LIFE CARE ANNUITIES (LCA) EMBEDDED IN A NOTIONAL DEFINED CONTRIBUTION (NDC) FRAMEWORK

BY

JAVIER PLA-PORCEL, MANUEL VENTURA-MARCO AND CARLOS VIDAL-MELIÁ

ABSTRACT

This paper examines the possibility of embedding public long-term care (LTC) insurance within the retirement pension system, i.e. introducing life care annuities into a notional defined contribution framework. To do this, we develop a multistate overlapping generations model that includes the so-called survivor dividend and give special attention to the assumptions made about mortality rates for dependent persons and LTC incidence rates, which largely determine the contribution rate assigned to LTC. The proposed model could be of interest to policymakers because it could be implemented without too much difficulty, it would universalize LTC coverage with a "fixed" cost, and it would discourage politicians from making promises about future LTC benefits without the necessary funding support.

KEYWORDS

LCA, NDC, pay-as-you-go, LTC insurance, social security.

1. INTRODUCTION

As Costa-Font and Font-Villalta (2009) point out, the ageing of the European population calls for insurance mechanisms to be extended to finance new social risks, such as that of needing long-term care (LTC) as one gets older.

Despite the inevitable uncertainty surrounding projections, De la Maisonneuve and Oliveira (2013) suggest there will be a rapidly rising trend in public health care expenditure over the next 50 years. Starting from around 6% of GDP in 2010, the combined public health and LTC expenditure for The Organisation for Economic Co-operation and Development (OECD) countries is projected to reach 9.5% in 2060 in the optimistic scenario assuming that policies will act more strongly than in the past to rein in some of the expenditure growth. In the worst scenario, which assumes no stepped-up policy action, spending could reach 14% of GDP.

Astin Bulletin 46(2), 331-363. doi: 10.1017/asb.2015.27

There are some very good reasons (Miyazawa *et al.*, 2000; Barr, 2010; Zuchandke *et al.*, 2010; Colombo *et al.*, 2011; Forder and Fernández, 2011; Colombo and Mercier 2012; Guillén and Comas-Herrera, 2012) for creating collective LTC coverage mechanisms to complement family and volunteer care arrangements:

- The cost of care can be high, thus placing a significant burden on users, especially those living on low incomes or with high levels of dependency.
- There are significant uncertainties about the need for LTC that individuals have to consider, especially the time when the need will come about, its duration and its intensity. It is understandable that they will want to cover this risk, but the costs can be high and access limited when covered exclusively by private insurance.
- Mechanisms for pre-paying and pooling LTC costs, such as LTC insurance, allowances and targeted assistance, provide an answer to high uncertainty and high cost.
- Social insurance systems give service users a right or an entitlement to a pre-defined level of support (in services or cash) depending on the person's need.
- The perception of financial security in relation to LTC needs to increase in all segments of the population.
- The introduction of public LTC insurance would probably increase efficiency in a moral hazard economy.

LTC as a contributory contingency has been provided in the German contributory pension system since the mid-1990s. Other OECD countries with public LTC arrangements include Japan (Campbell *et al.*, 2010), Korea (Chon, 2014), the Netherlands (Schut and Van den Berg, 2010) and Luxembourg (Colombo *et al.*, 2011).

The German contributory LTCI system is a very valuable benchmark to follow in the field of public LTC arrangements. The LTCI Act of 1994 (Rothgang, 2010) established social LTCI and mandatory private LTCI in Germany, which together cover almost the whole population. Statutory Health Insurance affiliates became members of the social LTCI scheme, and those with private health insurance were obliged to buy private LTCI. Social LTCI financing follows the pay-as-you-go (PAYG) principle. It is funded almost exclusively by contributions and, according to Colombo *et al.* (2011), retirees are also required to contribute depending on how much pension they receive. LTCI benefits are set by law, whereas private mandatory LTCI is a partially-funded scheme. Beneficiaries can choose between home care, day and night care and nursing home care. In home care, there is a choice between in-kind benefits for community care and cash benefits. Cash benefits are given directly to the dependent person, who can choose whether to pass the money on to a family caregiver[1].

With regard to the issue of linking the retirement and LTC contingencies, Chen (1994) proposed the creation of a social insurance program to provide basic LTC coverage by diverting a small portion of a retiree's social security

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

cash benefits for LTC. She called this trade-off plan "social security long-term care". Some years later, Chen (2001) provided more detail to her original proposal by suggesting a more widespread use of the insurance principle for both private and public sector programs, linking several pre-existing sources of funds in each sector so as to increase the efficiency with which these resources could be used. More specifically, her suggestion relies on building a three-legged stool for financing LTC that would begin with the creation of a compulsory social insurance program for a basic amount of LTC coverage. This program would then be supplemented on a voluntary basis by more private LTC insurance coverage and personal savings.

As a way of improving the diffusion of LTC insurance coverage, Pitacco (2002) proposed the establishment of an LTC insurance scheme embedded in the retirement pension system, specifically the introduction of enhanced pension annuities (EPA) funded with contributions. For a given amount of single premium, the "price" of the LTC coverage is a reduction in the amount of the initial retirement pension. Forder and Fernandez (2011) also suggest that linking LTC insurance to retirement pensions is a good way to extend coverage.

Costa-Font *et al.* (2014) observed that LTC finance needs to be considered as part of an overall retirement strategy rather than just a simple extension of health insurance, even if one can separate the goals of consumption smoothing (retirement) from insurance (LTC).

In the field of private insurance, as proposed by Warshawsky (2007, 2012) and Brown and Warshawsky (2013) for financial defined contribution schemes (FDCs), the life care annuity (LCA) is designed to deal with major problems in the currently separate markets for life annuities and LTCI. This is a prominent idea among specialists (Davidoff, 2009) because the combination of both insurance arrangements can alleviate problems not only of supply (selection) but also of demand (liquidity) in these markets. According to Pestieau and Ponthiere (2011), the problems of private LTC can be described using the concept of the LTC insurance puzzle[2]. For Spillman *et al.* (2003), a combined benefit simplifies and integrates two aspects of retirement planning often treated separately. It embodies a recognition that the potential for needing LTC is just one of the contingencies that retirement planning should take into account.

Ventura-Marco and Vidal-Meliá (2016) explore the idea of integrating old-age and permanent disability into a generic notional defined contribution (NDC) framework. An NDC scheme, according to a widely disseminated definition, is a PAYG system that deliberately mimics an FDC by paying an income stream whose present value over a person's expected remaining lifetime equals the amount accumulated at retirement[3]. Consequently, the idea of integrating retirement and LTC annuities comes naturally to actuarial thinking, especially after dealing with disability insurance.

Finally, demand for LTC is highly age-related, although elderly people are not the only target group. Less than 1% of those under 65 use LTC, while after that age the probability of LTC use increases rapidly. According to Colombo *et al.* (2011), pressures on LTC costs are anticipated to grow for at least four reasons: (1) the number of older people is increasing in many countries, (2) traditional family supports are being eroded due to fewer children, more women working and changing societal models which are likely to contribute to a decline in the availability of informal caregivers and lead to an increase in the need for paid care, (3) individuals are increasingly demanding better and more responsive social-care systems and (4) technological change enhances the possibilities of LTC services at home but may require care to be organized in a different way.

In short, it is difficult to hide the real importance of this topic. The future of LTC will involve greater demand and higher spending on services which, in line with actuarial principles, requires a good funding model. To put in another way (Colombo and Mercier, 2012), the right balance needs to be found between fair protection and financial sustainability in the long run[4], without shifting too large a financial burden onto future generations.

The aim of this paper is to explore the possibility of embedding a public LTC insurance scheme within the retirement pension system, specifically by introducing EPA into an NDC framework. As Pitacco (2013; 2014) explains, an EPA is a LCA paid as a pension benefit, in which the uplift is financed by a reduction (with respect to the basic retirement pension) of the benefit paid while the policyholder is healthy.

To achieve our aim, we develop a multistate overlapping generations model (MOLG) that includes the so-called survivor dividend (SD) (also known as inheritance gains), i.e. the distribution of the account balances of participants who do not survive to retirement to the accounts of surviving contributors on a birth cohort basis. Special attention is given to the assumptions made about mortality rates for dependent persons and LTC incidence rates, which largely determine the contribution rate assigned to LTC. The proposed model has many practical implications for policymakers because it could be implemented without too much difficulty, it would universalize LTC coverage with a "fixed" cost, and it would discourage politicians from making promises about future LTC benefits without the necessary funding support. As far as we know, the model proposed is an innovation in this field and we have been unable to find similar models in the economic literature.

The structure of the paper is as follows. After this introduction, Section 2 presents a multistate OLG model that integrates retirement and LTC annuities into a generic NDC framework. For the sake of clarity, this section is divided into three subsections dealing with (1) the determination of the year in which the system reaches a mature state, (2) the definition and determination of the SD, and (3) the effect that the introduction of the new contingency would have on the initial retirement pension and the contribution rate if it were decided to maintain the amount of the initial retirement pension. Section 3 shows a numerical illustration representing a generic NDC pension system with retirement and LTC annuities. The paper ends with the conclusions, possible directions for future research and an appendix in which the mortality rate for dependent persons used in the numerical example is detailed.

2. The model

This section extends the actuarial overlapping generations model (MOLG) developed by Boado-Penas and Vidal-Meliá (2014) and Vidal-Meliá *et al*. (2015) to include an LTC annuity. In our context, the MOLG can be defined as a multi-state non-homogeneous Markov process in which inhomogeneity is already given because we work with participants of different ages in the context of overlapping generations and also with different patterns of transitions between states for a given range of ages. However, this inhomogeneity does not fully apply when the economically active population increases or decreases over time.

The model incorporates insurance innovation into the NDC framework by integrating retirement and LTC annuities. Yakoboski (2002) states that, among LTC policyholders, the probability of becoming dependent and needing LTC is an integral part of retirement planning. Likewise, Murtaugh *et al.* (2001) observe that the risks of LTC needs and retirement (longevity) are negatively correlated, creating a selection-based supply-side complementarity that reinforces the demand-side argument for combining the two products. There is therefore increasing consensus among economists that bundling LTCI with illiquid annuities may broaden the appeal of both.

To a great extent, the model includes realistic demography (Bommier and Lee, 2003) insofar as it takes into account an age and health status schedule of mortality and the uncertainty that surrounds the timing of becoming dependent (LTC incidence rates by age). It also allows for changes in population and for a large number of generations of contributors and pensioners (active and dependent) to coexist at each moment in time.

The model brings an actuarial approach to the accounting framework for organizing, summarizing and interpreting data on transfer systems and the life cycle developed in Lee (1994); Willis (1988) and Arthur and McNicoll (1978), which to some degree inspired the models developed by Settergren and Mikula (2005); Boado-Penas *et al.* (2008); Vidal-Meliá *et al*. (2009); Vidal-Meliá and Boado-Penas (2013) and Ventura-Marco and Vidal-Meliá (2014).

In the model, the affiliates contribute to both retirement and LTC contingencies. The state of dependency is linked to retirement ages. There is a defined contribution rate (fixed over time), θ_a, to cover both contingencies. It is assumed that contributions and benefits are payable yearly in advance.

As too much specificity would further complicate the notation and calculations and reduce the transparency of the results, only one level of dependency is considered and dependent persons are thus assumed to be unable to recover their previous health status (active or autonomous)[5]. Becoming dependent means that the amount of the retirement pension is automatically increased by a certain percentage, ξ, to help to pay for care services, i.e. those dependent on care obtain additional cash to hire the required services as they see fit. The model uses LCA (Pitacco, 2014) in which LTC benefit is defined in terms of an uplift with respect to the basic pension (b). The basic pension b is paid out from

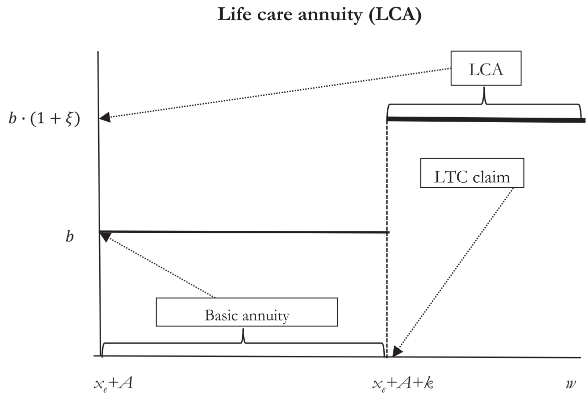

FIGURE 1: Life care annuity (LCA).

