theory of stochastic processes, there are likely other books better suited to that purpose.

The 42 chapters of the book can be grouped into 7 parts:

Part 1 introduces the basic tools for analysis stochastic processes, focusing on Brownian motion definition, construction and properties.
Part 2 looks at the theory of stochastic calculus, including stochastic integrals and Itô's formula.
Part 3 is concerned with jump processes based on foundations of stochastic processes.
Part 4 addresses Markov processes, their applications and transformations.
Part 5 outlines stochastic differential equations, with two important applications: the Black-Scholes formula for pricing of derivatives in financial mathematics and the Kalman-Bucy filter used in the US space program.
Part 6 considers (weak convergence of processes and semi-group theory) probability measures on metric space, the weak convergence of random variables taking values in a metric space, with applications to Gaussian processes, but also empirical processes.
Part 7 further examines Markov processes, their construction and some important examples.

The wealth of material provided is made manageable to read because the book is organised in fairly brief chapters which are well structured. Each chapter presents a rigorous exposition of the theory or its application and concludes with exercises to complete the topic. Key concepts and techniques can be learnt in each chapter. The mathematics is sufficiently elaborated through discussion and coherent arguments. Some chapters are concerned with a basic treatment whilst others expand the underlying ideas in greater depth which aid progressive learning. In this way the book can be used as a quick source of reference for various technical points.

Certain topics are treated in greater detail which is very helpful for anyone interested in acquiring a more thorough grounding of the specific techniques. For instance, the exploration of Markov processes is particularly well addressed: chapters 19–23 introduce the reader to Markov processes and their applications. Then chapters 36–42 extensively discuss Markov processes with their construction and the theory is reinforced by the use of two examples: diffusions on the real line and Levy processes. Also the Markov process is made more accessible to students by simplifying the stochastic notations used which also aids clarity.

The author maintains the reader's interest and assists the learning process through the inclusion of exercises and some worked examples which illustrate the methods used. At the end of the chapters exercises are included to encourage students to deepen understanding by testing their knowledge of the material learnt. Some notes at the end of the exercises point to another resource for further information for the students who find the topics challenging or wish to further expand their knowledge. The book could have benefited by grading the problems' difficulty and including some brief comments for working on some of the more demanding exercises. This would have made the book more accessible, in keeping with the book's stated aim.

One strength of the book is that whilst the main focus of it is on several topics related to stochastic processes and their applications, it also considers the required background material used. For instance, the appendices aid the reader's understanding through the inclusion of rigorous mathematical proofs of the probability theories needed in the main text. Also, some results from analysis, regular conditional probability, and Kolmogorov's extension theorem develop understanding of theories and concepts that are drawn upon but not elaborated in the main body of the book.

Stochastic processes are useful in actuarial science, most notably in the area of modelling, such as in Economic Scenario Generators (ESGs) and insurance models, and their potential for application in areas of growing importance such as risk management is exciting. However, the proportion of actuaries who need the level of mathematical knowledge of stochastic processes given in the book is likely to be a small one and restricted to those who particularly work in the development of stochastic models. So the book would probably not command a wider actuarial readership but rather a readership comprised of those who have a particular interest in stochastic processes and their applications with a mathematical background to be able to engage with the material.

In this book Richard Bass offers a greater emphasis on the theory of stochastic processes rather than its applications, although applications such as the famous Black-Scholes equations are considered. In conclusion, this book is an excellent treatment of the stochastic processes, its theory and some of its most important applications, though in terms of direct actuarial application the scope of the book is limited.

Philip Grundey

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