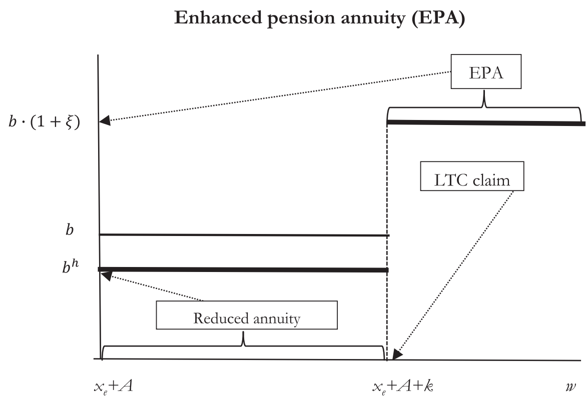

FIGURE 2: Enhanced pension annuity (EPA).

retirement onwards and is replaced by the LTC annuity benefit, $b \cdot (1+\xi)$, in the case of an LTC claim. The uplift can be financed during the whole accumulation period by contributions higher than those needed to purchase the basic pension b (see Figure 1).

The EPA is an LCA in which the uplift is financed by a reduction (with respect to basic pension b) of the benefit paid while the pensioner is healthy. Thus, the reduced benefit, b^h, is paid out as long as the retiree is healthy, while the uplifted benefit, $b \cdot (1+\xi)$, will be paid in the case of an LTC claim. Logically, $b^h < b < b \cdot (1+\xi)$ (see Figure 2).

The system does not provide a minimum pension and the age giving entitlement to retirement pension, $x_e + A$, is fixed. We also assume that participants' lives last $(\omega - 1 - x_e)$ periods, where $(\omega - 1)$ is the highest age to which it is possible to survive and x_e is the earliest age of entry into the system.

As regards the supplementary amount for dependency, it is assumed that the ages that give entitlement are to be found in age interval $[x_e + A + 1, \omega - 1]$.

The contribution base grows at an annual rate of g and the economically active population increases or decreases over time at an annual rate of γ, affecting all groups of contributors equally. Thus, the system's income from contributions (wage bill growth) also grows (decreases) at rate $G = (1 + g) \cdot (1 + \gamma) - 1$.

When the system reaches the mature state $t = \omega - x_e - A$ years from inception, A generations of contributors and $\omega - (x_e + A)$ generations of pensioners coexist at each moment in time. Being in a mature state means that the system is stabilized and paying benefits to all generations of retirement pensioners (active and dependent).

The model includes the so-called "survivor dividend", Vidal-Meliá *et al.* (2015), i.e. the account balances of participants who do not survive to retirement are distributed as inheritance capital to the accounts of the surviving participants on a birth cohort basis.

The initial retirement pension basically depends on the value of the accumulated notional account, the expected mortality of the cohort in the year the contributor reaches retirement, the expected LTC incidence rates by age, the stipulated percentage increase in the amount of the retirement pension if the retiree becomes dependent, the expected mortality of dependent persons and a notional imputed future indexation rate, α, i.e. pensions in payment increase or decrease at an annual rate of α.

The NDC system is considered to be in a mature state. As we will see later, the main implications of this are that it pays full benefits to all generations of retirement pensioners, the dependency ratio (Pensioners/Contributors), dr_t, stabilizes, and the financial ratio (Average Pension/Average Contribution Base), fr_t, is constant due to the fact that the average pension and average contribution base both evolve at the rate of variation in wages. Hence, the total contribution rate (θ_t) that ensures equality between contribution revenue and pension expenditure is constant over time. The explicit consideration of the SD[6] guarantees equivalence between the macro (balanced) contribution rate, θ_t, and the credited individual contribution rate, θ_a.

Now that the main assumptions have been given, for the sake of clarity this section is divided into three subsections dealing with (1) the determination of the year in which the system reaches a mature state, (2) the definition and determination of the SD and (3) the effect of introducing the new contingency on the initial retirement pension and its impact on the contribution rate if it were decided to maintain the amount of the initial retirement pension.

2.1. Description of the system and determination of the year in which it reaches a mature state

Figure 3 shows the relationships (transitions) between the various collectives (states) that will be separated in the model. The difference between this model and the one found in Boado-Penas and Vidal-Meliá (2014) (Figure 4) is that a

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

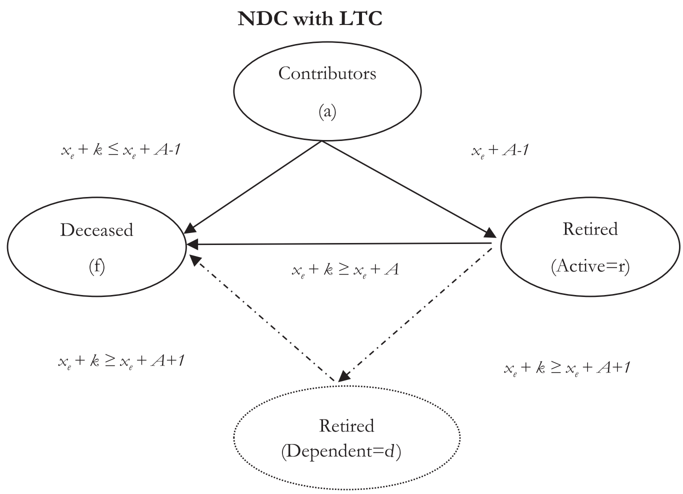

FIGURE 3: NDC with LTC.

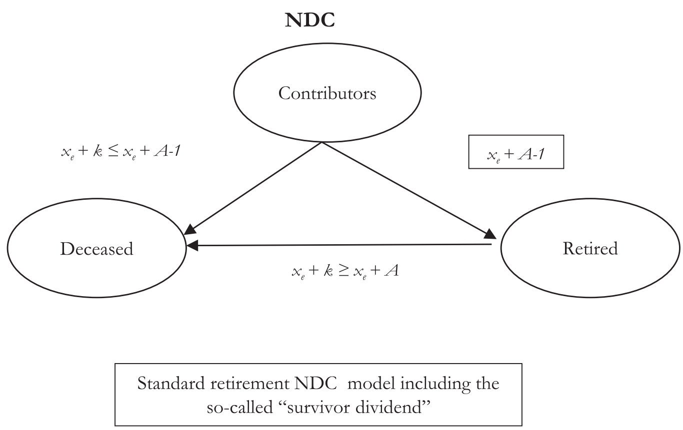

FIGURE 4: NDC.

new state-dependent — is introduced, along with the new relationships shown by dotted lines in the diagram. The model accounts for the health status of retired pensioners and distinguishes between individuals who are active, i.e. pensioners living independently at home or in sheltered accommodation, and dependent, i.e. pensioners needing help with ADLs.

We work with a simplified type of multiple state transition model (Haberman and Pitacco, 1999, and Pitacco, 2014), which is a probability model that describes a subject's movements between various states: contributors (active) (a), retired (healthy) (r), retired (dependent, i.e. care recipient) (d) and deceased (f). The model is close to actuarial practice (Montesquieu, 2012).

From a pension mathematics point of view, the model can be seen as a multi-state non-homogeneous Markov process, $\{S(t), t \in \mathbb{Z}^+\}$, where $S(t)$ is the random variable that represents the process state at time t with values in a finite state space $\mathcal{S} = \{a, r, d, f\}$ and a set of direct transitions $T = \{(a, r), (a, f), (r, d), (r, f), (d, f)\}$, where each transition is an ordered pair. Therefore, pair $(\mathcal{S}, T)$ is the multiple state model used.

2.1.1. *Yearly transition probabilities.* The discrete multi-state model, considering the time unit to be one year, can be expressed as a four-state non-homogeneous time-discrete Markov chain, where no more than one transition within the year is assumed.

The most important yearly transition probabilities, forming the stochastic one-year transition matrix, are:

$P^{aa}_{x_e+k}$: Probability that a contributor aged $x_e + k$ will reach age $x_e + k + 1$ being a contributor.
$P^{af}_{x_e+k}$: Probability that a contributor aged $x_e + k$ will die during the year.
$P^{ar}_{x_e+k}$: Probability that a contributor aged $x_e + k$ will be retired one year later.
$P^{rr}_{x_e+k}$: Probability that a retired person (active) aged $x_e + k$ will reach age $x_e + k + 1$ in the same state.
$P^{rd}_{x_e+k}$: Probability that a retired person (active) aged $x_e + k$ will reach age $x_e + k + 1$ in a state of dependency.
$P^{rf}_{x_e+k}$: Probability that a retired person (active) aged $x_e + k$ will die during the year.
$P^{dd}_{x_e+k}$: Probability that a retired person (dependent) aged $x_e + k$ will reach age $x_e + k + 1$ in the same state.
$P^{df}_{x_e+k}$ Probability that a retired person (dependent) aged $x_e + k$ will die during the year.

As we are working with Markov processes, from now on we apply classic recurrent Chapman–Kolmogorov equations to obtain the corresponding multi-year transition probabilities.

The demographic-financial structure at any moment t from the start of the system is given by formulas 1 to 24:

2.1.2. *Age.*

$$\overbrace{x_e, x_e+1, x_e+2, \ldots\ldots\ldots, x_e+A-1}^{\text{Contributors' ages}}, \underbrace{x_e+A,\ \overbrace{x_e+A+1, \ldots\ldots\ldots, w-1}^{\text{Pensioners' ages (dependency)}}}_{\text{Pensioners' ages (retirement)}} \tag{1}$$

The state of dependency implies having at least one year as a healthy retired, therefore the first age at which an individual could be dependent is $x_e + A + 1$.

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

2.1.3. *Number of contributors by age at time t.*

$$\begin{aligned}&l_{(x_e,t)} = l_{(x_e,1)} \cdot (1+\gamma)^{t-1}, l_{(x_e,+1,t)} = l_{(x_e+1,1)} \cdot (1+\gamma)^{t-1}\\&, \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots,\\&l_{(x_e+A-1,t)} = l_{(x_e+A-1,1)} \cdot (1+\gamma)^{t-1}\end{aligned} \tag{2}$$

where $l_{(x_e+k,1+k)} = l_{(x_e,1)} \cdot (1+\gamma)^k \cdot {}_kP_{x_e}$ with ${}_kP_{x_e}$ being the stable-in-time ratio between the number of individuals aged x_e and $x_e + k$ years. Stable ratios or probabilities include the decrements due to death associated with each age. It is a different matter when it comes to considering decrements or new entries due to migratory movements, these being included in parameter γ.

2.1.4. *Average wage (average contribution base) by age at time t.*

$$\begin{aligned}&y(x_e, t) = y(x_e, 1) \cdot (1+g)^{t-1},\\&y(x_e+1, t) = y(x_e+1, 1) \cdot (1+g)^{t-1},\\&, \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots,\\&y(x_e+A-1, t) = y(x_e+A-1, 1) \cdot (1+g)^{t-1}\end{aligned} \tag{3}$$

The demographic framework above implies that the age-wage structure only undergoes proportional changes. The slope of the age-wage structure is constant.

2.1.5. *Number of retired people (active) by age at time t.*

$$\begin{aligned}&l^r_{(x_e+A,t)} = l^r_{(x_e+A,1)} \cdot (1+\gamma)^{t-1},\\&l^r_{(x_e+A+1,t)} = l^r_{(x_e+A+1,2)} \cdot (1+\gamma)^{t-2}\\&, \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots,\\&l^r_{(w-1,t)} = l^r_{(w-1,t-1)} \cdot (1+\gamma)^1\end{aligned} \tag{4}$$

where $l^r_{(x_e+A+k,1+k)} = l^r_{(x_e+A,1)} \cdot {}_kP^{rr}_{x_e+A}$, with ${}_kP^{rr}_{x_e+A}$ being the probability that a retired individual (active) aged $x_e + A$ will reach age $x_e + A + k$ being active.

2.1.6. *Number of retired people by age at time t.*

$$\begin{aligned}&l^d_{(x_e+A+1,t)} = l^d_{(x_e+A+1,2)} \cdot (1+\gamma)^{t-2},\\&l^d_{(x_e+A+2,t)} = l^d_{(x_e+A+2,3)} \cdot (1+\gamma)^{t-3},\\&, \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots,\\&l^d_{(w-1,t)} = l^d_{(w-1,t-1)} \cdot (1+\gamma)^1\end{aligned} \tag{5}$$

where $l^d_{(x_e+A+k,1+k)} = l^r_{(x_e+A,1)} \cdot {}_kP^{rd}_{x_e+A}$, i.e. the number of retired (active) people of age $x_e + A$ in year $(t = 1)$ who after k periods are in a state of dependency. This can be calculated as the product of the initial group of retired people in activity and the probability that a retired person (active) aged $x_e + A$ will reach age $x_e + A + k$ as a dependent (${}_k P^{rd}_{x_e+A}$):

$$ {}_k P^{rd}_{x_e+A} = \sum_{t=1}^{k} \underbrace{{}_{t-1} P^{rr}_{x_e+A} \cdot P^{rd}_{x_e+A+t-1}}_{{}_{t-1/} P^{rd}_{x_e+A}} \cdot {}_{k-t} P^{dd}_{x_e+A+t} = \sum_{t=1}^{k} {}_{t-1/} P^{rd}_{x_e+A} \cdot \prod_{r=t}^{k-1} P^{dd}_{x_e+A+r} \quad (6) $$

with ${}_{t-1/} P^{rd}_{x_e+A}$ being the probability that a retired person (healthy) aged $x_e + A$ will reach age $x_e + A + t - 1$ in the same state but in year t will become dependent, i.e. the probability that a retired person (active) aged $x_e + A$ will become dependent at age $x_e + A + t - 1$.

2.1.7. *The yearly probability of dying for retired people (active and dependent).* From retirement age onwards, $x_e + k \geq x_e + A$, the yearly probability of dying for retired people (general retired population) can be calculated as a weighted average of the probabilities of dying for both collectives, the weighting being the LTC and active prevalence rates:

$$ \overbrace{q_{x_e+k}}^{Pensioners} = \underbrace{\lambda_{x_e+k}}_{LTC\ prevalence\ rate} \cdot \overbrace{P^{df}_{x_e+k}}^{Dependent\ persons} + \underbrace{(1 - \lambda_{x_e+k})}_{Active\ prevalence\ rate} \cdot \overbrace{P^{rf}_{x_e+k}}^{Active\ persons} \quad (7) $$

Formula (7) implies that the following probabilities also hold:

$$ \begin{gathered} \overbrace{P_{x_e+k}}^{Pensioners} = \overbrace{\frac{l^d_{(x_e+k,t)}}{l^r_{(x_e+k,t)} + l^d_{(x_e+k,t)}}}^{\lambda_{xe+k}} \cdot \overbrace{P^{dd}_{x_e+k}}^{Dependent\ persons} + \overbrace{\frac{l^r_{(x_e+k,t)}}{l^r_{(x_e+k,t)} + l^d_{(x_e+k,t)}}}^{(1-\lambda_{xe+k})} \cdot \overbrace{P^r_{x_e+k} + P^{rd}_{x_e+k}}^{Active\ persons} \\ P^r_{x_e+k} = P^{rr}_{x_e+k} + P^{rd}_{x_e+k} = 1 - P^{rf}_{x_e+k} \\ P_{x_e+k} + q_{x_e+k} = 1 \\ P^{dd}_{x_e+k} = 1 - P^{df}_{x_e+k} \end{gathered} \quad (8) $$

where:

λ_{x_e+k}: LTC prevalence rate for the group aged $x_e + k$, which is the ratio between the number of dependent persons and the total retired population (active+dependent persons) aged $x_e + k$. It is important to highlight that for the group aged $x_e + A$ the LTC prevalence rate is equal to 0.
q_{x_e+k}: Probability that a retired person aged $x_e + k$ will die within the year.
P_{x_e+k}: One year survival probability for a retired person aged $x_e + k$.
$P^r_{x_e+k}$: Probability that a retired person (active) aged $x_e + k$ will reach age $x_e + k + 1$ in the same state or as a dependent person.

It is well documented (Pitacco, 2012, but also Rickayzen, 2007, and Rickayzen and Walsh, 2002) that the mortality of disabled and dependent people contains

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

an "extra-mortality" term and can be represented either as a specific mortality (using the appropriate numerical tables or parametric mortality laws) or via adjustments to the standard age pattern of mortality. The "extra-mortality" term is very difficult to model and could have serious consequences if it were overestimated because this would mean an underestimation of the LTC liabilities, as LTC are "living" benefits. The opposite would be true if the mortality of dependent people were underestimated.

The average initial pension (with SD) for an individual aged $x + A$ in year t, enhanced by percentage ξ if the active person becomes dependent, $\bar{P}^{(r,d)}_{(x_e+A,t)}$, can be expressed as

$$\overline{P}^{(r,d)}_{(x_e+A,t)} = \frac{\overbrace{\theta_a \cdot \sum_{k=0}^{A-1} l_{(x_e+k,k+t-A)} \cdot y_{(x_e+k,k+t-A)} \cdot (1+G)^{A-k}}^{\text{Expenditure on pensions=Liabilities to new pensioners}=K^{acT}_{(x_e+A,t)}}}{l_{(x_e+A,t)} \cdot [\ddot{a}^{r\alpha}_{x_e+A} + {}^{\xi}A^{rd\alpha}_{x_e+A}]} = \frac{\overline{K}^{ac}_{(x_e+A,t)}}{\underbrace{\ddot{a}^{r\alpha}_{x_e+A}}_{\text{Active persons}} + \underbrace{{}^{\xi}A^{rd\alpha}_{x_e+A}}_{\text{Dependent persons}}}, \tag{9}$$

where

$\ddot{a}^{r\alpha}_{x_e+A}$: Present value at age $x_e + A$ of 1 monetary unit of a lifetime pension payable in advance while the individual is healthy, indexed at rate α with a technical interest rate equal to G^7.

$$\ddot{a}^{r\alpha}_{x_e+A} = \sum_{k=0}^{w-x_e-A-1} {}_kP^{rr}_{x_e+A} \cdot F^k \tag{10}$$

$F = [\frac{1+\alpha}{1+G}]$: An indexation factor which depends on α (indexation of pensions in payment) and G.

$K^{acT}_{(x_e+A,t)}$: Total accumulated notional capital at time t for all individuals who reach age $x_e + A$. The notional account is an accumulation of the contributions made, the survivor dividend distributed and the returns generated over the participant's working life.The survivor dividend means that account balances of participants who do not survive to retirement are distributed as inheritance capital to the accounts of survivors on a birth cohort basis.

$\overline{K}^{ac}_{(x_e+A,t)}$: Average accumulated notional capital at time t for individuals aged $x_e + A$.

$$\overline{K}^{ac}_{(x_e+A,t)} = \frac{K^{acT}_{(x_e+A,t)}}{l_{(x_e+A,t)}} \tag{11}$$

${}_kP^{rd}_{x_e+A}$: Probability that a retired person (active) aged $x_e + A$ will reach age $x_e + A + k$ in a state of dependency (this expression was developed earlier in Formula (6)).

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

${}^{\xi}A^{rd\alpha}_{x_e+A}$: Present actuarial value, for an active person aged $x_e + A$, of the LTC annuity or enhanced pension that supplements one monetary unit of the initial retirement pension with percentage ξ. The enhanced benefit is paid from the moment the active person becomes dependent and for as long as they remain in a state of dependency. The pension in payment is indexed at rate α and the present value is computed using a technical interest rate equal to G.

$$ {}^{\xi}A^{rd\alpha}_{x_e+A} = (1+\xi) \cdot \sum_{k=1}^{w-x_e-A-1} {}_{k-1/}P^{rd}_{x_e+A} \cdot \ddot{a}^{d\alpha}_{x_e+A+k} \cdot F^{k} \tag{12} $$

$\ddot{a}^{d\alpha}_{x_e+A+k}$: Present value at age $x_e + A$ of 1 monetary unit of a lifetime pension payable in advance while the individual remains dependent, indexed at rate α with a technical interest rate equal to G.

$$ \ddot{a}^{d\alpha}_{x_e+A+k} = \sum_{t=k}^{w-x_e-A-1} {}_{t-k}P^{dd}_{x_e+A+k} \cdot F^{k-t} \tag{13} $$

The particular case of $F = 1$, i.e. $\alpha = G$, is especially interesting because the average initial pension (Formula (9)) can be expressed using the life expectancy of active persons disaggregated into healthy and unhealthy life years:

$$ \overline{P}^{(r,d)}_{(x_e+A,t)} = \frac{\overline{K}^{ac}_{(x_e+A,t)}}{\underbrace{1+e^{rr}_{x_e+A}}_{\text{healthy life years}} + (1+\xi)\cdot \underbrace{e^{rd}_{x_e+A}}_{\text{unhealthy life years}}} = \frac{K^{acT}_{(x_e+A,t)}}{l_{(x_e+A,t)}\cdot\left[(1+e^{rr}_{x_e+A})+(1+\xi)\cdot\underbrace{\sum_{k=1}^{w-x_{e-A-1}} {}_{k-1}P^{rr}_{x_e+A}\cdot P^{rd}_{x_e+A+k-1}\cdot(1+e^{d}_{x_e+A+k})}_{e^{rd}_{x_e+A}}\right]}, \tag{14} $$

where

$e^{r}_{x_e+A} = e^{rr}_{x_e+A} + e^{rd}_{x_e+A}$: Life expectancy for active persons aged $x_e + A$. This can be broken down into the health status (active or dependent) they can expect to experience. It should be emphasized that this relationship is only true at the age of retirement.

$e^{rr}_{x_e+A}$: Dependency-free life expectancy (or "healthy life years") is defined as the number of years an active person aged $x_e + A$ is likely to spend free of activity limitation. The concept is also referred to as active life expectancy based on the ability to perform ADLs without human assistance.

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

$e^{rd}_{x_e+A}$: Dependency life expectancy (or "unhealthy life years") is defined as the number of years an active person aged $x_e + A$ is expected to spend with activity limitation.

$e^{d}_{x_e+A+k}$: Life expectancy for dependent persons aged $x_e + A + k$, assuming that $e^{d}_{x_e+A+k} < e^{r}_{x_e+A+k}$.

The amount of the initial retirement pension awarded to pensioners in year t is

$$\bar{P}^{(r,d)}_{(x_e+A,t)} = \bar{P}^{(r)}_{(x_e+A,t)}. \tag{15}$$

For the following years, $k \in \{1, 2, \ldots, \omega - 1\}$, the benefit will depend on the health status of the pensioner, i.e. whether the retiree is healthy, $S(x_e+A+k) = r$, or has become dependent, $S(x_e + A + k) = d$. For $k = 1$, we have

$$\bar{P}^{(r,d)}_{(x_e+A+1,t+1)} = \begin{cases} \bar{P}^{r}_{(x_e+A+1,t+1)} \text{ if } S(x_e + A + 1) = r|S(x_e + A) = r \\ \bar{P}^{d}_{(x_e+A+1,t+1)} \text{ if } S(x_e + A + 1) = d|S(x_e + A) = r \\ \quad 0 \text{ if } S(x_e + A + 1) = f|S(x_e + A) = r \end{cases} \tag{16}$$

and for $k \geq 2$ onwards

$$\bar{P}^{(r,d)}_{(x_e+A+k,t+k)} = \begin{cases} \bar{P}^{r}_{(x_e+A+k,t+k)} \text{ if } S(x_e + A + k) = r|S(x_e + A + k - 1) = r \\ \bar{P}^{d}_{(x_e+A+k,t+k)} \text{ if } S(x_e + A + k) = d|S(x_e + A + k - 1) = r \\ \bar{P}^{d}_{(x_e+A+k,t+k)} \text{ if } S(x_e + A + k) = d|S(x_e + A + k - 1) = d \end{cases}$$
$$\begin{cases} 0 \text{ if } S(x_e + A + k) = f|S(x_e + A + k - 1) = r \\ 0 \text{ if } S(x_e + A + k) = f|S(x_e + A + k - 1) = d \end{cases} \tag{17}$$

With population growth of $\gamma = 0$, once the individual joins the labour market they will continue working non-stop until retirement age. The only exit from the labour market is early death. Therefore, there are A different contribution pathways that will determine A different pensions, as contributors might be working for 1 year, 2 years..., A years.

$$l_{(x_e+A,t)} = \sum_{c=1}^{A} l_{(x_e+A,c,t)};\ K^{acT}_{(x_e+A,t)} = \sum_{c=1}^{A} K^{ac}_{(x_e+A,c,t)} \cdot l_{(x_e+A,c,t)} \tag{18}$$

$$\bar{K}^{ac}_{(x_e+A,t)} = \frac{\overbrace{\sum_{c=1}^{A} K^{ac}_{(x_e+A,c,t)} \cdot l_{(x_e+A,c,t)}}^{K^{acT}_{(x_e+A,t)}}}{l_{(x_e+A,t)}}, \tag{19}$$

where

$l_{(x_e+A,c,t)}$: Number of individuals who retire at age $x_e + A$ and have been contributing for the last c years at time t.

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

$K^{ac}_{(x_e+A,c,t)}$: Accumulated notional capital at time t for one individual aged $x_e + A$ who has been contributing for the last c years.

The average pension for individuals who retire at the ordinary retirement age, $\bar{P}^{(r,d)}_{(x_e+A,t)}$, is a weighted average of the A different pensions once settled. To determine this benefit, the system does not take into account the contributions made (if any) by the contributor before joining the scheme:

$$\bar{P}^{(r,d)}_{(x_e+A,t)} = \frac{\sum_{c=1}^{A} P^{(r,d)}_{(x_e+A,c,t)} \cdot l_{(x_e+A,c,t)}}{l_{(x_e+A,t)}}. \tag{20}$$

The total amount of pensions paid in year t is

$$\sum_{c=1}^{A} P^{(r,d)}_{(x_e+A,c,t)} \cdot l_{(x_e+A,c,t)} \cdot \left[\sum_{k=0}^{t-1} {}_kP^{rr}_{x_e+A} \cdot F^k + (1+\xi) \cdot \sum_{k=1}^{t-1} {}_kP^{rd}_{x_e+A} \cdot F^k \right] \tag{21}$$

In the financially sustainable NDC framework, spending on pensions has to be equal to the aggregate income from contributions according to balanced rate θ_t :

$$\underbrace{\theta_t \cdot \sum_{k=0}^{A-1} l_{(x_e+k,t)} \cdot y_{(x_e+k,t)}}_{\text{Revenues}} = \underbrace{\bar{P}^{(r,d)}_{(x_e+A,t)} \cdot \overbrace{\left[\sum_{k=0}^{w-xe-A-1} l^r_{(x_e+A+k,t)} \cdot F^k + (1+\xi) \cdot \left(\sum_{k=1}^{w-xe-A-1} l^d_{(x_e+A+k,t)} \cdot F^k \right) \right]}^{l_{(x_e+A,t)} \cdot [\ddot{a}^{r\alpha}_{x_e+A} + {}^{\xi}A^{rd\alpha}_{(x_e+A)}]}}_{\text{Expenditure on pensions}} \tag{22}$$

This leads to:

$$\underbrace{\theta_t \cdot \sum_{k=0}^{A-1} l_{(x_e+k,t)} \cdot y_{(x_e+k,t)}}_{\text{Aggregate contributions}} = \underbrace{\bar{P}^{(r,d)}_{(x_e+A,t)} \cdot l_{(x_e+A,t)} \cdot \left[\ddot{a}^{r\alpha}_{x_e+A} + {}^{\xi}A^{rd\alpha}_{x_e+A} \right]}_{\text{Liabilities to new pensioners}} = \overbrace{l_{(x_e+A,t)} \cdot \bar{K}^{ac}_{(x_e+A,t)}}^{K^{acT}_{(x_e+A,t)}} \tag{23}$$

It can therefore be said that the system's aggregate contributions at t are equivalent to the present actuarial value of the pensions awarded in that year (commitments that the system takes on with pensioners who have just retired), i.e. the accumulated notional capital of all the individuals who reach age $x_e + A$ in year t. This is in line with the Swedish NDC model, in which each monetary unit contributed is paid out in the form of retirement benefit.

Once the system reaches the mature state, the dependency ratio, dr_t, stabilizes. Meanwhile, the financial ratio, fr_t, is constant due to the fact that the average pension and average contribution base both evolve at the rate of variation in wages. Hence, the total contribution rate, θ_t, that ensures equality between contribution revenue and pension expenditure is constant over time. Consequently, the contribution rate, also called the macro contribution rate, is the product of the demographic dependency ratio and the financial ratio (the system's average replacement rate):

$$\theta_t = dr_t \cdot fr_t = \theta_{t+1} = \cdots = \theta = \frac{\bar{P}}{\bar{W}} \cdot \frac{R}{C} = \frac{\bar{P}^r \cdot (R - D) + \bar{P}^d \cdot D}{\bar{W} \cdot C}. \tag{24}$$

2.2. Definition and determination of the survivor dividend

All income from contributions is considered to be paid out in the form of retirement and LTC benefits, although not necessarily to the individual who made the contributions. The SD is the distribution of the account balances of participants who do not survive to retirement to the accounts of surviving contributors on a birth cohort basis, which is in line with the principle of actuarial fairness. In this aspect, we follow the current Swedish NDC model, but we could have followed other alternatives for distributing these inheritance gains[8].

Among the countries with NDC systems (Sweden, Latvia, Poland and Italy), only Sweden uses the SD to calculate the initial amount of the retirement pension. In the other three countries, no use of this money is identified and it implicitly becomes a component of general public revenues. In Poland and Latvia, these revenues provide a source of finance for other insurance commitments with no specified source, for example the legacy costs from the old system. Both countries decided to introduce funded components and as a result the revenue of the PAYG pillars were reduced by contributions being transferred to funded accounts, so the inheritance gains help to cover this double payment burden[9].

Following the notation introduced by Boado-Penas and Vidal-Meliá (2014), the mathematical expression for the accumulated SD at retirement age $(x_e + A)$ at time t for an individual who belongs to the initial group and has therefore contributed since entering the system, $D^{ac}_{(x_e+A,A,t)}$, is the difference between the credited capital, $K^{ac}_{(x_e+A,A,t)}$, which includes the contributions and indexation on contributions of members from the same cohort who died, and the individual credited notional capital, $K^{i}_{(x_e+A,A,t)}$:

$$\begin{aligned} D^{ac}_{(x_e+A,A,t)} &= K^{ac}_{(x_e+A,A,t)} - \underbrace{\theta_a \cdot \sum_{k=0}^{A-1} y_{(x_e+k,k+t-A)} \cdot (1+G)^{A-K}}_{K^{i}_{(x_e+A,A,t)}} \\ &= \sum_{k=0}^{A-1} D_{(x_e+k,k+t-A)} \cdot (1+G)^{A-k}. \end{aligned} \tag{25}$$

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

The accumulated dividend at the age of retirement, assuming that the contributor enters the system at age $x_e + s$, will be

$$D^{ac}_{(x_e+A,\,A-s,t)} = K^{ac}_{(x_e+A,\,A-s,t)} - \underbrace{\theta_a \cdot \sum_{k=0}^{A-1-s} y_{(x_e+k,k+t-A)} \cdot (1+G)^{A-K}}_{K^{i}_{(x_e+A,\,A,-s,t)}}$$

$$= \sum_{k=1}^{A-s} D_{(x_e+k,k+t-A)} \cdot (1+G)^{A-s-k}. \tag{26}$$

The average accumulated dividend at age $x_e + A$, taking into account the different A contribution profiles, can be calculated as follows:

$$\bar{D}^{ac}_{(x_e+A,t)} = \frac{\sum_{c=1}^{A} D^{ac}_{(x_e+A,c,t)} \cdot l_{(x_e+A,c,t)}}{l_{(x_e+A,t)}} = \bar{K}^{ac}_{(x_e+A,t)} - \bar{K}^{i}_{(x_e+A,t)} =$$

$$\frac{1}{l_{(x_e+A,t)}} \cdot \left[\overbrace{\theta_a \cdot \sum_{k=0}^{A-1} l_{(x_e+k,k+t-A)} \cdot y_{(x_e+k,k+t-A)} \cdot (1+G)^{A-k}}^{\text{Accumulated contributions (including deceased persons)}} - \underbrace{\theta_a \cdot \sum_{c=1}^{A} K^{i}_{(x_e+A,c,t)} \cdot l_{(x_e+A,c,t)}}_{\text{Accumulated contributions (survivors)}} \right] =$$

$$\frac{1}{l_{(x_e+A,t)}} \cdot \left[\theta_a \cdot \sum_{k=0}^{A-1} N_{(x_e+k,t)} \cdot y_{(x_e+k,t)} - \theta_\alpha \cdot \sum_{k=0}^{A-1} y_{(x_e+k,k+t-A)} \cdot (1+G)^{A-k} \cdot l_{(x_e+A,t)} \right]. \tag{27}$$

2.3. The cost of introducing the LTC contingency into the system and its effect on the initial retirement pension

The relationship between the credited contribution rate and the balanced rate according to (9) and (7) is

$$\underbrace{\frac{\theta_\alpha \cdot \overbrace{\sum_{k=0}^{A-1} l_{(x_e+k,k+t-A)} \cdot y_{(x_e+k,k+t-A)} \cdot (1+G)^{A-k}}^{\sum_{k=0}^{A-1} l_{(x_e+k,t)} \cdot y_{(x_e+k,t)}}}{l_{(x_e+A,t)} \cdot \left(\ddot{a}^{r\alpha}_{x_e+A} + {}^{\xi}A^{rd\alpha}_{x_e+A}\right)} \cdot l_{(x_e+A,t)} \cdot \left(\ddot{a}^{r\alpha}_{x_e+A} + {}^{\xi}A^{rd\alpha}_{x_e+A}\right)}_{\text{Expenditure on pensions}}$$

$$= \underbrace{\theta_t \cdot \sum_{k=0}^{A-1} l_{(x_e+k,t)} \cdot y_{(x_e+k,t)}}_{\text{System's revenues}} \tag{28}$$

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

Therefore, it is straightforward to observe that both rates coincide: $\theta_a = \theta_t$.

It would be interesting to study the impact of the introduction of the LTC contingency on the initial retirement pension. To do this, we need to compare the initial annuities awarded in both cases: the NDC scheme with a retirement annuity and the NDC scheme with retirement and LTC annuities. The accumulated notional capital at time t for the cohort of retired persons aged $x_e + A$ is the same under both schemes:

$$\theta_a \cdot \sum_{k=0}^{A-1} l_{(x_e+k,k+t-A)} \cdot y_{(x_e+k,k+t-A)} \cdot (1+G)^{A-k}. \tag{29}$$

The difference is given by the LTC coverage. If we define the following annuity factors:

$AF_{(x_e+A)} = \ddot{a}^{\alpha}_{x_e+A}$, the annuity factor of the NDC system without LTC coverage, and $AF^{\mathrm{LTC}}_{(x_e+A)} = \ddot{a}^{\alpha}_{x_e+A} + {}^{\xi}A^{rd\alpha}_{x_e+A}$, the annuity factor of the system with LTC coverage — the average initial pension in both systems can be expressed as

$$\begin{aligned} \overbrace{\bar{P}_{(x_e+A,t)}}^{\text{Annuity without LTC coverage}} &= \frac{\theta_a \cdot \sum_{k=0}^{A-1} l_{(x_e+k,k+t-A)} \cdot y_{(x_e+k,k+t-A)} \cdot (1+G)^{A-k}}{l_{(x_e+A,t)}} \cdot \frac{1}{AF_{(x_e+A)}}, \\ &> \\ \underbrace{\bar{P}^{(r,d)}_{(x_e+A,t)}}_{\text{Annuity with LTC coverage}} &= \frac{\theta_a \cdot \sum_{k=0}^{A-1} l_{(x_e+k,k+t-A)} \cdot y_{(x_e+k,k+t-A)} \cdot (1+G)^{A-k}}{l_{(x_e+A,t)}} \cdot \frac{1}{AF^{\mathrm{LTC}}_{(x_e+A)}}. \end{aligned} \tag{30}$$

Given that $\xi > 0$, it is easy to see that $AF_{(x_e+A)} < AF^{\mathrm{LTC}}_{(x_e+A)}$.

Under the NDC framework it is logical that the initial amount of the annuity with LTC coverage would be lower than the amount of the annuity without it. The so-called "coverage ratio", CR_t, is the link between both types of annuity. The difference in the amounts basically depends on the mortality ratio between dependent and active persons, i.e. the extra-mortality added for dependent persons, the probability of becoming dependent and the level of the enhanced pension to help to pay for LTC services, or in other words the value assigned to ξ. The coverage ratio indicates in present value the number of equivalent monetary units needed to determine the initial pension under the integrated scheme (retirement and LTC) for each monetary unit of the initial pension under the basic scheme (retirement only). If the equivalence is maintained, the integrated NDC scheme remains financially balanced given that the initial pension is reduced according to the coverage ratio. Under the above assumptions, the coverage ratio can be expressed as

$$CR_t = \frac{AF^{\mathrm{LTC}}_{(x_e+A)}}{AF_{(x_e+A)}} = 1 + \xi \cdot \underbrace{\frac{A^{rd\alpha}_{x_e+A}}{\ddot{a}^{r\alpha}_{x_e+A}}}_{\substack{\text{Weight of the LTC} \\ \text{contingency} = \mathrm{LTC}W_t}} = 1 + \xi \cdot \mathrm{LTC}W_t. \tag{31}$$

It is important to highlight that $({}^{\xi}A^{rd\alpha}_{x_e+A} > A^{rd\alpha}_{x_e+A})$, given that $\xi > 0$. In the integrated system, the initial pension is reduced according to the inverse of the coverage ratio:

$$\bar{P}_{(x_e+A,t)} \cdot \overbrace{\left[\frac{1}{1+\xi \cdot \mathrm{LTC}W_t}\right]}^{1/CR_t} = \bar{P}_{(x_e+A,t)} \cdot \left[1 - \overbrace{\frac{\xi \cdot \mathrm{LTC}W_t}{1+\xi \cdot \mathrm{LTC}W_t}}^{\substack{\text{Effect of the LTC} \\ \text{contingency on the} \\ \text{initial pension}}}\right] = \bar{P}^{(r,d)}_{(x_e+A,t)}. \tag{32}$$

An analysis of the coverage ratio gives us a better understanding of the key parameters that determine the amount of the initial pension when the LTC contingency is included:

1. The higher the value assigned to ξ, the lower the amount of the initial pension in the integrated scheme. It is easy to see that if $\xi = 0$, $CR_t = 1$ given that $(\ddot{a}^{\alpha}_{x_e+A} = \ddot{a}^{r\alpha}_{x_e+A} + {}^{0}A^{rd\alpha}_{x_e+A})$, i.e. the amount of the pension is not increased when the healthy retiree becomes dependent.
2. The higher the probability of becoming dependent for a given age, $P^{rd}_{x_e+A+t-1}$, and/or the higher the probability of survival for dependent persons, ${}_{k-t}P^{dd}_{x_e+A+t}$, the lower the amount of the initial pension. Under the assumption made for formula (7), it can be demonstrated that $\ddot{a}^{\alpha}_{x_e+A} = \ddot{a}^{r\alpha}_{x_e+A} + A^{rd\alpha}_{x_e+A}$, and thus the LTC ratio, $\mathrm{LTC}W_t = \frac{A^{rd\alpha}_{x_e+A}}{\ddot{a}^{\alpha}_{x_e+A}}$, expresses the actuarial cost that the LTC contingency represents out of the total costs (retirement and dependency), i.e. the higher the resulting value of the $\mathrm{LTC}W_t$, the lower the amount of the initial pension in the integrated scheme.
3. For the particular case of $F = 1$, i.e. $\alpha = G$, the content of the previous paragraph becomes even clearer given that the LTC ratio, $LTCW_t = \frac{e^{rd}_{x_e+A}}{1+e_{x_e+A}}$, can be expressed as a ratio of life expectancy depending on health status. The higher the number of expected "unhealthy life years", the lower the amount of the initial pension.

It is also worth thinking about what the new contribution rate, θ^*_a, should be in order to maintain the initial retirement pension. This new rate can be computed taking into account the following formula:

$$\bar{P}_{(x_e+A,t)} = \frac{\theta^*_a \cdot \sum_{k=0}^{A-1} l_{(x_e+k,k+t-A)} \cdot y_{(x_e+k,k+t-A)} \cdot (1+G)^{A-K}}{l_{(x_e+A,t)}} \cdot \frac{1}{\ddot{a}^{r\alpha}_{x_e+A} + {}^{\xi}A^{rd\alpha}_{x_e+A}}. \tag{33}$$

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

If the developed expression of $\bar{P}_{(x_e+A,t)}$ (the first part of formula (30)) is substituted into formula (33), it is easy to get θ_a^* as a function of CR_t :

$$\theta_a^* = \theta_a \cdot \overbrace{\left[\frac{\ddot{a}^{r\alpha}_{x_e+A} + {}^{\xi}A^{rd\alpha}_{x_e+A}}{\ddot{a}^{\alpha}_{x_e+A}}\right]}^{CR_t} = \theta_a \cdot CR_t = \theta_a \cdot \left(1 + \overbrace{\xi \cdot \mathrm{LTC}W_t}^{\substack{\text{Effect of the LTC} \\ \text{contingency on the} \\ \text{contribution rate}}}\right) \tag{34}$$

Finally, it would also be useful to analyse the effect of the SD on the system's financial equilibrium. If the amount of the pension is determined from the individual notional capital without considering the SD, then the new balanced contribution rate, θ_t^*, and the credited individual contribution rate, θ_a, are different because the retirement benefits are lower than they strictly could be (because the SD is not distributed among the survivors).

The relationship between both rates can be determined taking into account the notional capital accumulated (with and without the SD) at the retirement age ($x_e + A$) of an individual who belongs to the initial group and has therefore contributed since entering the system:

$$\begin{gathered}
\overbrace{K^{ac}_{(x_e+A,A,t)}}^{\text{with SD}} = \left[\left[\theta_a \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\ k+t-A)}\right] + \sum_{k=1}^{A} D_{(x_e+k,\ k+t-A)}\right] \cdot (1+G)^{A-k}, \\
\underbrace{K^{i}_{(x_e+A,A,t)}}_{\text{without SD}} = \theta_a \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\ k+t-A)} \cdot (1+G)^{A-k}.
\end{gathered} \tag{35}$$

The initial pension in each case will be

$$\begin{aligned}
\overbrace{\bar{P}_{(x_e+A,t)}}^{\text{with SD}} &= \frac{K^{ac}_{(x_e+A,A,t)}}{l_{(x_e+A,\ t)}} \cdot \frac{1}{\ddot{a}^{r\alpha}_{x_e+A} + {}^{\xi}A^{rd\alpha}_{x_e+A}}, \\
\underbrace{\bar{P}^{i}_{(x_e+A,t)}}_{\text{without SD}} &= \frac{K^{i}_{(x_e+A,A,t)}}{l_{(x_e+A,t)}} \cdot \frac{1}{\ddot{a}^{r\alpha}_{x_e+A} + {}^{\xi}A^{rd\alpha}_{x_e+A}}.
\end{aligned} \tag{36}$$

As already stated in Section 2.2, the individual credited capital with SD is the sum of the individual credited capital without SD plus the accumulated SD at retirement age, $K^{ac}_{(x_e+A,A,t)} = K^{i}_{(x_e+A,A,t)} + D^{ac}_{(x_e+A,A,t)}$, so the increase in the initial pension is due to the so-called dividend effect, as shown in the following expression:

$$\bar{P}_{(x_e+A,t)} = \bar{P}^{i}_{(x_e+A,t)} \cdot \left(1 + \overbrace{\frac{D^{ac}_{(x_e+A,A,t)}}{K^{i}_{(x_e+A,A,t)}}}^{De_t}\right), \tag{37}$$

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

which is the same result reached by Vidal-Meliá *et al.* (2015) for the classic NDC scheme and Ventura-Marco and Vidal-Meliá (2016) for the integrated NDC model with old-age and permanent disability.

The dividend effect can also be expressed as

$$De_t = \frac{\bar{P}_{(x_e+A,t)}}{\bar{P}^i_{(x_e+A,t)}} - 1 = \frac{\bar{K}^{ac}_{(x_e+A,t)}}{\bar{K}^i_{(x_e+A,t)}} - 1 = \frac{\theta_a}{\theta^*_t} - 1, \tag{38}$$

where θ^*_t is the new balanced contribution rate if the amount of the pension is determined from the individual notional capital without considering the SD.

3. Numerical illustration

This section shows the results obtained for a numerical example representative of the model developed in the previous section. For this, we use closed-form expressions. To be specific, we use three alternative sets of assumptions — a low-cost or optimistic scenario (I), a normal or central scenario (II) and a high-cost or pessimistic scenario (III) — and present the main values that make up the system's equilibrium. These include the contribution rates assigned to each contingency, the dependency ratio, the financial ratio, the effect of introducing the new contingency on the initial retirement pension and the impact on the contribution rate if it were decided to maintain the amount of the initial retirement pension. Special attention is given to the assumptions made about mortality rates for dependent persons and LTC incidence rates, which largely determine the contribution rate assigned to LTC.

The main assumptions made for this numerical example are:

- Individuals can join the labour market from age $x_e = 16$ onwards.
- The credited contribution rate, θ_a, is constant and equal to 16%.
- The fixed retirement age for all individuals is $x_e + A = 65$, i.e. the highest age that individuals can join the labour market is 64.
- The contribution bases, g, grow at an annual cumulative rate of 1.6%, and the economically active population of all ages, γ, grows at an annual rate of 1%.
- A realistic income profile is assumed, similar to the one used in Sweden when making an assumption about the average individual's life earnings. It is a long-observed concave income profile typical of developed countries. With this income pattern, yearly earnings increase more rapidly than the average (g) from ages 16 to 36, more slowly than the average from 37 to 51, remain constant in real terms from 52 to 58, and generally decrease from 59 to retirement.
- The retirement pension, once settled, is constant in real terms ($\alpha = 0$).
- ξ=1, i.e. becoming dependent means that the amount of the retirement pension doubles.

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

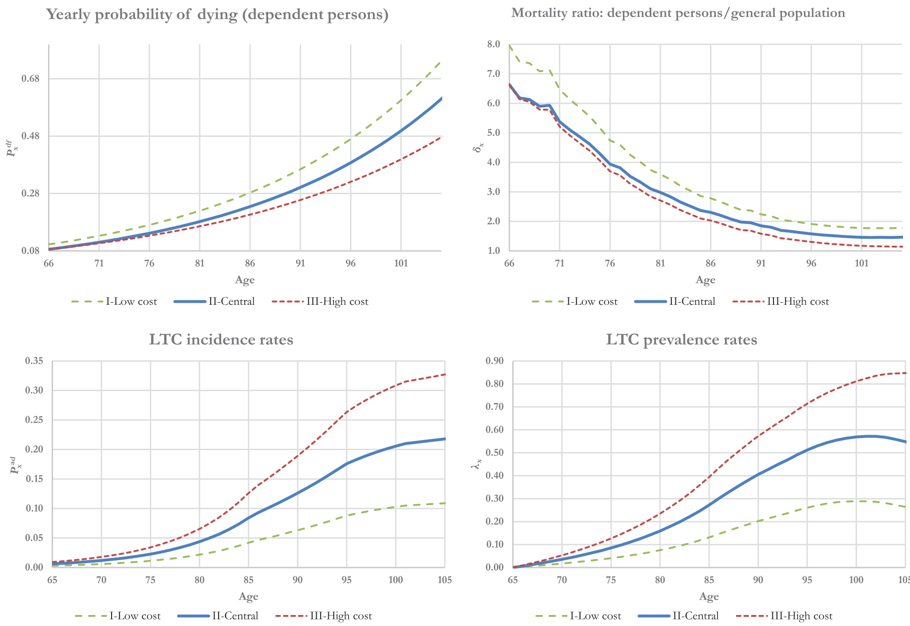

FIGURE 5: Morbidity (incidence and prevalence) and mortality rates. (Color online)

- The mortality table[10] used for the general population is the same as for Japan in 2009.
- The LTC incidence rates used are obtained from Helms (2003) and correspond to "Custodial Insurance, Japan".
- The mortality table used for dependent persons are derived from the data provided by Artís *et al.* (2007) (see Appendix).

The normal or central alternative (II) is based on these assumptions. Scenarios I and III are derived from Scenario II by modifying the LTC incidence rates and the yearly probabilities of dying for dependent persons:

Optimistic (I): The LTC incidence rates are 50% lower than those for the base scenario, while the yearly probability of dying for a dependent person aged 65 is 20% higher than in the base scenario and will also grow faster (5.20% annually) than in the base scenario (5.16%).
Pessimistic (III): The LTC incidence rates are 50% higher than those for the base scenario, while the yearly probability of dying for a dependent person aged 65 is the same as in the base scenario but will grow more slowly (4.50% annually) than in the base scenario (5.16%).

Figure 5 shows the morbidity and mortality rates under the three alternative scenarios broken down into four graphs:

1. Yearly probability of dying for dependent persons (top left). This is increasing with age for all three scenarios.
2. Mortality ratio: dependent persons/general population (top right). This shows the ratio between the mortality rates for dependent persons and the general population, which in general terms decreases with age. The extra-mortality for dependent persons is very noticeable, although much lower than the mortality rates reported by SOA (2011). Our assumption is closer to the French experience (Montesquieu, 2012) than the US experience.
3. The LTC incidence rates for each scenario (bottom left). Generally speaking the LTC incidence rate increases with the age of the individual[11]. LTC incidence rates express the probability of becoming dependent within the year and surviving as a dependent person until the end of the year. For this reason, the rate is smoothed for much older individuals because their probability of survival in a state of dependency is very low.
4. The age-specific LTC prevalence rates in the mature state that result from combining the mortality rates for dependent persons and the LTC prevalence rates previously assumed (bottom right). As expected, the rates for the pessimistic scenario are considerably higher than for the other scenarios. The average LTC prevalence rate, $\bar{\lambda}_x$ in Table 1 below, is 13.13% for the best estimate scenario and 19.03% and 6.39% for the pessimistic and optimistic scenarios respectively[12].

TABLE 1

NDC SYSTEM WITH RETIREMENT AND LTC ANNUITIES: SOME SELECTED VALUES.

Items	I-Low Cost	II-Normal	III-High Cost
$(\theta_t = \theta_a)$		16.00%	
θ_t^*		14.42%	
De_t		10.93%	
dr_t		0.3556	
fr_t		0.4499	
$\ddot{a}^{\alpha}_{x_e+A}$		14.86	
θ_t^r	15.21%	14.45%	13.84%
θ_t^d	0.79%	1.55%	2.16%
$\ddot{a}^{r\alpha}_{x_e+A}$	14.09	13.26	12.54
${}^{\xi}A^{rd\alpha}_{x_e+A}$	1.55	3.19	4.64
$(LTCW_t)$	5.20%	10.73%	15.60%
CR_t	1.05	1.11	1.16
$(\frac{CR_t-1}{CR_t})$	4.94%	9.69%	13.50%
θ_a^*	16.83%	17.72%	18.50%
$\bar{\lambda}_x$	6.39%	13.13%	19.03%
(P^d/P^a)	6.83%	15.12%	23.50%
Baseline Scenario with $G = (1.016)(1.01) - 1 = 0.0216$			

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

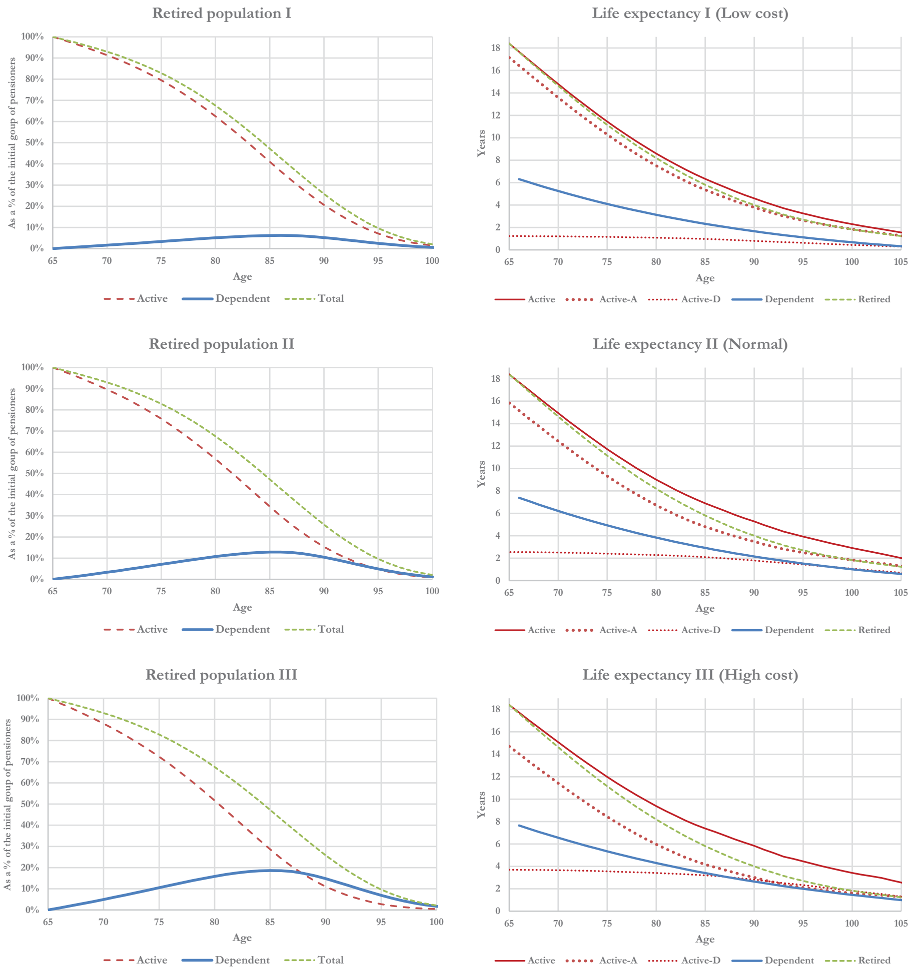

FIGURE 6: Evolution of the retired population and their life expectancy by age and health status under the three scenarios. (Color online)

Figure 6 (above) shows the evolution of the retired population and their life expectancy by age and health status for the three scenarios. It complements Figure 5 and contains six graphs. The evolution of the retired population — active persons, dependent persons and combined total — can be found on the left-hand side. In the high-cost scenario, the percentage of dependent persons by age reaches a peak of nearly 19% of the retired population for the group aged 85. These graphs are directed linked to the graph representing LTC prevalence rates in Figure 5. The ratio between dependent and active persons, (P^d / P^a) in Table 1 below, is 23.50% for the pessimistic scenario and 15.12% and 6.83% for the best-estimate and optimistic scenarios respectively.

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

The evolution of life expectancy for each scenario can be found on the right-hand side of Figure 6 below. These graphs show the total life expectancy for active persons (solid red line) broken down by health status (in activity (heavy dotted red line) and in a state of dependency (light dotted red line)), life expectancy for dependent persons and life expectancy for the general population (all the retired population). Life expectancy by age for the general population is the same for all three scenarios, so the change in life expectancy for dependent persons has a direct effect on life expectancy for the active[13]. In short, the higher the LTC incidence rates and the lower the mortality rates for dependent persons, the higher the LTC prevalence rates and the higher the cost of introducing the LTC contingency, as we can see in Table 1.

The five first items in Table 1 have the same value for all three scenarios. As expected, the balanced contribution rate (θ_t) coincides with the credited contribution rate (θ_α). Also for all three scenarios, if the SD had not been included when calculating the initial retirement pension, a discrepancy would have arisen between the credited contribution rate equal to 16% and the rate necessary to finance the pension, θ_t^* in this case 14.42%. The impact of the dividend effect, De_t, on the initial pension is not irrelevant, and the initial retirement pension rises by 10.93% using the Japanese mortality tables. The demographic ratio, dr_t, and the financial ratio, fr_t, also coincide for the three scenarios because the number of contributors and the level of wages by age do not change even though different scenarios are considered. And, although at first glance it may appear otherwise, the total number of pensioners and the average pension paid to beneficiaries (active and dependent) remain unchanged. To fully understand why both these ratios remain unchanged in all three alternatives, we need to revisit formula (7), which contains the key: the yearly probability of dying for retired people (general retired population) can be calculated as a weighted average of the probabilities of dying for both collectives, the weighting being the LTC and active prevalence rates.

If the mortality rates for the general population are the same, then obviously $\ddot{a}^{\alpha}_{x_e+A}$ (the present value at age $x_e + A$ of one monetary unit of a lifetime benefit payable in advance and indexed at rate $\alpha = 0$, with a technical interest rate equal to $G = 0.0216$) coincides for all three scenarios. Beginning with the seventh item, θ_t^r, the data vary for the three scenarios. The contribution rate assigned to enhance the retirement pension when active persons become dependent, θ_t^d, largely depends on the LTC incidence rates and mortality rates for the disabled by age, which determine the average LTC prevalence rate, $\bar{\lambda}_x$, for each scenario. The contribution rate assigned to LTC is nearly three times higher in the pessimistic scenario than in the optimistic (2.16% as opposed to 0.79%), but a similar ratio can also be found between the average LTC prevalence rates for the extreme scenarios.

So what effect does the introduction of the new contingency have on the amount of the initial pension? As can be seen in Table 1, $(\frac{CR_t-1}{CR_t})$, in order to maintain the system's financial equilibrium under the assumption of an enhanced pension of 100%, the initial pension for the best estimate scenario has to be 9.69% lower than before. For the pessimistic scenario, the reduction is 13.50%, while for the optimistic scenario it is only 4.94%.

If the aim is to leave the amount of the initial pension unchanged (as if the new contingency had not been introduced), the new contribution rate needed to maintain the system's financial equilibrium, θ_a^*, would have to be 17.7163%, i.e. it would need to be increased by 1.7163% to preserve the system's financial equilibrium. The increases for the pessimistic and optimistic scenarios would be 2.4963% and 0.8324% respectively.

4. Conclusion, discussion and future research

Demand for LTC is highly age-related and pressures on LTC costs are anticipated to grow. There are powerful rationales for creating collective coverage LTC mechanisms to complement family and volunteer care arrangements. It is a stylized fact that the future of LTC will involve more demand and more spending on services, and in line with actuarial principles, this requires a good funding model.

This paper has examined the possibility of embedding a public LTC insurance scheme within the retirement pension system, specifically by introducing LCAs into an NDC framework. A MOLG was developed and included the SD. Special attention was given to the assumptions made about mortality rates for dependent persons and LTC incidence rates, which largely determine the contribution rate assigned to LTC.

The generic NDC framework is inspired by the current Swedish NDC model, so we have followed the principle that each monetary unit contributed is paid out in the form of benefit. Our model relies on cash-for-care schemes and LTC insurance, i.e. combining retirement and LTC annuities using a contributory NDC framework will help to finance the costs incurred by retirement pensioners when they become dependent. The authors have considered LTC as a contingency exclusively linked to retirement, but we are fully aware that LTC policies are not restricted to the frail elderly and have multiple implications for society that go beyond the scope of this paper.

The model confirms that the SD has a sound financial basis that enables the balanced macro contribution rate applied to be the same as the individual credited rate in the integrated model. The main implication of this result is that, if the amount of the initial pension were determined by the individual notional capital without considering the SD, the balanced contribution rate and the credited rate would be different because the system's benefits would be lower than they could be.

The model also enables us to assess the cost of introducing the LTC contingency into the NDC retirement framework. This is computed from a double perspective: the reduction in the initial retirement pension needed to maintain the system's financial equilibrium, i.e. using EPA, and the compulsory increase in the contribution rate needed if it were decided to leave the amount of the initial retirement pension unchanged, i.e. using LCA. For a given framework, the burden of introducing the LTC contingency critically depends on the assumptions made about mortality rates for dependent persons and LTC incidence rates. It can therefore be said that the insurer (i.e. the state in our model) faces significant uncertainty regarding future costs for this contingency, and this means it would be important to periodically provide accurate data on relevant aspects.

Last but not least, it can be said that our model can easily be linked to real practices in social security policies and could be of interest to policy makers. To mention just a few positive features, it could be implemented without too much difficulty, it would help to mitigate individual risk, it would universalize LTC coverage with a “fixed” cost, it would make it easy to adapt the system to changing realities, it would discourage politicians from making promises about future LTC benefits without the necessary funding support, and it would encourage actuarial fairness and stimulate contributors’ interest in the LTC contingency.

To close, based on the model presented in the paper, at least three directions for future research can be identified:

- To adapt the actuarial balance sheet specifically designed for NDC systems to the new model with LTC and evaluate what impact the introduction of a minimum pension would have on the system’s financial equilibrium. NDC schemes should be supplemented with a minimum income (pension) guarantee.
- To extend the model to take into account different levels of LTC. In practice, various degrees of dependency are usually recognized and these have a direct effect on the amount of benefit paid. The most natural way to do this would be to extend the states shown in Figure 3, which would obviously involve a considerable increase in the complexity of the formulas to be obtained.
- To put the model into practice is by no means a minor topic and would call for a new paper that would need to thoroughly address at least the following issues: the transition rules from the old system to the integrated NDC framework, the issue of providing a minimum pension, the relationship between permanent disability and LTC, the updating of the annuity divisors and the statistical data needed to compute the real cost of dependency, the design of an appropriate yearly account statement containing individual pension information about retirement and LTC rights, and the advisability of adopting an automatic balance mechanism based on an actuarial balance sheet to adapt the system to changing realities.

ACKNOWLEDGEMENTS

The authors are grateful for the financial assistance received from the Spanish Ministry of the Economy and Competitiveness (Ministerio de Economía y

Competitividad) project ECO2012-36685. They would also like to thank seminar participants (at the universities of Alcalá de Henares, Barcelona, Valencia and Vienna) and in particular Francisco Navarro-Cabo for his help in calculating the numerical example and Peter Hall for his English support. We would like to give special thanks to Robert Holzmann, Ed Palmer and Ole Settergren for giving us the opportunity to research in the field of NDC pensions. The comments and suggestions made by Prof. Devolder and the two anonymous referees were extremely helpful in improving the paper. Any errors are entirely due to the authors.

Notes

1. A cash-for-care program aims to contribute to the costs of care, but without necessarily providing sufficient payment to buy all the care needed. Interested readers can consult the papers by Da Roit & Le Bihan (2010), Damiani *et al.* (2011), Moran *et al.* (2013) and Schut & Van den Berg (2010).
2. There is a very rich literature on these questions that goes beyond the scope of this paper. Interested readers can consult Brown *et al.* (2012), Costa-Font & Courbage (2014) or Courbage & Roudaut (2008) to name just a few.
3. For a general view of NDCs, see for example the papers by Auerbach & Lee (2011), Chłoń-Domińczak *et al.* (2012) and Holzmann *et al.* (2012). The actuarial aspects can be consulted in Devolder *et al.* (2012).
4. The funding model, as Costa-Font (2010) pointed out, has to be designed in conjunction with the prevalent social values in each country, an area that goes beyond the scope of this paper.
5. According to Pitacco (2014), the possibility of recovery can be ruled out given that this event has a very low probability of occurrence.
6. Ventura-Marco & Vidal-Meliá (2016) demonstrate, for a generic NDC model with two contingencies (retirement and disability), the crucial role played by the survivor dividend in maintaining the system's financial equilibrium.
7. To determine benefits, actuarial values are computed using the so-called "inception-annuity model", which is consistent with the MOLG framework. LTC incidence rates play an important role in this method. Alternative methods for computing the actuarial values, mainly based on the probability of being dependent, can be found in Pitacco (2014).
8. For example, distribution on a "population basis", but this method could have additional intragenerational effects which calls for further research beyond the scope of this paper.
9. See the papers by Chłoń-Domińczak, *et al.* (2012), Boado-Penas & Vidal-Meliá (2014) and Ventura-Marco & Vidal-Meliá (2015).
10. Only observed mortality rates are used and not the population structure by ages.
11. See the pattern of LTC incidence rates by age published by Broyles *et al.*, (2010). Van der Gaag *et al.*, (2014) show a much higher LTC incidence rate by age than the one we present in the pessimistic scenario.
12. The report by Mot *et al.* (2012) projected the (average) LTC prevalence rate within the 65+ population for four European countries (Spain, Germany, the Netherlands and Poland) in 2040. It predicted that it would be highest in Poland (39%) and lowest in the Netherlands (17%), only slightly lower than our pessimistic scenario.
13. See formula 7 and the values for the average probability of dying for dependent persons, $\bar{P}_x^{df}$, and active persons, $\bar{P}_x^{af}$, in Table 2 in the Appendix.
14. A similar pattern can be observed for the US experience (SOA (2011), Appendix J-7).
15. For example, for dependent persons aged 65 the mortality rate is 55.63 times higher than for the active, whereas for individuals aged 85 the mortality rate is "only" 5.18 times higher. The differences in mortality (Montesquieu, 2012) are much lower in the French experience. For a man aged 85, mortality during the 2nd year of dependency is "only" 2.75 times higher than for active lives.

REFERENCES

ARTHUR, W.B. and MCNICOLL, G. (1978) Samuelson, population and intergenerational transfers. *International Economic Review*, **19**(1), 241–246.

ARTÍS, M., AYUSO, M., GUILLÉN, M. and MONTEVERDE, M. (2007) Una estimación actuarial del coste individual de la dependencia en la población de mayor edad en España. *Estadística Española*, **49**(165), 373–402.

AUERBACH, A.J. and LEE, R.D. (2011) Welfare and generational equity in sustainable unfunded pension systems. *Journal of Public Economics*, **95**(1–2), 16–27.

BARR, N. (2010) Long-term care: A suitable case for social insurance. *Social Policy and Administration*, **44**(4), 359–374.

BOADO-PENAS, M.C., VALDÉS-PRIETO, S. and VIDAL-MELIÁ, C. (2008) An actuarial balance sheet for pay-as-you-go finance: Solvency indicators for Spain and Sweden. *Fiscal Studies*, **29**, 89–134.

BOADO-PENAS, M.C. and VIDAL-MELIÁ, C. (2014) Nonfinancial defined contribution pension schemes: Is a survivor dividend necessary to make the system balanced?. *Applied Economics Letters*, **21**(4), 242–247.

BOMMIER, A. and LEE, R.D. (2003) Overlapping generations models with realistic demography. *Journal of Population Economics*, **16**, 135–160.

BROWN, J. and FINKELSTEIN, A. (2011) Insuring long term care in the U.S. *Journal of Economic Perspectives*, **25**(4), 119–142.

BROWN, J., GODA, G.S. and MCGARRY, K. (2012) Long-term care insurance demand limited by beliefs about needs, concerns about insurers, and are available from family. *Health Affairs*, **31**(6), 1294–1302.

BROWN, J.R. and WARSHAWSKY, M. (2013) The life care annuity: A new empirical examination of an insurance innovation that addresses problems in the markets for life annuities and long-term care insurance. *The Journal of Risk and Insurance*, **80**(3), 677–703.

BROYLES, I., HAMMELMAN, E., TUMLINSON, A. and WEIER, S. (2010) Technical assumptions informing an interactive, web-based model of public long-term care insurance programs, avalere health, http://ltcpolicysimulator.org/LTC-Policy-Simulator-Technical-Paper.pdf.

CAMPBELL, J.C., IKEGAMI, N. and GIBSON, M.J. (2010) Lessons from public long-term care insurance in Germany and Japan. *Health Affairs*, **29**(1), 87–95.

CHEN, Y.-P. (1994) Financing long-term care: An intragenerational social insurance model. *The Geneva Papers on Risk and Insurance*, **19**(73), 490–495.

CHEN, Y.-P. (2001) Funding long-term care in the United States: The role of private insurance. *The Geneva Papers on Risk and Insurance*, **26**(4), 656–666.

CHŁOŃ-DOMIŃCZAK, A., FRANCO, D. and PALMER, E. (2012) The first wave of NDC — taking stock ten years plus down the road. In *NDC Pension Schemes in a Changing Pension World* (eds. R. Holzmann, E. Palmer, and D. Robalino), Chapter 2, Volume 1, pp. 31–84. Washington, DC: World Bank.

CHON, Y. (2014) The expansion of the Korean welfare state and its results – focusing on long-term care insurance for the elderly. *Social Policy & Administration*, **48**(6), 704–720.

COLOMBO, F. and MERCIER, J. (2012) Help wanted? fair and sustainable financing of long-term care services. *Applied Economic Perspectives and Policy*, **34**(2), 316–332.

COLOMBO, F.A., LLENA-NOZAL, J., MERCIER, J. and TADENS, F. (2011) Help wanted? providing and paying for long-term care. OECD Health Policy Studies, OECD Publishing. http://dx.doi.org/10.1787/9789264097759-en.

COSTA-FONT, J. (2010) Family ties and the crowding out of long-term care insurance. *Oxford Review of Economic Policy*, **26**(4), 691–712.

COSTA-FONT, J. and COURBAGE, C. (2014) *Crowding Out of Long-Term Care Insurance: Evidence from European Expectations Data*, Working Paper 4910. Munich, Germany: CESifo.

COSTA-FONT, J., COURBAGE, C. and ZWEIFEL, P. (2014) Policy dilemmas in financing long-term care in Europe. *LSE Health*, WP No. 36/2014.

COSTA-FONT, J. and FONT-VILLALTA, M. (2009) Does 'early purchase' improve the willingness to pay for LTC care insurance?. *Applied Economics Letters*, **16**(13), 1301–1305.

COURBAGE, C. and ROUDAUT, N. (2008) Empirical evidence of long-term care insurance purchase in France. *The Geneva Papers on Risk and Insurance—Issues and Practice*, **33**(4), 645–656.
DAMIANI, G., FARELLI, V., ANSELMI, A., SICURO, L., SOLIPACA, A., BURGIO, A., IEZZI, D.F. and RICCIARDI, W. (2011) Patterns of long term care in 29 European countries: Evidence from an exploratory study. *BMC Health Services Research*, 2011(11/316).
DA ROIT, B. and LE BIHAN, B. (2010) Similar and yet so different: Cash-for-care in six European countries' long-term care policies. *Milbank Quarterly*, **88**(3), 286–309.
DAVIDOFF, T. (2009) Housing, health, and annuities. *The Journal of Risk and Insurance*, **76**(1), 31–52.
DE LA MAISONNEUVE, C. and OLIVEIRA MARTINS, J. (2013) Public spending on health and long-term care: A new set of projections, OECD Economic Policy Papers, No. 6, OECD Publishing. DOI: 10.1787/5k44t7jwwr9x-en.
DEVOLDER, P., JANSSEN, J. and MANCA, R. (2012) Stochastic Methods for Pension Funds. ISTE Ltd and John Wiley & Sons, Inc. Great Britain and the United States.
FORDER, J. and FERNÁNDEZ, J.L. (2011) What works abroad? Evaluating the funding of long-term care: International perspectives, Report commissioned by Bupa Care Services, PSSRU Discussion Paper 2794, Canterbury: PSSRU.
GUILLÉN, M. and COMAS-HERRERA, A. (2012) How much risk is mitigated by LTC protection schemes? A methodological note and a case study of the public system in Spain. *The Geneva Papers on Risk and Insurance*, **37**, 712–724.
HABERMAN, S. and PITACCO, E. (1999) *Actuarial Models for Disability Insurance*. London: Chapman and Hall.
HELMS, F. (2003) Estimating LTC premiums using GEEs for pseudo-values. *Technische Universitat Munchen*, Zentrum Mathematik.
HOLZMANN, R., PALMER, E. and ROBALINO, D. (2012) *Non-financial Defined Contribution Pension Schemes in a Changing Pension World: Vol.* 1 *, Progress, Lessons, and Implementation*. Washington, DC: World Bank.
LEE, R. (1994) The formal demography of population aging, transfers, and the economic life cycle. In *Demography of Aging* (eds. L. G. Martin and S. H. Preston), pp. 8–49. Washington, DC: National Academy Press.
MIYAZAWA, K., MOUDOUKOUTAS, P. and TADASHI, Y. (2000) Is public long-term care insurance necessary?. *The Journal of Risk and Insurance*, **67**(2), 249–264.
MONTESQUIEU, L. (2012) Construction of biometric actuarial bases for long-term care insurance. SCOR report. http://www.scor.com/en/sgrc/life/long-term-care/item/2014.html?lout=sgrc.
MORAN, M., GLENDINNING, C., WILBERFORCE, M., STEVENS, M., NETTEN, A., JONES, K., MANTHORPE, J., KNAPP, M., FERNÁNDEZ, J.L., CHALLIS, D. and JACOBS, S. (2013) Older people's experiences of cash-for-care schemes: evidence from the english individual Budget pilot projects. *Ageing and Society*, **33**, 826–851.
MOT, E., FABER, R., GEERTS, J. and WILLEMÉ, P. (eds.) (2012) Performance of long-term care systems in Europe. ENEPRI Research Report-117. http://www.ancien-longtermcare.eu/.
MURTAUGH, C., SPILLMAN, B. and WARSHAWSKY, M. (2001) In sickness and in health: An annuity approach to financing long-term care and retirement income. *The Journal of Risk and Insurance*, **68**(2), 225–254.
PESTIEAU, P. and PONTHIERE, G. (2011) The long term care insurance puzzle. Chapter 2, pages 41–52, In *Financing Long term Care in Europe: Institutions, Markets and Models* (eds. J. Costa-Font and C. Courbage), Palgrave Macmillan. [Online] Available at: http://www.palgraveconnect.com/pc/doifinder/10.1057/9780230349193. (Accessed: 6 January 2015).
PITACCO, E. (2002) LTC insurance in Italy. Paper presented at the XXVII ICA - Cancun, 2002 Health Seminar on Critical Issues in Managing Long-Term Care Insurance.
PITACCO, E. (2012) Mortality of disabled people (January 26, 2012). Available at SSRN: http://ssrn.com/abstract=1992319 or http://dx.doi.org/10.2139/ssrn.1992319.
PITACCO, E. (2013) Biometric risk transfers in life annuities and pension products: A survey. CEPAR Working Paper 2013/25, 2013. http://www.cepar.edu.au/media/123434/25_biometric_risk_transfers in_life_annuities_and_pension_products.pdf.
PITACCO, E. (2014) *Health Insurance. Basic Actuarial Models*. EAA Series. Springer.

RICKAYZEN, B.D. (2007) An analysis of disability-linked annuities. Faculty of Actuarial Science and Insurance, Cass Business School, City University, London. Actuarial Research Paper No. 180. http://www.cass.city.ac.uk/__data/assets/pdf_file/0018/37170/180A RP.pdf.

RICKAYZEN, B.D. and WALSH, D.E.P. (2002) A multi-state model of disability for the United Kingdom: Implications for future need for long-term care for the elderly. *British Actuarial Journal*, 8 (Part II, 36), 341–393.

ROTHGANG, H. (2010) Social insurance for long-term care: An evaluation of the German model. *Social Policy and Administration*, **44**(4), 436–460.

SCHUT, F. and VAN DEN BERG, B. (2010) Sustainability of comprehensive universal long-term care insurance in the netherlands. *Social Policy and Administration*, **44**(4), 411–435.

SETTERGREN, O. and MIKULA, B.D. (2005) The rate of return of pay-as-you-go pension systems: A more exact consumption-loan model of interest. *The Journal of Pensions Economics and Finance*, **4**(2), 115–138.

Society of Actuaries (SOA) (2011), Long term care experience committee intercompany study 1984–2007. http://www.soa.org/research/experience-study/ltc/research-ltc-study-1984-report.aspx.

SPILLMAN, B, MURTAUGH, C. and WARSHAWSKY, M. (2003) Policy implications of an annuity approach to integrating long-term care financing and retirement income. *Journal of Aging and Health*, **15**(1), 45–73.

VAN DER GAAG, N., BIJWAARD, G., DE BEER, J. and BONNEUX L. (2014) Forecasting long-term care need of elderly using a multistate projection model. NIDI Working Paper no. 2014/2. The Hague: NIDI.

VENTURA-MARCO, M. and VIDAL-MELIÁ, C. (2014) An actuarial balance sheet model for defined benefit pay-as-you-go pension systems with disability and retirement contingencies. *ASTIN Bulletin*, **44**(2), 367–415.

VENTURA-MARCO, M. and VIDAL-MELIÁ, C. (2016) Integrating retirement and permanent disability in NDC pension schemes. Applied Economics, 48(12), 1081–1102, DOI:10.1080/00036846.2015.1093084 Published online: 30 Sep 2015.

VIDAL-MELIÁ, C. and BOADO-PENAS, M.C. (2013) Compiling the actuarial balance for pay-as-you-go pension systems. Is it better to use the hidden asset or the contribution asset?. *Applied Economics*, **45**(10), 1303–1320.

VIDAL-MELIÁ, C., BOADO-PENAS, M.C. and NAVARRO-CABO, F. (2015) Notional defined contribution pension schemes: Why does only Sweden distribute the survivor dividend?. *Journal of Economic Policy Reform*, Published online: 15 Apr 2015. **DOI**: 10.1080/17487870.2015.1028547.

VIDAL-MELIÁ, C., BOADO-PENAS, M.C. and SETTERGREN, O. (2009) Automatic balance mechanisms in pay-as-you-go pension systems. *The Geneva Papers on Risk and Insurance: Issues and Practice*, **33**(4), 287–317.

WARSHAWSKY, M.J. (2007) The life care annuity - a proposal for an insurance product innovation to simultaneously improve financing and benefit provision for long-term care and to insure the risk of outliving assets in retirement. Georgetown University - Long-Term Care Financing Project. Working Paper No. 2. http://ltc.georgetown edu/forum/2warshawsky061107.pdf.

WARSHAWSKY, M.J. (2012) *Retirement Income: Risks and Strategies*. Cambridge, MA: MIT Press.

WILLIS, R. (1988) Life cycles, institutions and population growth: A theory of the equilibrium interest rate in an overlapping-generations model. pp. 106–138 in R.D. Lee, W.B. Arthur, and G. Rodgers, eds., Economics of Changing Age Distributions in Developed Countries. Oxford: Clarendon Press.

YAKOBOSKI, P.J. (2002) Understanding the motivations of long-term care insurance owners: The importance of retirement planning, *Benefits Quarterly* (Second Quarter) **18**(2), 16–21.

ZUCHANDKE, A., REDDEMANN, S., KRUMMAKER, S. and V. D. SCHULENBURG, J.-M. G. (2010) Impact of the introduction of the social long-term care insurance in Germany on financial security assessment in case of long-term care need. *The Geneva Papers on Risk and Insurance - Issues and Practice*, **35**(4), 626–643.

JAVIER PLA-PORCEL
Actuarial Department, SOS Seguros y Reaseguros, S.A.,

Calle Ribera del Loira 4-6, 28042 Madrid. (Spain)
Email: javier.pla@internationalsos.com

MANUEL VENTURA-MARCO
Department of Financial Economics and Actuarial Science,
University of Valencia, Avenida de los Naranjos s.n., 46022 Valencia. (Spain)
Email: manuel.ventura@uv.es

CARLOS VIDAL-MELIÁ (Corresponding author)
Department of Financial Economics and Actuarial Science, University of Valencia,
Avenida de los Naranjos s.n., 46022 Valencia. (Spain)
Email: carlos.vidal@uv.es

APPENDIX: MORTALITY RATES FOR DEPENDENT PERSONS

For Montesquieu (2012), this assumption is a complex one. A classic mortality table depends on age and gender. The mortality assumption for a dependent person requires that we take into account the age at which the state of dependency began, as life expectancy in this case varies according to the cause of the dependency, which is itself correlated with the age of becoming dependent. The main causes of dependency have quite variable durations: relatively short (cancer…), average (rheumatism, cardiovascular disease…) or much longer, up to 10 years (neurological problems, senile dementia). The mortality rates for the first year of dependency do not increase strictly with age, but decrease until an age of about 75. In fact until age 75 there is a preponderance of illnesses like cancer, which play out over relatively short periods, whereas after this age the illnesses follow longer courses[14]. From the second year of dependency onwards, the age factor takes over again. As the years of dependency pass, the mortality curve flattens out, the influence of the state of dependency diminishes in favour of that of age, long-term diseases predominate and mortality approaches general mortality.

A report by the SOA (2011) shows that the mortality rate for dependent persons is about 25 times higher than for active persons[15]. Our assumption is closer to the French experience than the US experience. To complete the necessary data for computing the numerical illustration, our starting point is the mortality rates for dependent persons provided in Artís *et al.* (2007). Their data are adjusted to an exponential function, $\alpha e^{\beta x}$, where α and β are constant parameters and x represents the age the dependent person has reached. Table 2 (below) shows some selected values for the mortality assumptions made for dependent persons and their implications for the active. The first item, $e_{65}^{rr}(\%)$, indicates the "healthy life years" for an active person aged 65, and in parentheses the percentage of their life expectancy which is likely to be spent free of activity limitation under the three alternatives.

The second item, $e_{65}^{rr}(\%)$, indicates the "unhealthy life years" for an active person aged 65, and in parentheses the percentage of their life expectancy which is likely to be spent with some type of activity limitation under the three alternatives.

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press

It is worth bearing in mind that the sum of items 1 and 2 is a fixed value (18.40 years) under the 3 alternatives, given that the mortality table for the general population is the same for everyone. Finally, e_{66}^{d} is life expectancy for dependent persons aged 66, $\bar{\delta}_x$ is the average dependent person/general population mortality ratio, $\bar{P}_x^{df}$ is the average mortality rate for dependent persons, and $\bar{P}_x^{af}$ is the average mortality rate for the active retired.

TABLE 2

MORTALITY ASSUMPTIONS: SOME SELECTED VALUES.

Items	I-Low Cost	II-Normal	III-High Cost
$e_{65}^{rr}(\%)$	17.16 (93.26)	15.85 (86.15)	14.71 (79.94)
$e_{65}^{rd}(\%)$	1.24 (6.74)	2.55 (13.85)	3.69 (20.06)
e_{66}^{d}	6.31	7.39	7.65
$\bar{\delta}_x$	5.03	4.18	3.98
$\bar{P}_x^{df}$	0.1919	0,1592	0.1458
$\bar{P}_x^{af}*$	0.0386	0.0278	0.0185
Base Scenario with $G = (1.016)(1.01) - 1 = 0.0216$			

https://doi.org/10.1017/asb.2015.27 Published online by Cambridge University Press